University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 1

Distributed Wednesday, January 5 2011.

Due Wednesday, January 12 2011, in class.

All answers should be accompanied by proofs.

1. [8 marks] Let L be any language over the alphabet Σ. Consider the languages L_1 and L_2 defined by

$$\begin{aligned} L_1 &:= \overline{L^*} \\ L_2 &:= \Sigma^* \, \overline{L} \, \Sigma^* \, . \end{aligned}$$

Which of the following symbols

$$=, \ \neq, \ \subsetneq, \ \subseteq, \ \supseteq, \ \supsetneq$$

can be placed between L_1 and L_2 to make an identity that holds for all L?

Note: $\subseteq$ means "subset, including equality" and $\subsetneq$ means "subset, not including equality", and similarly for $\supseteq$ and $\supsetneq$.

2. [12 marks] On pages 2 and 92–93 of the course textbook you can find the definition of "subsequence" and the operation on languages $\text{sub}(L)$: a word x is a subsequence of a word y if x can be obtained from y by striking out 0 or more letters. For example, `gem` is a subsequence of `enlightenment`. Furthermore,

$$\text{sub}(L) = \{x \ : \ \text{there exists } y \in L \text{ such that } x \text{ is a subsequence of } y\}.$$

Which of the following are identities? Prove your answer.

   (a) $\text{sub}(L_1 \ \cup \ L_2) = \text{sub}(L_1) \ \cup \ \text{sub}(L_2)$

   (b) $\text{sub}(L_1 L_2) = \text{sub}(L_1)\text{sub}(L_2)$

   (c) $\text{sub}(L^*) = \text{sub}(L)^*$

3. [10 marks] On page 2 of the course textbook you can find the definition of "prefix" and "suffix": a word x is a prefix of a word y if there exists z (possibly empty) such that $y = xz$.

A word x is a suffix of a word y if there exists a z (possibly empty) such that $y = zx$.

We can extend this definition to languages by writing

$$\begin{aligned}
\text{pref}(L) &= \{x \ :\ \text{there exists } y \in L \text{ such that } x \text{ is a prefix of } y\} \\
\text{suff}(L) &= \{x \ :\ \text{there exists } y \in L \text{ such that } x \text{ is a suffix of } y\}
\end{aligned}$$

Prove that for all languages L over the alphabet Σ, either $\text{pref}(\ \overline{\text{suff}(L)}\ ) = \Sigma^*$ or $\text{pref}(\ \overline{\text{suff}(L)}\ ) = \emptyset$.