University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 4

Distributed Wednesday, January 26 2011.
Due Wednesday, February 2 2011, in class.

All answers should be accompanied by proofs.

1. [10 marks] Which of the following assertions are true for all languages L_1, L_2?

   (a) $(L_1/L_2)L_2 \subseteq L_1$;

   (b) $L_1 \subseteq (L_1/L_2)L_2$.

2. [10 marks] Let $L_1, L_2 \subseteq \Sigma^*$ be languages. In class we defined the (right) quotient L_1/L_2. But there is also a (left) quotient defined by

   $$L_1 \backslash L_2 = \{x \in \Sigma^* \ : \ \text{there exists } y \in L_2 \text{ such that } yx \in L_1\}.$$

   Prove or disprove: if L_1 is regular, and L_2 is any language, then $L_1 \backslash L_2$ is regular.

3. [10 marks] Consider the shuffle operation shuff as defined in Example 3.3.8, page 57. If $\text{shuff}(L, \{0\})$ is regular, need L be regular?