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Event-Based Distributed State Estimation for Distrite-Time Stochastic Systems with Deception Attacks over Sensor Networks

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Abstract: This paper addresses H_∞ state estimation problem for a class of discrete-time stochastic delayed systems subject to deception attacks over sensor networks. We are interested in the scenario where the sensor output and estimator output are transmitted to its corresponding estimator and other estimators, respectively, over two different digital channels. To cope with the problem of constrained bandwidth of the network, an event-triggering communication scheme is designed in both sensor-to-estimator channel and estimator-to-estimator channel. Meanwhile, we consider there is an attacker that possibly corrupts the measurement output. Under these circumstances, we aim to design the state estimator such that the exponential mean-square stability with H_∞ performance is satisfied. By virtue of Lyapunov-Krasovskii functional and stochastic analysis, explicit estimator parameters are obtained through solving a set of linear matrix inequalities (LMIs). Finally, a simulation example is established to illustrate the feasibility and effectiveness of the proposed distributed estimation method.

1 Introduction

Wireless sensor networks (WSNs) have gained significant achievements in the fields of environmental monitoring, industrial automation, battlefield surveillance, distributed robotics, and target tracking. Generally, a WSN comprises low power devices that integrate computing, sensing with wireless communication. It is worth noting that, constrained energy resources and communication bandwidth result in limitation in traditional centralized applications. Compared with centralized systems, distributed systems have unique advantages such as lower communication burden, better fault-tolerance and better scalability [1, 2], and thus become hot research issues. As a fundamental problem in WSNs, distributed state estimation has been attracting considerable attentions [3–8]. Among the existing methods, distributed $H\infty$ state estimation gains more and more attention for its efficiency in handling exogenous disturbances, see, e.g., [9–11].

It should be noted that, as the limited communication bandwidth and energy supplies are common constraints in WSNs, the traditional sampled data-based strategies may be somewhat resource wasting since sometimes communication is unnecessary when there is no great fluctuation of sampled information. To tackle such limitation, the so-called event-triggering scheme has been introduced. Unlike the traditional time-triggering scheme with which data transmission is implemented at a predefined time sequence, event-triggering scheme is more intelligent as the information is only transmitted when its value is vital for the guarantee of performance and stability property. Fruitful results have been achieved in control and estimation fields. To mention a few, in [13], an event-triggered condition is presented for multi-agent control system, and the data is updated only when the event-triggered function is violated. In [14], a stochastic event-triggered scheme is proposed which preserves the Gaussian property of innovation process, and in [15], a dynamic event-triggered scheme is designed which could result in larger average interevent times. However, most of the existing works about distributed estimation only focus on either the sensor-to-estimator channel or estimator-to-estimator channel, which means it is possible to further release the communication burden. It should be noted that [16] and [17] used a variance-constrained approach to implement the event-triggered scheme in both channels, and we are looking for other methods to analyze the problem. Such an efficient way to reduce communication burden has not been fully investigated yet, which constitutes one motivation of this work.

The communication between sensors in a network plays a key role for estimation performance, as it provides a bridge for sensors to exchange and obtain more information. Besides the common network-induced constraints, such as time-delay [18], packet dropout[19], cyber-attacks has gradually attracted researchers' attention due to its significant effects on estimation performance and practical demands. Recently many researchers have studied different kinds of attack strategy, see, e.g. [20, 21], which demonstrates that research in the field of resisting cyber-attacks is still on the way. The cyber-attacks could be roughly divided into two categories: namely, denial of service (DOS) attacks and deception attacks, both of which would cause great damages to the system. The deception attacks may arise from natural random factors in a harsh environment or humanoid malicious signals which are often set to be random in order to avoid exposure. There are also results that restrain the effects of deception attacks, see, e.g. [22–25]. In these works, [22] and [23] investigate distributed state estimation of the continuous-time model. In [24] and [25], discrete-time system with deception attacks is studied. However, the plant model they analyzed is not general enough. Therefore, distributed state estimation with deception attacks for the discrete-time case has not been fully investigated yet, which is the second motivation of this paper.

In addition, it is known that time delay and stochastic nonlinearity inherently exist in many practical systems, and may result in degraded performance or instability in the network system. So far, a great deal of effort has been devoted to filter and stabilization control problems for delayed system [19, 27–31]. For example, [19] studies distributed state estimation problem for a constant-delay system, [30] investigates event-triggered H_∞ filtering for delayed neural networks. Motivated by the aforementioned facts, in this paper, our objective is to propose a H_∞ state estimator for stochastic time-varying delayed system with event-triggering mechanism in both channels, with deception attacks also considered.

Motivated by the discussions above, the main contributions of this paper are as follows:

(i) The distributed state estimation problem has been addressed for discrete-time stochastic systems subject to deception attacks with event-triggering mechanism.

(ii) Both sensor-to-estimator and estimator-to-estimator channels are considered in an event-triggering scheme to better reduce the communication burden with the performance guaranteed.
(iii) A unified distributed estimator framework is presented to solve the problem in the presence of event-triggered scheme, time-varying delays, deception attacks, stochastic nonlinearity and external disturbance simultaneously. Sufficient conditions are proposed to reach the $H\infty$ performance index.

The remainder of this paper is structured as follows. In Section 2, some basic concepts on graph theory are presented and the problem is formulated in detail. In Section 3, by virtue of Lyapunov-Krosovskii functional and stochastic analysis, sufficient conditions are derived in the form of LMI, and the estimator gains are obtained by the results of LMI. Section 4 provides an example to illustrate the effectiveness of the designed method. Finally, conclusion is drawn in section 5.

Notations: $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $\mathbb{E}\{x\}$ represents the expectation of x and $\mathbb{E}\{x|y\}$ represents the expectation of x conditioned on y. Prob means the probability of the event. $\mathrm{col}_{\mathcal{N}}\{Z\}$ denotes an $\mathcal{N}$-block column vector $[Z^T\ ...\ Z^T]^T$ and $\mathrm{col}_{\mathcal{N}}^i\{Z_i\}$ denotes an $\mathcal{N}$-block column vector $Z_1^T\ ...\ Z_{\mathcal{N}}^T$. $\mathrm{diag}_{\mathcal{N}}\{Z\}$ stands for a diagonal matrix with $\mathcal{N}$ blocks $Z,...Z$ and $\mathrm{diag}_{\mathcal{N}}^i\{Z_i\}$ represents a diagonal matrix with $\mathcal{N}$ blocks $[Z_1,...Z_{\mathcal{N}}]$. $l_2[0,\infty)$ denotes the space of square summable vector functions. The Kronecker product is denoted as $\otimes$. Matrices are assumed to have compatible dimensions if they are not explicitly specified. Other notations are standard if not mentioned.

2 Problem formulation and Preliminaries

In this paper, we assume that there are N nodes in a sensor network, each of which comprises a sensor to monitor the plant and an estimator to estimate the state of the plant. N cooperative nodes are spatially distributed over a wireless communication network. The network topology is represented by an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{H})$ of order N with the sets of nodes $\mathcal{V}=\{1,2,...N\}$. $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ denotes the edge set of paired sensor nodes, and $\mathcal{H}=[h_{ij}]\in\mathbb{R}^{N\times N}$ denotes the adjacency matrix with non-negative element $h_{ij}>0$ if $(i,j)\in\mathcal{E}$, and $h_{ij}=0$ otherwise. Selfloops are excluded in this paper, i.e. $h_{ii}=0, i\in\mathcal{V}$. Node j is called a neighbor of node i if $h_{ij}>0$, and the set of neighbors of node i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$. The Laplacian matrix of the digraph $\mathcal{G}$ is set as $\mathcal{L}=\mathcal{W}-\mathcal{H}$, where $\mathcal{W}=\mathrm{diag}_{\mathcal{N}_i}\{w_i\}$, with the diagonal element $w_i=\sum_{j\in\mathcal{N}_i}h_{ij}$.

