1 VAE derivation of loss

$$\log(p(x)) =$$

$$\log\left(\sum_z p(x|z)p(z)\right) =$$

$$\log\left(\sum_z p(x|z)p(z)\frac{q(z|x)}{q(z|x)}\right) \geq$$

$$\mathbb{E}_{z\sim q(z|x)} \log\left(\frac{p(x|z)p(z)}{q(z|x)}\right) =$$

$$\mathbb{E}_{z\sim q(z|x)} \log\left(p(x|z)\right) + \mathbb{E}_{z\sim q(z|x)} \log\left(\frac{p(z)}{q(z|x)}\right) =$$

$$\mathbb{E}_{z\sim q(z|x)} \log\left(p(x|z)\right) - \mathbb{E}_{z\sim q(z|x)} \log\left(\frac{q(z|x)}{p(z)}\right) =$$

$$\mathbb{E}_{z\sim q(z|x)} \log\left(p(x|z)\right) - \mathrm{KL}\left(q(z|x)||p(z)\right)$$

We want to maximize the above so we want to minimize

$$-\mathbb{E}_{z\sim q(z|x)} \log\left(p(x|z)\right) + \mathrm{KL}\left(q(z|x)||p(z)\right)$$

Useful formula when we assume that $p(z) \sim \mathcal{N}(0,1)$ and $q(z|x) \sim \mathcal{N}(\mu,\sigma)$

$$\mathrm{KL}\left(q(z|x)||p(z)\right) = \frac{1}{2}\left(\sigma^2 - \log\left(\sigma^2\right) + \mu^2 - 1\right)$$