

University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2013
Problem Set 3

Distributed Tuesday, January 22 2013.

Due Tuesday, January 29 2013, in class.

All answers should be accompanied by proofs.

1. [10 marks] Must every primitive word of length > 1 have at least two unbordered conjugates? Prove or disprove.

2. [10 marks] Consider the equation in words $x\text{III}y = z^2$, where III is the perfect shuffle. Describe all solutions to this equation. Hint: there are separate cases depending on whether $|x|, |y|$ are both even or both odd.

(By “describe all solutions” we mean come up with a characterization of the solutions that is somewhat analogous to the descriptions in Theorems 2.3.2 and 2.3.3 of the course text, giving necessary and sufficient conditions for x, y, z .)

3. [10 marks] Call a word w *uneven* if every nonempty subword has the property that at least one letter appears an odd number of times. For example, **abac** is uneven.

- (a) [5 marks] Show that if w is an uneven word over an alphabet with k letters, then $|w| < 2^k$.

- (b) [5 marks] Prove that the bound in (a) is sharp, by exhibiting an uneven word of length $2^k - 1$ over every alphabet of size $k \geq 1$.