

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 COMPENSATORS

1.1 Lag-Lead Design

1.1.1. Using the frequency response method, design a lag-lead compensator for the unity feedback system given

$$G(S) = \frac{K(s+7)}{s(s+5)(s+15)} \quad (1.1.1.1)$$

The following specifications must be met: Peak overshoot = 15%, settling time = 0.1 second and velocity error constant = 1000 Use second order approximation.

Solution: Figure: 1.1.1 models the equivalent of compensated closed loop system. Velocity

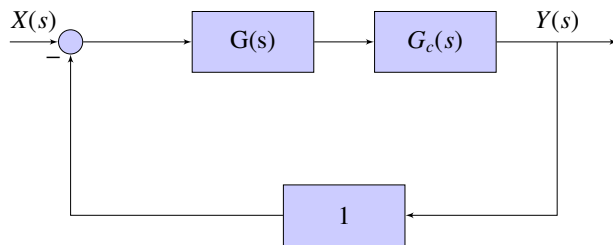


Fig. 1.1.1

error constant

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad (1.1.1.2)$$

$$\lim_{s \rightarrow 0} s \frac{K(s+7)}{s(s+5)(s+15)} = 1000 \quad (1.1.1.3)$$

$$\Rightarrow K = 10714 \quad (1.1.1.4)$$

Bode plot of G(s) for the value of K The

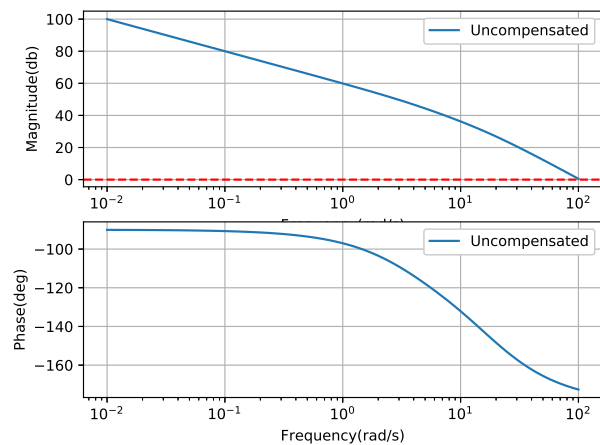


Fig. 1.1.1

following code verifies the result.

```
codes/ee18btech11012/ee18btech11012_1.py
```

Relation between %OS and Damping ratio

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{(\pi)^2 + (\ln(\%OS/100))^2}} \quad (1.1.1.5)$$

$$\Rightarrow \zeta = 0.517 \quad (1.1.1.6)$$

Phase Margin for a Damping ratio is given by

$$\phi_m = 90^\circ - \arctan\left(\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta}\right) \quad (1.1.1.7)$$

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$$\Rightarrow \phi_m = 53.17^\circ \quad (1.1.1.8)$$

For an additional 5° for lag compensation, Phase margin is

$$\phi_m = 53.17^\circ + 5^\circ = 58.17^\circ \quad (1.1.1.9)$$

Note : Adding 5° phase angle to compensate the phase angle contribution of the lag compensator. Bandwidth frequency is given by

$$\omega_{BW} = \omega_n (\sqrt{(1 - 2\zeta^2)} + \sqrt{4\zeta^4 - 4\zeta^2 + 2}) \quad (1.1.1.10)$$

where

$$\omega_n = \frac{4}{T_s \zeta} \quad (1.1.1.11)$$

Given settling time = 0.1 sec then

$$\omega_n = 77.37 \text{ rad/sec} \quad (1.1.1.12)$$

then

$$\omega_{BW} = 96.91 \text{ rad/sec} \quad (1.1.1.13)$$

1.1.2. Designing Lag-Lead Compensator $G_c(s)$

Solution: General lag-lead compensator

$$G_c(s) = \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\gamma T_2}} \right) \quad (1.1.2.1)$$

- Choose the new phase-margin frequency

$$\omega_{Pm} = 0.8\omega_{BW} = 77.53 \text{ rad/sec} \quad (1.1.2.2)$$

- At this phase-margin frequency, Phase angle is -170.52° .
- Then the contribution required from the lead is

$$\phi_{max} = 58.17 - (180 - 170.52) = 48.69^\circ. \quad (1.1.2.3)$$

- Now Using the relation

$$\phi_{max} = \sin^{-1} \left(\frac{1 - \beta}{1 + \beta} \right) \quad (1.1.2.4)$$

then we get

$$\beta = 0.142 \quad (1.1.2.5)$$

- Lag Compensator Design: The Compensator must have a dc gain of unity to retain the value of K_v that we have already designed

by setting $K = 10714$.

