Test

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1 Expansion

1.1 from time to scale factor

We have two kinds of "time" in cosmology since our Universe are expanding.

If we focus on the coordinate, the metric is written as (c=1)

$$\mathrm{d}s^2 = \mathrm{d}t^2 + a^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \tag{1}$$

$$a\,\mathrm{d}x \tag{2}$$

By scaling the time axis, we can get the conformal time, which is equalized to the space coordinates

$$\mathrm{d}s^2 = a^2(t)(\mathrm{d}\tau^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \tag{3}$$

The scale factor can be solved from the **Friedmann equation**, [1]

$$\left\{ (\frac{\dot{a}}{a}) \right\}^2 = \frac{8\pi G}{3}\rho \tag{4}$$

with **the model of energy-momentum** contained in the Universe [2]

$$\frac{\rho(t)}{\rho_{\mathrm{cr}}} = \sum_{s=\gamma,\mathrm{m},\nu,\mathrm{DE}} \Omega_s a(t)^{-3(1+w_s)} \tag{5}$$

This model assumes a constant of equation of state.

$$\frac{\rho(t)}{\rho_{\mathrm{cr}}} = \Omega_{\mathrm{R}}\left(\frac{a}{a_0}\right)^{-4} + \Omega_{\mathrm{M}}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{\Lambda} + \Omega_{\mathrm{K}}\left(\frac{a}{a_0}\right)^{-2} \tag{6}$$

just solve the differencial equation

$$\dot{a} = H_0 a_0 \sqrt{\Omega_R\left(\frac{a}{a_0}\right)^{-2} + \Omega_M\left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda\left(\frac{a}{a_0}\right)^{2} + \Omega_K} \tag{7}$$

$$t = \int_0^{a_1} \frac{\mathrm{d}a}{H_0 a_0 \sqrt{\Omega_R\left(\frac{a}{a_0}\right)^{-2} + \Omega_M\left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda\left(\frac{a}{a_0}\right)^{2} + \Omega_K}} \tag{8}$$

[1] also deriveing the Friedmann equations from Einstein field equation is not that trival, see another block in preparation..

[2] Friedmann equation itself has assumptions about the energy-momentum, it's ideal fluid which can be parameterized by only ρ and P..

It's not linear, so need numerical solution.

But it's OK to see some exceptions. matter donimated:

$$\dot{a} = H_0(a_0\Omega_M)^{\frac{1}{2}}a^{-\frac{1}{2}} \tag{9}$$

then

$$t = \frac{2}{3H_0(a_0\Omega_M)^{\frac{1}{2}}}a^{\frac{3}{2}} \tag{10}$$

assumed $a(t=0)=0$.

2 HUbble follow

$$v_{\mathrm{H}} = \dot{a}\chi = H_0(a_0\Omega_M)^{\frac{1}{2}}a^{-\frac{1}{2}} \tag{11}$$

LaTeX

$$\mathrm{H}\,. \tag{12}$$

$$\mathrm{H}\,|\psi\rangle \tag{13}$$