CS222/CS122C: Principles of Data Management

UCI, Fall 2019
Notes #06

B+ trees

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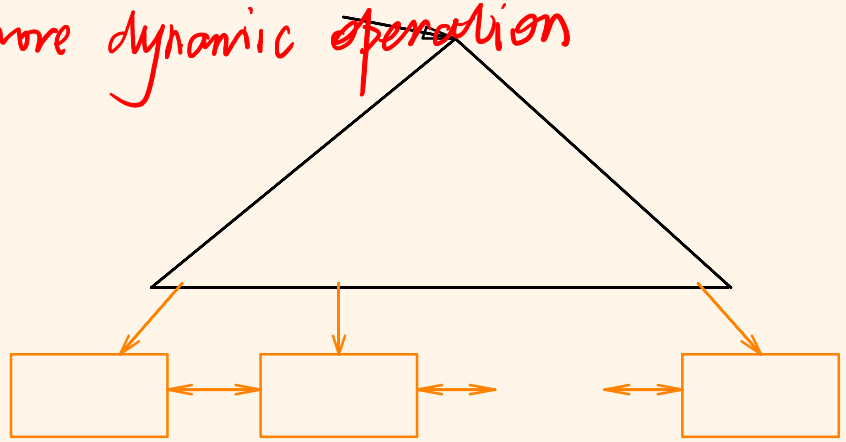
B+ Tree: Most Widely Used Index!

- Insert/delete at $\log_F N$ cost; keep tree *height-balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains **d** <= *m* <= 2**d** entries. The (mythical) **d** is called the *order* of the B+ tree.
- Supports equality and range-searches efficiently.

two types of node
{ root node
  intermediate node

Suppose more dynamic operation

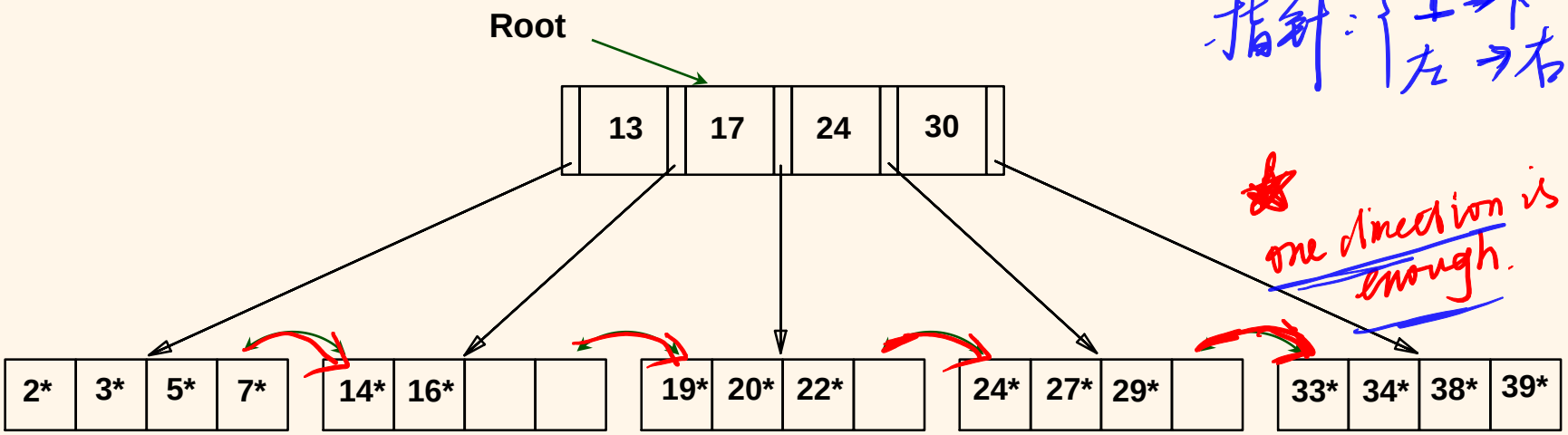


Index Entries
(Direct search)

Data Entries
("Sequence set")

Example B+ Tree

- Search begins at root, and key comparisons direct the search to a leaf (as in ISAM).
- *Ex:* Search for 5*, 15*, all data entries >= 24*, ...



Based on the search for 15, we know it is not in the tree!*

B+ Trees in Practice

- Typical order: 100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top level(s) in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

