University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 10

Distributed Wednesday, March 16 2011.
Due Wednesday, March 23 2011, in class.

All answers should be accompanied by proofs.

1. [10 marks] Let S be a subset of $\mathbb{N}^2$. Show that $\{a^i b^j \ : \ (i,j) \in S\}$ is context-free if and only if S is semilinear.

   Is the analogous result true for $\{a^i b^j c^k \ : \ (i,j,k) \in S\}$ and $S \subseteq \mathbb{N}^3$?

2. [10 marks] Consider the following grammar

$$\begin{aligned} S &\rightarrow AC \mid BD \mid b \\ A &\rightarrow BC \mid AS \mid a \\ B &\rightarrow DA \mid c \\ C &\rightarrow BB \mid a \\ D &\rightarrow CA \mid a \end{aligned}$$

   Determine if $aaabcc \in L(G)$ using the CYK algorithm, and if it is, give a parse tree. Show your work.

3. [10 marks] Suppose L is a CFL that is inherently ambiguous. Then by the definition, for every context-free grammar G with $L = L(G)$, at least one word in L has at least two different parse trees in G. Show that in fact *infinitely* many words in L must have at least two different parse trees.