University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2013
Problem Set 1

Distributed Tuesday, January 8 2013.

Due Tuesday, January 15 2013, in class.

All answers should be accompanied by proofs. In all problems the underlying alphabet Σ is assumed to be finite.

1. [10 marks] Show that for every infinite string $\mathbf{w}$ there must be some letter a and some finite string x such that axa appears infinitely often as a subword of $\mathbf{w}$.

2. [10 marks] Let u, v be strings of the same length, and let $d(u, v)$ be the number of positions on which u and v differ; that is, the number of indices i for which $u[i] \neq v[i]$. For example, $d(\texttt{seven}, \texttt{three}) = 4$. Show that for all strings x, y we have $d(xy, yx) \neq 1$. Note that x and y need not be of the same length here.

   Hint: There is a very simple solution that can be hard to find, and a lengthy solution that is easy to find.

3. [10 marks] An infinite string $\mathbf{x} = a_0 a_1 a_2 \cdots$ is said to be *recurrent* if every finite subword that occurs in $\mathbf{x}$ occurs infinitely often in $\mathbf{x}$. An infinite string is *mirror invariant* if for every finite subword w of $\mathbf{x}$, the reversed string w^R is also a subword of $\mathbf{x}$.

   (a) Show that an infinite string $\mathbf{x}$ is recurrent if and only if every finite subword that occurs in $\mathbf{x}$ occurs at least twice.

   (b) Show that if an infinite string is mirror invariant, then it is recurrent.