} ⇒ in buffer pool

B^+ tree: Each page not too full nor empty

leaf page (k, RID) → 10 Bytes → $\frac{4096}{10} \approx 400$

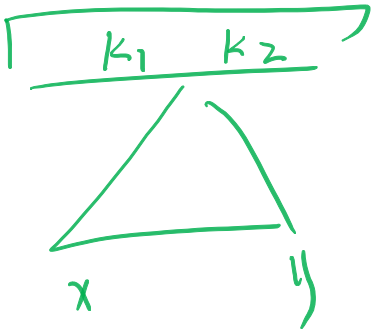
k: 4 RID: 6

intermediate page. (k, ptr) → 8 Bytes → $\frac{4096}{8} \approx 512$

k: 4 ptr: 4

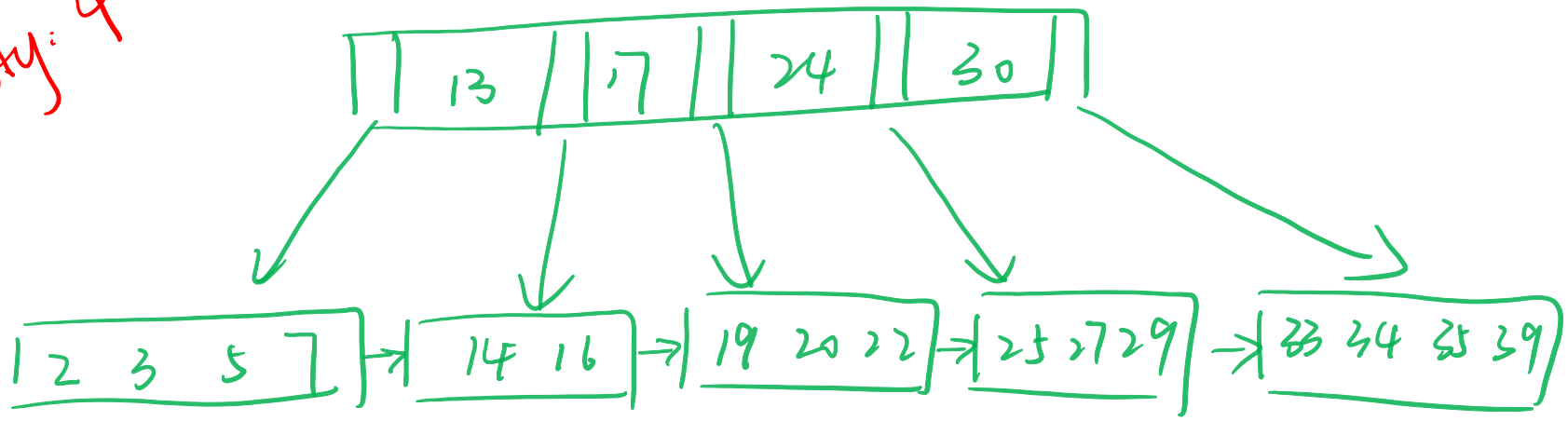
# of Entries: leaf page. 200 – 400

intermediate page 256 – 512



$k_1 \le x < y < k_2$

Node capacity: 4

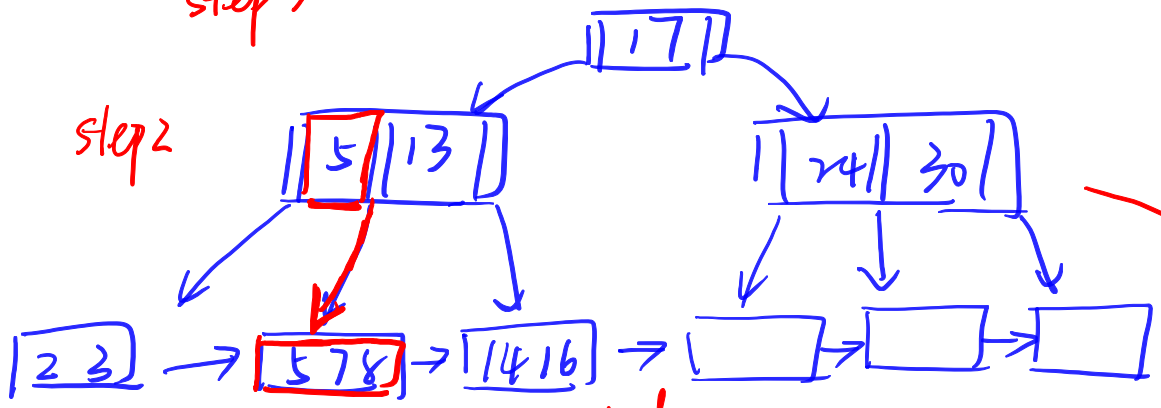


insert 8:

step 3

step 2

step 1



new inserted

Each level: keep balanced.