Consider the plant described by a discrete-time stochastic nonlinear system

$$\begin{cases} x(k+1) &= Ax(k)+A_d x\left(k-\tau(k)\right)+f\big(x(k),\vartheta(k)\big) \\ & \quad +B\omega(k) \\ z(k) &= Mx(k) \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^{n_x}$ is the state vector which can not be observed directly, $z(k)\in\mathbb{R}^{n_z}$ is the output to be estimated, $\omega(k)$ is the external disturbance vector belonging to $l_2[0,\infty)$. $\tau(k)$ represents the time-varying delay of the system, which satisfies

$$\tau_m\le\tau(k)\le\tau_M,\quad k=1,2,\dots \tag{2}$$

where the positive integers τ_m and τ_M are known bounds of the delay.

The function $f\big(x(k),\vartheta(k)\big)$ with $f\big(0,\vartheta(k)\big)=0$ is a stochastic nonlinear function satisfying the following properties:

$$\mathbb{E}\left\{f\big(x(k),\vartheta(k)\big)|x(k)\right\}=0, \tag{3}$$

$$\mathbb{E}\left\{f^T\big(x(k),\vartheta(k)\big)f\big(x(j),\vartheta(j)\big)|x(k)\right\}=0,\quad k\neq j, \tag{4}$$

$$\begin{aligned}\mathbb{E}\left\{f^T\big(x(k),\vartheta(k)\big)f\big(x(k),\vartheta(k)\big)|x(k)\right\}& \\ \le\beta^2x^T(k)R^TRx(k).&\end{aligned} \tag{5}$$

The equation (5) assumes the bounds of the stochastic nonlinearity, which is sensible in many practical examples. β and R are previously known constants, and ϑ_k represents the Gaussian white noise.

The model of N sensors under deception attack is given as follows [25]:

$$\begin{cases} y_i(k)=\bar{y}_i(k)+\alpha(k)\nu(k) \\ \bar{y}_i(k)=C_ix(k) \\ \nu(k)=-\bar{y}_i(k)+\theta_i(\bar{y}_i(k)) \end{cases} \tag{6}$$

where $\bar{y}_i(k)\in\mathbb{R}^{n_y}$ is the measurement output in the normal and safe situation, $y_i(k)$ is the actual measurement that may be under attack, $\nu(k)$ is caused by the attacker, and $\theta_i(\bar{y}_i(k))$ is the malicious signal resulting in the deviation of measurement. In many practical situation, the ability of attackers is limited due to its own limitation or sensor's intelligence and robustness given by human through setting some intelligent algorithms. In other words, attackers either fail to implement an attack or cause a limited degree of bias. We assume that $\theta_i(\bar{y}_i(k))$ satisfies the nonlinear vector-valued function $\theta_i(\cdot):\mathbb{R}^{n_y}\to\mathbb{R}^{n_y}$. The nonlinear function is assumed to be continuous and satisfy the following sector-bounded condition:

$$[\theta_i(\bar{y}_i(k))-H_1\bar{y}_i(k)]^T[\theta_i(\bar{y}_i(k))-H_2\bar{y}_i(k)]\le 0 \tag{7}$$

for all $\bar{y}_i(k)\in\mathbb{R}^{n_y}$, where H_1 and H_2 are real matrices of appropriate dimensions. The stochastic variable $\alpha(k)\in\mathbb{R}$ is a Bernoulli distributed sequence taking values on 0 or 1 with

$$\mathrm{Prob}(\alpha(k)=1)=\bar{\alpha},\quad \mathrm{Prob}(\alpha(k)=0)=1-\bar{\alpha}, \tag{8}$$

where $\bar{\alpha}$ is a known positive constant, and $\bar{\alpha}$ is assumed to be independent of $\vartheta(k)$ and $x(0)$.

It is well known that event-triggering communication scheme is often utilized to reduce the communication burden of network. As to the distributed state estimation problem for some sensor networks, the sensors are not smart or strong enough to process the data efficiently. Therefore, the usual strategy is that each sensor, which implements the periodic sampling rule, transmits its measurement output to its corresponding estimator, and the estimators carry out effective calculation. When the sensor measurement fluctuates very slightly, the strategy would definitely cause wastage of energy consumption. In addition, one of the superiority of the sensor network is the availability of valuable information from other nodes via data transmission. Unfortunately, frequent communication between estimators would cause unnecessary wastage of communication resource as well. To alleviate the communication burden as much as possible, both sensor-to-estimator and estimator-to-estimator channels are considered, under which only important information that is vital to the estimation performance would be transmitted. The communication scheme over the network is shown in Fig1.

The event-triggering condition of sensor-to-estimator channel is set as follows:

$$\begin{aligned}\Big(y_i(\varphi_{k-1}^i)-y_i(k)\Big)^T\Omega_{1,i}\Big(y_i(\varphi_{k-1}^i)-y_i(k)\Big)>& \\ \sigma_iy_i^T(k)\Omega_{1,i}y_i(k),&\end{aligned} \tag{9}$$

where φ_{k-1}^i is the latest time instant when sensor i sends the measurement to estimator i at $k-1$ time instant, $\sigma_i>0$ is the predefined trigger threshold, and $\Omega_{1,i}$ is the predefined weighting matrix with appropriate dimensions.

As to estimator-to-estimator channel, estimator i would transmit the information to its neighbors when the following condition is

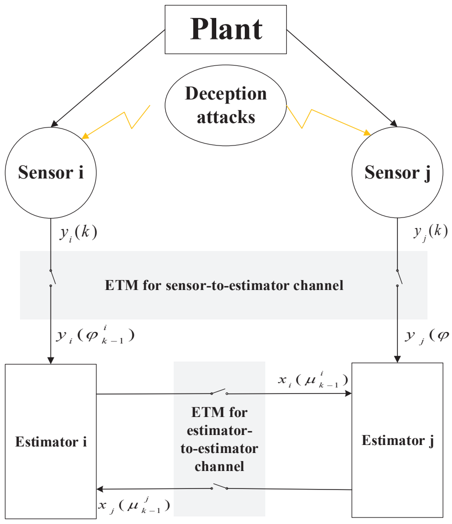

Fig. 1: Overall event-triggering mechanism (ETM) among the sensors and estimators.

satisfied:

$$\left(\hat{x}_i(k) - \hat{x}_i(\mu^i_{k-1})\right)^T \Omega_{2,i} \left(\hat{x}_i(k) - \hat{x}_i(\mu^i_{k-1})\right) > \delta_i \mathbb{Y}_i^T(k)\Omega_{2,i}\mathbb{Y}_i(k), \tag{10}$$

where

$$\mathbb{Y}_i(k) = \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_j(\mu^j_{k-1}) - \hat{x}_i(\mu^i_{k-1})), \tag{11}$$

and $\mu^i_{k-1}$is the latest time instant when estimator i transmits its data to its neighbors at $k-1$ time instant, $\delta_i > 0$ is the predefined trigger threshold, and $\Omega_{2,i}$ is the predefined weighting matrix with appropriate dimensions. The corresponding update rules of φ^i_k and μ^i_k are interpreted as

$$\varphi^i_k = \begin{cases} k, & \text{if } y_i(k) \text{ is transmitted} \\ \varphi^i_{k-1}, & \text{otherwise} \end{cases} \tag{12}$$

and

$$\mu^i_k = \begin{cases} k, & \text{if } \hat{x}_i(k) \text{ is transmitted} \\ \mu^i_{k-1}, & \text{otherwise} \end{cases} \tag{13}$$

