Assignment 2

Due: Thursday, February 4 (at the beginning of class)

1. Let $\Sigma = \{0, 1, 2\}$, and consider the following three languages over Σ:

$$A_1 = \{x2y \; : \; x, y \in \{0, 1\}^*, \; |x| = |y|\} ,$$
$$A_2 = \{x2y \; : \; x, y \in \{0, 1\}^*, \; x \neq y\} ,$$
$$A_3 = \{xy \; : \; x, y \in \{0, 1\}^*, \; x \neq y\} ,$$
$$A_4 = \{xy \; : \; x, y \in \{0, 1\}^*, \; |x| = |y| \text{ and } x \neq y\} .$$

   Determine whether each of the languages A_1, A_2, A_3 and A_4 is regular or not, and prove that your answers are correct.

2. Give context-free grammars for each of the languages A_1, A_2, A_3 and A_4 defined in question 1.

3. Let $\Sigma = \{0, 1\}$, and define a language $A \subseteq \Sigma^*$ as follows:

   $$A = \{x \in \Sigma^* \; : \; x \text{ contains the substrings } 01 \text{ and } 10 \text{ an equal number of times}\} .$$

   For example, 10001001 is in A, because

   (i) it contains the substring 01 two times: 10001001
   (ii) and it contains the substring 10 two times: 10001001

   Determine which of the following statements is true:

   (a) A is regular.
   (b) A is not regular, but it is context-free.
   (a) A is not context-free.

   Give a detailed proof of whichever statement you select.

4. Suppose α is a real number satisfying $\alpha > 1$. Define a language

   $$E_\alpha = \left\{ 0^{\lfloor \alpha^n \rfloor} \; : \; n \in \mathbb{N} \right\} .$$

   Prove that E_α is not context-free.

5. Let $\Sigma = \{0, 1\}$ and let $A \subseteq \Sigma^*$ be a given regular language. Define a new language $B \subseteq \Sigma^*$ as

   $$B = \{x \in \Sigma^* \; : \; xx \in A\} .$$

   Is B necessarily a regular language? If your answer is "yes," prove that this is so, and if your answer is "no," give a counter-example.