Relational Algebra

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CS 348
Introduction to Database Management
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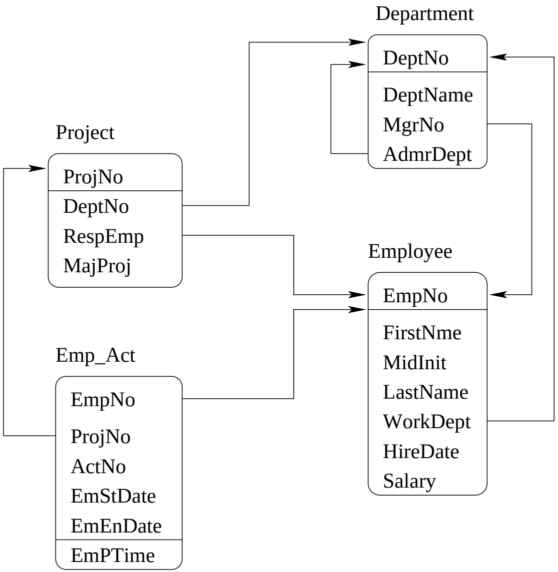
Database Schema Used in Examples



Relational Algebra

- the relational algebra consists of a set of *operators*
- relational algebra is *closed*
    - each operator takes as input zero or more relations
    - each operator defines a single output relation in terms of its input relation(s)
    - relational operators can be composed to form expressions that define new relations in terms of existing relations.

Some Relational Operators

$$
\begin{array}{rcl}
E & ::= & R \\
& | & \sigma_{condition}(E) \\
& | & \pi_{attributes}(E) \\
& | & \rho(R(\overline{F}, E)) \\
& | & E_1 \times E_2 \\
& | & E_1 \cup E_2 \\
& | & E_1 - E_2
\end{array}
$$

R is a relation name and E is a relational algebra expression

Some Relational Operators (cont'd)

- Relation Name: R
- Selection: $\sigma_{condition}(E)$
    - result schema is the same as E's
    - result instance includes a subset of the tuples of E
- Projection: $\pi_{attributes}(E)$
    - result schema includes only the specified attributes
    - result instance would have as many tuples as E, except that duplicates are eliminated

Some Relational Operators (cont'd)

- Rename: $\rho(R(\overline{F}, E))$
  - $\overline{F}$ is a list of terms of the form $oldname \rightarrow newname$
  - returns the result of E with columns renamed according to $\overline{F}$.
  - remembers the result as R for future expressions
- Product: $E_1 \times E_2$
  - result schema has all of the attributes of E_1 and all of the attributes of E_2
  - result instance includes one tuple for every pair of tuples (one from each expression result) in E_1 and E_2
  - sometimes called cross-product or Cartesian product
  - renaming is needed when E_1 and E_2 have common attributes

Cross Product Example

R

AAA	BBB
a_1	b_1
a_2	b_2
a_3	b_3

S

CCC	DDD
c_1	d_1
c_2	d_2

$R \times S$

AAA	BBB	CCC	DDD
a_1	b_1	c_1	d_1
a_2	b_2	c_1	d_1
a_3	b_3	c_1	d_1
a_1	b_1	c_2	d_2
a_2	b_2	c_2	d_2
a_3	b_3	c_2	d_2

Select,Project,Product Examples

- Note: Use *Emp* to mean the Employee relation, *Proj* the project relation
- Find the last names and hire dates of employees who make more than $100000.

$$\pi_{LastName,HireDate}(\sigma_{Salary>100000}(Emp))$$

- For each project for which department E21 is responsible, find the name of the employee in charge of that project.

$$\pi_{ProjNo,LastName}(\sigma_{DeptNo=E21}(\sigma_{RespEmp=EmpNo}(Emp \times Proj)))$$

Joins

- Natural join ($R \bowtie S$) is a very commonly used operator which can be defined in terms of selection, projection, and Cartesian product.
- The result of $R \bowtie S$ can be formed by the following steps
  1. form the cross-product of R and S
  2. eliminate from the cross product any tuples that do not have matching values for all pairs of attributes common to R and S
  3. eliminate any duplicate attributes
- Natural join is special case of *equijoin*, a common and important operation.

$$Proj \bowtie_{(\mathrm{RespEmp}=\mathrm{EmpNo})} Emp$$

Example: Natural Join

- Consider the natural join of the Project and Department tables, which have attribute DeptNo in common
  - the schema of the result will include attributes ProjName, DeptNo, RespEmp, MajProj, DeptName, MgrNo, and AdmrDept
  - the resulting relation will include one tuple for each tuple in the Project relation (why?)

Set-Based Relational Operators

- Union ($R \cup S$):
  - schemas of R and S must be "union compatible"
  - result includes all tuples that appear either in R or in S or in both
- Difference ($R - S$):
  - schemas of R and S must be "union compatible"
  - result includes all tuples that appear in R and that do not appear in S
- Intersection ($R \cap S$):
  - schemas of R and S must be "union compatible"
  - result includes all tuples that appear in both R and S
- Union Compatible:
  - Same number of fields.
  - 'Corresponding' fields have the same type

Relational Division

X

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_1	b_2	c_2
a_2	b_1	c_1
a_2	b_1	c_2
a_2	b_2	c_2
a_3	b_1	c_1

S

B	C
b_1	c_1
b_1	c_2
b_2	c_2

X/S

A
a_1
a_2

Division is the Inverse of Product

R

A
a_1
a_2

S

B	C
b_1	c_1
b_1	c_2
b_2	c_2

$R \times S$

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_1	b_2	c_2
a_2	b_1	c_1
a_2	b_1	c_2
a_2	b_2	c_2

$(R \times S)/S$

A
a_1
a_2

Algebraic Equivalences

- This:

$$\pi_{ProjNo,LastName}(\sigma_{DeptNo=E21}(\sigma_{RespEmp=EmpNo}(E \times P)))$$

- is equivalent to this:

$$\pi_{ProjNo,LastName}(\sigma_{DeptNo=E21}(E \bowtie_{RespEmp=EmpNo} P))$$

- is equivalent to this:

$$\pi_{ProjNo,LastName}(E \bowtie_{RespEmp=EmpNo} \sigma_{DeptNo=E21}(P))$$

- is equivalent to this:

$$\pi_{ProjNo,LastName}(\; (\; \pi_{LastName,EmpNo}(E)) \bowtie_{RespEmp=EmpNo} (\; \pi_{ProjNo,RespEmp}(\sigma_{DeptNo=E21}(P))))$$

- More on this topic later when we discuss database tuning...

Relational Completeness

Definition (Relationally Complete)

A query language that is at least as expressive as relational lagebra is said to be *relationally complete*.

- The following languages are all relationally complete:
  - safe relational calculus
  - relational algebra
  - SQL
- SQL has additional expressive power because they capture duplicate tuples, unknown values, aggregation, ordering, ...