We Study QFTs, with conformal symmetry in two dimmensions.

2d is interesting because the conformal group is infinite dimmensional (i.e we have infinite symmetry).

Specifically the papper discusses the following topics related to CFTs:

1. Stress tensor

2. Ward identities and conformal families

3. Conformal properties of the operator algebra

4. Degenerate conformal families

5. Minimal theories.

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1. Stress-Energy-Momentum tensor.

Stress tensor in QFT is defined as

$$T^{\mu\nu}$$

Words & Sentences

Conformal blocks:

Conformal blocks are used to express N-point functions in terms of operator product expansions. Specifically they are used in combination with 3 point structure constants.

Take for example the 4-point function.

$$\langle V_1 V_2 V_3 V_4 \rangle = \sum_s{}' C_{12s} C_{s34} \mathcal{F}_s^{(s\text{-channel})}$$

$$= \sum_t{}' C_{14t} C_{t23} \mathcal{F}_t^{(t\text{-channel})}$$

$$= \sum_u{}' C_{13u} C_{24u} \mathcal{F}_u^{(u\text{-channel})}$$

Laurent Series and the Virasoro algebra

Something profound.

Feynman diagrams are in position space
position space.

No loops

Transformation rules?

(crossing symmetry e.g
s-channel = t-channel
= u-channel)

Conformal bootstrap - using
crossing symmetry to find symmetry)

Modular bootstrap. (Put CFT on a torus)

(Ghosts = states with vanishing norm)

In 2D

$$T_a{}^a(\xi) = 0 \iff \xi^a \to \lambda \xi^a$$

$$\frac{\delta S}{\delta g_{\mu\nu}} \equiv T^{\mu\nu}, \quad (1.4)$$

$\delta g_{\mu\nu}$ compute for
a scale transform

Do this calculation.

Laurent → Taylor
from $-\infty$ to ∞

Di Franscesco Chapter 4

A Conformal transformation $x \to x'$ acting on a d-dimensional metric tensor $g_{\mu\nu}(x)$ is by definition a transformation that changes the metric up to some scale, i.e.:

$$g'_{\mu\nu}(x') = \Lambda(x) g_{\mu\nu}(x)$$

Starting by investigating infinitesmal transformations

$$x^\mu \to x'^\mu = x^\mu + \epsilon(x^\mu)$$

$$g_{\mu\nu} \to g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

$$\frac{\partial x^\alpha}{\partial x'^\mu} = \frac{\partial}{\partial x'^\mu}(x'^\alpha - \epsilon^\alpha)$$

$$= \delta^\alpha_\mu - \partial_\mu \epsilon^\alpha$$

$$\Rightarrow g_{\mu\nu}' = (\delta_\mu^\alpha - \partial_\mu \epsilon^\alpha)(\delta_\nu^\beta - \partial_\nu \epsilon^\beta) g_{\alpha\beta}$$

$$= \delta_\mu^\alpha (\delta_\nu^\beta - \partial_\nu \epsilon^\beta) g_{\alpha\beta}$$

$$- \partial_\mu \epsilon^\alpha (\delta_\nu^\beta - \partial_\nu \epsilon^\beta) g_{\alpha\beta}$$

$$= g_{\mu\nu} - \partial_\nu \epsilon_\mu - \partial_\mu \epsilon_\nu - \partial_\mu \epsilon^\alpha \partial_\nu \epsilon_\alpha$$

$$\approx g_{\mu\nu} - (\partial_\nu \epsilon_\mu + \partial_\mu \epsilon_\nu)$$

$$= g_{\mu\nu} - 2\partial_{(\nu}\epsilon_{\mu)}$$

Since

$$g'(x') = \Lambda(x) g_{\mu\nu}(x)$$

We can establish that

$$2\partial_{(\nu}\epsilon_{\mu)} = f(x) g_{\mu\nu}$$

Tracing over both sides we get

$$(2\partial_\mu \epsilon^\mu) = f(x) g_\mu{}^\mu$$

$$f(x) = \frac{2\partial_\mu \epsilon^\mu}{g_\mu{}^\mu} \qquad (g_\mu{}^\mu = d)$$

if $g_\mu{}^\mu = \eta_\mu{}^\mu$

$= \mathrm{diag}(1, 1, \dots, 1)$

$$f(x) = \frac{2}{d} \partial_\rho \epsilon^\rho$$

$$\partial_\rho(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) = \partial_\rho(f(x) g_{\mu\nu})$$

Permuting

$$\partial_\mu(\partial_\nu \epsilon_\rho + \partial_\rho \epsilon_\nu) = \partial_\mu(f(x) g_{\nu\rho})$$

$$\partial_\mu \partial_\nu \epsilon_\rho + \partial_\mu \partial_\rho \epsilon_\nu = \partial_\mu f(x) + \partial_\mu g_{\nu\rho}$$

for $g_{\nu\rho} = \eta_{\nu\rho}$:

$$\partial_\mu \eta_{\nu\rho} = 0$$

$$\partial_\mu \partial_\nu \epsilon_\rho + \partial_\mu \partial_\rho \epsilon_\nu = \partial_\mu f(x) \eta_{\nu\rho}$$

$$\eta_{\nu\rho} \partial_\mu f + \eta_{\mu\rho} \partial_\nu f - \eta_{\mu\nu} \partial_\rho f$$

$$= (\partial_\mu \partial_\nu \epsilon_\rho + \cancel{\partial_\mu \partial_\rho \epsilon_\nu}) + (\partial_\nu \partial_\mu \epsilon_\rho + \cancel{\partial_\nu \partial_\rho \epsilon_\mu})$$

$$- (\cancel{\partial_\rho \partial_\mu \epsilon_\nu} + \cancel{\partial_\rho \partial_\nu \epsilon_\mu})$$

$$= 2\partial_\mu \partial_\nu \epsilon_\rho$$

Contracting with $\eta^{\mu\nu}$

$$2\partial_\mu \partial^\mu \epsilon_\rho = \partial_\rho f + \partial_\rho f - d\,\partial_\rho f$$

$$= (2-d)\partial_\rho f$$

$$2\partial_\nu \partial^2 \epsilon_\rho = (2-d)\partial_\nu \partial_\rho f$$

$$\partial^2(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) = \partial^2(\eta_{\mu\nu} f)$$

$$2\,\partial^2 \partial_\mu \epsilon^\mu = \partial^2 f$$

$$2\,\partial^2 \partial_\mu \epsilon_\nu \eta^{\nu\mu} = \partial^2 f$$

$$2\partial^2 \partial_\mu \epsilon_\nu = \eta_{\nu\mu} \partial^2 f$$

$$(2-d)\partial_\mu \partial_\nu f = \eta_{\nu\mu} \partial^2 f$$

$$(2-d)\partial^2 f = \partial^2 f$$

$$(1-d)\partial^2 f = 0$$

$$b_{\mu\nu} + b_{\nu\mu} = \frac{2}{d} b^\lambda{}_\lambda \eta_{\mu\nu}$$

sym tensor

$$\eta^{\mu\nu}(b_{\mu\nu} + b_{\nu\mu}) = \frac{2}{d} b^\lambda{}_\lambda$$

$$b^\mu{}_\mu + b^\nu{}_\nu = \frac{2}{d} b^\lambda{}_\lambda$$

$$2b^\mu{}_\mu = \frac{2}{d} b^\lambda{}_\lambda$$

$$1 = \frac{1}{d} \Rightarrow d = 1,$$

For a infinite-Conformal transformation

We have shown that

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} - \frac{2}{d}\partial_\rho \epsilon^\rho g_{\mu\nu}$$