

RC 2023/24, Exam, theory

Every problem is worth the same amount of points. Every problem should be solved on a separate sheet of paper. Write your name on every sheet of paper. You can use a calculator and a ruler. You have 60 minutes to solve the problems.

Problem 1

Consider the non-linear dynamical system given by the following equations:

$$\dot{x} = y - x^3 + 3x^2 \quad (1)$$

$$\dot{y} = -x - 2y^3 + y \quad (2)$$

where \dot{x} and \dot{y} represent the time derivatives of x and y respectively.

Tasks:

1. **Find the Fixed Points:** Determine all the fixed points of this system.
2. **Linearize the System:** For each fixed point found in part (1), linearize the system around that point by computing the Jacobian matrix at each fixed point.

$$J = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix}$$

Problem 2

Consider a robot with three joints and three links, resembling the one encountered in Lab 13. The first and last joints are revolute, while the second joint is prismatic. The table below provides some of the possible configurations $(\theta_1, l_2, \theta_3)$ of the robot along with the corresponding positions of the robot's end effector:

θ_1	l_2	θ_3	x	y	z
0	0	0	0	1	0
0	0.5	0	0	1.5	0
0	0.5	90	0.5	1	0
90	0.5	90	0	1	0.5
180	0.5	90	?	?	?
180	0.5	180	?	?	?

We use degrees for the angles and meters for the lengths.

Tasks:

1. **Missing values:** Find the missing values in the table above (write it on paper).
2. **Forward kinematics:** Compute the forward kinematics of the robot end effector.

Problem 3

Let's assume that we have a pinhole camera with the following parameters:

- Focal length: $f = 1$
- Principal point: $c = (0, 0)$
- Image resolution: 640×480

The camera, in world frame, is located at the position $p = (0, 0, 0)$ and is oriented in the direction of the z axis.

The camera sees a point at the following pixel coordinates: $p = (320, 240)$. z coordinate of the point in the world frame is $z = 5$.

Tasks:

1. **Compute the 3D coordinates:** Compute the 3D coordinates of the point in the world frame.
2. **Move the camera:** What would be the pixel coordinates of the point if the camera was moved to the position $p = (0, 0, 1)$ remaining oriented in the direction of the z axis?

Assume that the camera is calibrated and that the distortion is negligible.