Contents

Preface		<i>page</i> xi
1	Basics	1
1.1	Shapes	2
1.1.1	Spaces and Maps	3
1.1.2	Manifolds	6
1.1.3	Complexes	7
1.2	Feature Size and Sampling	8
1.2.1	Medial Axis	10
1.2.2	Local Feature Size	14
1.2.3	Sampling	16
1.3	Voronoi Diagram and Delaunay Triangulation	18
1.3.1	Two Dimensions	19
1.3.2	Three Dimensions	22
1.4	Notes and Exercises	23
	Exercises	24
2	Curve Reconstruction	26
2.1	Consequences of ε -Sampling	27
2.2	Crust	30
2.2.1	Algorithm	30
2.2.2	Correctness	32
2.3	NN-Crust	35
2.3.1	Algorithm	35
2.3.2	Correctness	36
2.4	Notes and Exercises	38
	Exercises	39

viii Contents

3	Surface Samples	41
3.1	Normals	43
3.1.1	Approximation of Normals	43
3.1.2		45
3.1.3	Edge and Triangle Normals	47
3.2	Topology	50
3.2.1	Topological Ball Property	50
3.2.2	Voronoi Faces	53
3.3	Notes and Exercises	57
	Exercises	57
4	Surface Reconstruction	59
4.1	Algorithm	59
4.1.1	Poles and Cocones	59
4.1.2	Cocone Triangles	62
4.1.3	Pruning	64
4.1.4	Manifold Extraction	66
4.2	Geometric Guarantees	70
4.2.1	Additional Properties	72
4.3	Topological Guarantee	73
4.3.1	The Map ν	73
4.3.2	Homeomorphism Proof	75
4.4	Notes and Exercises	76
	Exercises	78
5	Undersampling	80
5.1	Samples and Boundaries	80
5.1.1	Boundary Sample Points	81
5.1.2	Flat Sample Points	82
5.2	Flatness Analysis	83
5.3	Boundary Detection	87
5.3.1	Justification	88
5.3.2	Reconstruction	89
5.4	Notes and Exercises	90
	Exercises	91
6	Watertight Reconstructions	93
6.1	Power Crust	93
6.1.1	Definition	94
612	Proximity	97

	Contents	ix
6.1.3	Homeomorphism and Isotopy	99
6.1.4	Algorithm	101
6.2	Tight Cocone	104
6.2.1	Marking	105
6.2.2	Peeling	107
6.3	Experimental Results	109
6.4	Notes and Exercises	111
	Exercises	111
7	Noisy Samples	113
7.1	Noise Model	113
7.2	Empty Balls	115
7.3	Normal Approximation	119
7.3.1	Analysis	119
7.3.2	Algorithm	122
7.4	Feature Approximation	124
7.4.1	Analysis	126
7.4.2	Algorithm	130
7.5	Notes and Exercises	131
	Exercises	132
8	Noise and Reconstruction	133
8.1	Preliminaries	133
8.2	Union of Balls	136
8.3	Proximity	142
8.4	Topological Equivalence	144
8.4.1	Labeling	145
8.4.2	Algorithm	147
8.5	Notes and Exercises	149
	Exercises	150
9	Implicit Surface-Based Reconstructions	152
9.1	Generic Approach	152
9.1.1	Implicit Function Properties	153
9.1.2	Homeomorphism Proof	154
9.2	MLS Surfaces	155
9.2.1	Adaptive MLS Surfaces	156
9.3	Sampling Assumptions and Consequences	159
9.3.1	Influence of Samples	162
9.4	Surface Properties	166

x Contents

9.4.1	Hausdorff Property	167
9.4.2	Gradient Property	170
9.5	Algorithm and Implementation	172
9.5.1	Normal and Feature Approximation	172
9.5.2	Projection	173
9.6	Other MLS Surfaces	175
9.6.1	Projection MLS	175
9.6.2	Variation	176
9.6.3	Computational Issues	176
9.7	Voronoi-Based Implicit Surface	179
9.8	Notes and Exercises	180
	Exercises	181
10	Morse Theoretic Reconstructions	182
10.1	Morse Functions and Flows	182
10.2	Discretization	185
10.2.1	Vector Field	185
10.2.2	Discrete Flow	187
10.2.3	Relations to Voronoi/Delaunay Diagrams	190
10.3	Reconstruction with Flow Complex	192
10.3.1	Flow Complex Construction	192
10.3.2	Merging	193
10.3.3	Critical Point Separation	194
10.4	Reconstruction with a Delaunay Subcomplex	196
10.4.1	Distance from Delaunay Balls	196
10.4.2	Classifying and Ordering Simplices	198
10.4.3	Reconstruction	201
10.4.4	Algorithm	202
10.5	Notes and Exercises	205
	Exercises	205
Bibliog	graphy	207
Index		213