Trajectory Optimization for Multi-lane Platoon Formation with Undefined Configurations

Siwen Yang, Yunwen Xu, Dewei Li, and Chen Zhang

Key Laboratory of System Control and Information Processing, Ministry of Education of China
Shanghai Jiao Tong University, China

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Motivation

Multi-lane Platoon Formation

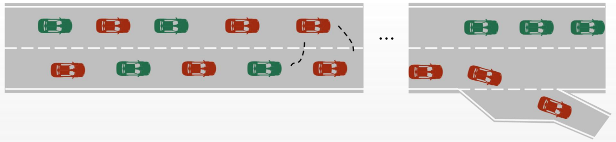
To assign and regulate vehicles scattered in different lanes to platoons

Overarching Motivation

- Reduce the delays of vehicles (at the vehicle level)
- Save the time consumed to form platoons (at the system level)
- Improve maneuverability by optimizing platoon configuration

Platooning

source: HORIBA MIRA & GMV NSL



Platoon formation

Background

- **Strategy-based Methods**

  Vehicle sorting for platoon formation[1] and platoon management[2]. Platoons are formed through several basic platooning maneuvers like tail merging

  - Relatively easy to implement online
  - Incapable of considering the optimal performance of the traffic flow

- **Optimization-based Methods**

  Optimize the platoons' configuration[3] or vehicles' trajectory[4] for platoon formation

  - Lack for consideration of the efficiency of platoon formation
  - Predefined and fixed configuration can reduce mobility and adaptability

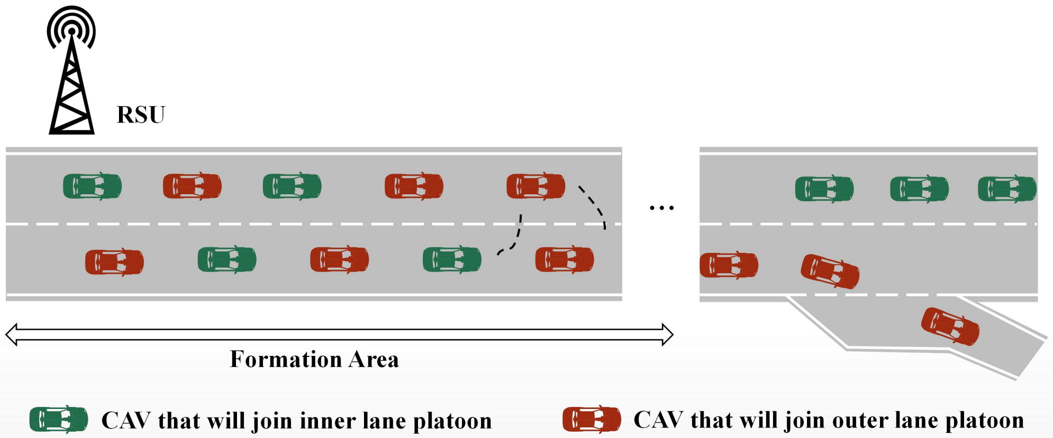
[1] R. Hall and C. Chin, "Vehicle sorting for platoon formation: Impacts on highway entry and throughput," Transportation Research Part C: Emerging Technologies, vol. 13, no. 5–6, pp. 405–420, Oct. 2005.
[2] M. Amoozadeh, H. Deng, C.-N. Chuah, H. M. Zhang, and D. Ghosal, "Platoon management with cooperative adaptive cruise control enabled by VANET," Vehicular Communications, vol. 2, no. 2, pp. 10–123, Apr. 2015.
[3] J. Heinovski and F. Dressler, "Platoon Formation: Optimized Car to Platoon Assignment Strategies and Protocols," in 2018 IEEE Vehicular Networking Conference (VNC), Taipei, Taiwan, Dec. 2018, pp. 1–8.
[4] R. Firoozi, X. Zhang, and F. Borrelli, "Formation and reconfiguration of tight multi-lane platoons," Control Engineering Practice, vol. 108, p. 104714, Mar. 2021.

