Question5

1. The definition is:

$$L_r \;=\; \{x \in \{e,n,w,s\}^* \mid \\ x\ contains\ equal\ number\ of\ n\ and\ s, and\ equal\ number\ of\ e\ and\ w\}$$

2. $\forall p$, take the string $e^p n^p w^p s^p$, then partitioned it into $uvxyz$ with $|vxy| \leq p$:
then $uvxyz$ can be partitioned into this ways:
(1), vxy are all in n^p or s^p, in this case, uv^2xy^2z are not in L_r due to unequal number of n and s, or e and w
(2), xy have the samce character, this can be seqrate into two cases:
    v have different character from xy: in this case, v can be $e..e, n..n, w..w$, corresonding xy can be $n..w, w..w, s..s$. So uv^2xy^2 will fail for breaking both the balance in e and w, n and s. so it is not in L_r
    v = a..ab..b, a is the character, b is the character that xy both have: in this case, v can be $e..en..n, n..nw..w, w..ws..s$, corresponding xy is $n..n, w..w, s..s$. So uv^2xy^2 will fail for breaking both the balance in e and w, n and s. so it is not in L_r.
(3), vx have the same character, this can be separate into to cases:
    y have different character from xy: in this case, y can be $n..n, w..w, s..s$, corresonding vx can be $e..e, n..n, w..w$. So uv^2xy^2 will fail for breaking both the balance in e and w, n and s. so it is not in L_r
    y = a..ab..b, a is the character, a is the character that vx both have: in this case, y can be $e..en..n, n..nw..w, w..ws..s$, corresponding xy is $e..e, n..n, w..w$. So uv^2xy^2 will fail for breaking both the balance in e and w, n and s. so it is not in L_r.
Therefore, by discussing this cases of separating uvxyz, we proved that this is not context free.

3. The grammar is as followed:

$$\begin{array}{ll} G \rightarrow AG|\epsilon & G' \rightarrow BG'|\epsilon \\ A \rightarrow E|W|\epsilon|n|s & B \rightarrow N|S|\epsilon|e|w \\ E \rightarrow eGw & N \rightarrow nG's \\ W \rightarrow wGe & S \rightarrow sG'n \end{array}$$