

University of Waterloo  
CS 462 — Formal Languages and Parsing  
Winter 2013  
Problem Set 2

*Distributed Tuesday, January 15 2013.*

*Due Tuesday, January 22 2013, in class.*

All answers should be accompanied by proofs.

1. [10 marks] Call a language  $L$  *commutative* if for all  $x, y \in L$  we have  $xy = yx$ . Show that  $L$  is commutative if and only if there exists a word  $w$  such that  $L \subseteq w^*$ .
2. [10 marks] Suppose  $A = \{x\}$  is a language consisting of one nonempty word and  $B = \{y, z\}$  is a language consisting of two (distinct) words, such that  $AB = BA$ . Show that  $x, y, z$  are all powers of the same word.
3. [10 marks] We say that  $x$  is an *abelian square* if it can be written in the form  $yy'$ , where  $|y| = |y'|$  and  $y'$  is a permutation of  $y$ . An example in English is **reappear**, since **pear** is a permutation of **reap**.

Let  $x$  be a word over the alphabet  $\{0, 1\}$ . Show that  $x$  is an abelian square if and only if there exists a word  $y$  such that  $x \in \text{shuff}(y, y^R)$ . Here  $\text{shuff}$  means (not necessarily perfect) shuffle, described on page 57 of the text.