

University of Waterloo
CS 798 — Mathematical Foundations of Computer
Networking
Winter 2012
Assignment
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1. Problem 1

1. Compute the linear combination of the vectors $[1 \ 4 \ 2 \ 5]$, $[4 \ 7 \ 9 \ 1]$, $[3 \ 4 \ 9 \ 2]$, and $[8 \ 3 \ 6 \ 3]$ with the weights 0.1, 0.4, 0.5, and 2.0 respectively.
2. Are the vectors $[1 \ 4 \ 9 \ 3]$, $[2 \ 9 \ 18 \ 5]$, and $[2 \ 8 \ 18 \ 6]$ linearly independent? Why or why not?
3. What is the dimension of a vector space spanned by the vectors $[1 \ 0 \ 0 \ 0]$, $[0 \ 1 \ 0 \ 0]$, $[0 \ 0 \ 1 \ 0]$, and $[0 \ 0 \ 0 \ 1]$?
4. Is the system $AX = [1000]^T$ homogeneous?

2. Problem 2

1. Write down the coefficient matrix corresponding to the system
$$\begin{aligned} 3x + 2y + z &= 4 \\ y + z &= 3 \\ x + 2y &= 7 \end{aligned}$$
2. What two row operations on rows 1 and 3 would result in the elimination of x ?
3. What is the rank of the matrix $\begin{pmatrix} 3 & 7 & -1 \\ 6 & 14 & -3 \\ 6 & 14 & -2 \end{pmatrix}$?
4. Compute the determinant of the matrix in Problem 3.
5. Is this matrix singular? If not, find its inverse.

3. Problem 3

1. What 3x3 matrix scales the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ to $\begin{pmatrix} 3a \\ 2b \\ 9c \end{pmatrix}$?
2. What should be the structure of an $n \times n$ matrix so that it scales the k^{th} element of a vector x , i.e. x_k by the constant a_k ?
3. Compute the eigenvalues and the corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 9 \\ 4 & 1 \end{pmatrix}$