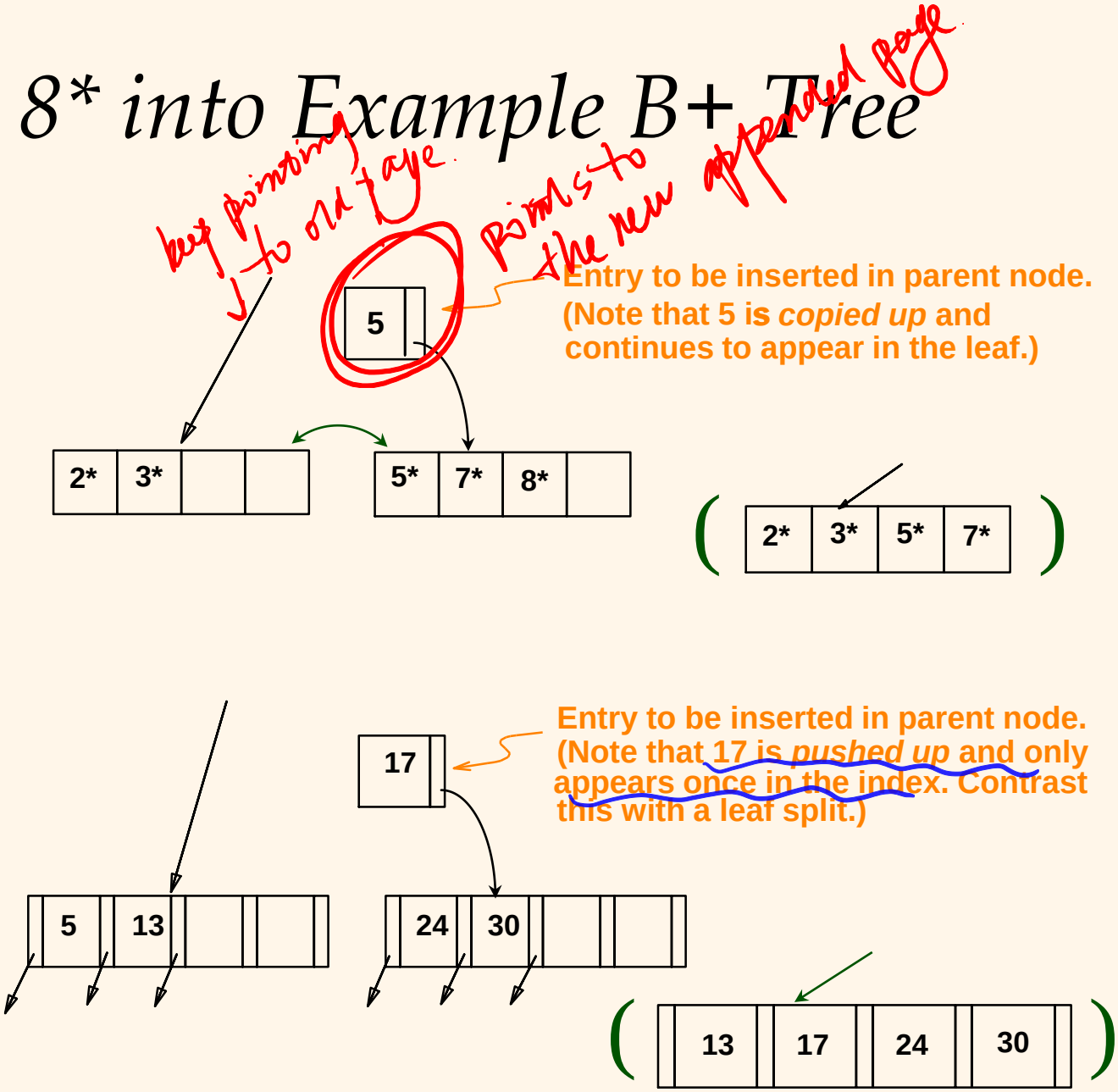
split because no enough space to insert 8:

Inserting a Data Entry into a B+ Tree

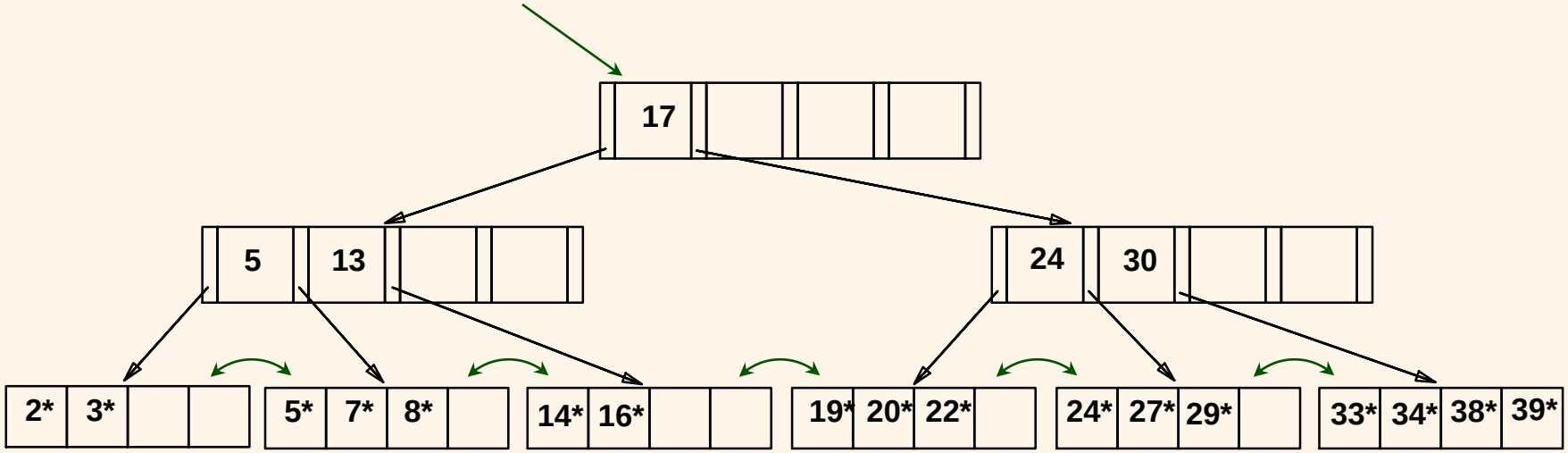
- Find correct leaf *L* (using a search).
- Put data entry onto *L*.
  - If *L* has enough space, *done*! (Most likely case!)
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries "evenly" and **copy up** L2's low key.
    - Insert new index entry pointing to *L2* into parent of *L*.
- This can happen recursively.
  - To split an index node, redistribute entries evenly but **push up** the middle key. (Contrast with leaf splits!)
- Splits "grow" tree; root split increases its height.
  - Tree growth: gets *wider* or *one level taller at top.*

Inserting 8 into Example B+ Tree*

- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.
- Note difference between *copy-up* and *push-up;* be sure you understand the reasons for this!



