Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Lag-Lead Compensator Designing

1.0.1. Using frequency response method, design a lag-lead compensator for the unity feedback system given

$$G(S) = \frac{K(s+7)}{s(s+5)(s+15)} \tag{1.0.1.1}$$

The following specifications must be met: Peak overshoot = 15%, settling time = 0.1 second and velocity error constant = 1000 Use second order approximation.
Solution: Fig.1.0.1; models the equivalent of compensated closed loop system.

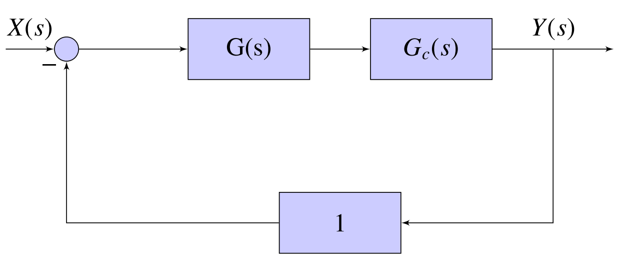

Fig. 1.0.1

Velocity error constant

$$K_v = \lim_{s\to 0} sG(s) \tag{1.0.1.2}$$

$$\lim_{t\to 0} s\frac{K(s+7)}{s(s+5)(s+15)} = 1000 \tag{1.0.1.3}$$

$$\implies K = 10714 \tag{1.0.1.4}$$

Bode plot of G(s) for the value of K

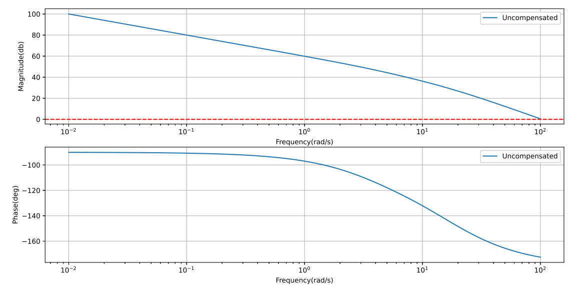

Fig. 1.0.1

The following code verifies the result.

```
codes/ee18btech11012/ee18btech11012_1.py
```

Relation between %OS and Damping ratio

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{(\pi)^2 + (\ln(\%OS/100))^2}} \tag{1.0.1.5}$$

$$\implies \zeta = 0.517 \tag{1.0.1.6}$$

Phase Margin for a Damping ratio is given by

$$\phi_m = 90^\circ - \arctan(\frac{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}{2\zeta} \tag{1.0.1.7}$$

$$\implies \phi_m = 53.17^\circ \tag{1.0.1.8}$$

For an additional 5°for lag compensation,Phase margin is

$$\phi_m = 53.17^\circ + 5^\circ = 58.17^\circ \tag{1.0.1.9}$$

Note : Adding 5°phase angle to compensate the phase angle contribution of the lag compensator.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Bandwidth frequency is given by

$$\omega_{BW} = \omega_n(\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}) \tag{1.0.1.10}$$

where

$$\omega_n = \frac{4}{T_s\zeta} \tag{1.0.1.11}$$

Given settling time = 0.1 sec then

$$\omega_n = 77.37 rad/sec \tag{1.0.1.12}$$

then

$$\omega_{BW} = 96.91 rad/sec \tag{1.0.1.13}$$

1.0.2. Designing Lag-Lead Compensator Gc(s)
Solution: General lag-lead compensator

$$G_c(s) = \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right) \tag{1.0.2.1}$$

- Choose the new phase-margin frequency

$$\omega_{Pm} = 0.8\omega_{BW} = 77.53 rad/sec \tag{1.0.2.2}$$

- At the new phase-margin frequency,the phase angle is -170.52°.
- Then the conribution required from the lead is

$$\phi_{max} = 58.17 - (180 - 170.52) = 48.69^\circ. \tag{1.0.2.3}$$

- Now Using the relation

$$\phi_{max} = \sin^{-1}(\frac{1-\beta}{1+\beta}) \tag{1.0.2.4}$$

then we get

$$\beta = 0.142 \tag{1.0.2.5}$$

- Lag Compensator Design:The Compensator must have a dc gain of unity to retain the value of Kv that we have already designed by setting K = 10714.