$$z_{clag} = \frac{\omega_{Pm}}{10} = \frac{77.53}{10} = 7.753 \quad (1.1.2.6)$$

$$p_{clag} = z_{clag} * \beta = 1.102 \quad (1.1.2.7)$$

Gain in the lag compensator is

$$K_{clag} = \frac{P_{clag}}{z_{clag}} = 0.1421 \quad (1.1.2.8)$$

- Hence the lag compensator transfer function is

$$G_{clag}(s) = \frac{0.1421(s + 7.753)}{s + 1.102} \quad (1.1.2.9)$$

- Lead Compensator Design: DC gain for this must be unity.

Relations to find T and β : The Compensator's magnitude at the phase margin frequency ω_{max}

$$|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}} \quad (1.1.2.10)$$

$$T = \frac{1}{\omega_{max} \sqrt{\beta}} \quad (1.1.2.11)$$

So, To find transfer function

$$z_{lead} = \frac{1}{T_2} = \omega_{Pm} * \sqrt{\beta} = 29.92 \quad (1.1.2.12)$$

$$p_{lead} = \frac{z_{lead}}{\beta} = 205.74, K_{lead} = \frac{P_{lead}}{z_{lead}} = 7.04 \quad (1.1.2.13)$$

- Thus lead compensator transfer function is

$$G_{lead} = \frac{7.04(s + 29.22)}{s + 205.74} \quad (1.1.2.14)$$

- So the overall compensator transfer function is

$$G_c(s) = G_{clag}(s)G_{lead}(s) \quad (1.1.2.15)$$

$$G_c(s) = \frac{1.000384(s + 7.753)(s + 29.23)}{(s + 1.102)(s + 205.7)} \quad (1.1.2.16)$$

1.1.3. Verifying Lag-lead Compensator using Plots

Solution: Magnitude and Phase plot The following code

codes/ee18btech11012/ee18btech11012_2.py

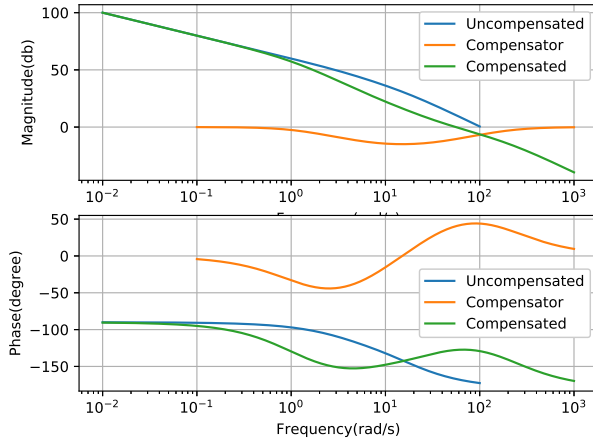


Fig. 1.1.3

1.1.4. Verifying in time domain

Solution: Time response for a unit step function The following code can be verified

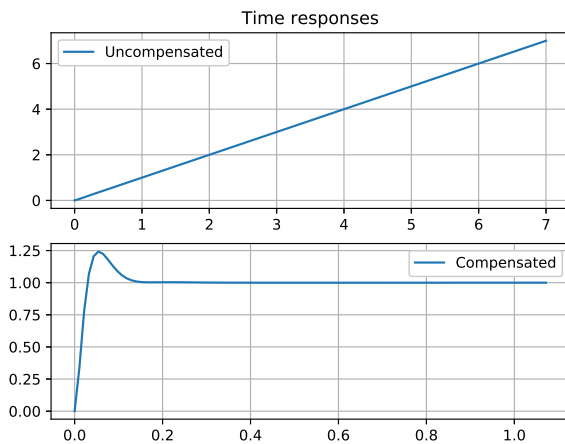


Fig. 1.1.4

```
codes/ee18btech11012/ee18btech11012_3.py
```

1.1.5. Verifying the designed lag-lead compensator

Solution:

Parameter Specification	Proposed	Actual
Phase Margin	53.17°	53.3994°
K_v	1000	1023.67
Phase Margin frequency	77.53	55.5874

TABLE 1.1.5: Comparing the desired and obtained results