Contribution

- A platoon formation strategy that simultaneously optimizes the undefined vehicle sequence in newly formed platoons and vehicle trajectories
- A multi-objective optimization that balances the time consumption of platoon formation and the vehicles' traveling speeds
- A Fibonacci search algorithm with an embedded mixed integer linear programming (MILP) model to solve our model

Problem Statement

- A formation area is set upstream of an off-ramp, and the goal is to coordinate CAVs with the same destination to form platoons separately in two lanes
- Several vehicles need to change lanes to join the platoon, and the vehicles' longitudinal sequence in new platoons and the trajectories of all vehicles before conducting lane-changing are to be optimized



Nomenclatures

- **General notations**: the most commonly used symbols
- **Traffic parameters**: significant sets and individuals
- **Platoon parameters**: information about the platoons to be formed
- **Vehicle trajectory parameters**: to be adjusted in advance according to traffic rules and demands
- **Control variables**: trajectories that vehicles should track

General notations	
T	Number of time intervals in a planning horizon
Δt	Length of time interval
l	Lane index
ω	Vehicle index
t	Time step index
Traffic parameters	
$\mathbf{L}$	Set of lanes
$\mathbf{\Omega}$	Set of all vehicles entering the formation area in this round of detection and optimization
N	Total number of vehicles in $\mathbf{\Omega}$
$\mathbf{\Omega}_l$	Set of vehicles in lane l, $\mathbf{\Omega}_l \subseteq \mathbf{\Omega}$
ω_l	Index of the leading vehicle in $\mathbf{\Omega}_l$
p_ω	Index of the preceding vehicle of vehicle ω
$\overline{x}_l(t)$	Position of the preceding vehicle of vehicle ω_l at time step t
Platoon parameters	
$\mathbf{\Omega}_l^{\mathrm{c}}$	Set of vehicles which intend to merge into lane l
$\mathbf{\Omega}_l^{\mathrm{p}}$	Set of vehicles in lane-changing platoon of lane l; that is, $\mathbf{\Omega}_l^{\mathrm{p}} = \mathbf{\Omega}_l \cup \mathbf{\Omega}_l^{\mathrm{c}}$
vehicle trajectory parameters	
$a_{\min}$	Minimum allowed acceleration
$a_{\max}$	Maximum allowed acceleration
$\Delta a_{\max}$	Maximum allowed acceleration variation between consecutive time steps
$v_{\min}$	Minimum allowed velocity
$v_{\max}$	Maximum allowed velocity
t_{gap}	Minimum allowed time headway gap
t_{TTC}	Minimum allowed time-to-collision
k_{sep}	The proportional coefficient between the maximum allowed separation velocity and relative distance of two vehicles
d_{follow}	Maximum distance between vehicles so that the rear vehicle is considered as a following vehicle
d_{safe}	Minimum allowed safe distance between vehicles; otherwise, it is considered that a collision will occur
k	The weight factor of the time consumption term
Control variables	
$a_\omega(t)$	Acceleration of vehicle ω at time step t
$v_\omega(t)$	Velocity of vehicle ω at time step t
$x_\omega(t)$	Position of vehicle ω at time step t

MPFUC: Objective function

$$\min_{T, a_\omega(t)} \frac{-\sum_{\omega \in \mathbf{\Omega}} \sum_{t=0}^{T} v_\omega(t)}{N(T+1)} + k \cdot T\Delta t \qquad (1)$$

The planning horizon T and acceleration of vehicles a_ω serve as decision variables

- The first term is the average velocity of all vehicles
- The second is the time consumed by this formation process
- The coefficient k is the weight factor of the time term to adjust priority in optimization

MPFUC: Constraints

Kinematic Constraints

- The dynamic equations

$$x_\omega(t) - x_\omega(t-1) = v_\omega(t-1)\Delta t + \frac{1}{2}a_\omega(t-1)\Delta t^2, \forall \omega \in \mathbf{\Omega};\ \forall t = 1, \ldots, T \tag{2}$$

$$v_\omega(t) - v_\omega(t-1) = a_\omega(t-1)\Delta t, \forall \omega \in \mathbf{\Omega};\ \forall t = 1, \ldots, T \tag{3}$$

- Limits on acceleration, velocity, and change of acceleration

$$a_{\min} \leq a_\omega(t) \leq a_{\max}, \forall \omega \in \mathbf{\Omega};\ \forall t = 1, \ldots, T \tag{4}$$

$$v_{\min} \leq v_\omega(t) \leq v_{\max}, \forall \omega \in \mathbf{\Omega};\ \forall t = 1, \ldots, T \tag{5}$$

$$|a_\omega(t) - a_\omega(t-1)| \leq \Delta a_{\max}, \forall \omega \in \mathbf{\Omega};\ \forall t = 1, \ldots, T \tag{6}$$