In this paper, the structure of the distributed state estimator is chosen as follows:

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + A_d\hat{x}_i\left(k-\tau(k)\right) \\ \qquad +K_i\left(y_i(\varphi^i_{k-1}) - C_i\hat{x}_i(k)\right) \\ \qquad +G_i\sum_{j\in\mathcal{N}_i} a_{ij}\left(\hat{x}_j(\mu^j_{k-1}) - \hat{x}_i(\mu^i_{k-1})\right) \\ \hat{z}_i(k) \quad = M\hat{x}_i(k) \end{cases} \tag{14}$$

where $\hat{x}_i(k)$ is the state estimation computed by estimator i, $\hat{z}_i(k)$ is the output of node i and represents an estimation of the objective output signal $z(k)$, K_i and G_i are the estimator gain matrices of node i to be determined.

Remark 1. *This study proposes a distributed estimation scheme based on Luenberger-like observer in combination with consensus strategy. The Luenberger-like observer term is a correction based on measured information, where the innovations are calculated according to the latest sensor's information at $k-1$ time instant. The second source of information comes from the estimates sent by the neighbors, letting the estimates of all the nodes reach a consensus and improve the estimation performance.*

Remark 2. $y_i(\varphi^i_{k-1})$ *in (12) means the latest obtained measured information at $k-1$ time instant, i.e., the newest information we can use is that from $k-1$ time instant. For estimator i, $y_i(\varphi^i_{k-1})$ and $y_i(\varphi^i_k)$ are both available. The choice of $y_i(\varphi^i_{k-1})$ rather than $y_i(\varphi^i_k)$ is with intrinsic motivations and reasons.*

(i)When sensor i does not transmit the measurement at k time instant, $y_i(\varphi^i_{k-1})$ is equivalent to $y_i(\varphi^i_k)$. When sensor i transmits its measurement, it means that the degree of deviation between $y_i(\varphi^i_{k-1})$ and $y_i(k)$ exceeds some threshold, induced by the plant's normal dynamics and/or malicious deception attacks. Confronted with the uncertain environment that may be under attack, more reliable information, i.e. $y_i(\varphi^i_{k-1})$, is selected, as we can get a constraint for $y_i(\varphi^i_{k-1})$ from the event-triggering condition at k time instant. Although $y_i(\varphi^i_{k-1})$ seems delayed compared with $y_i(\varphi^i_k)$, we can still guarantee the estimation performance as shown in the following results.
(ii)This choice brings great convenience in the design of unified distributed estimator framework. Exponential stability and H_∞ performance index can be analyzed on this basis of this selection.
(iii)An additional potential advantage is that this choice is more robust to the transmission delay problem. As is well known, the process of transmitting from sensor to estimator needs certain time. When encountering unfortunate factors that lengthen transmitting time, $y_i(\varphi^i_k)$ may be not available, which would lead to system mistakes.

We find similar ideas in [12], which uses a time-delay approach to settle down the networked control problems and allows the transmission delays to be larger than the sampling intervals. The reasons for the choice of $\hat{x}_i(\mu^i_{k-1})$ is similar to that of $y_i(\varphi^i_{k-1})$ and thus omitted here.

For notational simplicity, we denote

$$\begin{aligned}
&e_i(k) = x_k - \hat{x}_i(k), && \tilde{z}_i(k) = z_k - \hat{z}_i(k), \\
&\xi_i(k) = y_i(\varphi^i_{k-1}) - y_i(k), && \varrho_i(k) = \hat{x}_i(k) - \hat{x}_i(\mu^i_{k-1}), \\
&\bar{x}(k) = \mathrm{col}_{\mathcal{N}}\left\{x(k)\right\}, && \tilde{z}(k) = \mathrm{col}^i_{\mathcal{N}}\left\{\tilde{z}_i(k)\right\}, \\
&e(k) = \mathrm{col}^i_{\mathcal{N}}\left\{e_i(k)\right\}, && \xi(k) = \mathrm{col}^i_{\mathcal{N}}\left\{\xi_i(k)\right\}, \\
&\varrho(k) = \mathrm{col}^i_{\mathcal{N}}\left\{\varrho_i(k)\right\}, && \theta(k) = \mathrm{col}^i_{\mathcal{N}}\left\{\theta_i(k)\right\}, \\
&\bar{f} = \mathrm{col}_{\mathcal{N}}\left\{f\right\}, && L_1 = \mathcal{L}\otimes I_n, \\
&\bar{A} = \mathrm{diag}_{\mathcal{N}}\left\{A\right\}, && \bar{A}_d = \mathrm{diag}_{\mathcal{N}}\left\{A_d\right\}, \\
&\bar{B} = \mathrm{col}_{\mathcal{N}}\left\{B\right\}, && \bar{C} = \mathrm{diag}^i_{\mathcal{N}}\left\{C_i\right\}, \\
&\bar{K} = \mathrm{diag}^i_{\mathcal{N}}\left\{K_i\right\}, && \bar{G} = \mathrm{diag}^i_{\mathcal{N}}\left\{G_i\right\}, \\
&\tilde{\alpha} = \alpha - \bar{\alpha}.
\end{aligned}$$

Combining (1), (6) and (14), the filtering error dynamics could be obtained:

$$\begin{aligned} e(k+1) = \; & \left(\bar{A} - \bar{K}\bar{C} - \bar{G}L_1\right)e(k) + \bar{A}_d e\left(k-\tau(k)\right) \\ & +\bar{\alpha}\bar{K}\bar{C}\bar{x}(k) + \tilde{\alpha}\bar{K}\bar{C}\bar{x}(k) - \bar{K}\xi(k) \\ & -\bar{\alpha}\bar{K}\theta(k) - \tilde{\alpha}\bar{K}\theta(k) - \bar{G}L_1\varrho(k) \\ & +\bar{f} + \bar{B}\omega(k). \end{aligned} \tag{15}$$

Letting $\eta(k) = [\bar{x}^T(k)\; e^T(k)]^T$, the augmented filtering system is obtained as follows

$$\begin{aligned} \eta(k+1) = \; & \bar{\mathcal{A}}\eta(k) + \tilde{\mathcal{A}}\eta(k) + \bar{\mathcal{A}}_d\eta\left(k-\tau(k)\right) \\ & -K_1\xi(k) - \bar{\alpha}K_1\theta(k) - \tilde{\alpha}K_1\theta(k) \\ & -L_2\varrho(k) + \bar{I}\bar{f} + \bar{\mathcal{B}}\omega(k), \end{aligned} \tag{16}$$

where

$$\bar{\mathcal{A}} = \begin{bmatrix} \bar{A} & 0 \\ \bar{\alpha}\bar{K}\bar{C} & \bar{A}-\bar{K}\bar{C}-\bar{G}L_1 \end{bmatrix},$$
$$\tilde{\mathcal{A}} = \begin{bmatrix} 0 & 0 \\ \tilde{\alpha}\bar{K}\bar{C} & 0 \end{bmatrix}, \ \bar{\mathcal{A}}_d = \begin{bmatrix} \bar{A}_d & 0 \\ 0 & \bar{A}_d \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0 \\ \bar{K} \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0 \\ \bar{G}L_1 \end{bmatrix},$$
$$\bar{I} = \begin{bmatrix} I_{N\times n_x} \\ I_{N\times n_x} \end{bmatrix}, \qquad \bar{\mathcal{B}} = \begin{bmatrix} \bar{B} \\ \bar{B} \end{bmatrix}.$$

Before proceeding further, we introduce the following definition.