$$z_{clag} = \frac{\omega_{Pm}}{10} = \frac{77.53}{10} = 7.753 \tag{1.0.2.6}$$

$$p_{clag} = z_{clag} * \beta = 1.102 \tag{1.0.2.7}$$

Gain in the lag compensator is

$$K_{clag} = \frac{p_{clag}}{z_{clag}} = 0.1421 \tag{1.0.2.8}$$

- Hence the lag compensator transfer function is

$$G_{clag}(s) = \frac{0.1421(s+7.753)}{s+1.102} \tag{1.0.2.9}$$

- Lead Compensator Design:Here also we must maintain unity DC gain so for that consider
Important Relations to find T and β: The Compensator's magnitude at the phase margin frequency ω_{max}

$$|G_c(j\omega_{max})| = \frac{1}{\sqrt{\beta}} \tag{1.0.2.10}$$

$$T = \frac{1}{\omega_{max}\sqrt{\beta}} \tag{1.0.2.11}$$

So,To find transfer function

$$z_{lead} = \frac{1}{T_2} = \omega_{Pm} * \sqrt{\beta} = 29.92 \tag{1.0.2.12}$$

$$p_{lead} = \frac{z_{lead}}{\beta} = 205.74, K_{lead} = \frac{p_{lead}}{z_{lead}} = 7.04 \tag{1.0.2.13}$$

- Thus lead compensator transfer function is

$$G_{lead} = \frac{7.04(s+29.22)}{s+205.74} \tag{1.0.2.14}$$

- So the overall compensator tranfer function is

$$G_c(s) = G_{clag}(s)G_{lead}(s) \tag{1.0.2.15}$$

$$\implies G_c(s) == \frac{1.000384(s+7.753)(s+29.23)}{(s+1.102)(s+205.7)} \tag{1.0.2.16}$$

- Verifying Lag-lead Compensator using Plots
Solution: Magnitude and Phase plot
The following code

```
codes/ee18btech11012/ee18btech11012_2.
    py
```

1.0.3. Verifying in time domain
Solution: Time response for a unit step function
The following code can be verified

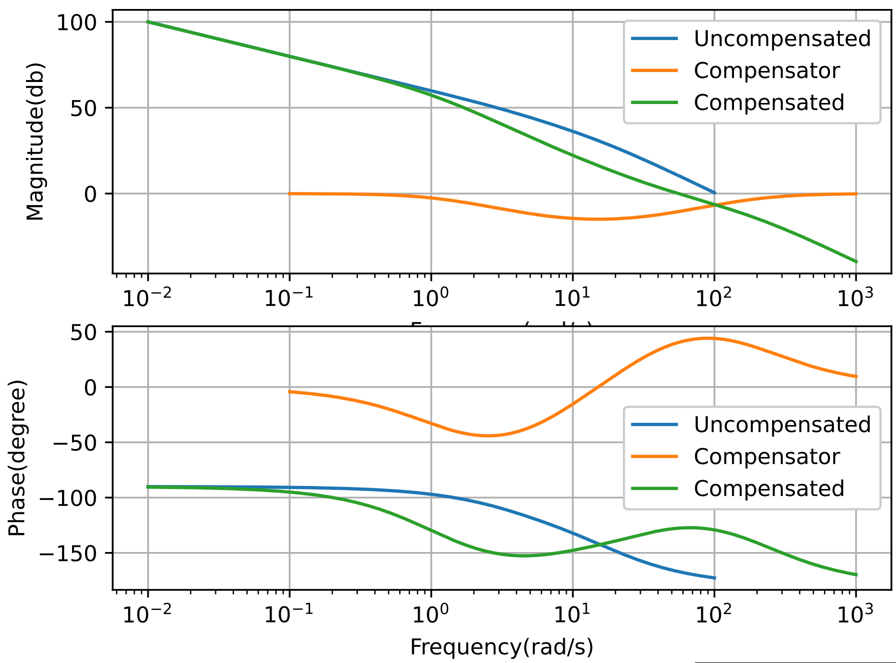

Fig. 1.0.2

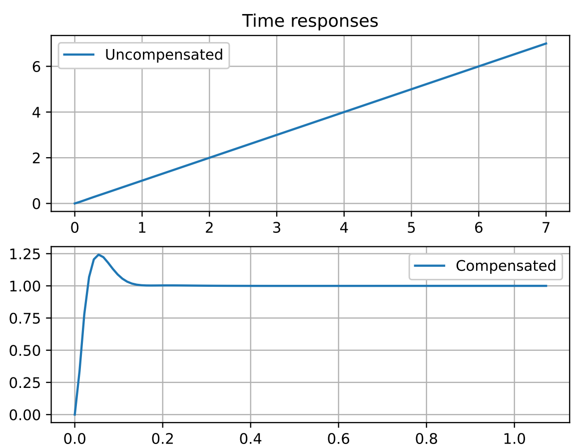

Fig. 1.0.3

codes/ee18btech11012/ee18btech11012_3.py

1.0.4. **Result :** The below is the summary for the designed lag-lead compensator

Parameter Specification	**Proposed**	**Actual**
Phase Margin	53.17°	53.3994°
K_v	1000	1023.67
Phase Margin frequency	77.53	55.5874

TABLE 1.0.4: Comparing the desired and obtained results