Title

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目录

Problem Set

Problem 2. “Sandage-Loeb test” . (3 credits)

The so-called “Sandage-Loeb test” offers a direct measurement of the expansion rate of the Universe. For a light ray that was emitted at emission time t_e and later observed at time t_0, the redshift

$$1 + z = \frac{a\left(t_0\right)}{a\left(t_e\right)},$$

where $a\left(t\right)$ is the scale factor at the time t. If an observer keeps observing for a (*very large*) duration of time dt_0, then there will be a change of observed redshift dz.

(1) Derive the change rate dz/dt_0 in terms of H_0, z, and Hubble parameter $H(z)$. [Hint: as the time of observation t_0 changes to $t_0 + dt_0$, we observe light that was also emitted at a later time $t_e + dt_e$. What is the relation of dt_e and dt_0? You may consider a wavelet of light rays with the length $c\ dt_e$ at the time of emission. It will be redshifted to the length $c\ dt_0$ at the time of observation.]

(2) In the ΛCDM model, the Hubble parameter is

$$H(z) = H_0\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3 + +\Omega_K(1+z)^2 + +\Omega_\gamma(1+z)^4}.$$

Assume $\Omega_K = 0$, $\Omega_\gamma \approx 0$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70$ km/s/Mpc. You observe a galaxy at $z = 1$. Determine how long (in years) you will have to keep observing the galaxy in order to see its redshift change $|dz| = 10^{-6}$. Is $dz < 0$ or $dz > 0$?

1 宇宙成分组成

1.1 who

1.1.1 哈勃参数

$$\begin{aligned}\vec{v}(t) = \frac{d\vec{x}}{dt} &= x\left(t_0\right)\frac{1}{a\left(t_0\right)} da/dt \\ &= x\left(t_0\right)\frac{a(t)}{a\left(t_0\right)}\frac{da/dt}{a(t)}\end{aligned}$$

定义哈勃参数 $H(t) \equiv \frac{da/dt}{a} = \frac{\dot{a}}{a}$，可以得到哈勃定律 $\vec{V}(t) = \vec{x}(t)H(t)$，所以 H_0 就是今天的哈勃参数。

因为历史原因，$H_0 = 100h^{-1}$ km s^{-1} Mpc^{-1} ，$h \simeq 0.7$ ，更精确的测量出现 Hubble tension 问题。

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$$\mathrm{GeV} M_{\mathrm{Pl}} \left\langle \hat{a}^{\dagger} \right\rangle \{1 \cdots n\} a \implies b\ddot{a} \rightarrow 0 \tag{1}$$

$$\hbar \notin \mathbb{R}$$

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2022 Spring student seminar

2 PNG21cm

2.1 bias meaurement

$$\delta_{x_{\mathrm{HI}}}(\boldsymbol{x}) = \frac{x_{\mathrm{HI}}(\boldsymbol{x})}{\bar{x}_{\mathrm{HI}}} - 1 \tag{2}$$

$$\delta_{\rho_{\mathrm{HI}}}(\boldsymbol{x}) = \delta_m(\boldsymbol{x}) + \delta_{x_{\mathrm{HI}}}(\boldsymbol{x}) + \delta_m(\boldsymbol{x})\delta_{x_{\mathrm{HI}}}(\boldsymbol{x}) \tag{3}$$

$$H = \frac{L_{\text{box}}}{N_{\text{cell}}} \tag{4}$$

$$\delta_{\rho_{\text{HI}}}(\boldsymbol{x}) = b_1\delta_m(\boldsymbol{x}) + \frac{1}{2}b_2\delta_m^2(\boldsymbol{x})$$

2.2 Theoritical prediction of bispectrum

$$x_{\text{HII}} = 1 - x_{\text{HI}} \tag{5}$$

$$b_{\text{HII}}^{\text{G}} = \frac{1 - b_1 x_{\text{HI}}}{x_{\text{HII}}} \tag{6}$$

3 “星系与宇宙”助教笔记

3.1 有趣的问题

Q1: 为什么宇宙的年龄是哈勃常数的倒数?

- 解弗里德曼方程,
  - 物质主导的宇宙解出 $a \propto t^{\frac{2}{3}}$ ，那么 $H = \frac{\dot{a}}{a} = \frac{2}{3t}$ ，在今天取值 $t_0 = \frac{2}{3H_0}$。
  - 辐射主导的宇宙解出 $a \propto t^{\frac{1}{2}}$ ，那么 $H = \frac{\dot{a}}{a} = \frac{1}{2t}$ ，在今天取值 $t_0 = \frac{1}{2H_0}$。
  - generally, $a \propto t^{\frac{2}{3(w+1)}}$ ，那么 $H = \frac{\dot{a}}{a} = \frac{2}{3(w+1)t}$ ，在今天取值 $t_0 = \frac{2}{3(w+1)H_0}$。
- 定性地可以理解为：宇宙以多项式的形式膨胀 $a \propto t^n$ （在大多数时间是的，inflation 的时候不是），$H = \frac{\dot{a}}{a} \propto \frac{1}{t}$

QN:

- 牛顿力学在均匀无限大宇宙中存在矛盾：$\nabla \cdot \mathbf{g} = -4\pi G\rho$，$\mathbf{g} \equiv 0$, while $\rho \neq 0$

3.2 宇宙学部分参考书目推荐：

- Astrophysics for Physicists–Choudhuri.pdf Chp 10, 11,15(?)