- The theory part of the exam will be held from 9:00 AM to 10:00 AM and is worth 20 points.
- The practical part will be held from 10:20 AM to 1:30 PM and is worth 30 points.
- For the theoretical part of the exam, you are allowed to refer to your personal written notes. (no iPads)
- During the practical part of the exam, you are permitted to use your own equipment and consult online documentation. However, using sites such as Stack Overflow is not allowed.
- There are four tasks in theory part to be completed. Each task must be solved on a separate piece of paper and signed by you to confirm its completion.

Theory problems:

1. We have two identical pinhole cameras with focal lengths 5 and optical centers $x = 300$, $y = 200$. Cameras have no distortions.
   (a) (1 point) What is the camera matrix?
   (b) (2 points) Let's assume two cameras' optic axes are parallel and the second camera is shifted by `x=0,y=1,z=0` with respect to the first camera. What is the pixel disparity (distance in pixels) between the points on images that correspond to a point in the 3D space with coordinates `(5, 5, 5)` in the first camera's coordinate frame.

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2. Recall from the classes that we can model population growth using the so called logistic model:

$$\dot{x} = x(P_{max} - x)$$

   where x is the population size and P_{max} is the population limit above which the environment becomes resource scarse.

   Now consider the more complicated situation when two species are competing with each other. In this situation the change in population depends also on the amount of interactions between two populations:

$$\dot{x} = x(P_{max} - x) - a \cdot g(x, y)$$

   where y is the current population size of the second species, a is some parameter and g is some function of x and y describing the interactions between species.

   Your task is to use the above model for a competition between rabbits and wolves - population sizes denoted by r and w respectively. You have to:
   (a) (1 point) write the ODEs describing the system assuming that:
   - the number of interactions between two populations is simply the number of (rabbit, wolf) pairs that can be created in the environment
   - P_{max} is equal to 4 for rabbits and 3 for wolves
   - a is equal to 2 for rabbits and 1 for wolves
   - your state is $\vec{x} = [r, w]^T$

   (b) (1 point) find fixed points of the created system
   (c) (3 points) linearize dynamics of the system around those fixed points

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3. Given the visual description of the kinematic chain (figure 1), consisting of:
   1. prismatic joint attached to base with actuation d_1 from 0 to 1 right
   2. link: a_1 right, a_2 up, a_3 right
   3. revolute joint with actuation θ from 0 to 2π
   4. link: a_4 up, a_5 right
   5. prismatic joint with actuation d_2 from 0 to 1 downwards
   6. link: a_6 down, ending with the end-effector

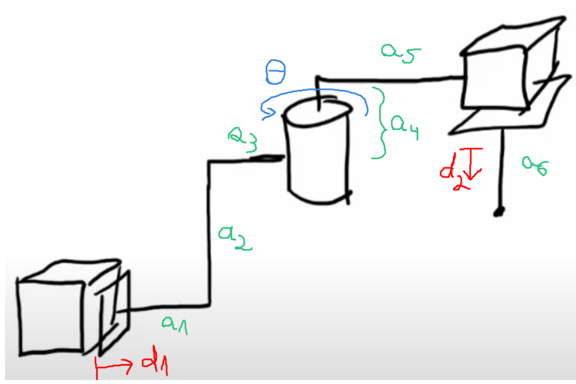

Figure 1: Kinematic chain for problem 2

do the following:

(a) (1 point) Find the forward kinematics $FK(d_1, \theta, d_2)$ of the robot

(b) (1 point) Find the workspace of the robot with fixed $d_1 = 0$

(c) (1 point) Find the inverse kinematics

$$IK$$

of the robot with fixed $d_1 = 0$, i.e. whenever $FK(0, \theta, d_2) = (x, y, z)$ we have $IK(x, y, z) = (\theta, d_2)$

(d) (1 point) Assign frames to the joints of the kinematic chain using the DH-convention

(e) (2 points) Create the DH-table for the kinematic chain including the actuation of joints

4. In this task we have a robot that is equipped with a 3D camera, and the camera's frame is represented by the coordinate system C. The global coordinate system is represented by the coordinate system W. The position and orientation of the camera (transformation from W to C) is given by a homogeneous transformation matrix H_C^W.

   (a) (2 points) Suppose that a point P has coordinates P^C in the camera frame C. Write the expression for the point P^W, that is the same point P expressed in the world frame. Use homogeneous coordinates.

   (b) (2 points) In this step you cannot use the matrix inverse operator M^{-1} directly. Use your knowledge about the inverse of the homogeneous transformation matrix.

   The point P is the same one as in the previous step. Now there is also a point Q. The difference between the two points is given by the vector δ^W in the global frame W, meaning $P^W + \delta^W = Q^W$ (all in the global frame W). Write an expression for the coordinates of the second point Q in the camera frame C, that is Q^C.

   Express the result using P^C, δ^W and component blocks of H_C^W.

   (c) (2 points) Use the result from two previous points to find the coordinates of point Q in the camera frame, assuming the following values:

$$H_C^W = \begin{bmatrix} \cos(45°) & -\sin(45°) & 0 & 1 \\ \sin(45°) & \cos(45°) & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^C = \begin{bmatrix} p_x^C \\ p_y^C \\ p_z^C \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\delta^W = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$