For two different fluctuation analysis method, we both take multifractality into consideration, so that " Multifractal detrended cross-correlation analysis " will be DCCA in short and " Detrended partial cross-correlation analysis " DPXA in short in the latter sections. We admit at $H\_{2, \cdot}$ is the measuring point for long-term cross-correlation of the return series.(cite) For both Period \Rmnum{1} and Period \Rmnum{2}, the Hurst exponents are larger than 0.5, indicating a strong persistent long-term correlation in the three pair of time series.

---------------------COMPARE--------------------------------

(Graph at point h=2 a-b-c a/b/c in period1-2

(not include \tau in discussion

Here we applied two different fluctuation analysis method to calculate generalized Hurst exponents. For normal prospect of time series analysis, we consider the Hurst exponent at order q=2, which reveals the long-term cross-correlation of the time series. In both DCCA and DPXA method, we observe an increase of Hurst exponents in three pairs of data from Period \Rmnum{1} to Period \Rmnum{2}. However, Hurst exponents calculated by DPXA has a greater increment. Since DPXA removes the common external force from time series that analyzed, it may infer that crude oil is the common external force. With the common external force subtracted in DPXA, we obtained a stronger long-term correlation, implying the existence of nucleus common factors, which confirm our suggestion for a complicated and not measurable common external force.

In empirical point of view, dominant factor always plays an important role in cross-correlation.

Since A-data has the same size of bulk carrier, the shipping line of B-data has the same starting point and C-data does not show a dominant factor, we expected that $H\_A>H\_B>H\_C$. We observed that Hurst exponents from DPXA fit the empirical prediction, however, that from DCCA have $H\_A>H\_C>H\_B$ which contradicts with empirical prediction. Under this circumstance, DCCA may not produce a empirically correct answer for Hurst exponents, so that DPXA appears to be a more advanced way of analysis. Despite the measuring point at q=2, the curves of general Hurst exponents generated by DPXA and DCCA are quite similar during the same period, hence DPXA is an effective way of improving DCCA.

Based on Eq.X, we can conclude that if the multifractal spectrum appears as a point, it is monofractal. As a result, the width of the multifractal spectrum can be employed to estimate the strength of multifractality(cite), with which we can quantitatively analyze the origin of multifractality. Apparently, in each multifractal spectrum, the curve is not a point which is a proof of the character of multifractality. Due to the limited size of data, we have to consider finite size effect to precisely analyze the effect part of the strength of multifractality.

The ratio of effective part is an import criterion for estimating multifractality. From Period \Rmnum{1} to Period \Rmnum{2}, the great increment of effective part relies on the decrease of linear correlation part and the increase of non-linear correlation part Furthermore, we observe that percentage of effective part in DPXA is larger of that in DCCA, persuading the effectiveness of DPXA. Also, linear correlation part calculated by DCCA and DPXA is close in Period \Rmnum{1}, however, distinguishable in period, so that in Period \Rmnum{1} common external forces mainly have impact on surrogated time series, and in period 2 linear correlation part is also infected.

---------------------MULTIFRACTAL-----------------------------

The Hurst exponents decrease with the order q with a non-linear relation as shown in Figure 1, 2, 3, 4, which indicates the fractal characteristic in the cross-correlation of the three pair of time series. We can also observe that the Hurst exponents of B-data and C-data exchange their relative magnitude at some points, where q-order is smaller(negative) in Period \Rmnum{1} and larger(positive) in Period \Rmnum{2}, meanwhile the Hurst exponents of A-data hold their relative position. In addition, it appears to be a stronger non-linear characteristic in Period \Rmnum{2} than in Period \Rmnum{1}, this phenomena can be explained as the expansion of fractal characteristic, which may be attributed to the economic fluctuation after financial crisis. But a nucleus common factor might still exist before and after the financial crisis, which still does not have an accurate data set to describe or too complicated to be stated. This type of common external force may also be the reason for the enlarged difference of Hurst exponent between Period \Rmnum{1} and Period \Rmnum{2}.

It has been clearly stated that two diﬀerent types of multifractality may exist in a time series data, namely multifractality due to long-term cross-correlations of the ﬂuctuations which is composed of the $\d\a\_{LM}$ (linear) and $\d\a\_{NL}$(nonlinear) correlation parts and multifractality due to a fat-tailed probability distribution function of the values in the series($\d\a\_{PDF}$).(cite)

As we calculate every part of $\d\a$ of DCCA and DPXA, we conclude that the $\d\a\_{EFF}$(effective part) of $\d\a$ mainly consists of $\d\a\_{NL}$(non-linear part), and $\d\a\_{NL}$ in DPXA is less than that in DCCA, which reveals a clearer trend after common external forces are wiped off. The increment of $\d\a\_{NL}$ of time series with strong dominant common factors (A-data, B-data) is larger than that of a relatively weak pair of time series(C-data) after financial crisis. In addition, B-data has the greater increment than A-data. Comprehensive causation behind this result can be length of the shipping line, type of harbor, or even the random choice of surrogating.

Another segment of $\d\a\_{EFF}$ is $\d\a\_{PDF}$, resulting from the probability distribution of cross-correlation function. As for data produced by random walk, central limit theorem states that the probability distribution is normal distribution if given large size of sample. Since we take log returns for all data, we get a log-normal distribution for original data which is a fat-tailed distribution. Obviously, none of those return data are random, which satisfies the result of Hurst exponents. As $\d\a\_{PDF}$ increases after financial crisis, the trend of global trend of the cross-correlation function is enhanced, which explains the increase of Hurst exponent. With the common external force subtracted in DPXA, the randomness caused by crude oil price is also reduced. Therefore, $\d\a\_{PDF}$ in DPXA is larger than that in DCCA

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Renyi exponent只要写在其中一篇里就可以了

From Eq. (7), we calculate and obtain the Renyi exponent $\tau(q)$ for three pairs of data that

varies with q varying from -5 to 5. The Renyi exponent $\tau(q)$ shows the the shape of

nonlinear curves both in Period \Rmnum{1} and Period \Rmnum{2} , which can also confirm the multifractal cross-correlation between BDTI return series.

Origin-random-surrogated 的三列表格放在compare里加上一列effective part 和占比；另一系列表格中去掉占比。

Compare里面不放origin-random-surrogated的图