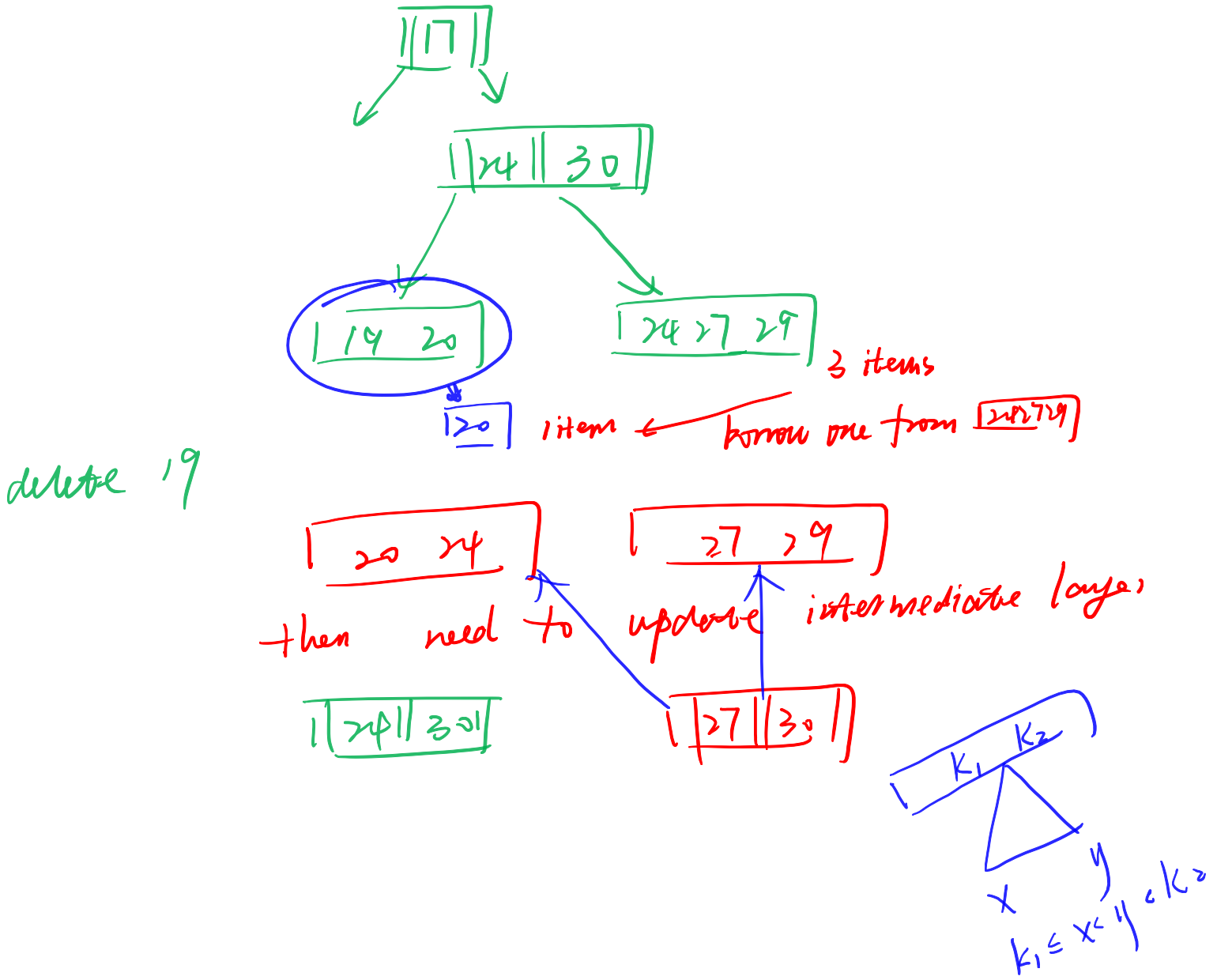
*Example B+ Tree After Inserting 8**



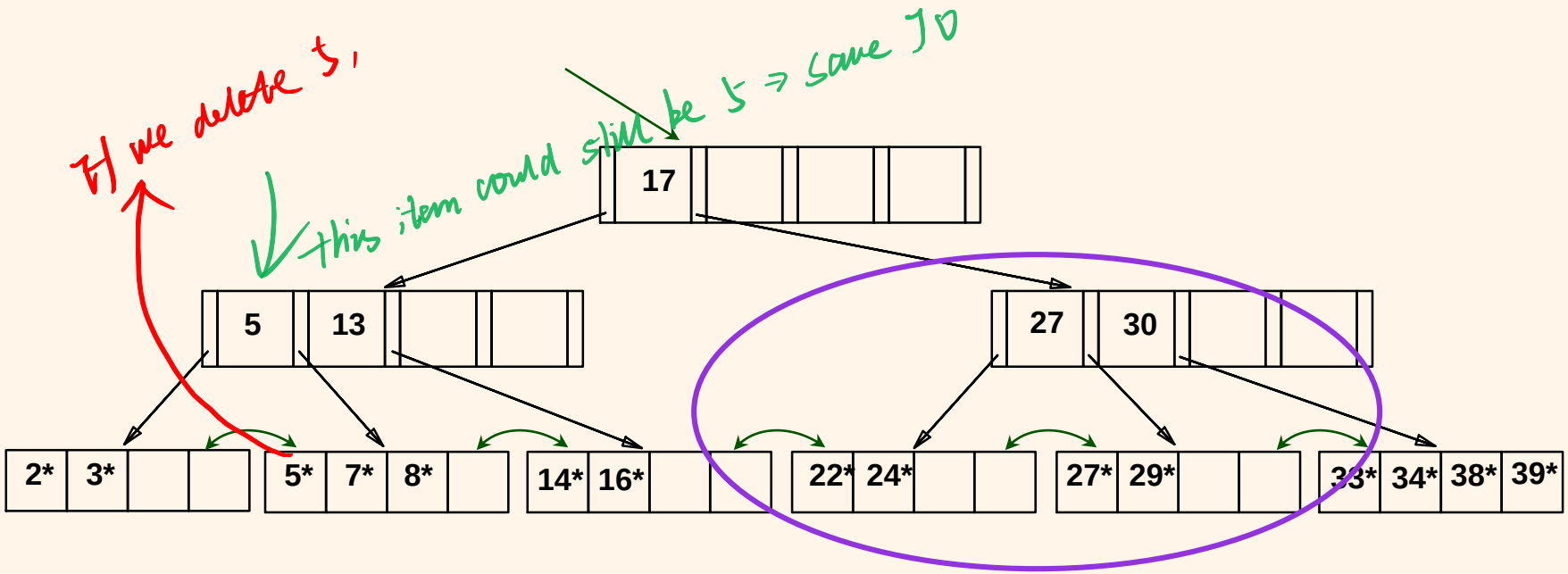
- Notice that root was split, leading to increase in height.
- In this example, could avoid split by redistributing entries; however, not usually done in practice. (Q: Why is that?)

Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is still at least half-full, *done!*
  - If L has only **d-1** entries,
    1. Try to redistribute, borrowing from *sibling* (*adjacent node with same parent as L*).
    2. If re-distribution fails, *merge* *L* and sibling.
- If merge occurred, must delete search-guiding entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.


17
24 30
14 20
24 27 29
20
3 items
1 item
borrow one from 24 27 29
delete 19
20 24
27 29
then need to update
intermediate layer
24 30
27 30
K1 K2
x
y
K1 ≤ x < y < K2

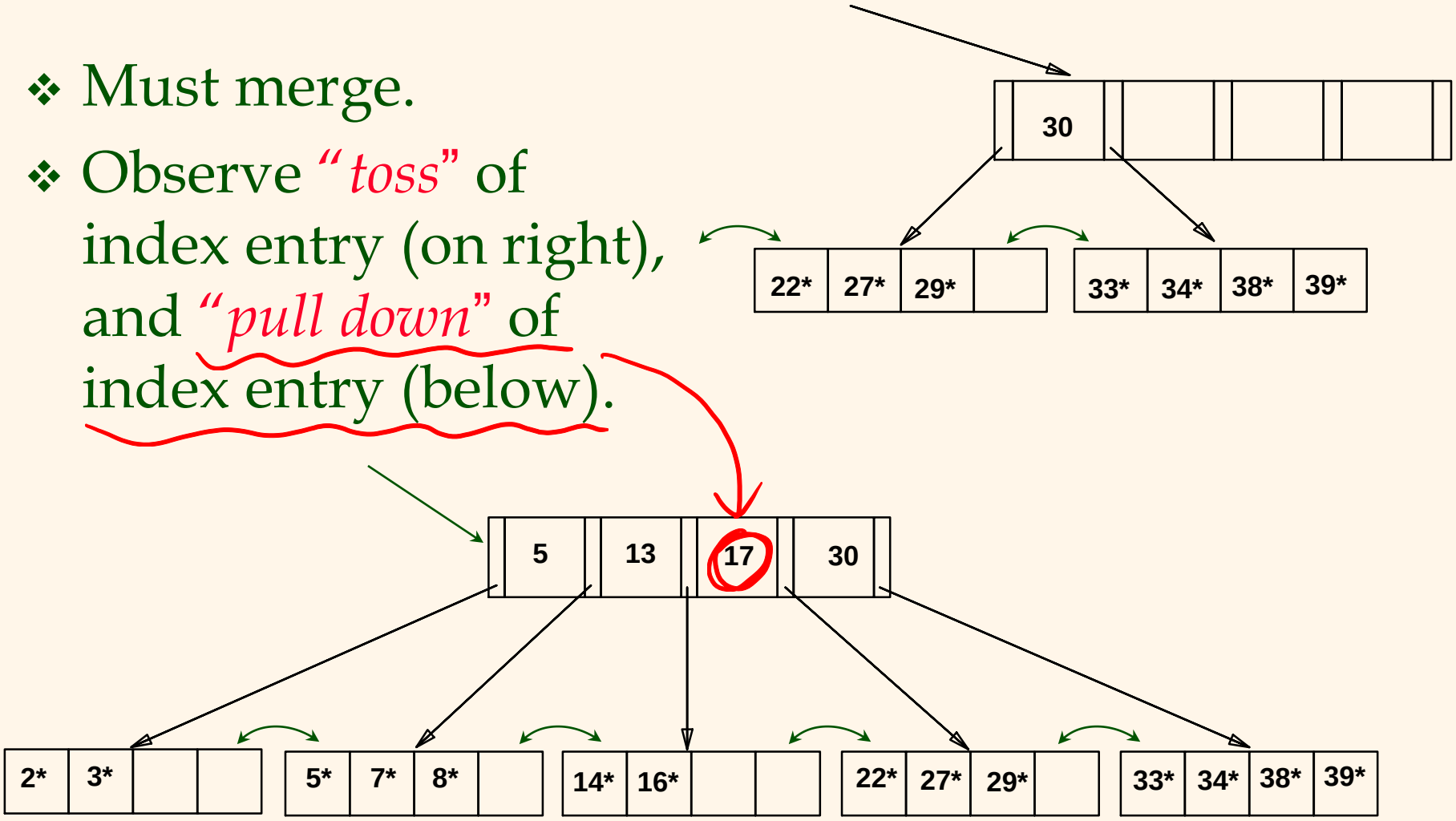
Example Tree After (Inserting 8, Then) Deleting 19* and 20* ...*



- Deleting 19* is easy.
- Deleting 20* is done with redistribution. Notice how new middle key is *copied up*.

