The destinctions of Symmetry.

In this section we Investigate the effect of a general QFT action functional. $S[\phi, \partial_\mu \phi]$.

The transformations acts on the position as well as the fields

$$x \to x'$$
$$\phi(x) \to \phi'(x')$$

The post-transform field is then a function of the previous field. * Note that in many QFTs it is common to work in momentum space. This is not the case for us, and we will treat almost every situation in position space. *

This perspective on transformations is often refered to as active transformations, as opposed to passive transformations; where transformations are treated as coordinate transformations.

The action (S) transforms as the following under the general transform.

$$S \to S' = \int d^d x \; \mathcal{L}(\phi'(x), \partial_\mu \phi'(x))$$

change of integration variable.

$$= \int d^d x' \; \mathcal{L}(\phi'(x'), \partial'_\mu \phi'(x'))$$

$$= \int d^d x' \; \mathcal{L}(\mathcal{F}(\phi(x)), \partial'_\mu \mathcal{F}(\phi(x)))$$

Jacobiane

$$= \int \left|\frac{\partial x'}{\partial x}\right| d^d x \; \mathcal{L}\left(\mathcal{F}(\phi(x)), \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} \mathcal{F}(\phi(x))\right)$$

Ex. 1 Translation

$$x' = x + a$$

$$\phi'(x+a) = \phi(x)$$

$$S = \int \left|\frac{\partial x'}{\partial x}\right| d^d x \, \mathcal{L}\left(f(\phi(x)), \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} f(\phi(x))\right)$$

$$\left|\frac{\partial x'}{\partial x}\right| = 1, \quad \frac{\partial x^\nu}{\partial x'^\mu} = \delta_\mu^\nu$$

$$S = \int d^d x \, \mathcal{L}\left(f(\phi(x)), \partial_\nu f(\phi(x))\right)$$

Remark 1 The action is invariant if not explicitly dependent on pos.

Ex. 2 Lorentz transformation

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu \,, \qquad \Phi'(\Lambda x) = L_\Lambda \phi(x)$$

$$S = \int \left|\frac{\partial x'}{\partial x}\right| d^d x \, \mathcal{L}\left(f(\phi(x)), \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} f(\phi(x))\right)$$

$$\left|\frac{\partial x'}{\partial x}\right| = \det\left(\frac{\partial x'^\mu}{\partial x^\nu}\right) = \det\left(\Lambda^\mu{}_\rho \delta^\rho_\nu\right)$$

$$= \det\left(\Lambda^\mu{}_\nu\right) \, ?$$

By unitarity

Make net of confusion Lorentz group, boost + rotations. Rotations are obv unitary but are boosts?

Ex. 3 Scale transform

We consider the transformation caused by a set of infinitesimal parameters $\{\omega_a\}$. Specifically, the transformations we are referring to are

$$x^\mu \to x'^\mu = x^\mu + \omega_a \frac{\delta x^\mu}{\delta \omega_a}$$

$$\phi'(x') = \phi(x) + \omega_a \frac{\delta F}{\delta \omega_a}(x) \qquad *$$

It is common to define Generators (G_a) caused by a symmetry transform as follows

Note definition

$$\frac{\delta \phi(x)}{\delta \omega} = \delta_\omega \phi(x) := \Phi'(x) - \Phi(x) := -i\omega_a G_a \Phi(x)$$

We then note that we can expand the field $\Phi(x)$ around the x' point in terms of $\{\omega_a\}$

$$\Phi(x) = \Phi(x') + \frac{\delta}{\delta \omega_a}\phi(x')(\omega_a - x')$$

$$= \phi(x') + \frac{\partial \phi(x')}{\partial x'^\mu}\frac{\partial x'^\mu}{\delta \omega_a}(\omega_a - x')$$

$$= \phi(x') + \frac{\delta x'^\mu}{\delta \omega_a}\partial_\mu \phi(x')\,(\omega_a - x')$$

$$= \phi(x') + \frac{\delta x'^\mu}{\delta \omega_a}\partial_\mu \phi(x')$$

$$\Phi(x) = \Phi(x') + \frac{\delta}{\delta \omega_a}\phi(x')(\omega_a - x')$$

$$= \phi(x') + \delta_{\omega_a}\phi(x')\left(\omega_a - x - \omega_a \frac{\delta x^\mu}{\delta \omega_a}\right)$$

Explain the confusion and go on. We get according to di Francesco

$$\phi'(x') = \phi(x) + \omega$$