Definition 1. *The augmented system in (16) is said to be exponentially mean-square stable, if with $\omega(k) = 0$, there exist constants $\kappa > 0$ and $0 < \varepsilon < 1$ such that*

$$\mathbb{E}\left\{\|\eta(k)\|^2\right\} \leq \kappa\varepsilon^k\mathbb{E}\left\{\|\eta(0)\|^2\right\}, \forall\, \eta(0) \in \mathbb{R}^n, k \in \mathbb{R}^+.$$

The aim of the following work is to design the estimator parameter K_i and $G_i (i = 1, 2, ...N)$ such that the following requirements are satisfied simultaneously:

(R1) The augmented filtering system (16) with $\omega(k) = 0$ is exponentially mean-square stable;
(R2) Under zero initial condition, for a given disturbance attenuation level $\gamma > 0$ and all non-zero $\omega(k)$, the output estimation error $\tilde{z}(k)$ satisfies the following H_∞ performance index

$$\frac{1}{N}\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\tilde{z}(k)\|^2\right\} \leq \gamma^2\sum_{k=0}^{\infty}\|\omega(k)\|^2. \tag{17}$$

3 Main results

In this section, we aim at searching for suitable estimator parameters that satisfy (R1) and (R2). Stochastic analysis techniques are utilized to solve the problem.

Theorem 1. *Consider the discrete-time stochastic system (1) and sensors (6) and event-triggered functions (9) (10). For the given estimator parameters $\bar{K}$ and $\bar{G}$ as well as attenuation level $\gamma > 0$, the dynamics of estimation error is exponentially mean-square stable and satisfies the H_∞ performance index (17), if there exist two positive definite matrices $P > 0$, $Q > 0$, scalars $\lambda_i > 0 (i = 1, 2, 3, 4)$ satisfying*

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & 0 & \Lambda_{17} \\ * & \Lambda_{22} & \Lambda_{32} & \Lambda_{42} & \Lambda_{52} & 0 & \Lambda_{72} \\ * & * & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & 0 & \Lambda_{37} \\ * & * & * & \Lambda_{44} & \Lambda_{45} & 0 & \Lambda_{47} \\ * & * & * & * & \Lambda_{55} & 0 & \Lambda_{57} \\ * & * & * & * & * & \Lambda_{66} & 0 \\ * & * & * & * & * & 0 & \Lambda_{77} \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned}\Lambda_{11} &= \bar{\mathcal{A}}^T P\bar{\mathcal{A}} + \bar{\alpha}(1-\bar{\alpha})\mathcal{A}_1^T P\mathcal{A}_1 - P + Q \\ &\quad + (\tau_M - \tau_m)Q + \lambda_1(1-\bar{\alpha})E_1^T\bar{C}^T\sigma\Omega_1\bar{C}E_1 \\ &\quad + \lambda_2 E_2^T L_1^T\delta\Omega_2 L_1 E_2 - \lambda_3 E_1^T\bar{C}^T\bar{H}_1^T\bar{H}_2\bar{C}E_1 \\ &\quad + \lambda_4\beta_2 E_1^T\bar{R}^T\bar{R}E_1 + \frac{1}{N}E_2^T\bar{M}^T\bar{M}E_2,\end{aligned}$$

$$\begin{aligned}
&\Lambda_{12} = \bar{\mathcal{A}}^T P\bar{\mathcal{A}}_d, \qquad \Lambda_{13} = -\bar{\mathcal{A}}^T PK_1, \\
&\Lambda_{14} = -\bar{\mathcal{A}}^T PL_2 + \lambda_2 E_2^T L_1^T\delta\Omega_2 L_1, \\
&\Lambda_{15} = -\bar{\alpha}\bar{\mathcal{A}}^T PK_1 + \frac{1}{2}\lambda_3 E_1^T\bar{C}^T(\bar{H}_1 + \bar{H}_2)^T, \\
&\Lambda_{17} = \bar{\mathcal{A}}^T P\bar{\mathcal{B}}, \qquad \Lambda_{22} = \bar{\mathcal{A}}_d^T P\bar{\mathcal{A}}_d - Q, \\
&\Lambda_{23} = -\bar{\mathcal{A}}_d^T PK_1, \qquad \Lambda_{24} = -\bar{\mathcal{A}}_d^T PL_2, \\
&\Lambda_{25} = -\bar{\alpha}\bar{\mathcal{A}}_d^T PK_1, \qquad \Lambda_{27} = \bar{\mathcal{A}}_d^T P\bar{\mathcal{B}}, \\
&\Lambda_{33} = K_1^T PK_1 - \lambda_1\Omega_1, \ \Lambda_{34} = K_1^T PL_2, \\
&\Lambda_{35} = \bar{\alpha}K_1^T PK_1, \qquad \Lambda_{37} = -K_1^T P\bar{\mathcal{B}}, \\
&\Lambda_{44} = L_2 PL_2 + \lambda_2 L_1^T\delta\Omega_2 L_1 - \lambda_2\Omega_2, \\
&\Lambda_{45} = \bar{\alpha}L_2^T PK_1, \qquad \Lambda_{47} = -L_2 P\bar{\mathcal{B}}, \\
&\Lambda_{55} = \bar{\alpha}K_1^T PK_1 + \lambda_1\bar{\alpha}\sigma\Omega_1 - \lambda_3 I, \\
&\Lambda_{57} = -\bar{\alpha}K_1^T P\bar{\mathcal{B}}, \qquad \Lambda_{66} = \bar{I}^T P\bar{I} - \lambda_4 I, \\
&\Lambda_{77} = \bar{\mathcal{B}}^T P\bar{\mathcal{B}} - \gamma^2 I,
\end{aligned}$$

and

$$E_1 = \begin{bmatrix} I & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & I \end{bmatrix},$$
$$\mathcal{A}_1 = \begin{bmatrix} 0 & 0 \\ \bar{K}\bar{C} & 0 \end{bmatrix}.$$

proof: First, let us handle some stochastic terms. One has

$$\begin{aligned} &\mathbb{E}\left\{\tilde{\alpha}^2\right\} = \bar{\alpha}(1-\bar{\alpha}), \quad \mathbb{E}\left\{(1-\alpha)^2\right\} = 1-\bar{\alpha} \\ &\mathbb{E}\left\{\alpha(1-\alpha)\right\} = 0, \quad \mathbb{E}\left\{\alpha^2\right\} = \bar{\alpha} \end{aligned}. \tag{19}$$

Choose a Lyapunov-Krasovskii functional as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{20}$$

where

$$V_1(k) = \eta^T(k)P\eta(k), \tag{21}$$

$$V_2(k) = \sum_{i=k-\tau(k)}^{k-1}\eta^T(i)Q\eta(i), \tag{22}$$

$$V_3(k) = \sum_{j=k-\tau_M+1}^{k-\tau_m}\sum_{i=j}^{k-1}\eta^T(i)Q\eta(i). \tag{23}$$

