The Apollo Missions

I Introduction

In order to make informed decisions about sending people to the moon, understanding the conditions that the rocket and astronauts will face is crucial. Specifically, this report will discuss the gravitational potential due to the earth, the gravitational potential due to the earth and moon system, the forces on the Apollo 11 Command module due to the earth and moon, and finally the predicted performance of the Saturn V rocket. Calculations, and graphs to analyze these concepts were done in a program written in python. Python has the ability to work with functions, graph, perform mathematical operations on numbers and functions, as well as scientific calculations. Specifically for graphing, one library that can be used is matplotlib. This allows for object oriented plotting to make graphs. In this case matplotlib was used to make a plot of a one variable function, color gradients, contour maps, and vector fields. In order to perform certain mathematical calculations a library called numpy was used. This allows calculations such as taking logs, absolute values, square roots, and more. Finally, a function from the scipy library was used to mathematically calculate a definite integral.

II The Gravitational Potential of the Earth-Moon System

The gravitational potential of the Earth-Moon system is the potential energy that the earth and moon will exert on other objects. The potential of of a mass M a distance r from it is $-\frac{GM}{r}$ where G is the gravitational constant. To find the gravitational potential from the Earth-Moon system we add together the potential from the Earth and the Moon. Throughout space the absolute value of the potential from the Earth-Moon system looks like Figure 1, where everything is being measured from the center of the earth. Where the larger green spot is the earth and the smaller green/blue spot is the moon. In reality, this potential is negative, however that is only because of the convention set that infinitely far away from a mass gives you 0 potential. Thus anything closer is a negative potential. However, in this case the magnitude works just as well to visualize the potential.

III The Gravitational Force of the Earth-Moon System

Any body with a mass will exert a gravitational force on other masses. The gravitational force of one mass M_1 on another mass m_2 is given by $-G\frac{M_1 m_2}{|\mathbf{r}_{21}|^2}\hat{r}_{21}$ where **r** is the vector pointing from the position of m_2 to M_1. In order to find the force that the Earth-Moon system will exert on the Apollo 11 at different points in space, we add the force from the moon on the rocket to the force from the earth on the rocket. In this case we say M_1 is the earth or moon, and m_2 is the rocket. The vector field of the net force looks like Figure 2, where everything is measured from the position of the center of the earth. Figure 2 shows that depending where the rocket is, the net force will pull the rocket towards the earth or toward the moon with different strengths. This is important to know so that we can figure out the safest way to get the astronauts to the moon without crashing into the moon, or being stuck in the orbit of earth and never reaching the moon, or being pulled back to earth.

IV Projected Performance of the Saturn V Stage 1

In order to make sure the Saturn V Stage 1 can safely carry the capsule it is important to understand what we expect the performance to be based on different factors. The rocket works by ejecting fuel which pushes the rocket forward. Eventually once all the fuel is burned the rocket won't be actively being propelled forward, although it may continue to move, just not at an accelerated speed. This time is modeled by $T = \frac{m_0 - m_f}{m_{dot}}$ where m_0 is the initial wet mass of the rocket, m_f is the final mass of the rocket after all the fuel has been burned and m_{dot} is the rate of change of the mass. This time was calculated to be 157.7 seconds. Thus the rocket is projected to be propelled forward for 157.7 seconds. The final altitude of the rocket can be calculated as well by integrating the change in velocity over time from 0 seconds to the time the rocket runs out of fuel. The change in velocity is modeled by $v_e ln(\frac{m_0}{m(t)}) - gt$ where v_e is the fuel exhaust velocity, $m(t) = m_0 - m_{dot}t$ is the mass over time, g is the gravitational constant near the surface of the earth, and t is time. The final altitude is projected to be 74.1 kilometers. These numbers can be compared with the distance we want the rocket to reach and how long we want the fuel to burn for. From that we can change different properties to try and achieve different results.

V Discussion and Future Work

Some approximations we have made are that we have only considered forces and potential from the earth and moon on the capsule. Our calculations also do not match exactly the recorded results as the measured total burn time was 160 seconds, vs the projected 157.7 seconds, and the measured final altitude was 70 km vs the projected 74.1 km. One reason the projected time is an underestimate might be that the rate that the fuel burns is not constant as we estimated. One reason the projected altitude is an overestimate might be because we are not considering the drag force on the rocket. In order to make these calculations more accurate we should also consider the error in any of the measurements, since all measured quantities have some error associated with them. Then our calculations should include error propagation and we can more accurately understand what we project the rocket and capsule to do.

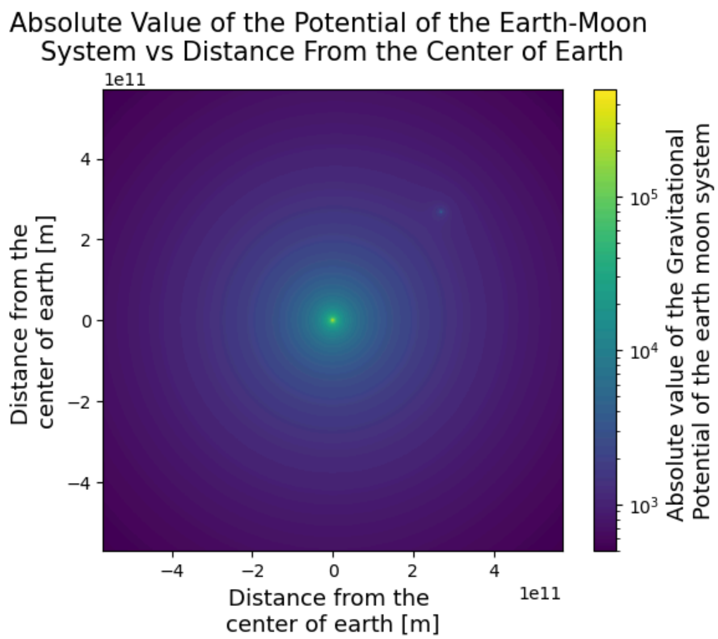

Figure 1: The Absolute Value of the Potential of the Earth-Moon System

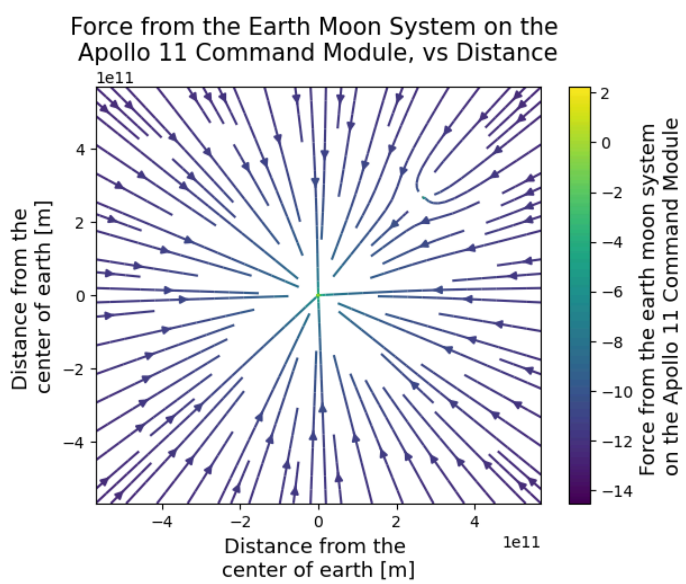

Figure 2: The Force From the Earth-Moon System on the Apollo 11 command module