Titolo

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Abstract

Introduzione

1 Expansion

1.1 from time to scale factor

We have two kinds of "time" in cosmology since our Universe are expanding.

If we focus on the coordinate, the metric is written as

$$ds^2 = dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right)$$

By scaling the time axis, we can get the conformal time, which is equalized to the space coordinates

$$ds^2 = a(t)\left(d\tau^2 + dx^2 + dy^2 + dz^2\right)$$

The scale factor can be solved from the **Friedmann equation**,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

with **the model of energy-momentum** contained in the Universe

$$\frac{\rho(t)}{\rho_{\mathrm{cr}}} = \sum_{s=\gamma,\mathrm{m},\nu,\mathrm{DE}} \Omega_s a(t)^{-3(1+w_s)}$$

This model assumes a constant

$$= \Omega_R\left(\frac{a}{a_0}\right)^{-4} + \Omega_M\left(\frac{a}{a_0}\right)^{-3} + \Omega_\Lambda + \Omega_K\left(\frac{a}{a_0}\right)^{-2}$$

where the second line is for ΛCDM model.

In ΛCDM model, just solve the differencial equation

$$\dot{a} = H_0\sqrt{\Omega_R\left(\frac{a}{a_0}\right)^{-2} + \Omega_M\left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda\left(\frac{a}{a_0}\right)^{2} + \Omega_K}$$

Friedmann equation itself has assumptions about the energy-momentum, it's ideal fluid which can be parameterized by only ρ and P..

also deriveing the Friedmann equations from Einstein field equation is not that trival, see another block in preparation..

It's not linear, so need numerical solution.

But it's OK to see some exceptions.

Propositions 1.1

matter donimated:

$$\dot{a} = H_0\left(a_0\Omega_M\right)^{\frac{1}{2}} a^{-\frac{1}{2}}$$

then

$$t = \frac{2}{3H_0 \left(a_0 \Omega_M\right)^{\frac{1}{2}}} a^{\frac{3}{2}}$$

assumed $a\left(t=0\right)=0$.

LaTeX

mathbb command is used to convert uppercase and lowercase letters to blackboard-bold in terms of shape, as $\mathbb{AC}$.