University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2013
Problem Set 3

Distributed Tuesday, January 22 2013.

Due Tuesday, January 29 2013, in class.

All answers should be accompanied by proofs.

1. [10 marks] Must every primitive word of length > 1 have at least two unbordered conjugates? Prove or disprove.

2. [10 marks] Consider the equation in words $x \text{Ш} y = z^2$, where Ш is the perfect shuffle. Describe all solutions to this equation. Hint: there are separate cases depending on whether $|x|, |y|$ are both even or both odd.

   $if|x| = |y| = 2 * ithenletx = xaxbandy = yaybwhere|xa| = |xb| = |ya| = |yb| = i$ $x \text{Ш} y = xa \text{Ш} yaxb \text{Ш} yb = zz$ $xa \text{Ш} ya = z = xb \text{Ш} yb$ xa = xb and ya = yb

   $if|x| = |y| = 2 * i + 1thenletx = xaxcxbandy = yaycybwhere|xa| = |xb| = |ya| = |yb| = iand|xc| = |yc| = 1$ $x \text{Ш} y = xa \text{Ш} yaxcycxb \text{Ш} yb = zz$ xa xc = yc yb and ya = xb

   (By "describe all solutions" we mean come up with a characterization of the solutions that is somewhat analogous to the descriptions in Theorems 2.3.2 and 2.3.3 of the course text, giving necessary and sufficient conditions for x, y, z.)

3. [10 marks] Call a word w *uneven* if every nonempty subword has the property that at least one letter appears an odd number of times. For example, `abac` is uneven.

   (a) [5 marks] Show that if w is an uneven word over an alphabet with k letters, then $|w| < 2^k$.

   (b) [5 marks] Prove that the bound in (a) is sharp, by exhibiting an uneven word of length $2^k - 1$ over every alphabet of size $k \geq 1$. construction

   let w be longest uneven word for sigma and —sigma— = k + 1 let w' be longest uneven word for sigma s and —sigma s— = k

   so find repetition