MPFUC: Constraints

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Collision Avoidance Constraints

- For non-leading vehicles in every lane

$$x_{p_\omega}(t) - x_\omega(t) \geq v_\omega t_{\text{gap}}, \forall \omega \in \mathbf{\Omega}_l, \omega \neq \omega_l; \forall l \in \mathbf{L};\ \forall t = 1, \ldots, T \tag{7}$$

- For leading vehicles in every lane

$$\bar{x}_l(t) - x_{\omega_l}(t) \geq v_{\omega_l}(t) t_{\text{gap}},\ \forall l \in \mathbf{L};\ \forall t = 1, \ldots, T \tag{8}$$

- For lang-changing vehicles

$$\begin{aligned} &\bar{x}_{\omega'}(T) - x_\omega(T) \geq v_\omega(T) t_{\text{gap}} \text{ or} \\ &\bar{x}_{\omega'}(T) - x_\omega(T) \leq - v_{\omega'}(T) t_{\text{gap}},\ \forall \omega \in \mathbf{\Omega}_l^c, \omega' \in \mathbf{\Omega}_l;\ \forall l \in \mathbf{L} \end{aligned} \tag{9}$$

MPFUC: Constraints

Safety Constraints for Lane-changing and Platooning

$$\Delta v^T_{\omega,\omega'} = v_\omega(T) - v_{\omega'}(T), \;\; \Delta x^T_{\omega,\omega'} = x_\omega(T) - x_{\omega'}(T)$$

- The longitudinal velocity of a lane-changing vehicle should be greater than the following vehicle

$$\Delta v^T_{\omega,\omega'} \geq \begin{cases} 0, \; 0 \leq \Delta x^T_{\omega,\omega'} \leq d_{\text{follow}} \\ -\infty, \text{otherwise} \end{cases}, \forall \omega \in \boldsymbol{\Omega}^c_l, \omega' \in \boldsymbol{\Omega}^p_l, \omega \neq \omega'; \forall l \in \mathbf{L} \tag{10}$$

- For the safety and stability of the platoon formation and platooning

$$\begin{cases} \Delta v^T_{\omega,\omega'} \leq (-\Delta x^T_{\omega,\omega'} - d_{\text{safe}}) \dfrac{1}{t_{\text{TTC}}}, \Delta x^T_{\omega,\omega'} < -d_{\text{safe}} \\ \Delta v^T_{\omega,\omega'} = 0, \; -d_{\text{safe}} \leq \Delta x^T_{\omega,\omega'} \leq d_{\text{safe}} \\ \Delta v^T_{\omega,\omega'} \geq -(\Delta x^T_{\omega,\omega'} - d_{\text{safe}}) \dfrac{1}{t_{\text{TTC}}}, \Delta x^T_{\omega,\omega'} > d_{\text{safe}} \end{cases}, \forall \omega, \omega' \in \boldsymbol{\Omega}^p_l, \omega \neq \omega'; \forall l \in \mathbf{L} \tag{11}$$

$$\begin{cases} \Delta v^T_{\omega,\omega'} \geq (\Delta x^T_{\omega,\omega'} + d_{\text{safe}}) k_{\text{sep}}, \Delta x^T_{\omega,\omega'} < -d_{\text{safe}} \\ \Delta v^T_{\omega,\omega'} \leq (\Delta x^T_{\omega,\omega'} - d_{\text{safe}}) k_{\text{sep}}, \Delta x^T_{\omega,\omega'} > d_{\text{safe}} \end{cases}, \forall \omega, \omega' \in \boldsymbol{\Omega}^p_l, \omega \neq \omega'; \forall l \in \mathbf{L} \tag{12}$$

MPFUC: Optimization Problem and Solution Approach

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$$\min_{T, a_\omega(t)} \frac{-\sum_{\omega \in \boldsymbol{\Omega}} \sum_{t=0}^{T} v_\omega(t)}{N(T+1)} + k \cdot T\Delta t \tag{13}$$

$$s.t. \quad (2) - (12)$$

- A nonlinear and combinatorial optimization problem (MINLP)
- T as a decision variable will influence the number of summation terms in the objective function and the number of active constraints in formulas (2) – (12) that are applied at multiple time steps

