An extension of switching model

Using semiparametric empirical likelihood method

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Outline

Introduction

Consider following regime switching model:

$$y_t = x_t\beta_1 + \delta + \epsilon_t \qquad \text{(Regime 1)}$$

$$y_t = x_t\beta_2 + \epsilon_t \qquad \text{(Regime 2)}$$

by using class indicator,

$$y_t = x_t\beta + \delta I_t + \epsilon_t$$

Introduction

- Review : switching model

- Assumptions in Lee, Porter(1984,[4])

  1. we cannot observe I_t, but have "imperfect regime classification indicator" w_t : w_t is a measure of I_t
  2. w_t is independent of $\epsilon_{1t}, \epsilon_{2t}$ conditional on I_t
  3. The distribution of w_t conditional on I_t is as follows

$$P(w_t = 1 | I_t = 1) = p_{11}$$

$$P(w_t = 1 | I_t = 0) = p_{01}$$

  4. $\epsilon_{it} \sim N(0, \sigma_i^2)$

Introduction

- Empirical likelihood estimation

- Empirical likelihood is a nonparametric estimatino procedure for MLE.
- When we have a sample $\{z_i\}$, and model with moment condition $Eg(z_i, \theta) = 0$, the nonparametric log-likelihood estimation maximizes

$$\ell_{NP}(p, \theta) = \sum_{i=1}^{n} \log p_i \qquad s.t. \quad \sum_{i=1}^{n} g(z_i, \theta) p_i = 0$$

To avoid high-dimensionality of parameter, the solution can be calculated by maximizing following profiled likelihood :

$$\ell(\theta) = \max_{p_1, \cdots, p_n} \ell_{NP}(p) \quad s.t. \quad \sum_{i=1}^{n} p_i = 1, \sum_{i=1}^{n} p_i g(z_i, \theta) = 0$$

Introduction

- Literature review

- F. Zou, J. P. Fine and B. S. Yandell(2002, [6]) : mixtures of distribution with known mixing proportions with following relation

$$E \log \frac{g(x)}{f(x)} = x\beta$$

- Song Xi Chen and Jing Qin(2006, [2]) : mixtures of known class indicator and some missing value of class indicator.

$$\mu_1 = E(x|I = 1)$$

$$\mu_2 = E(x|I = 0)$$

$$E(x) = \mu_1\pi + \mu_2(1 - \pi)$$

Introduction

- Literature review

- Tatiana Benaglia, Didier Chauveau, David R. Hunter (2009,[1]) : proposed EM-like algorithm. Nonparametric version of EM-algorithm using kernerl density estimation
- Pierre Vandekerkhove (2013) : Estimation of a semiparametric mixture of regressions model. One component is entirely known, and the other is unknown.

Switching model with empirical likelihood method

Switching model with empirical likelihood method

- Case 1 : Full information for sample separation is available(class indicator is known)

- Model :

$$y_i = \beta' x_i + \delta I_i + u_i \qquad i = 1, 2, \cdots, n$$

$$E\left[u_i | x_i, I_i\right] = 0$$

I_i is observed class indicator, and the moment condition is

$$E\left[y_i - \delta - \beta' x_i | x_i, I_i = 1\right] = 0$$

$$\Rightarrow E\left[x_i'(y_i - \delta - \beta' x_i) | I_i = 1\right] = E\left[g_1(z_i, \theta) | I_i = 1\right] = 0$$

$$E\left[y_i - \beta' x_i | x_i, I_i = 0\right] = 0$$

$$\Rightarrow E\left[x_i'(y_i - \beta' x_i) | I_i = 0\right] = E\left[g_2(z_i, \theta) | I_i = 1\right] = 0$$

where $g_1(z_i, \theta) = x_i'(y_i - \delta - \beta' x_i)$, $g_2(z_i, \theta) = x_i'(y_i - \beta' x_i)$.

Switching model with empirical likelihood method

- Case 1 : Full information for sample separation is available(class indicator is known)

- Empirical likelihood function to be maximized :

$$
\begin{aligned}
L(p_1, \cdots, p_n, q_1, \cdots, q_n, \lambda; I_i) &= \max_{p_i, q_i} \prod_{i=1}^{n} (\lambda p_i)^{I_i} \left((1-\lambda)q_i\right)^{(1-I_i)} \\
&= \max_{p_i, q_i} \prod_{(i:I_i=1)} \lambda p_i \prod_{(i:I_i=0)} (1-\lambda) q_i
\end{aligned}
$$

such that

$$
\begin{aligned}
\sum_{(i:I_i=1)} p_i &= \sum_{(i:I_i=0)} q_i = 1 \\
\sum_{(i:I_i=1)} p_i g_1(z_i, \theta) &= \sum_{(i:I_i=0)} q_i g_2(z_i, \theta) = 0
\end{aligned}
$$

Switching model with empirical likelihood method

- Case 1 : Full information for sample separation is available(class indicator is known)

- Firstly, maximize $\log L(p, q, \lambda, \theta)$ w.r.t. $p_1, \cdots, p_n, q_1, \cdots, q_n, \lambda$, given θ.

  Then, from FOCs,

$$p_i = \frac{1}{n_1\left(1 + \gamma_1' g_1(z_i, \theta)\right)} \qquad i = 1, 2, \cdots, n$$

$$q_i = \frac{1}{n_2\left(1 + \gamma_1' g_2(z_i, \theta)\right)} \qquad i = 1, 2, \cdots, n$$

$$\lambda = \frac{n_1}{n_1 + n_2}$$

  γ_1, γ_2 is Lagrange multipliers, and n_k is the number of obs. from regime $k(k = 1, 2)$.

- Next, maximize profiled log-likelihood

$$\ell(\theta) = \sum_{(i:I_i=1)} \log\left(\frac{1}{n_1\left(1 + \gamma_1' g_1(z_i, \theta)\right)}\right) + \sum_{(i:I_i=0)} \log\left(\frac{1}{n_2\left(1 + \gamma_1' g_2(z_i, \theta)\right)}\right)$$

Switching model with empirical likelihood method

- Case 1 : Full information for sample separation is available(class indicator is known)

- Asymptotic properties :
  - The asymptotic properties of the typical empirical liklihood estimator was proved by Owen(1990, [5]).
  - Chen, Qin(2006, [2]) consider this case.
- Asymptotic inference using empirical likelihood ratio statistic :

$$r = -2\left[\sup_{(\theta : R(\theta)=0)} \ell(\theta) - \ell(\hat{\theta}_{EL})\right]$$

- Moment condition

$$E(x_i'(y_i - \beta' x_i - \delta\lambda)) = E(g(z_i, \theta)) = 0$$

- Empirical likelihood function to be maximized :

$$L(p, q, \lambda) = \max_{p_i, q_i} \prod_{i=1}^{n} [\lambda p_i + (1 - \lambda) q_i]$$

such that

$$\sum_{i=1}^{n} [\lambda p_i + (1 - \lambda) q_i] = 1$$

$$\sum_{i=1}^{n} [\lambda p_i + (1 - \lambda) q_i] \, g(z_i, \theta) = 0$$

Future works

- Case 3 : Imperfect information for sample separation is available

- Empirical likelihood function to be maximized :

$$L(p,q,\lambda) = \max_{p_i,q_i} \prod_{i=1}^{n} [\lambda p_i p_{11} + (1-\lambda) q_i p_{01}]^{w_i}$$
$$\cdot [\lambda p_i (1-p_{11}) + (1-\lambda) q_i (1-p_{01})]^{(1-w_i)}$$

References

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