0.1 TODO List

0.1.1 Classic Dimensionality Reduction

- 1. Learn about the following:
 - (a) PCA
 - (b) MDS
 - (c) isomap
- 2. Maybe need to implement some of them
- 3. Know the difference from a practical aspect

0.1.2 Clustering

- 1. Need to learn about the different approaches
 - (a) Variational
 - (b) Spectral
 - (c) Hierarchical
 - (d) Density thresholding
 - (e) Mode seeking
 - (f) Valley Seeking
- 2. Need to know the specific applications
- 3. Need to delve into one of them to see if possible for improvement

0.2 Clustering

Point cloud (with coordinates)

Distance / dissimilarity matrix

Note: this seems to be a different idea of the distance function used in TDA, which is called lens in that context.

 $Barcode \rightarrow merge tree \rightarrow dendrogram$

0.2.1 Mode Seeking Paradigm

Problems:

- 1. Noisy estimator
- 2. Neighborhood graph

Solutions:

1. Be proactive: smooth

2. Be reactive: merge clusters after clustering This leads to "topological persistence"

Persistence for Model Seeking: Probability density function f

- Nested family (filtration) of of inverse images, or superlevel-sets $f^{-1}([t,+\infty))$ for t from $+\infty$ to $-\infty$
- \bullet Track evolution of "topology"

0.3 Topological Persistence

Persistence diagram shows the "persistence" of the topological features. Slight perturbation causes slight difference in persistence diagrams.

0.4 Mapper