➡ Numerous binary variables and redundant constraints need to be introduced

MPFUC: Optimization Problem and Solution Approach

$$\min_{T,a_\omega(t)} \frac{-\sum_{\omega\in\boldsymbol{\Omega}}\sum_{t=0}^{T} v_\omega(t)}{N(T+1)} + k\cdot T\Delta t \qquad\qquad \min_{a_\omega(t)} \frac{-\sum_{\omega\in\boldsymbol{\Omega}}\sum_{t=0}^{T} v_\omega(t)}{N(T+1)}$$

$s.t.$ (2) – (12) $\quad$ T fixed $\quad$ $s.t.$ (2) – (12)

MINLP ➡ **MILP**

- The nonlinear constraints (9) – (12) with fixed T can be easily linearized by introducing fewer binary variables
- The multi-objective optimization problem (13) can be viewed as a line search problem on a univariate function J of integer variable T

$$\begin{cases} J(T) = -v_{\text{mean}} + k\cdot T\Delta t \\ v_{\text{mean}} = \min\limits_{a_\omega(t)} -\sum_{\omega\in\boldsymbol{\Omega}}\sum_{t=0}^{T} v_\omega(t)/N(T+1) \\ \text{subject to (2) – (8) and linearized (9) – (12)} \end{cases}$$

Algorithm 1. Fibonacci search algorithm with embedded MILP model to solve (13).

input: Traffic, platoon, and vehicle trajectories parameters in Table I; minimum and maximum allowed time consumption (calculated as integer multiples of Δt): $T_{\min}$ and $T_{\max}$
output: Optimal planning horizon for platoon formation T^* and the corresponding optimal trajectories $[x_\omega^*, v_\omega^*, a_\omega^*]$

Compute the smallest integer n satisfying $F_n > T_{\max} - T_{\min} + 2$ by iteration: $F_0 = 0$ $F_1 = 1$ and $F_{k+1} = F_k + F_{k-1}$
$\lambda = T_{\min} + F_{n-2} - 1$, $\mu = T_{\min} + F_{n-1} - 1$
Evaluate $J_\lambda := J(\lambda)$ and $J_\mu := J(\mu)$, save the corresponding optimized $[x_{\omega,\lambda}, v_{\omega,\lambda}, a_{\omega,\lambda}]$ and $[x_{\omega,\mu}, v_{\omega,\mu}, a_{\omega,\mu}]$
while $F_{n-2} > 1$ **do**
  if $J_\lambda > J_\mu$ **then**
    $\lambda \leftarrow \mu$, $J_\lambda \leftarrow J_\mu$
    $\mu \leftarrow \mu + F_{n-4}$
    if $\mu \leq T_{\max}$ **then**
      $J_\mu \leftarrow J(\mu)$, save the optimized $[x_{\omega,\mu}, v_{\omega,\mu}, a_{\omega,\mu}]$
    else
      $J_\mu \leftarrow M$ // M is a sufficiently large value
    end if
  else
    $\mu \leftarrow \lambda$, $J_\mu \leftarrow J_\lambda$
    $\lambda \leftarrow \lambda - F_{n-4}$
    $J_\lambda \leftarrow J(\lambda)$, save the optimized $[x_{\omega,\lambda}, v_{\omega,\lambda}, a_{\omega,\lambda}]$
  end if
  $n \leftarrow n - 1$
end while
if $J_\lambda > J_\mu$ **then**
  $T^* \leftarrow \mu$, $[x_\omega^*, v_\omega^*, a_\omega^*] \leftarrow [x_{\omega,\mu}, v_{\omega,\mu}, a_{\omega,\mu}]$
else
  $T^* \leftarrow \lambda$, $[x_\omega^*, v_\omega^*, a_\omega^*] \leftarrow [x_{\omega,\lambda}, v_{\omega,\lambda}, a_{\omega,\lambda}]$
end if
return T^*, $[x_\omega^*, v_\omega^*, a_\omega^*]$

