

(Time duration: One hour)

01. Find the following limits.

i. $\lim_{x \rightarrow 2} \left(\frac{x^3 - x^2 - x - 2}{x - 2} \right)$

ii. $\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 7x - 1}{x^2 + 5} \right)$

iii. $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+4} - 2}{x(x+1)} \right)$

iv. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

v. $\lim_{x \rightarrow \infty} \frac{2 - \cos x}{x+3}$

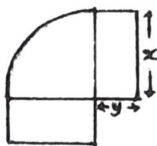
02. Find $\frac{dy}{dx}$. (Give your answer in the simplest form)

i. $y = (3x^2 - 1)(x^2 + 5x + 2)$

ii. $y = \frac{x^2 + 6}{2x - 7}$

iii. $2x^2 + 3xy + 5y^2 = 0$

03. The figure shows a flowerbed. Its shape is a quarter of a circle of radius x meters with two equal rectangles attached to it along its radii. Each rectangle has length equal to x meters and width equal to y meters.



Given that the area of the flowerbed is 4 m^2 .

i. Show that

$$y = \frac{16 - \pi x^2}{8x}$$

ii. Hence show that the perimeter P meters of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

iii. Use calculus to find the minimum value of P .

iv. Find the width of each rectangle when the perimeter is a minimum.