

University of Waterloo  
CS 462 — Formal Languages and Parsing  
Winter 2011  
Problem Set 3

*Distributed Wednesday, January 19 2011.*

*Due Wednesday, January 26 2011, in class.*

All answers should be accompanied by proofs.

1. [10 marks] What is the largest power of a word that you can find in the decimal expansion of  $\pi$ ? Use a search engine to locate digits of  $\pi$  and any method to search for powers, give the power, its location in  $\pi$ , and the URL you used. Note: I was able to find a 9th power without any programming at all, and just 5 minutes with a search engine. Can you do better?
2. [10 marks] A word  $w$  is a *conjugate* of a word  $x$  if  $w$  can be obtained from  $x$  by cyclically shifting the letters. For example, **enlist** is a conjugate of **listen**.
  - (a) [7 marks] Let  $y, z$  be palindromes. Show that if at least one of  $|y|, |z|$  is even, then some conjugate of  $yz$  is a palindrome.
  - (b) [3 marks] Give an example to show that if both  $|y|$  and  $|z|$  are odd, then it is possible to have no conjugate of  $yz$  be a palindrome.
3. [10 marks] Prove the following improvement on Theorem 2.3.6 in the course text. You can use the same idea as in the proof of that theorem.

Let  $x$  and  $y$  be nonempty words.

Show that  $x^\alpha = y^\beta$  for some fractional exponents  $\alpha, \beta$  satisfying  $\alpha + \beta \leq \alpha\beta$  iff  $xy = yx$ .