CS 798 – Winter 2014
Assignment 2
Due: February 26, 2014

Problem 1

You are given a fair die and a biased die. Let X be a random variable representing the value shown face up when a die is rolled. For the fair die, $P(X = i) = 1/6$, $i = 1, 2, \ldots, 6$; and for the biased die, $P(X = 1) = 3/4$ and $P(X = i) = 1/20$, $i = 2, 3, \ldots, 6$. You have no way of telling which die is biased. Suppose you select a die at random and upon rolling it, you get a "5", what is the probability that you have selected the biased die?

Problem 2

Let X be a random variable representing the number of times a coin is tossed until for the first time the same result appears twice in succession. It is assumed that the tosses are independent of each other, and for each toss, $P(\text{head}) = p$ and $P(\text{tail}) = q = 1 - p$.

(a) Give an expression for $P(X = 6)$.

(b) For the special case $p = q = 0.5$, find the probability mass function of X.

Problem 3

X and Y are exponentially distributed random variables with parameters λ and μ, respectively. Suppose X and Y are independent. Derive an expression for $P(X > Y)$.

Problem 4

Let m and σ^2 be, respectively, the mean and variance of a random variable X. Express $E[(X - b)^2]$ as a function of b, m, and σ^2.

Problem 5

Suppose $Y = X_1 + X_2$ where X_1 and X_2 are independent random variables. X_1 has Poisson distribution with parameter λ_1 and X_2 has Poisson distribution with parameter λ_2.

(a) Show that Y also has Poisson distribution, with parameter $\lambda_1 + \lambda_2$.

(b) Using the results in (a), show that the conditional distribution $P(X_1 = k \mid Y = n)$ is Binomial.

Problem 6

X and Y are continuous random variables, both uniformly distributed between 0 and L. Suppose X and Y are independent. Let $D = |X - Y|$. Derive an expression for $E[D]$.

Problem 7

Suppose $Y = X_1 + X_2, \ldots + X_N$ where $X_1, X_2, \ldots, X_N$ are independent and identically distributed random variables with mean E[X] and variance var(X). N, the number of X_i's in the sum, is also a random variable; the mean and variance of N are E[N] and var(N), respectively. Derive analytic expressions for E[Y] and var(Y).

Problem 8

Suppose $X_1, X_2, \ldots, X_n$ are independent random variables.

(a) Let $T = \max(X_1, X_2, \ldots, X_n)$. Express $F_T(x)$ as a function of $F_{Xi}(x)$, $i = 1, 2, \ldots, n$. [Hint: Interpret $P(T \leq x)$.]

(b) Let $T = \min(X_1, X_2, \ldots, X_n)$. Express $F_T(x)$ as a function of $F_{Xi}(x)$, $i = 1, 2, \ldots, n$. [Hint: Interpret $P(T > x)$.] Show further that if each of the X_i's has exponential distribution, T is also exponential.