Mearsuring Power Spectrum

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Refers to the Chapter 7 of Donghui Jeong's thesis (Jeong, 2010).

Contents

Notations

In this article, superscript 'g' means grid.

1 from simulations

1.1 Distribute particles onto the regular grid

If there are N_p particles in a simulation box, the particle number density is

$$n_\mathrm{p}(\mathbf{x}) = \sum_{i=1}^{N_\mathrm{p}} \delta^\mathrm{D}(\mathbf{x} - \mathbf{x}_i), \tag{1}$$

In order to apply the FFT, we have to assign the particle number density onto each point in the regular grid, called Particle Assignment Scheme (PAS)[1]. To do this we define a *shape function*, $S(\mathbf{r})$, which describe how the particle mass distribute like a PSF. The three common choice of PAS is

1. the Nearest-Grid-Point (NGP): $S_\mathrm{NGP}(\mathbf{r}) = \delta^\mathrm{D}(\mathbf{r})$, $p = 1$;
2. the cloud-in-cell (CIC): $S_\mathrm{CIC}(\mathbf{r}) = \mathcal{T}_H(\mathbf{r})$, as shown in Eq. 4; $p = 2$;
3. the Triangular-Shape-Cloud (TSC) scheme.

After the particles are distributed to $\mathbf{r}$, each point in the grid will occupy a value which is the intergral of the 'cell' around it, as

$$n^\mathrm{g}(\mathbf{x}^\mathrm{g}) = \sum_{i=1}^{N_\mathrm{p}} \int_{|\mathbf{x}'_j - \mathbf{x}^\mathrm{g}_j| < H/2} \frac{\mathrm{d}^3 x'}{H^3} S(\mathbf{x}' - \mathbf{x}_i), \tag{2}$$

Using the top-hat function $\mathcal{T}(x)$ to represent the intergral space,

$$\mathcal{T}(x) = \begin{cases} 1, & \text{if} |x| < 1/2, \\ 1/2, & \text{if} |x| = 1/2, \\ 0, & \text{if otherwise.} \end{cases} \tag{3}$$

[1]Or called mass assignment scheme (MAS) in pylians.

Furthermore, normalize the top-hat function as

$$\mathcal{T}_H(x) \equiv \frac{1}{H}\mathcal{T}\left(\frac{x}{H}\right) = \begin{cases} 1/H, & \text{if} |x| < H/2, \\ 1/(2H), & \text{if} |x| = H/2, \\ 0, & \text{if otherwise,} \end{cases} \tag{4}$$

and the 3D top-hat function is $\mathcal{T}_H(\mathbf{x}) = \prod_{j=1}^{3} \mathcal{T}_H(x_j)$.

Then the Eq. 2 can be written as

$$n^{\mathrm{g}}(\mathbf{x}^{\mathrm{g}}) = \sum_{i=1}^{N_{\mathrm{p}}} \int \mathrm{d}^3x'\, \mathcal{T}_H(\mathbf{x}' - \mathbf{x}^{\mathrm{g}}) S(\mathbf{x}' - \mathbf{x}_i). \tag{5}$$

Window function

The continuous number density field is $n(\mathbf{x})$, the number density in grid is a sampling of $n(\mathbf{x})$, we define a *window function* to describe the sampling progress as

$$n^{\mathrm{g}}(\mathbf{x}^{\mathrm{g}}) = \int_V \mathrm{d}^3x'\, n(\mathbf{x}') W(\mathbf{x}^{\mathrm{g}} - \mathbf{x}'). \tag{6}$$

The window function maps the continuous field to the grid field.

Here we assume $n(\mathbf{x}) = n_{\mathrm{p}}(\mathbf{x})$ since we are mearsure the power spectrum of the particle distribution. It will introduce some shot noise which will be discussed in (?).

Put Eq. 1 into Eq. 6, and compare with Eq. 5, we have

$$W(\mathbf{x}^{\mathrm{g}} - \mathbf{x}_i) = \int \mathrm{d}^3x'\, \mathcal{T}_H(\mathbf{x}' - \mathbf{x}^{\mathrm{g}}) S(\mathbf{x}' - \mathbf{x}_i). \tag{7}$$

rewrite as

$$\mathbf{x}'' = \mathbf{x}^{\mathrm{g}} - \mathbf{x}', \quad W(\mathbf{r}) = \int \mathrm{d}^3x''\, \mathcal{T}_H(\mathbf{x}'') S(\mathbf{r} - \mathbf{x}''). \tag{8}$$

or

$$W = \mathcal{T}_H \otimes S. \tag{9}$$

to density contranst

The density contrast $\delta \equiv n/\bar{n} - 1$ is then

$$\delta^{\mathrm{g}}(\mathbf{x}^{\mathrm{g}}) = \int_V \mathrm{d}^3x'\, \delta(\mathbf{x}') W(\mathbf{x}^{\mathrm{g}} - \mathbf{x}'), \tag{10}$$

where we adopt that the window function is normalized as $\int_V \mathrm{d}^3x\, W(\mathbf{x}) = 1$.

That is

$$\delta^{\mathrm{g}}(\mathbf{x}^{\mathrm{g}}) = [\delta \otimes W](\mathbf{x}^{\mathrm{g}}). \tag{11}$$

After Fourier transformation, we have

$$\delta^{\mathrm{g}}(\mathbf{k}^{\mathrm{g}}) = \delta(\mathbf{k}^{\mathrm{g}}) W(\mathbf{k}^{\mathrm{g}}). \tag{12}$$

1.1.1 3D Window functions

$$W(\mathbf{r}) = W(r_1) W(r_2) W(r_3), \tag{13}$$

$$W(\mathbf{k}) = \left[\mathrm{sinc}\left(\frac{\pi k_1}{2k_{\mathrm{N}}}\right)\mathrm{sinc}\left(\frac{\pi k_2}{2k_{\mathrm{N}}}\right)\mathrm{sinc}\left(\frac{\pi k_3}{2k_{\mathrm{N}}}\right)\right]^p, \tag{14}$$

where $\mathrm{sinc}(x) = \frac{\sin x}{x}$.

1.2 Pk estimate

Direct Sampling

1.3 Deconvolve window function

1.4 Subtract shot noise

References

Jeong D., 2010, PhD thesis