Numerical Experiments

- **Parameters in the case studies**
  - $a_{\min} = -3\ \mathrm{m/s^2}$, $a_{\max} = 3\ \mathrm{m/s^2}$, $\Delta a_{\max} = 2\ \mathrm{m/s^2}$, $v_{\min} = 0$, $v_{\max} = 22\ \mathrm{m/s}$, $t_{\mathrm{gap}} = 1.2\ \mathrm{s}$, $t_{\mathrm{TTC}} = 4\ \mathrm{s}$, $k_{\mathrm{sep}} = 0.5\ \mathrm{s^{-1}}$, $d_{\mathrm{safe}} = 7\ \mathrm{m}$, $d_{\mathrm{follow}} = 30\ \mathrm{m}$, $\Delta t = 1\ \mathrm{s}$
  - Ten vehicles are considered, with five vehicles in the inner lane and the outer lane, respectively

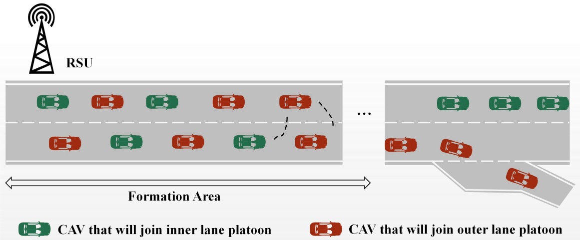
    $\boldsymbol{\Omega}_0 = \{1, 2, 3, 4, 5\}$, $\boldsymbol{\Omega}_1 = \{6, 7, 8, 9, 10\}$

- **Experimental scenarios setting**
  - Specific Scenario: five vehicles need to change lanes $\boldsymbol{\Omega}_1^c = \{2, 5\}$, $\boldsymbol{\Omega}_0^c = \{6, 7, 9\}$

    $x_\omega(0) = [133, 104, 71, 38, 19, 127, 100, 77, 35, 0]$

    $v_\omega(0) = [16, 17, 18, 19, 15, 22, 19, 15, 22, 19]$

    $a_\omega(0) = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

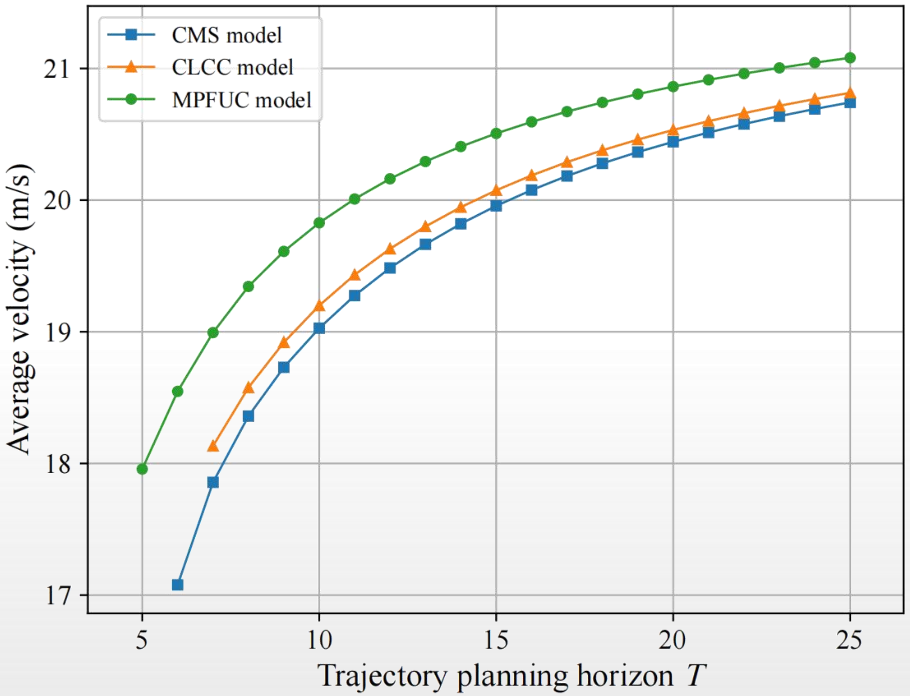


  - Randomly Generated Scenarios: experiments with thirty groups of randomly generated CAVs' initial states and target lanes

Numerical Experiments

- Results Comparison: For the sake of fairness and comparability, we conduct the numerical experiment with a series of preset planning horizons T

