University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2011
Problem Set 2

Distributed Wednesday, January 12 2011.

Due Wednesday, January 19 2011, in class.

All answers should be accompanied by proofs.

1. [10 marks] Suppose $A = \{x\}$ is a language of one word and $B = \{y, z\}$ is a language of two (distinct) words, such that $AB = BA$. Show that x, y, z are all powers of the same word.

2. [10 marks] Let w be a binary word that is primitive. Show that $\overline{w}$ is a conjugate of w if and only if there exists u such that $w = u\overline{u}$. Here $\overline{w}$ denotes the word w where every 0 is changed to 1 and vice versa.

3. [10 marks] We say that x is an *abelian square* if it can be written in the form yy', where $|y| = |y'|$ and y' is a permutation of y. An example in English is `reappear`, since `pear` is a permutation of `reap`.

   Let x be a binary word. Show that x is an abelian square if and only if there exists a word y such that $x \in \text{shuff}(y, y^R)$. Here shuff means (not necessarily perfect) shuffle, described on page 57 of the text.