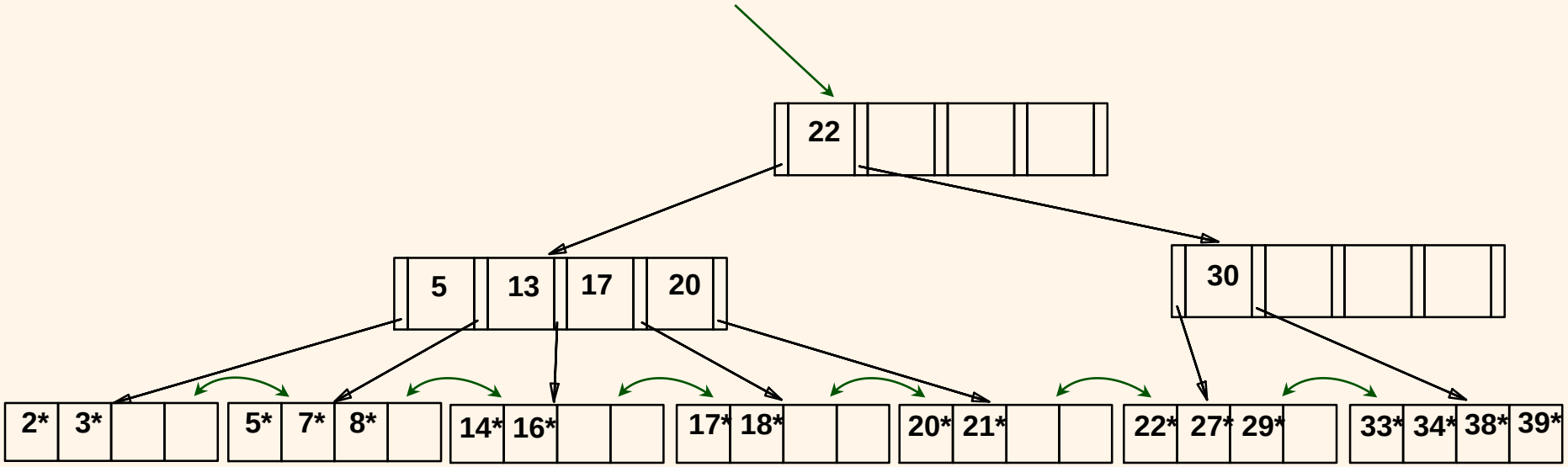
... And Then Deleting 24*

- Must merge.
- Observe "*toss*" of index entry (on right), and "*pull down*" of index entry (below).



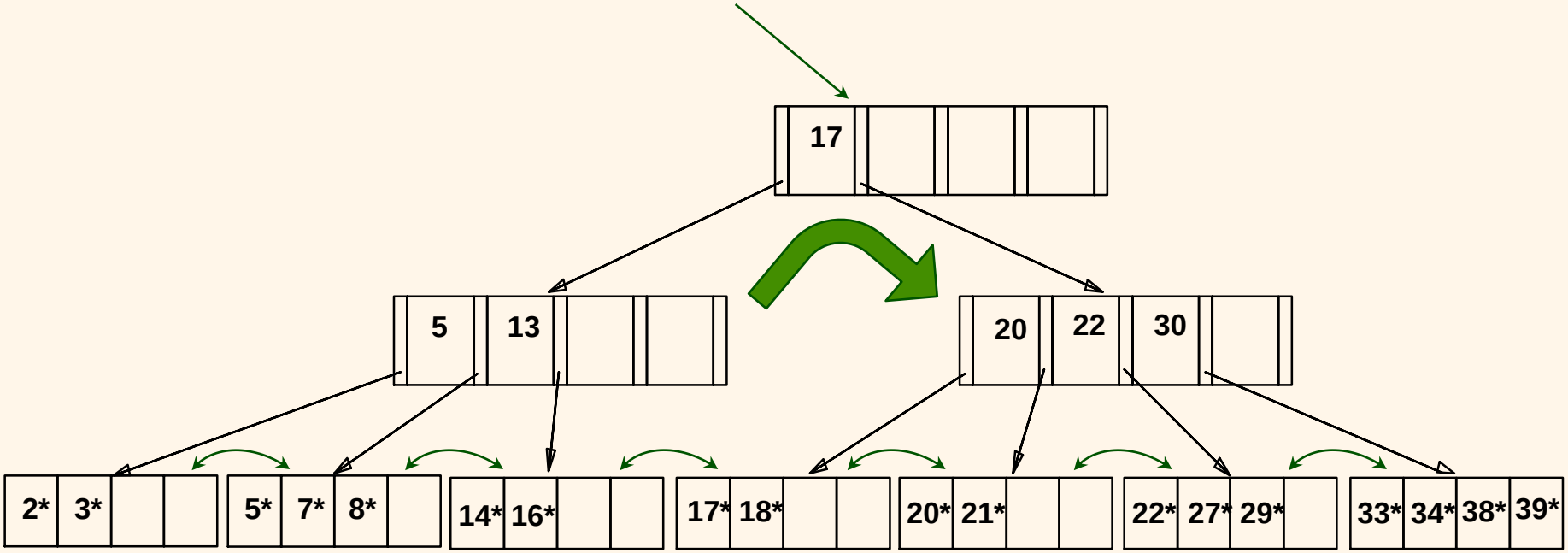
Example of Non-leaf Redistribution

- New/different example B+ tree is shown below *during deletion* of 24*
- In contrast to previous example, can redistribute entry from left child of root to right child.