  - In the specific scenario



  - In thirty randomly generated scenarios

Planning horizon T	Model performance	MPFUC	CLCC[5]	CMS[6]
6	Average velocity (m/s)	18.21	17.15	17.52
	Feasible times	30	12	9
8	Average velocity (m/s)	19.01	18.01	17.90
	Feasible times	30	30	28
10	Average velocity (m/s)	19.54	18.72	18.61
	Feasible times	30	30	30
15	Average velocity (m/s)	20.30	19.73	19.68
	Feasible times	30	30	30
20	Average velocity (m/s)	20.71	20.27	20.23
	Feasible times	30	30	30

[5] X. Hu and J. Sun, "Trajectory optimization of connected and au_x0002_tonomous vehicles at a multilane freeway merging area," Transportation Research Part C: Emerging Technologies, vol. 101, pp. 111–125, Apr. 2019.
[6] Y. Xie, H. Zhang, N. H. Gartner, and T. Arsava, "Collaborative merging strategy for freeway ramp operations in a connected and autonomous vehicles environment," Journal of Intelligent Transportation ystems, vol. 21, no. 2, pp. 136–147, Mar. 2017.

Numerical Experiments

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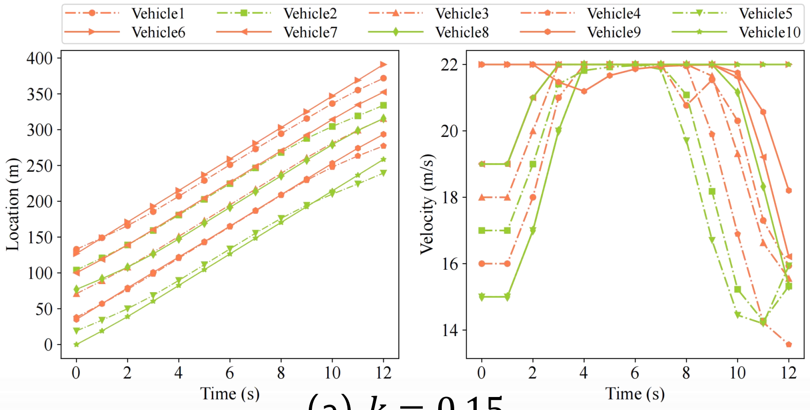
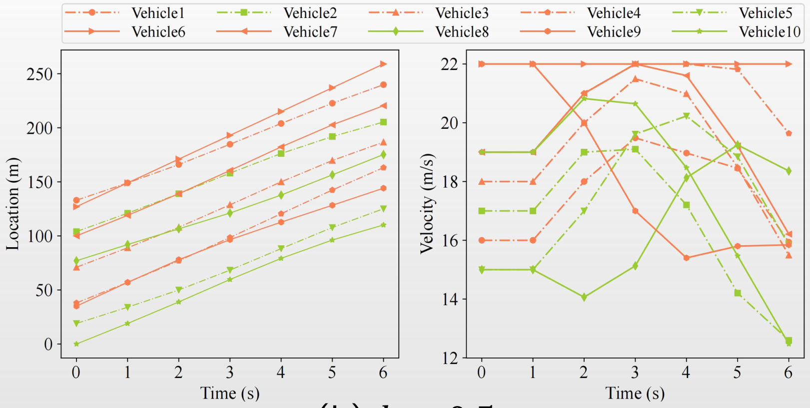
- Performance Analysis
  - Position and velocity profiles of all vehicles with different weight factor k



(a) $k = 0.15$



(b) $k = 0.5$

  - Multi-objective optimization results with different weight factor k

Weight factor k	Average velocity (m/s)	Time consumption (s)
0	21.23	30
0.03	21.15	27
0.05	20.91	21
0.07	20.67	17
0.1	20.41	14
0.15	20.16	12
0.3	19.34	8
0.5	18.55	6
1	17.96	5

Conclusions

1. We introduce an optimization-based trajectory planning method for multi-lane platoon formation. It optimizes the configurations of newly formed platoons and the longitudinal trajectories of all vehicles simultaneously.
2. A multi-objective optimization is formulated to consider the time consumed before changing lanes in the platoon formation process and the vehicles' average velocity.
3. The nonlinear optimization with an alterable planning horizon is efficiently solved by a Fibonacci search algorithm with an embedded MILP model.
4. Numerical experiments results show that the proposed method performs better compared to benchmark methods by optimizing platoons' configurations and balancing the vehicle delay and platoon formation efficiency.