Assignment 1

Due: Tuesday, January 19 (at the beginning of class)

Assume that $\Sigma = \{0, 1\}$ for every question on this assignment.

1. Prove that there are countably many finite languages over Σ.

2. For each positive integer n define the language A_n over Σ as follows:

$$A_n = \{x \in \Sigma^* \,:\, |x| \geq n \text{ and the } n\text{-th symbol from the end of } x \text{ is } 0\}.$$

   To make clear what is meant by "the n-th symbol from the end", the fourth symbol from the end of the following string has been underlined:

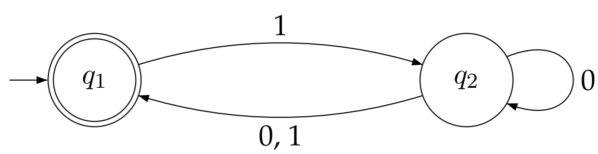
$$0010111\underline{0}100$$

   Given that this symbol is 0, it therefore holds that $00101110100 \in A_4$.

   Give a DFA for A_3 that has the minimum number of states required for such a DFA.

   You do not need to prove that the number of states in your DFA is minimal, and if your DFA has more states than is necessary you will partial (but not full) credit for this question.

3. Consider the following NFA N:



   Give an NFA with just two states that recognizes the language $\overline{L(N)}$ over Σ.

4. Decide whether the following statements are true or false, and defend your answer: give a short proof for each statement you believe is true, and give a counter-example to each statement you believe is false.

   a. Let $A \subseteq \Sigma^*$. If A^* is regular then A is also regular.
   b. Let $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$ be non-regular languages. Then AB is also non-regular.
   c. For every choice of languages A, B, and C over Σ it holds that $A(B \cap C) = AB \cap AC$.
   d. Let $A \subseteq \Sigma^*$ be a regular language. Then there exists a DFA M with an even number of states such that $A = L(M)$.

5. Let us say that a string x is obtained from a string w by *deletions* if it is possible to remove zero or more symbols from w so that just the string x remains.

   For example, the following strings can all be obtained from 0110 by deletions:

   $$\varepsilon,\ 0,\ 1,\ 00,\ 01,\ 10,\ 11,\ 010,\ 011,\ 110,\ \text{and } 0110$$

   Suppose $A \subseteq \Sigma^*$ is a given regular language, and define

   $$B = \{x \in \Sigma^* \,:\, x \text{ is obtained from some string } w \in A \text{ by deletions}\}\,.$$

   Prove that B is regular.