Day: 14 Siva Reddy Bodapati 15·07·2021

Sat

Polynomial Partial derivatives

⇒ ① $f(x,y) = x^2 + y^2$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2)$$

(constant to x derivative)

$$= 2x + 0$$

$$\frac{\partial f}{\partial x} = 2x$$

⇒ ② $f(x,y) = x^2 + y^2$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2)$$

$$= 0 + 2y$$

$$\frac{\partial f}{\partial y} = 2y$$

⇒ ③ $f(x,y) = x^2 y^2$

$$\frac{\partial f}{\partial y} = ?$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 y^2)$$

$$= x^2 2y$$

$$\frac{\partial f}{\partial y} = 2x^2 y$$

⇒ ④ $f(x,y) = \frac{x^2}{y^2}$

$$\frac{\partial f(x,y)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x^2}{y^2}\right)$$

$$= \frac{1}{y^2}\frac{\partial}{\partial x}(x^2)$$

$$= \frac{1}{y^2} \cdot 2x$$

$$\frac{\partial f(x,y)}{\partial x} = \frac{2x}{y^2}$$

(5) $\Rightarrow f(x,y) = \dfrac{x^2+2x}{3x+y^3}$

we use Chain rule

$\boxed{\frac{\partial}{\partial x} f(g(x)) = f'g(x)\, g'(x)}$

$\frac{\partial}{\partial y} \cancel{f(g(y))}$

$= \cancel{f'(y)\, g'(y)}$

(or)

Quotient Rule

$\dfrac{f(x)}{g(x)} = \dfrac{f'g - g'f}{g^2}$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x^2+2x}{3x+y^3}\right)$$

$$= (x^2+2x)\,\frac{\partial}{\partial y}\left(\frac{1}{3x+y^3}\right)$$

$$= (x^2+2x)\,\frac{\partial}{\partial y}\left((3x+y^3)^{-1}\right)$$

$$= (x^2+2x)\cdot(-1)(3x+y^3)^{-1-1}\,\frac{\partial}{\partial y}(3x+y^3)$$

$$= (x^2+2x)\cdot(-1)\,(3x+y^3)^{-2}\cdot(3y^2)$$

$$= (x^2+2x)\cdot\frac{-1}{(3x+y^3)^2}\cdot 3y^2$$

$$= -\frac{(x^2+2x)\cdot 3y^2}{(3x+y^3)^2}$$

Trigonometry.

(1) $f(x,y) = \cos(x,y)$

$\frac{\partial f}{\partial x} = ?$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\cos(xy)) = -\sin(xy)\cdot\frac{\partial}{\partial x}(xy)$$

$$= -\sin(xy)\cdot y$$

$$\boxed{\frac{\partial f}{\partial x} = -y\sin(xy)}$$

(2) $f(x,y) = \sin(xy) \Rightarrow \frac{\partial f}{\partial y} = ?$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\sin(xy)$$

$$= \cos(xy)\,\frac{\partial}{\partial y}(xy)$$

$$\boxed{\frac{\partial f}{\partial y} = x\cos(xy)}$$

$\Rightarrow$ ③ $f(x,y) = \cos(x^2+4y)$

$\frac{\partial f}{\partial y} = ?$ $\quad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \cos(x^2+4y)$

$= -\sin(x^2+4y) \cdot \frac{\partial}{\partial y}(x^2+4y)$

$= -4 \cdot \sin(x^2+4y)$

$$\boxed{\frac{\partial f}{\partial y} = -4 \cdot \sin(x^2+4y)}$$

④ $f(x,y) = \sin\left(\sqrt{\frac{x}{y}}\right)$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \sin\left(\sqrt{\frac{x}{y}}\right)$

$= \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \frac{\partial}{\partial x}\left(\left(\frac{x}{y}\right)^{1/2}\right)$

$= \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \frac{1}{2}\left(\frac{x}{y}\right)^{1/2-1}$

$= \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \frac{1}{2}\left(\frac{x}{y}\right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial x}\left(\frac{x}{y}\right)$

$= \cos\sqrt{\frac{x}{y}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{x}{y}}} \cdot \frac{1}{y}$

$$\cancel{\boxed{\frac{\partial f}{\partial x} \Rightarrow = \frac{1}{2} \cdot \cos\sqrt{\frac{x}{y}}}}$$

$$\cancel{\boxed{\frac{\partial f}{\partial x} = \frac{1}{2y} \cdot \frac{1}{\sqrt{\frac{x}{y}}} \cdot \cos\left(\sqrt{\frac{x}{y}}\right)}}$$

$\frac{2}{\sqrt{2}}$
$\sqrt{2}$
$\frac{1}{\sqrt{\sqrt{2}}}$
$\sqrt{2}$
$\sqrt{2}$

$\Rightarrow \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \frac{\partial}{\partial x}\left(\left(\frac{x}{y}\right)^{1/2}\right)$

$= \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \sqrt{\frac{1}{y}} \cdot \frac{\partial}{\partial x}\left((x)^{1/2}\right)$

$= \cos\left(\sqrt{\frac{x}{y}}\right) \cdot \sqrt{\frac{1}{y}} \cdot \frac{1}{2} \cdot \frac{}{\sqrt{x}}$

$$\boxed{\frac{\partial f}{\partial x} = \frac{1}{2}\sqrt{\frac{1}{y}} \cdot \frac{1}{\sqrt{x}} \cdot \cos\sqrt{\frac{x}{y}}}$$

Logarithms

⇒ ① $f(x,y) = y \ln x$

$$\frac{\partial f}{\partial y} = \frac{\partial (y \ln x)}{\partial y}$$

$$= \frac{\partial}{\partial y}(y \cdot \ln x) = \ln x \cdot 1$$

$$\frac{\partial f}{\partial y} = \ln x$$

⇒ ② $f(x,y) = \ln(x) - x\ln(-y)$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\ln(x) - x\ln(-y)\right)$$

$$= \frac{1}{x} - \ln(-y) \cdot 1$$

$$\left(\frac{\partial f}{\partial x}\right) = \frac{1}{x} - \ln(-y)$$

③ $f(x,y) = \frac{\ln(x)}{y}$

$$\frac{\partial f}{\partial y} = \cancel{\frac{\partial}{\partial y}\left(\frac{\ln(x)}{y}\right)} = \frac{\partial}{\partial y}\left(\frac{\ln(x)}{y}\right)$$

$$\frac{\partial f}{\partial y} = \ln(x)\frac{\partial}{\partial y}(y^{-1})$$

$$= \ln(x) \quad -1\ y^{-2}$$

$$\frac{\partial f}{\partial y} = -\frac{\ln(x)}{y^2}$$

④ $f(x,y) = y^2 - \ln(x^2y+1)$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(y^2 - \ln(x^2y+1)\right)$$

$$= 2y - \frac{\partial}{\partial y}\ln(x^2y+1)$$

$$= 2y - \left(\frac{1}{x^2y+1} \cdot (x^2)\right)$$

$$\frac{\partial f}{\partial y} = 2y - \frac{x^2}{x^2y+1}$$