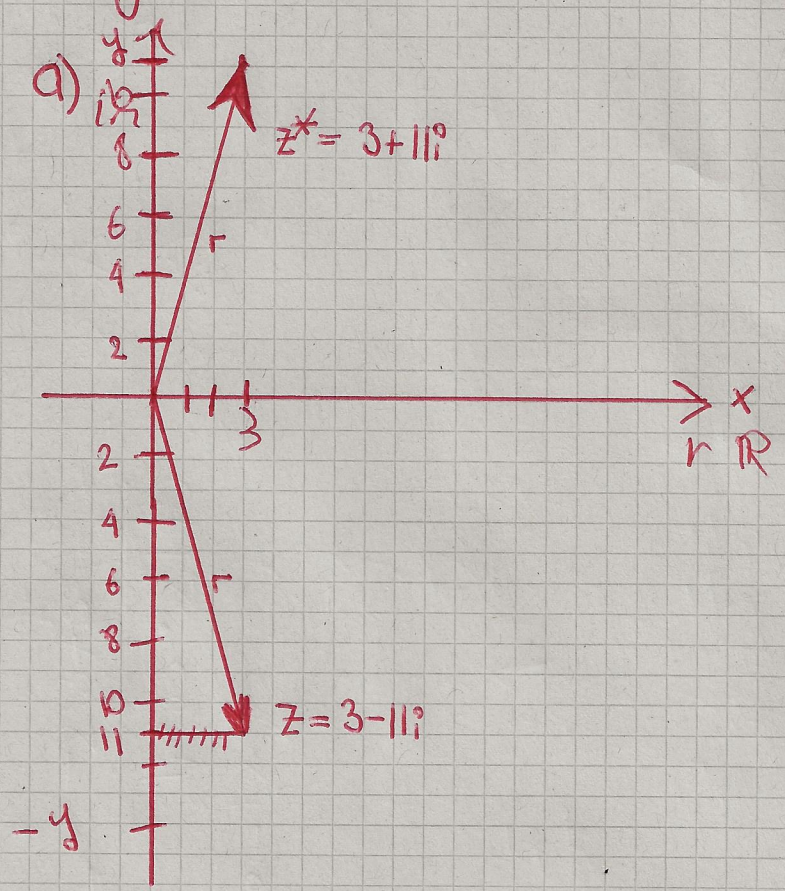
Übungsserie 11

Aufgabe 1)

$z = 3 - 11i$

a)



$r = \sqrt{x^2 + y^2} = \sqrt{9 + 121} = \sqrt{130}$

$\approx 11{,}4018$

$\cos\varphi = x/r = 3/11{,}4018 \approx 0{,}2631$

$\sin\varphi = 11/11{,}4018 \approx 0{,}9648$

$\varphi = -1{,}3045$

Trigonometrische Form

~~$z = 11{,}4018(0{,}2631 + i \cdot 0{,}9648)$~~

$z = 11{,}4018(\cos\varphi + i\sin\varphi)$

$z^* = 11{,}4018(\cos\varphi - i\sin\varphi)$

Exponentialform:

$z = 11{,}4018 \cdot e^{i \cdot 1{,}3}$

$z^* = 11{,}4018 \cdot e^{+i \cdot 1{,}3}$

Aufgabe 1b)

~~4~~ $\cos(-40°) + i \cdot \sin(-40°)$

$r = 4$; ~~y~~ $= r \cdot \sin(\varphi)$; ~~x~~ $= r \cdot \cos(\varphi)$

$y = -2{,}5712$; $x = 3{,}0642$

1) $3{,}0642 - 2{,}5712i$

~~3)~~ $-3 + 1{,}5i$

$(1{,}7321 + 3{,}0642 - 3) + i(-2{,}5712 + 1 + 1{,}5)$

$z = 1{,}7963 - 0{,}0712\,i$

$z^* = 1{,}7963 + 0{,}0712i$

$2e^{i30°}$ $\quad \varphi = 30°$, $r = 2$

$y = r \cdot \sin(\varphi)$, $x = r \cdot \cos(\varphi)$

$x = 1{,}7321$ $\quad y = 1$

2) $1{,}7321 + 1i$

S2

$\varphi_1 = \arctan\left(\frac{y}{x}\right) = 0{,}1636 \quad \varphi_2 = \arctan\left(\frac{-2}{1}\right) = -1{,}1072$

Aufgabe 1c

$r_1 = \sqrt{2^2+1^2} = \sqrt{5} \quad r_2 = \sqrt{1^2+(-2)^2} = \sqrt{5}$

$$z_1 = \frac{2+i}{1-2i} = \frac{\sqrt{5}\cdot e^{i\cdot 0{,}4636}}{\sqrt{5}\cdot e^{i\cdot(-1{,}1072)}} = \underline{e^{i(1{,}5708)}}$$

$z_3 = \underline{4e^{i\,0{,}5236}}$ $\quad 30° \rightarrow 0{,}5236\ \text{rad}$

$z_2 = 2e^{-i\pi/3} \Rightarrow \underline{2e^{-i\,1{,}0472}}$

$z_1^* = \underline{e^{i(-1{,}5708)}}$

$$\frac{z_1^* \cdot z_3}{0{,}5\, z_2} \Rightarrow \frac{e^{i(-1{,}5708)} \cdot 4e^{i\,0{,}5236}}{0{,}5 \cdot 2e^{-i\cdot 1{,}0472}} = \frac{4e^{i(-1{,}0472)}}{e^{-i\cdot(1{,}0472)}} =$$

$$\frac{4}{1}\left(e^{i(-1{,}0472+1{,}0472)} = 4e^{0} = \underline{\underline{4}}\right.$$

Aufgabe 1d)

$(1-\sqrt{2}i)^3$

$r \rightarrow \sqrt{x^2+y^2} \Rightarrow \sqrt{1^2+2} = \sqrt{3}$

$\varphi \Rightarrow \arctan\left(\frac{-2}{1}\right) \Rightarrow -0{,}9553$

$z = \sqrt{3}\,e^{i(-0{,}9553)}$

$z \cdot z \cdot z \Rightarrow$ [illegible]

$r = \sqrt{3}^3 \Rightarrow 3\cdot\sqrt{3}$

$\cdot\ e^{i(-0{,}9553\cdot 3)} \Rightarrow \underline{\underline{3\cdot\sqrt{3}\,e^{i(-2{,}866)}}}$