Serie 5

Donnerstag, 24. Oktober 2024 13:14

Aufgabe 1

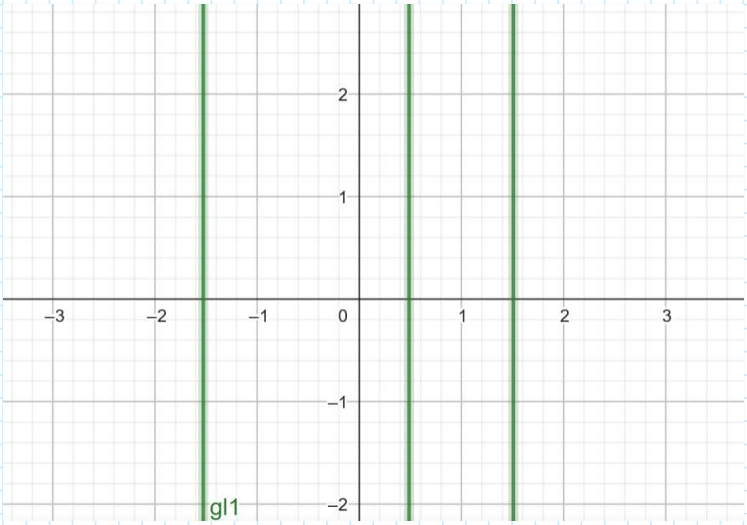
3 Nullstellen

$[-2,-1]$, $[0,1]$, $[1,2]$

$$e^{x^2} + x^{-3} = 10$$

$$f(x) = e^{x^2} + x^{-3} - 10$$

$$f'(x) = e^{x^2} \cdot 2x - 3x^{-4}$$



Newtonverfahren:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Startwert	Resultat
$x_0 = 2$	1,7950
$x_1 = 1,7950$	1,6250
$x_2 = 1,6250$	1,5308
$x_3 = 1,5308$	1,5086
$x_4 = 1,5086$	1,5076
$x_5 = 1,5076$	1,5076

vereinfachtes Newtonverfahren:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}$$

Startwert	Resultat
$x_0 = 2$	1,7950
$x_1 = 1,7950$	1,7251
$x_2 = 1,7251$	1,6802
$x_3 = 1,6802$	1,6479
$x_4 = 1,6479$	1,6235
$x_5 = 1,6043$	1,5889
$x_6 = 1,5889$	1,5764
$x_7 = 1,5764$	1,5660

Startwert	Resultat
$x_0 = 0,5$	0,4847
$x_1 = 0,4847$	0,4856
$x_2 = 0,4856$	0,4856
$x_3 = 0,4856$	0,4856

Startwert	Resultat
$x_0 = 0,5$	0,4847
$x_1 = 0,4847$	0,4857
$x_2 = 0,4857$	0,4856
$x_3 = 0,4856$	0,4856
$x_4 = 0,4856$	0,4856

Sekantenverfahren:

$x_0 = 1$ $x_1 = 1,2$

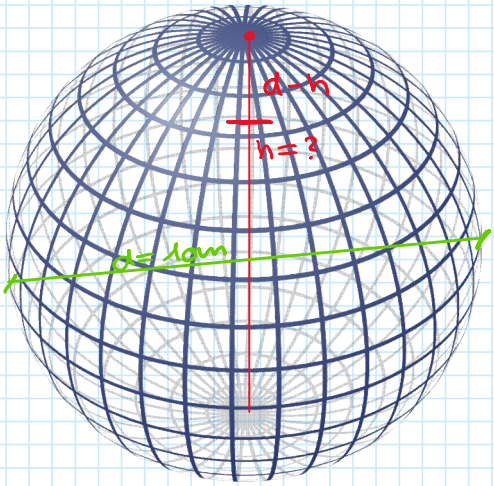
$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{} \cdot f(x_n)$$

$x_0 = \dots \quad x_1 = \dots$

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \cdot f(x_n)$$

Startwert	Resultat
$x_0 = 1{,}0$	
$x_1 = 1{,}2$	2,1621
$x_2 = 2{,}1621$	1,2488
$x_3 = 1{,}2488$	1,2912
$x_4 = 1{,}2912$	1,6565

Aufgabe 2



$V_{max} = 471\,m^3$ Wasser

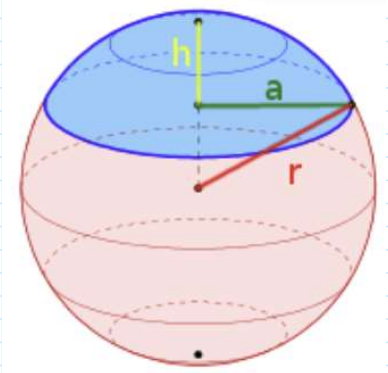
$d = 10\,m \qquad r = 5\,m$

$h = ? \qquad h_0 = 9\,m$

Fehlertoleranz 10^{-3}

$$V = \frac{4}{3}\pi \cdot r^3 = \frac{1}{6} \cdot \pi \cdot d^3$$

$$= \frac{4}{3}\pi \cdot 5^3 = \underline{\underline{523{,}599\,m^3}}$$



$$V_{Kugelsegment} = \frac{h^2 \cdot \pi}{3} \cdot (3 \cdot r - h)$$

$$f(h) = \frac{h^2 \cdot \pi}{3} \cdot (3 \cdot r - h) - V_{Kugelsegment}$$

$$f'(h) = 2h \cdot r \cdot \pi - h^2 \cdot \pi$$

Newtonverfahren:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Startwert	Resultat
$h_0 = 9m$	$7{,}658$
$h_1 = 7{,}658$	$8{,}015$
$h_2 = 8{,}015$	$8{,}037$
$h_3 = 8{,}037$	$8{,}037$