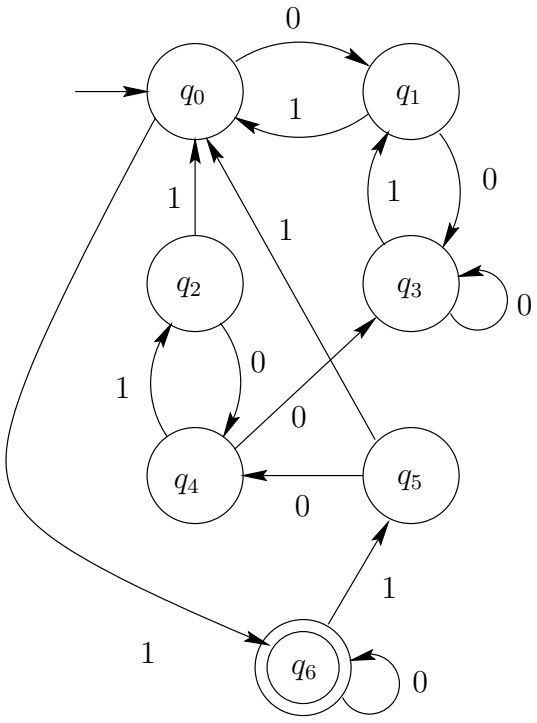
University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 7

Distributed Wednesday, February 16 2011.

Due Wednesday, March 2 2011, in class.

All answers should be accompanied by proofs.

1. [10 marks] Consider the following DFA. Using any algorithm, find the minimal equivalent DFA. Show all your steps.



2. [10 marks] Let L be the language that contains the base-10 representations of the non-negative integers which are not squares (of integers). Thus

$$L = \{2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, \ldots\}.$$

Find the minimal elements (for the subsequence order) for L. Use this to give an expression for $\sup(L)$.

3. [20 marks]

(a) [10 marks]

Let n be an integer ≥ 1, and let $F_n \subseteq \{\mathtt{0}, \mathtt{1}, \mathtt{2}, \mathtt{3}, \mathtt{4}\}^*$ be defined as follows:

$$F_n = \{\mathtt{3}\ \mathtt{0}^{i_1}\ \mathtt{1}\ \mathtt{0}^{i_2}\ \mathtt{1}\ \cdots \mathtt{1}\ \mathtt{0}^{i_n}\ \mathtt{2}^k\ \mathtt{0}^{i_k}\ \mathtt{4}\ :\ 1 \leq k \leq n,\ 1 \leq i_j \leq n\}.$$

For example,

$$\begin{aligned} F_2 \ = \ & \{\mathtt{3010204}, \mathtt{30102204}, \mathtt{300102004}, \mathtt{300102204}, \\ & \mathtt{30100204}, \mathtt{3010022004}, \mathtt{3001002004}, \mathtt{30010022004}\}. \end{aligned}$$

Show that F_n can be accepted by a 2DFA using $O(n)$ states. Please provide a detailed description of your 2DFA, explaining exactly what it does, but it is not necessary to draw a complete transition diagram.

(b) [10 marks]

Using the Myhill-Nerode theorem, show that any DFA for the language F_n defined in part (a) has n^n states.