


Test

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1 Expansion

1.1 from time to scale factor

We have two kinds of “time” in cosmology since our Universe are expanding.

If we focus on the **coordinate**, the metric is written as ($c = 1$)

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \quad (1)$$

By scaling the time axis, we can get the **conformal time**, and have an “appearantly” Minkovski spacetime

$$ds^2 = a^2(t)(-d\tau^2 + dx^2 + dy^2 + dz^2) \quad (2)$$

The **scale factor** can be solved from the **Friedmann equation**, ¹

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad (3)$$

with **a model of energy-momentum** contained in the Universe like ²

$$\frac{\rho(t)}{\rho_{\text{cr}}} = \sum_{s=\gamma, \text{m}, \nu, \text{DE}} \Omega_s a(t)^{-3(1+w_s)} \quad (4)$$

This model assumes a constant of equation of state.

In our Λ CDM model (reserve curvature)

$$\frac{\rho(t)}{\rho_{\text{cr}}} = \Omega_{\text{R}} \left(\frac{a}{a_0}\right)^{-4} + \Omega_{\text{M}} \left(\frac{a}{a_0}\right)^{-3} + \Omega_{\Lambda} + \Omega_{\text{K}} \left(\frac{a}{a_0}\right)^{-2} \quad (5)$$

¹also deriving the Friedmann equations from Einstein field equation is not that trivial, see another block in preparation..

²Friedmann equation itself has assumptions about the energy-momentum, it's **ideal fluid** which can be parameterized by only ρ and P ..

just solve the differential equation

$$\dot{a} = H_0 a_0 \sqrt{\Omega_R \left(\frac{a}{a_0}\right)^{-2} + \Omega_M \left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda \left(\frac{a}{a_0}\right)^2 + \Omega_K} \quad (6)$$

$$t(a_1) = \int_0^{a_1} \frac{da}{H_0 a_0 \sqrt{\Omega_R \left(\frac{a}{a_0}\right)^{-2} + \Omega_M \left(\frac{a}{a_0}\right)^{-1} + \Omega_\Lambda \left(\frac{a}{a_0}\right)^2 + \Omega_K}} \quad (7)$$

It's not linear, so need numerical solution. See code/expansion.ipynb

But it's OK to see some exceptions.

Block 1. *matter dominated:*

$$\dot{a} = H_0 (a_0 \Omega_M)^{\frac{1}{2}} a^{-\frac{1}{2}} \quad (8)$$

then

$$t = \frac{2}{3H_0 (a_0 \Omega_M)^{\frac{1}{2}}} a^{\frac{3}{2}} \quad (9)$$

assumed $a(t=0) = 0$.

1.2 to cosmology redshift and Hubble parameter

It will be easy to use **redshift** and **Hubble parameter $H(z)$** in the calculations.

Define redshift as

$$\frac{a}{a_0} = \frac{1}{1+z} \quad (10)$$

And turn Eq. (3) to

$$H(a) \equiv \frac{\dot{a}}{a} = H_0 \sqrt{\frac{\rho(a)}{\rho_{\text{cr}}}} \equiv H_0 E(a) \quad (11)$$

Eq. (5) turns out

$$E(a) = \sqrt{\Omega_R \left(\frac{a}{a_0}\right)^{-4} + \Omega_M \left(\frac{a}{a_0}\right)^{-3} + \Omega_\Lambda + \Omega_K \left(\frac{a}{a_0}\right)^{-2}} \quad (12)$$

$$E(z) = \sqrt{\Omega_R (1+z)^4 + \Omega_M (1+z)^3 + \Omega_\Lambda + \Omega_K (1+z)^2} \quad (13)$$

1.3 Hubble sphere

Hubble sphere is where the Hubble flow velocity equals to the speed of light.

$$v_H = \dot{a}\chi \quad (14)$$

The comoving distance

$$\chi = \int_{a_1}^{a_0} \frac{da}{a^2 H(a)} \quad (15)$$

Hubble sphere as a function of time should be

$$\chi = \frac{c}{aH(a)} \quad (16)$$

L^AT_EX

$$H. \tag{17}$$

$$H|\psi\rangle \tag{18}$$

citation example[[Lyth and Liddle\(2009\)](#)]

参考文献

[Lyth and Liddle(2009)] D. H. Lyth and A. R. Liddle, *The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure* (Cambridge University Press, Cambridge, UK, 2009).