

$$Ax = b$$

a) $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-1} & 0 & 10^{-4} \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} -1 & 0 & 2 \cdot 10^4 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \cdot 10^4 \end{pmatrix}$

$\text{cond}(A) \cdot \|A^{-1}\| = 3 \cdot 20001 = \underline{\underline{60003}}$

b) $\frac{\|\tilde{x} - x\|_\infty}{\|x\|_\infty} \Rightarrow 1\% \rightarrow 0,01$ $\|b\|_\infty = \max \frac{1}{\epsilon}$

$$\|b - \tilde{b}\| \Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & \epsilon \end{pmatrix} \geq \frac{\epsilon}{\max}$$

$$0,01 \leq 60003 \cdot \frac{\epsilon}{1} = \epsilon = \underline{\underline{\frac{0,01}{60003}}}$$

c) $A \circ x = b$ $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-1} & 0 & 10^{-4} \end{pmatrix} \circ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{array}{l} x_1 = 1 - 2x_3 \\ x_2 = 1 \\ x_3 = (-10^{-1}) / 10^{-4} \end{array}$

$$\begin{array}{lcl} x_1 = 1 - 2x_3 \\ x_2 = 1 \\ x_3 = -1 \\ \hline \end{array} = \begin{array}{lcl} x_1 = 1 + 2x_1 \\ x_2 = -1 \\ x_3 = -1 \\ \hline \end{array} = \begin{array}{lcl} x_1 = -1 \\ x_2 = 1 \\ x_3 = 1 \\ \hline \end{array}$$

$A \circ \tilde{x} = \tilde{b}$ $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-1} & 0 & 10^{-4} \end{pmatrix} \circ \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \frac{0,01}{60003} \end{pmatrix} = \begin{array}{l} \tilde{x}_1 = -0,99666633 \\ \tilde{x}_2 = 1 \\ \tilde{x}_3 = 0,9983342 \end{array}$

~~$\max \frac{\|\tilde{x} - x\|_\infty}{\|x\|_\infty} = \frac{1,667 \cdot 10^{-4}}{1}$~~

$$\begin{array}{r} + 0,99666683 \\ - 1 \\ \hline 1 - 0,9983342 \end{array}$$

$$\underline{\underline{= 0,0016658}}$$

$$\begin{pmatrix} -0,00333317 \\ 0 \\ 0,0016658 \end{pmatrix} = \underline{\underline{0,00333317}}$$

b) $\text{cond}(A) = 60003$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix}, \quad F = (1e-7)$$

$$\tilde{A} = \begin{pmatrix} 1+F & F & 2+F \\ 0 & 1+F & F \\ 10^{-4}F & F & 10^{-4}+F \end{pmatrix}$$

$$\frac{\|A - \tilde{A}\|}{\|A\|} \approx 1,000000001946 \cdot 10^7$$

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$$0.01 \leq \frac{60'003}{(1-60003) \cdot \frac{\|A - \tilde{A}\|}{\|A\|}} \cdot \left(\frac{\|A - \tilde{A}\|}{\|A\|} + \epsilon \right) = \underline{6,568} \cdot 10^{-8}$$