Calculating the difference of $V(k)$ along the trajectory of system (16) with $\omega(k) = 0$ and taking the mathematical expectation, we get the following results:

$$\mathbb{E}\left\{\Delta V(k)\right\} = \mathbb{E}\left\{\Delta V_1(k)\right\} + \mathbb{E}\left\{\Delta V_2(k)\right\} + \mathbb{E}\left\{\Delta V_3(k)\right\}, \tag{24}$$

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_1(k)\Big\} = \mathbb{E}\Big\{V_1(k+1) - V_1(k)\Big\} \\
&= \mathbb{E}\Big\{\big[W_1 + \tilde{\mathcal{A}}\eta(k) - \tilde{\alpha}K_1\theta(k) + \bar{I}\bar{f}\big]^T P\big[W_1 + \tilde{\mathcal{A}}\eta(k) \\
&\quad - \tilde{\alpha}K_1\theta(k) + \bar{I}\bar{f}\big] - \eta^T(k)P\eta(k)\Big\} \\
&= \mathbb{E}\Big\{W_1^T PW_1 + \tilde{\alpha}^2\theta^T(k)K_1^T PK_1\theta(k) \\
&\quad + \eta^T(k)\tilde{\mathcal{A}}^T P\tilde{\mathcal{A}}\eta(k) + \bar{f}^T\bar{I}^T P\bar{I}\bar{f} - \eta^T(k)P\eta(k)\Big\} \\
&= \mathbb{E}\Big\{W_1^T PW_1 + \bar{\alpha}(1-\bar{\alpha})\theta^T(k)K_1^T PK_1\theta(k) \\
&\quad + \bar{\alpha}(1-\bar{\alpha})\eta^T(k)\mathcal{A}_1^T P\mathcal{A}_1\eta(k) + \bar{f}^T\bar{I}^T P\bar{I}\bar{f} \\
&\quad - \eta^T(k)P\eta(k)\Big\},
\end{aligned} \tag{25}$$

IET Research Journals, pp. 1–8

where

$$W_1 = \bar{\mathcal{A}}\eta(k) + \bar{\mathcal{A}}_d\eta\left(k-\tau(k)\right) - K_1\xi(k) - \bar{\alpha}K_1\theta(k) - L_2\varrho(k),$$

and

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_2(k)\Big\} = \mathbb{E}\Big\{V_2(k+1) - V_2(k)\Big\}\\
&= \mathbb{E}\Big\{\sum_{i=k+1-\tau(k+1)}^{k}\eta^T(i)Q\eta(i) - \sum_{i=k-\tau(k)}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
&\le \mathbb{E}\Big\{\eta^T(k)Q\eta(k) - \eta^T(k-\tau(k))Q\eta(k-\tau(k))\\
&\qquad + \sum_{i=k+1-\tau_M}^{k-\tau_m}\eta^T(i)Q\eta(i)\Big\},
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_3(k)\Big\} = \mathbb{E}\Big\{V_3(k+1) - V_3(k)\Big\}\\
&= \mathbb{E}\Big\{\sum_{j=k-\tau_M+2}^{k+1-\tau_m}\sum_{i=j}^{k}\eta^T(i)Q\eta(i) - \sum_{j=k-\tau_M+1}^{k-\tau_m}\sum_{i=j}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
&= \mathbb{E}\Big\{\sum_{t=k-\tau_M+1}^{k-\tau_m}\sum_{i=t+1}^{k}\eta^T(i)Q\eta(i) - \sum_{j=k-\tau_M+1}^{k-\tau_m}\sum_{i=j}^{k-1}\eta^T(i)Q\eta(i)\Big\}\\
&= \mathbb{E}\Big\{\sum_{j=k-\tau_M+1}^{k-\tau_m}\Big(\eta^T(k)Q\eta(k) - \eta^T(j)Q\eta(j)\Big)\Big\}\\
&= \mathbb{E}\Big\{(\tau_M - \tau_m)\eta^T(k)Q\eta(k) - \sum_{j=k-\tau_M+1}^{k-\tau_m}\eta^T(j)Q\eta(j)\Big\}.
\end{aligned} \tag{27}$$

From the event-triggering condition (9) in sensor-to-estimator channel, we have

$$\begin{bmatrix} \bar{x}(k) \\ \theta(k) \end{bmatrix}^T \begin{bmatrix} (1-\alpha)\bar{C}^T \\ \alpha I \end{bmatrix} \sigma\Omega_1 \begin{bmatrix} (1-\alpha)\bar{C} & \alpha I \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ \theta(k) \end{bmatrix} > \xi^T(k)\Omega_1\xi(k). \tag{28}$$

Notice that $\bar{x}(k) = E_1\eta(k)$. Taking expectations on both sides of (28), we get

$$\mathbb{E}\Big\{\eta_1^T(k)W_2\eta_1(k) - \xi^T(k)\Omega_1\xi(k)\Big\} > 0, \tag{29}$$

where

$$\eta_1(k) = \begin{bmatrix} \eta(k) \\ \theta(k) \end{bmatrix},$$

$$W_2 = \begin{bmatrix} (1-\bar{\alpha})E_1^T\bar{C}^T\sigma\Omega_1\bar{C}E_1 & 0 \\ 0 & \bar{\alpha}\sigma\Omega_1 \end{bmatrix}.$$

It follows from the event-triggering condition (10) that

$$\begin{aligned}
&\mathbb{E}\Big\{[L_1E_2\eta(k) + L_1\varrho(k)]^T\delta\Omega_2[L_1E_2\eta(k) + L_1\varrho(k)]\\
&\qquad - \varrho^T(k)\Omega_2\varrho(k)\Big\} > 0,
\end{aligned} \tag{30}$$

where $E_2 = \begin{bmatrix} 0 & I \end{bmatrix}$, $e(k) = E_2\eta(k)$. In addition, it follows from (7) that

$$\eta_1^T(k)W_3\eta_1(k) > 0, \tag{31}$$

where

$$W_3 = \begin{bmatrix} -E_1^T\bar{C}^T\bar{H}_1^T\bar{H}_2\bar{C}E_1 & * \\ \frac{1}{2}(\bar{H}_1 + \bar{H}_2)^T\bar{C}E_1 & I \end{bmatrix}.$$

Furthermore, we can get from (5) that

$$\beta^2\eta^T(k)E_1^T\bar{R}^T\bar{R}E_1\eta(k) - \bar{f}^T\bar{f} > 0, \tag{32}$$

where $\bar{R} = \mathrm{diag}_{\mathcal{N}}\{\mathrm{R}\}$.

