Test

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目录

1 Expansion **1**
1.1 from time to scale factor . . . 1
1.2 to cosmology redshift and Hubble parameter . . . 2
1.3 Hubble sphere . . . 2
1.4 Light Cone . . . 3
1.5 Event Horizon . . . 3
1.6 Particle Horizon . . . 3
1.6.1 optical horizon . . . 3

1 Expansion

1.1 from time to scale factor

We have two kinds of "time" in cosmology since our Universe are expanding.

If we focus on the coordinate, the metric is written as ($c = 1$)

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \tag{1}$$

By scaling the time axis, we can get the conformal time, and have an "appearantly" Minkovski spacetime

$$\mathrm{d}s^2 = a^2(t)(-\mathrm{d}\tau^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \tag{2}$$

The scale factor can be solved from the **Friedmann equation**, [1]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{3}$$

with **a model of energy-momentum** contained in the Universe like [2]

$$\frac{\rho(t)}{\rho_{\mathrm{cr}}} = \sum_{s=\gamma,\mathrm{m},\nu,\mathrm{DE}} \Omega_s a(t)^{-3(1+w_s)} \tag{4}$$

[1]also deriveing the Friedmann equations from Einstein field equation is not that trival, see another block in preparation..

[2]Friedmann equation itself has assumptions about the energy-momentum, it's ideal fluid which can be parameterized by only ρ and P..

This model assumes a constant of equation of state.

In our ΛCDM model (reserve curvature)

$$\frac{\rho(t)}{\rho_{\mathrm{cr}}} = \Omega_{\mathrm{R}}\left(\frac{a}{a_0}\right)^{-4} + \Omega_{\mathrm{M}}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{\Lambda} + \Omega_{\mathrm{K}}\left(\frac{a}{a_0}\right)^{-2} \tag{5}$$

just solve the differencial equation

$$\dot{a} = H_0 a_0 \sqrt{\Omega_R\left(\frac{a}{a_0}\right)^{-2} + \Omega_M\left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda\left(\frac{a}{a_0}\right)^{2} + \Omega_K} \tag{6}$$

$$t(a_1) = \int_0^{a_1} \frac{\mathrm{d}a}{H_0 a_0 \sqrt{\Omega_R\left(\frac{a}{a_0}\right)^{-2} + \Omega_M\left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda\left(\frac{a}{a_0}\right)^{2} + \Omega_K}} \tag{7}$$

It's not linear, so need numerical solution. See code/expansion.ipynb

But it's OK to see some exceptions.

Block 1. *matter donimated:*

$$\dot{a} = H_0(a_0\Omega_M)^{\frac{1}{2}} a^{-\frac{1}{2}} \tag{8}$$

then

$$t = \frac{2}{3H_0(a_0\Omega_M)^{\frac{1}{2}}} a^{\frac{3}{2}} \tag{9}$$

assumed $a(t=0)=0$.

1.2 to cosmology redshift and Hubble parameter

It will be easy to use redshift and Hubble parameter $H(z)$ in the calculations.

Define redshift as

$$\frac{a}{a_0} = \frac{1}{1+z} \tag{10}$$

And turn Eq. (3) to

$$H(a) \equiv \frac{\dot{a}}{a} = H_0\sqrt{\frac{\rho(a)}{\rho_{\mathrm{cr}}}} \equiv H_0 E(a) \tag{11}$$

Eq. (5) turns out

$$E(a) = \sqrt{\Omega_{\mathrm{R}}\left(\frac{a}{a_0}\right)^{-4} + \Omega_{\mathrm{M}}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{\Lambda} + \Omega_{\mathrm{K}}\left(\frac{a}{a_0}\right)^{-2}} \tag{12}$$

$$E(z) = \sqrt{\Omega_{\mathrm{R}}(1+z)^4 + \Omega_{\mathrm{M}}(1+z)^3 + \Omega_\Lambda + \Omega_{\mathrm{K}}(1+z)^2} \tag{13}$$

1.3 Hubble sphere

Hubble sphere is where the Hubble flow velocity equals to the speed of light.

$$v_{\mathrm{H}} = \dot{a}\chi \tag{14}$$

Hubble sphere as a function of time should be

$$\chi_{\mathrm{H}} = \frac{c}{aH(a)} \tag{15}$$

1.4 Light Cone

Light cone is defined as $\mathrm{d}s^2 = 0$, so

$$\mathrm{d}t^2 = a^2(t)\mathrm{d}\chi^2 \tag{16}$$

$$\mathrm{d}t = a(t)\mathrm{d}\chi \tag{17}$$

$$\chi_{\mathrm{lc}}(a_1) = \int_{t_1}^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_{a_1}^{a_0} \frac{\mathrm{d}a}{a\dot{a}} = \int_{a_1}^{a_0} \frac{\mathrm{d}a}{a^2 H(a)} \tag{18}$$

1.5 Event Horizon

just like light cone but end at $a = \infty$.

$$\chi_{\mathrm{EH}}(a_1) = \int_{0}^{t_1} \frac{\mathrm{d}t}{a(t)} = \int_{0}^{a_1} \frac{\mathrm{d}a}{a\dot{a}} = \int_{0}^{a_1} \frac{\mathrm{d}a}{a^2 H(a)} \tag{19}$$

1.6 Particle Horizon

Particle horizon is the trajectory that a photon start from $\chi = 0$ at t_i (in 'our model', $t_i = a_i = 0$).

$$\chi_{\mathrm{PH}}(a_1) = \int_{t_i}^{t_1} \frac{\mathrm{d}t}{a(t)} = \int_{a_i}^{a_1} \frac{\mathrm{d}a}{a\dot{a}} = \int_{a_i}^{a_1} \frac{\mathrm{d}a}{a^2 H(a)} \tag{20}$$

1.6.1 optical horizon

The optical horizon due to recombination $z(t_r) \approx 1100$.

$$d_{\mathrm{opt}}(t) = a(\tau)(\tau - \tau_r) = a(t)\int_{t_r}^{t} \frac{\mathrm{d}t'}{a(t')} \tag{21}$$

LaTeX

$$\mathrm{H}\,. \tag{22}$$

$$\mathrm{H}\,|\psi\rangle \tag{23}$$

citation example[Lyth and Liddle(2009)]

参考文献

[Lyth and Liddle(2009)] D. H. Lyth and A. R. Liddle, *The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure* (Cambridge University Press, Cambridge, UK, 2009).