After Redistribution

- Intuitively, entries are redistributed by "*pushing through*" the splitting entry in the parent node.



Prefix Key Compression

- It's important to increase tree fan-out. (Why?)
- Key values in index entries only `direct traffic'; we can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Then can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete logic must be suitably modified.

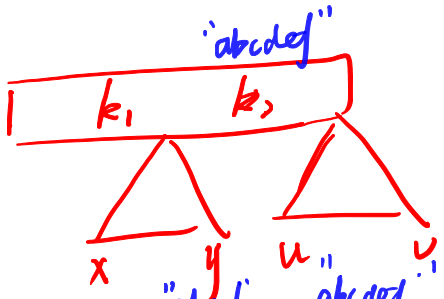
Compression { int, double, string

for B^+ tree, ⇒ need $<, =$ operation for its data type.

delta compression: store delta value

int: | 57 62 63 70 73 | ⇒ | 57 5 1 7 3 |

string:



$k_1 \le x < y < k_2 \le u < v$

range for k_2

k_2 "abcdef" → "abcde" → $y < k_2 \le u$

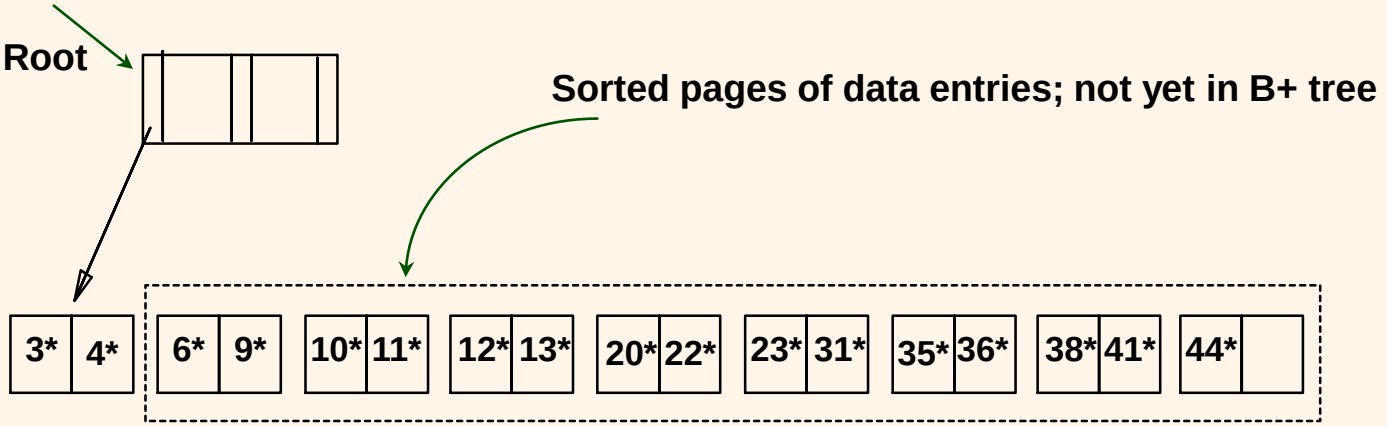
remove tail compress

{ suffix compression

prefix compression

Bulk Loading of a B+ Tree

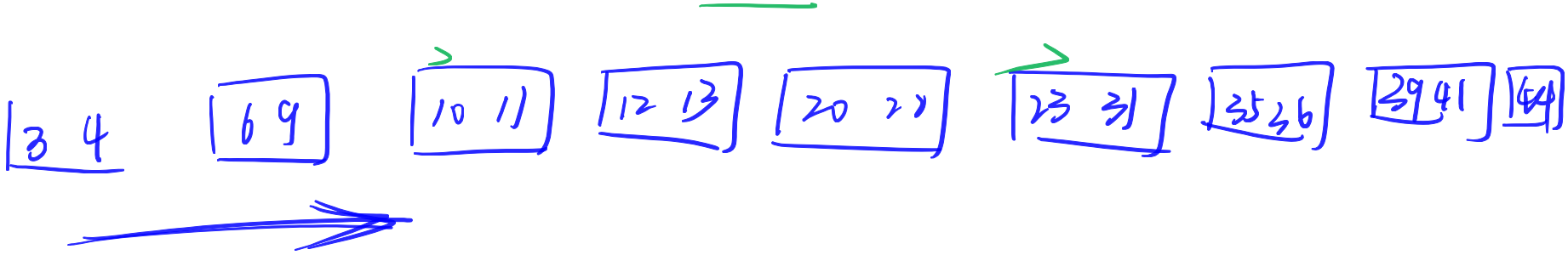
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- *Bulk Loading* can be done much more efficiently!
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk load:

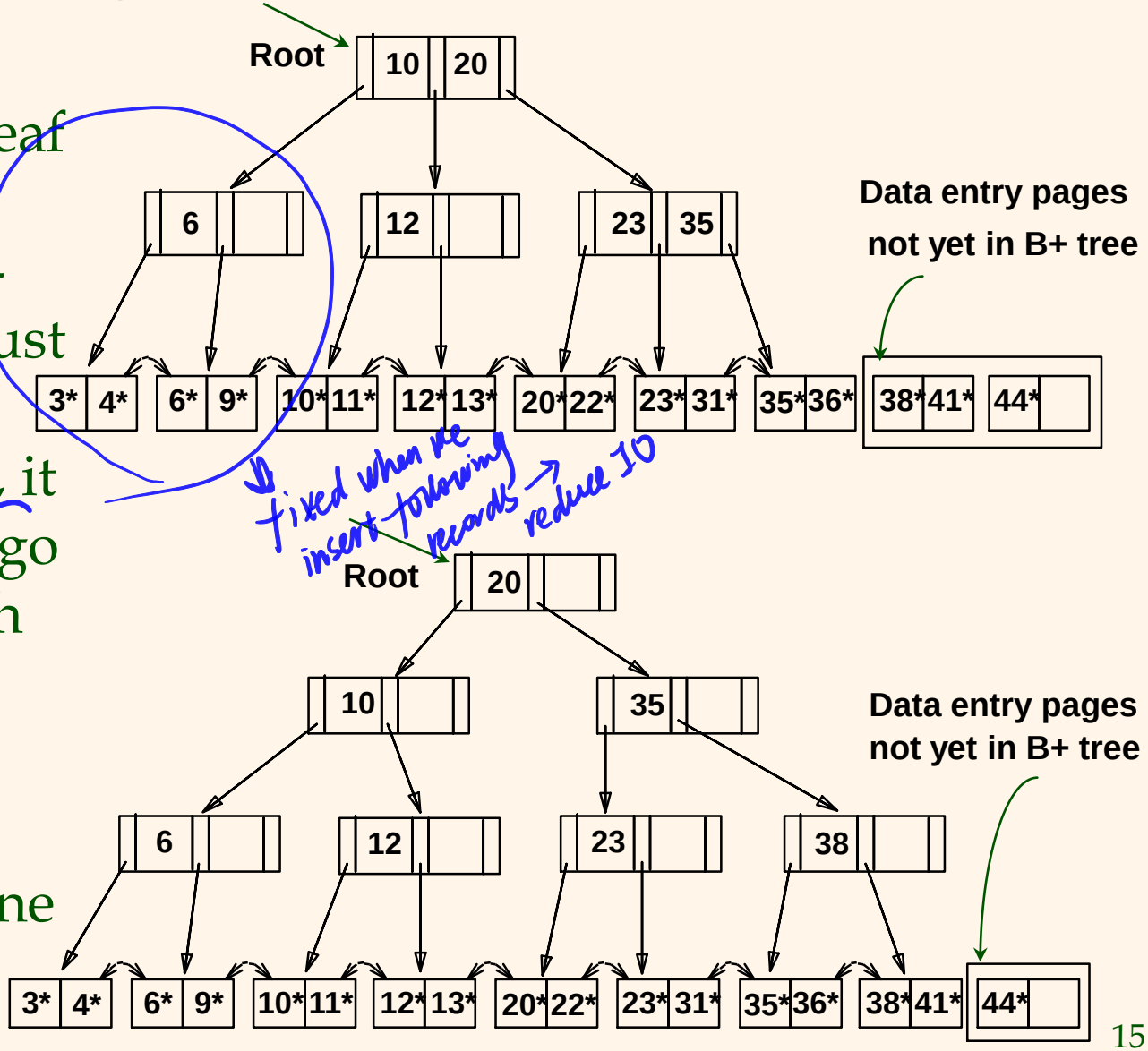
1) Create table
   Insert record 1M records ⇒ Bulk load.
   create index for 1M records (already have information about 1M records)

2) create table with an idex
   insert record → this means each time insert on record
   → insert index



Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!



Summary of Bulk Loading

- Option 1: multiple inserts.
  - Slow.
  - Does not give storage layout of leaves.
  - Also leaves a wake of half-filled pages.
- Option 2: *Bulk Loading*
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked).
  - Can control "fill factor" on pages.
  - Can optimize non-leaf splits more than shown.

A Note on B+ Tree "Order"

- (Mythical) *order* (**d**) concept replaced by physical space criterion in practice ("*at least half-full*").
  - Index pages can typically hold many more entries than leaf pages (RIDs vs. PIDs).
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.
- ISAM is a static structure.
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

Summary (Cont'd.)

- Typically, 67% occupancy on average.
- Generally preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
- If data entries are data records, splits can change rids!

- Key compression increases fanout, reduces height.
- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- Most widely used index in database management systems because of its versatility. Also one of the most heavily optimized components of a DBMS.

Project 3 extra credit :

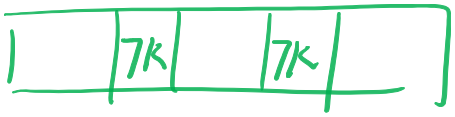
- Duplicate key :

Comp (Id, sal, ...)

Duplicate sal key

{ (7K, ...)
(7K, ...)
(7K, ...) }

1) Intermediate Node.



Leaf Node

Both Intermediate and leaf Node could be duplicate.

Solution : key: (sal) → (sal, RID)

this is unique

2) (key, list of RIDs with same key)

implement list