Substituting (25)-(27), (29)-(32) into (24) leads to

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V(k)\Big\}\\
&\le \mathbb{E}\Big\{\big[\bar{\mathcal{A}}\eta(k) + \bar{\mathcal{A}}_d\eta\left(k-\tau(k)\right) - K_1\xi(k) - \bar{\alpha}K_1\theta(k)\\
&\quad - L_2\varrho(k)\big]^T P\big[\bar{\mathcal{A}}\eta(k) + \bar{\mathcal{A}}_d\eta\left(k-\tau(k)\right) - K_1\xi(k)\\
&\quad - \bar{\alpha}K_1\theta(k) - L_2\varrho(k)\big] + \bar{\alpha}(1-\bar{\alpha})\theta^T(k)K_1^TPK_1\theta(k)\\
&\quad + \eta^T(k)\mathcal{A}_1^TP\mathcal{A}_1\eta(k) + \bar{f}^T\bar{I}^TP\bar{I}\bar{f} - \eta^T(k)P\eta(k)\Big\}\\
&\quad + \eta^T(k)Q\eta(k) - \eta^T\left(k-\tau(k)\right)Q\eta\left(k-\tau(k)\right)\\
&\quad + (\tau_M - \tau_m)\eta^T(k)Q\eta(k) - \lambda_1\xi^T(k)\Omega_1\xi(k)\\
&\quad + \lambda_1\eta_1^T(k)W_2\eta_1(k) - \lambda_2\varrho^T(k)\Omega_2\varrho(k)\\
&\quad + \lambda_2[L_1E_2\eta(k) + L_1\varrho(k)]^T\delta\Omega_2[L_1E_2\eta(k) + L_1\varrho(k)]\\
&\quad + \lambda_3\eta_1^T(k)W_2\eta_1(k) - \lambda_4\bar{f}^T\bar{f}\\
&\quad + \lambda_4\beta^2\eta^T(k)E_1^T\bar{R}^T\bar{R}E_1\eta(k)\Big\}\\
&= \mathbb{E}\Big\{\bar{\eta}^T(k)\bar{\Lambda}\bar{\eta}(k)\Big\},
\end{aligned} \tag{33}$$

where

$$\bar{\eta}(k) \triangleq [\eta^T(k)\ \eta^T(k-\tau(k))\ \xi^T(k)\ \varrho^T(k)\ \theta^T(k)\ \bar{f}^T],$$

$$\bar{\Lambda} \triangleq \begin{bmatrix} \bar{\Lambda}_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & 0 \\ * & \Lambda_{22} & \Lambda_{32} & \Lambda_{42} & \Lambda_{52} & 0 \\ * & * & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & 0 \\ * & * & * & \Lambda_{44} & \Lambda_{45} & 0 \\ * & * & * & * & \Lambda_{55} & 0 \\ * & * & * & * & * & \Lambda_{66} \end{bmatrix}$$

with $\bar{\Lambda}_{11} = \Lambda_{11} - \frac{1}{N}E_2^T\bar{M}^T\bar{M}E_2$. Condition (18) is sufficient to make $\bar{\Lambda} < 0$, thus

$$\mathbb{E}\Big\{\Delta V(k)\Big\} \le -\lambda_{\min}(-\bar{\Lambda})\big\|\bar{\eta}(k)\big\|^2. \tag{34}$$

It is obvious to see that the augmented system with $\omega(k) = 0$ is exponentially mean-square stable. As to the H_∞ performance index, we introduce the following inequality:

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V(k) + \frac{1}{N}\|\tilde{z}(k)\|^2 - \gamma^2\|\omega(k)\|^2\Big\}\\
&\le \mathbb{E}\Big\{\bar{\eta}^T(k)\bar{\Lambda}\bar{\eta}(k) + 2\eta^T(k)\bar{\mathcal{A}}^TP\bar{\mathcal{B}}\omega(k)\\
&\quad + 2\eta^T(k-\tau(k))\bar{\mathcal{A}}_d^TP\bar{\mathcal{B}}\omega(k) - 2\xi^T(k)K_1^TP\bar{\mathcal{B}}\omega(k)\\
&\quad - 2\bar{\alpha}\theta^T(k)K_1^TP\bar{\mathcal{B}}\omega(k) - 2\varrho^T(k)L_2^TP\bar{\mathcal{B}}\omega(k)\\
&\quad + \omega^T(k)\bar{\mathcal{B}}^TP\bar{\mathcal{B}}\omega(k) + \frac{1}{N}\eta^T(k)E_2^T\bar{M}^T\bar{M}E_2\eta(k)\\
&\quad - \gamma^2\omega^T(k)\omega(k)\Big\},
\end{aligned} \tag{35}$$

which leads to

$$\mathbb{E}\Big\{\Delta V(k) + \frac{1}{N}\|\tilde{z}(k)\|^2 - \gamma^2\|\omega(k)\|^2\Big\} \le \mathbb{E}\Big\{\tilde{\eta}^T(k)\Lambda\tilde{\eta}(k)\Big\} < 0. \tag{36}$$

with $\tilde{\eta}^T(k) = [\bar{\eta}^T(k)\ \omega^T(k)]$. Summing up (36) from 0 to ∞ under zero-initial condition, we can get

$$\frac{1}{N}\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\tilde{z}(k)\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty}\|\omega(k)\|^2,$$

which completes the proof of theorem 1. After guaranteeing the mean-square stability and H_∞ performance, we are in a position to design the distributed estimator. Explicit estimator parameters are obtained by the following theorem.

Theorem 2. *Let the disturbance attenuation level γ be given. Considering the discrete-time stochastic system (1) and sensors (6) and event-triggered function (9) (10), the dynamics of estimation error is exponentially mean-square stable and satisfies H_∞ performance constraint, if there exist two positive definite matrices $P > 0$, $Q > 0$, positive constant scalars $\lambda_i > 0 (i = 1, 2, 3, 4)$ and the matrices X and Y satisfying that*

$$\begin{aligned} P = diag\{P_1, P_2, ..., P_{2N}\} > 0,\\ \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \end{aligned} \tag{37}$$

where

$$\Pi_{11} = \begin{bmatrix} \Sigma_{11} & 0 & 0 & \Sigma_{14} & \Sigma_{15} & 0 & 0 \\ * & \Sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55} & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 \\ * & * & * & * & * & * & \Sigma_{77} \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & 0 \\ \Gamma_{21} & 0 & 0 & 0 \\ \Gamma_{31} & 0 & 0 & 0 \\ \Gamma_{41} & 0 & 0 & 0 \\ \Gamma_{51} & 0 & \Gamma_{53} & 0 \\ 0 & 0 & 0 & \Gamma_{64} \\ \Gamma_{71} & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{bmatrix},$$

$$\begin{aligned}
\Sigma_{11} &= -P + (\tau_M - \tau_m + 1)Q + \lambda_1(1-\bar{\alpha})E_1^T\bar{C}^T\sigma\Omega_1\bar{C}E_1\\
&\quad \lambda_2 E_2^T L_1^T \delta\Omega_2 L_1 E_2 - \lambda_3 E_1^T\bar{C}^T\bar{H}_1^T\bar{H}_2\bar{C}E_1\\
&\quad \lambda_4\beta^2 E_1^T\bar{R}^T\bar{R}E_1 + \frac{1}{N}E_2^T\bar{M}^T\bar{M}E_2,\\
\Sigma_{22} &= -Q,\ \Sigma_{33} = -\lambda_1\Omega_1,\ \Sigma_{44} = -\lambda_2\Omega_2,\\
\Sigma_{55} &= \lambda_1\bar{\alpha}\sigma\Omega_1 - \lambda_3 I,\ \Sigma_{66} = -\lambda_4 I,\\
\Sigma_{77} &= -\gamma_2 I,\ \Sigma_{14} = \lambda_2 E_2^T L_1^T\delta\Omega_2 L_1,\\
\Sigma_{15} &= \frac{1}{2}\lambda_3 E_1^T\bar{C}^T(\bar{H}_1 + \bar{H}_2)^T,\\
\Gamma_{11} &= \bar{\mathcal{A}}_0^T P - \tilde{C}^T E_3^T X^T - \tilde{L}_1^T E_4^T Y^T,\\
\Gamma_{12} &= \sqrt{\bar{\alpha}(1-\bar{\alpha})}\tilde{C}^T E_5^T X^T,\ \Gamma_{21} = \bar{\mathcal{A}}_d^T P,\\
\Gamma_{31} &= -E_7^T X^T,\ \Gamma_{41} = -L_1^T E_6^T Y^T,\\
\Gamma_{51} &= -\bar{\alpha}E_7^T X^T,\ \Gamma_{53} = \sqrt{\bar{\alpha}(\bar{\alpha})}E_7^T X^T,\\
\Gamma_{64} &= \bar{I}^T P,\ \Gamma_{71} = \bar{\mathcal{B}}^T P,
\end{aligned}$$

$$\begin{aligned}
E_3 &= \begin{bmatrix} 0 & 0 \\ \alpha I & I \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix},\\
E_5 &= \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, E_6 = \begin{bmatrix} 0 \\ I_{N\times n_x} \end{bmatrix},\\
E_7 &= \begin{bmatrix} 0 \\ I_{N\times n_y} \end{bmatrix}.
\end{aligned} \tag{38}$$

