University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 5

Distributed Wednesday, February 2 2011.

Due Wednesday, February 9 2011, in class.

All answers should be accompanied by proofs.

1. [10 marks] Suppose $L \subseteq \Sigma^*$ is a regular language. Show that

$$2L := \{a_1 a_1 a_2 a_2 \cdots a_k a_k \ : \ \text{each } a_i \in \Sigma \text{ and } a_1 a_2 \cdots a_k \in L\}$$

   is regular.

2. [10 marks] Let $h : \Sigma^* \to \Sigma^*$ be a morphism, and let $L \subset \Sigma^*$ be a language. Define

$$h^{-*}(L) = \bigcup_{i \geq 0} h^{-i}(L),$$

   where by $h^{-i}(L)$ we mean $\overbrace{h^{-1}(h^{-1}(\cdots h^{-1}}^{i \text{ times}}(L)))$. Show that if L is regular, so is $h^{-*}(L)$.

3. [10 marks]

   (a) [2 marks – the easy part] Let $h : \{a, b\}^* \to \{0\}^*$ be the morphism defined by $h(a) = h(b) = 0$. Let $L \subseteq \{a, b\}^*$ be a language. If $h(L)$ is regular, need L be regular?

   (b) [8 marks – the harder part] Now let $\Sigma = \{a, b, c\}$. Let $h_{a,b} : \Sigma^* \to (\Sigma \cup \{0\})^*$ be the morphism defined by $h(a) = h(b) = 0$ and $h(c) = c$. Thus $h_{a,b}$ maps both a and b to 0 and leaves the remaining letter, c, unchanged. In a similar way define $h_{a,c}$ and $h_{b,c}$.

   Now let $L \subseteq \Sigma^*$ be a language, and define $A := h_{b,c}(L)$, $B := h_{a,c}(L)$, and $C := h_{a,b}(L)$. If A, B, and C are all regular, need L be regular?