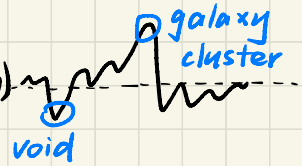
standard perturbation theory (SPT)

$$\delta = \frac{\delta\rho}{\bar{\rho}} = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \dots \text{ (Taylor Expansion)}$$



$\delta \ll 1$

large scale : linear regime - SPT works to $k \sim 0.1\,\mathrm{Mpc}$

middle : mild-linear - SPT break down EFT to $k \sim 0.5\,\mathrm{Mpc}$ for CDM, $k \sim 0.2\,\mathrm{Mpc}$ for galaxy

small : non-linear - unknown physics

$\theta = \nabla \cdot \vec{u}$ (velocity) continuity: $\delta' + \nabla[(1+\delta)\vec{u}] = 0$ ($' \to \frac{d}{d\tau}$, conformal time)

euler: $u_i' + H u_i + u_j \partial_i u_i = -\partial \Phi_i - \frac{1}{\rho}\partial_j(\rho \sigma_{ij})$ (H: Hubble parameter)

linear regime: collisionless pefect fulid $\sigma_{ij} = 0$

$\delta \ll 1$. $u_i \ll 1$

$$\Rightarrow u' + Hu = -\nabla\Phi$$

$$\delta' + \theta = 0$$

SPT: $\delta'(k) + \theta(k) = S_\alpha(\vec{k}, \tau)$ source terms

$\theta'(k) + H\theta(k) + \frac{3}{2}\Omega_m H^2 \delta(k) = S_\beta(\vec{k}, \tau)$ $S_{\alpha,\beta}(\delta, \theta, \alpha)$ (α: coupling)

$\delta = \sum \delta^{(i)} a^i$ $\theta = H \sum a^i \theta^{(i)}$

use first order δ, θ to get $\delta^{(2)}, \theta^{(2)}$

then use the second order to get $\delta^{(3)}, \theta^{(3)}$

and step by step...

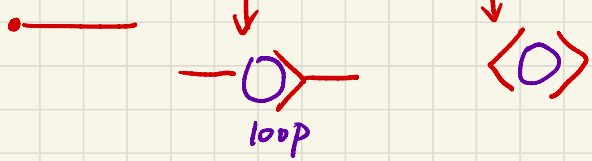
power spectrum: $\langle \delta\delta \rangle = \langle (\delta^1 + \delta^2 + \delta^3 + \dots)(\delta^1 + \delta^2 + \delta^3 + \dots) \rangle$

$$= \langle \delta^1\delta^1 + 2\delta^1\delta^2 + 2\delta^1\delta^3 + \delta^2\delta^2 + \dots \rangle$$

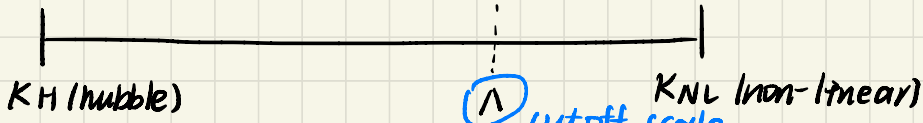
WICK Theorem: $= \langle \delta^1\delta^1 \rangle + 2\langle \delta^1\delta^2 \rangle + 2\langle \delta^1\delta^3 \rangle + \langle \delta^2\delta^2 \rangle + \dots$

$\langle \delta \rangle = 0$ $= \langle \delta\delta \rangle + 2\cancel{\langle \delta \cdot \delta \cdot \delta \rangle} + 2\langle \delta \cdot \delta \cdot \delta \cdot \delta \rangle + \dots$

$$= \underbrace{\langle \delta\delta \rangle}_{P_{lin}} + \underbrace{2\langle \delta \cdot \delta^3 \rangle + \langle \delta^2 \delta^2 \rangle}_{P_{1\text{-}loop}}$$



Effective field Theory (EFT)



$\rho^L = W_\Lambda \rho$ $\qquad \rho = \rho^L + \rho^S$ (long waves + small scale)

$\Phi^L = W_\Lambda \Phi$

$\delta_L' + \partial_i [(1+\delta_L) v_{\rho,i}] = 0$

$\rho_L [v_{Li}' + H v_i^L + v_{Lij} \partial_i v_{Li}] + \rho_L \partial_i \Phi^L = \partial_j [\sigma_{ij}]_\Lambda$

σ_{ij} : depends on Λ. containing all small scale things.

$\sigma_{ij}(\Phi, \Lambda, v_L, \ldots) = c_1 T_1 + c_2 T_2 + \cdots$

EFT: imperfect fluid. velocity pressure

↳ ignorance: c_i

$$P_{g,l}(k) = P^{lin}_{g,l}(k) + P^{1\text{-}loop}_{g,l}(k) + \underbrace{P^{ctr}_{g,l}(k)}_{\text{UV counterterms}} + \underbrace{P^{sto}_{g,l}(k)}_{\text{statistic error}}$$

rise from small scale

cosmological perturbation theory → LSS

primodial density perturbation

composited of CDM

fluid dynamics ⟸ collisionless, non-realistic

Expanding universe $\Longleftrightarrow$ fluid dynamics

$\Omega_m, H, \Omega_\Lambda$ $\qquad\qquad$ ρ_m