If the above-mentioned inequality is solvable, then the distributed estimator parameter can be obtained by $\bar{K} = E_7^T P^{-1} X E_7$, and $\bar{G} = E_6^T P^{-1} Y E_6$.

proof: In order to eliminate the nonlinear terms in linear matrix equality problem, we set $X = P\tilde{K}$, $Y = P\tilde{G}$. We decompose Λ into two terms.

$$\Lambda = \Pi_{11} + FDF^T \tag{39}$$

where

$$\begin{aligned}
F^T &= \begin{bmatrix} \bar{\Gamma}_{11} & \bar{\Gamma}_{12} & 0 & 0 \\ \bar{\Gamma}_{21} & 0 & 0 & 0 \\ \bar{\Gamma}_{31} & 0 & 0 & 0 \\ \bar{\Gamma}_{41} & 0 & 0 & 0 \\ \bar{\Gamma}_{51} & 0 & \bar{\Gamma}_{53} & 0 \\ 0 & 0 & 0 & \bar{\Gamma}_{64} \\ \bar{\Gamma}_{71} & 0 & 0 & 0 \end{bmatrix},\\
D &= -I_4 \otimes P,\\
\bar{\Gamma}_{11} &= \bar{\mathcal{A}}_0^T P - \tilde{C}^T E_3^T \tilde{K}^T P - \tilde{L}_1^T E_4^T \tilde{G}^T P,\\
\bar{\Gamma}_{12} &= \sqrt{\bar{\alpha}(1-\bar{\alpha})}\tilde{C}^T E_5^T X^T, \quad \bar{\Gamma}_{21} = \bar{\mathcal{A}}_d^T P,\\
\bar{\Gamma}_{31} &= -E_7^T \tilde{K}^T P, \qquad \bar{\Gamma}_{41} = -L_1^T E_6^T \tilde{G}^T P,\\
\bar{\Gamma}_{51} &= -\bar{\alpha}E_7^T \tilde{K}^T P, \qquad \bar{\Gamma}_{53} = \sqrt{\bar{\alpha}(1-\bar{\alpha})}E_7^T \tilde{K}^T P,\\
\bar{\Gamma}_{64} &= \bar{I}^T P, \qquad \bar{\Gamma}_{71} = \bar{\mathcal{B}}^T P.
\end{aligned} \tag{40}$$

Substituting $X = P\tilde{K}$, $Y = P\tilde{G}$ into F^T, we see that F^T transfers into Π_{12}. Taking use of Schur complement, equation (37) in Theorem 2 is obtained.

Remark 3. *The parameters σ_i and δ_i in event-triggering conditions play a key role in balancing data transmission and estimation performance. If the parameters tend to zero, it turns to conventional sampled data-based case. Smaller parameter σ_i in sensor-to-estimator channel gives rise to more availability to measurement information, and smaller parameter δ_i leads to more frequent communication with neighbors. Thus adjusting these parameters can achieve a trade-off between network load and performance index.*

Remark 4. *Regarding the state estimation problem for discrete-time system with deception attacks, our work is an extension of [24]. In this paper, time-varying delay and stochastic nonlinearity existing in the system are considered, which is more general than that of [24]. We also implement event-triggering mechanism in both channels, leading to further reduction of communication burden. Besides, in the presence of external disturbance, estimation performance index can still be guaranteed. By constructing a Lyapunov-Krasovskii functional, sufficient conditions have been established under which estimation problem is transformed into a feasibility problem of a few LMIs that can be easily solved using available software package.*

4 Simulation example

In order to validate the effectiveness of the proposed method of event-triggered distributed estimators for discrete-time stochastic delayed non-linear system with deception attacks, a simulation example is presented.

The sensor network topology is represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{H})$ with the set of nodes $\mathcal{V} = \{1, 2, 3, 4\}$, and edges $\mathcal{E} = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), (4, 1)\}$. The adjency matrix is given as follows:

$$\mathcal{H} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The parameters of target plant and sensor are given as follows:

$$\begin{aligned}
&A = \begin{bmatrix} 0.7 & -0.4 \\ 0.5 & -0.2 \end{bmatrix}, \ A_d = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
&B = \begin{bmatrix} 0.03 \\ 0.02 \end{bmatrix}, \ M = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}, \\
&C_1 = \begin{bmatrix} 0.57 & -0.31 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.60 & -0.33 \end{bmatrix}, \\
&C_3 = \begin{bmatrix} 0.61 & -0.27 \end{bmatrix}, \ C_4 = \begin{bmatrix} 0.64 & -0.29 \end{bmatrix}, \\
&\bar{\alpha} = 0.2.
\end{aligned}$$

The stochastic nonlinearity is chosen as

$$\begin{aligned}
f(x(k), \vartheta(k)) = \begin{bmatrix} 0.03 \\ 0.06 \end{bmatrix} \times \Big(&0.2\mathrm{sgn}(\mathrm{x}^1(\mathrm{k}))\mathrm{x}^1(\mathrm{k})\vartheta^1(\mathrm{k}) \\
&+ 0.12\mathrm{sgn}(\mathrm{x}^2(\mathrm{k}))\mathrm{x}^2(\mathrm{k})\vartheta^2(\mathrm{k})\Big).
\end{aligned}$$

Therefore, the parameters in (5) can be chosen as

$$\beta^2 = 0.03^2 + 0.06^2, \ R = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.12 \end{bmatrix}.$$

The malicious signal is set to be

$$\begin{cases} \theta_1(\bar{y}_1(k)) = \sin(0.5\bar{\mathrm{y}}_1(\mathrm{k})) \\ \theta_2(\bar{y}_2(k)) = \sin(0.6\bar{\mathrm{y}}_2(\mathrm{k})) \\ \theta_3(\bar{y}_3(k)) = \sin(-0.6\bar{\mathrm{y}}_3(\mathrm{k})) \\ \theta_4(\bar{y}_4(k)) = \sin(0.7\bar{\mathrm{y}}_4(\mathrm{k})) \end{cases}$$

and it is easy to verify that the setting with $H_1 = 0.7$, $H_2 = -0.6$ satisfies (7). The time-varying delay is chosen as $\tau(k) = 3 + sin(k\pi)$, and we can easily get $\tau_m = 2$, $\tau_M = 4$. In the example, the disturbance attenuation level is set as $\gamma = 0.95$. The event-triggered parameters are selected as $\Omega_1(k) = I, \Omega_2(k) = I$, and the thresholds are $\sigma_i = 0.4 (i = 1, 2, 3, 4)$, $\delta_i = 0.3(i = 1, 2, 3, 4)$.Using the Matlab software, a set of solutions in Theorem 2 is given as follows:

$$\begin{aligned}
&K_1 = \begin{bmatrix} 0.1908 \\ 0.1405 \end{bmatrix}, \ K_2 = \begin{bmatrix} 0.2657 \\ 0.2256 \end{bmatrix}, \\
&K_3 = \begin{bmatrix} 0.2871 \\ 0.2489 \end{bmatrix}, \ K_4 = \begin{bmatrix} 0.2829 \\ 0.2451 \end{bmatrix}, \\
&G_1 = \begin{bmatrix} 0.0617 & 0.0255 \\ 0.0593 & 0.0332 \end{bmatrix}, \ G_2 = \begin{bmatrix} 0.0443 & 0.0198 \\ 0.0402 & 0.0269 \end{bmatrix}, \\
&G_3 = \begin{bmatrix} 0.0422 & 0.0166 \\ 0.0372 & 0.0235 \end{bmatrix}, \ G_4 = \begin{bmatrix} 0.0259 & 0.0110 \\ 0.0226 & 0.0156 \end{bmatrix}.
\end{aligned}$$

Select the external disturbance as $\omega(k) = \frac{1}{0.1k+1} sin(2k)$. To illustrate the effectiveness of reducing the communication burden and maintaining estimation performance, we run three simulation tests, which are called test A, B, C. In test A, we implement event-triggering mechanism in both channels and the system is under deception attacks. Test B assumes there is no event-triggering mechanism in sensor-to-estimator channel, i.e., time-driven mechanism is utilized, with deception attacks considered. Test C assumes no attack and no event-triggering mechanism in sensor-to-estimator channel. Simulation results are plotted as follows. Figs 2 and 3 show the state and the estimation of four nodes. The output estimation is shown in Fig 4. Figs 5 and 6 show the events of test A in sensor-to-estimator channel and estimator-to-estimator channel, respectively.

From the above simulation results, we can see that, despite the appearance of deception attacks, the estimation performance curves of three tests are very closed, which means that the effect of deception attacks has been inhibited. Moreover, data transmission is effectively reduced with the sacrifice of certain estimation performance, as shown in Fig 5. We can adjust the parameters in event-triggering conditions to realize an ideal trade-off between communication cost and estimation performance, and when parameters tend to zero, the event-triggering mechanisms turn to time-driven mechanism. The simulation result has confirmed the effectiveness of the method presented in this paper.

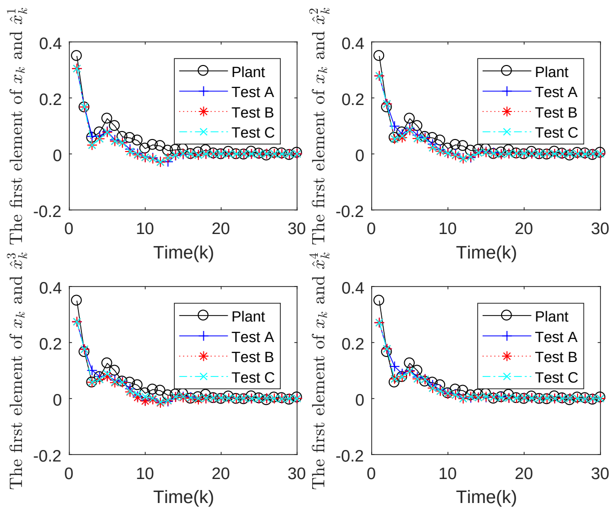

Fig. 2: The first element of x_k and its estimation of four nodes.

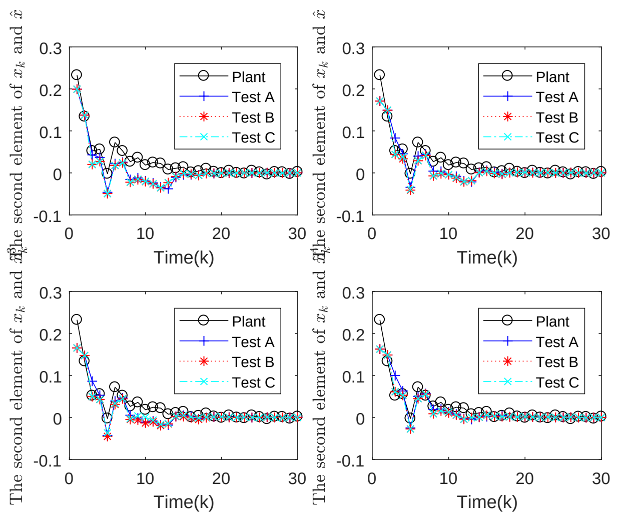

Fig. 3: The second element of x_k and its estimation of four nodes.

5 Conclusion

This paper investigates H_∞ state estimation problem for a class of stochastic time-varying delayed system with deception attacks. In order to reduce the network burden and energy consumption, event-triggered communication scheme has been employed in both

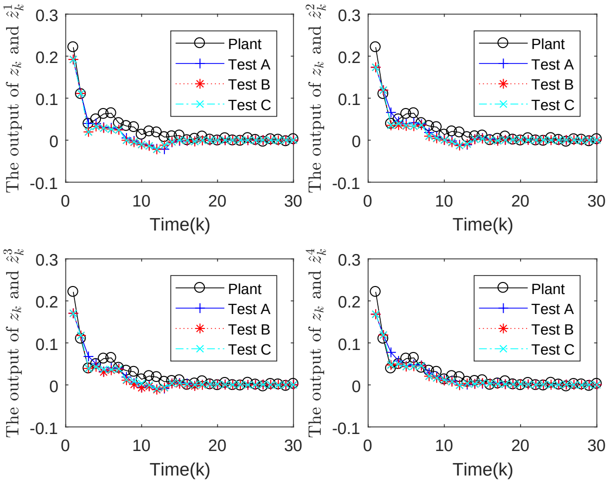

Fig. 4: Output and its estimation of four nodes.

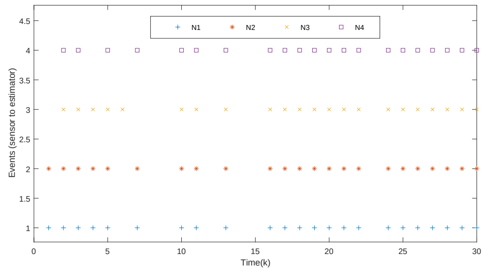

Fig. 5: Event-triggering times in sensor-to-estimator channel.

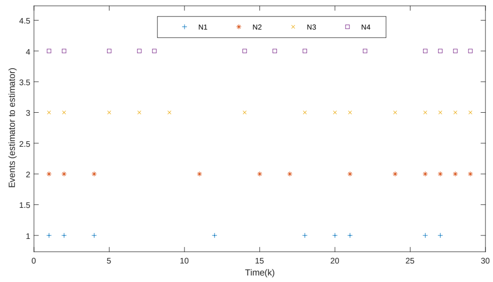

Fig. 6: Event-triggering times in estimator-to-estimator channel.

sensor-to-estimator channel and estimator-to-estimator channel. By constructing Lyapunov-Krosovskii functional, delay-dependent conditions are presented to guarantee the exponential mean-square stability of estimation, as well as the H_∞ performance index. The final estimator parameter could be derived by solving a feasibility problem using Matlab software. Finally, a numerical example has been provided to demonstrate the effectiveness of the proposed method.

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