There are some issues that you should be able to easily address:

1) The matrix A is not STRICTLY diagonally dominant. Check for example row 4. Is Gauss-Seidel still convergent? Why?

I made a miscalculation to wrongly conclude that A is strictly dominant. It’s still convergent. As I wrote in the edited report, I checked the spectral radius of is less than 1 (L is the lower triangular part of A), which is the necessary and sufficient condition for convergence of Gauss-Seidel method.

2) In order to say if A is not ill-conditioned you need to check the condition number of A. If you only take the lower triangular part, then you say the the lower triangular part is not ill-conditioned.

I wrongly assumed that if A is not ill-conditioned and symmetric, then the lower triangular part of A is also not ill-conditioned. I corrected this by directly stating the condition number of is relatively small.

3) How can you say the matrix is not positive definite? Did you check?

I made huge mistake on this. I made some typos in matlab code. It’s actually positive definite, after checking with matlab chol function.

4) Are you sure you could not use the conjugate gradient method? We did not cover this, but since you mention it, how can you support your claim.

Again, I made a huge mistake on this. If A were not positive definite, then conjugate gradient and steepest descent could not be used. Conjugate gradient method and steepest descent are indeed usable. With a 1e-4 residual tolerance, conjugate gradient method converges in 4 iterations, and steepest descent converges in 56 iterations.

5)You claim: “Therefore, we do not need to worry much about the speed, or convergence rate, of the method, and thus Gauss-Seidel method, though definitely not among the faster methods, satisfies our need”. What is the order of convergence of Gauss-Seidel? Can other methods be used to solve this problem? Are they faster? In which sense?

Jacobi method is not good, since the spectral radius of the iteration matrix is greater than 1. JOR is usable with proper choice of , as stated by theorem 4 of chapter 4.2 of Quateroni. SOR is convergent. And conjugate gradient and steepest descent methods are also convergent.

Actually, conjugate gradient method is probably a better choice to solve the problem. Since it’s guaranteed to converge in 15 (dimension of the system) iterations, which is very satisfying. The convergence of Gauss-Seidel method, as demonstrated in many exercises, is highly dependent on the initial guess and the matrix A. Typically, conjugate gradient requires less iterations than Gauss-Seidel, JOR, SOR, and steepest descent.

Nonetheless, in this question Gauss-Seidel is certainly also usable. As stated in the report, we reach 1e-5 residual tolerance after 15 iterations, which is a considerably small and satisfying number. And to keep consistency with my original report, I still use Gauss-Seidel in the final submission.

6) Format your plot such that the labels, the title, and the numbers are clearly readable.

I reformatted some of the plots.

7) You show that at every level the pressure decreases. The different levels are separated by a distance Lm. Can you take this into account?

I’m not sure what you mean by this. I stated that , demonstrating that is less than . And in fact, we already take Lm into account when constructing the linear system.

8) Your arguments on the symmetry of the systems are pertinent but not enough to guarantee the same pressure for all nodes at every level. What happens if p31 = 1?

If p31=1, then the binary tree is no longer balanced, and the system is not symmetric. Nonetheless, I elaborated on what I mean by the symmetry of the system.