Mine Crafting

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I Introduction

Using a test mass and recording how long it takes to reach the bottom of a shaft can be a very useful tool when it comes to measuring depth. Depending on the level of precision needed in each measurement, different elements that impact the fall time must be included or can be omitted. This report will discuss the fall time of a 1 kg test mass and how depth-dependent gravity, drag, and the Coriolis force impact the object's fall time. This report will also go more in depth to discuss how the mass falls if the mine shaft were to extend all the way through the earth, if the mine shaft was on the moon, and the relationship between the density of the body the object is falling through and the fall time. Finally, the report will discuss whether using a test mass and measuring the fall time is a good way to determine the vertical depth of a shaft. In this case most of the methods used to compute these answers and relationships were found by solving differential equations. To actually carry out the calculations a program written in the programming language python was used. In order to do this, different python libraries were used to assist with numerical calculations, integration, solving differential equations and making plots.

II Calculation of Fall Time

In calculating the fall time of this 1 kg test mass, three cases were tested. The first case was where we assumed a constant force due to gravity as the mass fell, and no drag force. In this case, the mass took about 28.6 seconds to reach the bottom of a 4 km shaft.

The second case was where we assumed a depth dependent force due to gravity, but still no drag. In this case the object still reached the bottom of the shaft at about 28.6 seconds. In this case the force due to gravity was related to how far into the earth the mass had fallen. Thus as the mass falls deeper down the shaft the force due to gravity decreases as there is less mass from the earth below the test mass. This causes the mass to take a longer time to reach the bottom, however when only looking at the fall time to 0.1 second precision, there is no difference in the fall times for a depth dependent force due to gravity and a constant force due to gravity.

Finally, the last case tested was with a depth dependent force due to gravity and a drag force. In this case the mass took about 84.3 seconds to reach the bottom of the shaft. In this case along with the same depth dependent force due to gravity used in the second case, we also considered the drag force. We knew the mass's terminal velocity was 50 m/s, so we used that to calibrate the drag coefficient, which

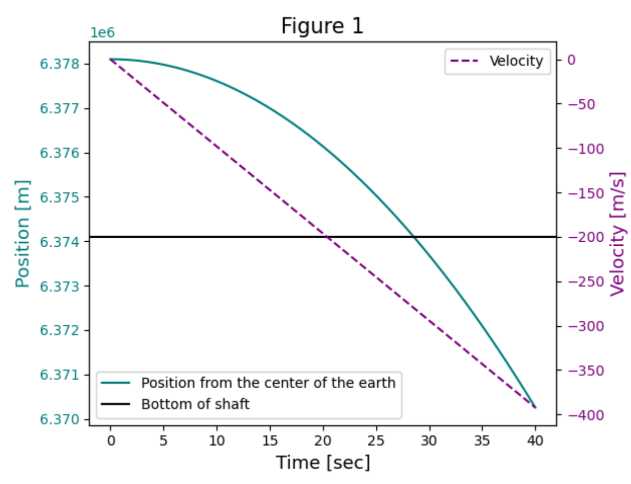

Figure 1: Position and velocity of a test mass falling with a depth dependent force due to gravity and no drag.

impacts the drag force. We looked for a drag coefficient that would cause the velocity to level off at -50 m/s. -50 m/s was used instead of 50 m/s solely because the coordinate system defined had the negative y direction being the direction that the mass was falling in.

III Feasibility of Depth Measurement Approach

While incorporating the drag force and a depth-dependent force due to gravity help to give a more accurate description of the fall time, it is also important to consider the Coriolis force. This is a force that is due to the rotation of the earth. This force adds an additional force component in the y direction, but it also adds a force in the x direction. Where the positive y direction is from the center of the earth pointing up the mineshaft, and the positive x direction points from the center of the earth to the east, and the shaft is located on the equator. Here we considered two cases, one without drag, and one with drag, both with a depth dependent force due to gravity. In the case with no drag, if the mine was infinitely wide, but only had a depth of 4 km, the mass would hit the bottom of the shaft at 28.6 seconds. However, if the shaft was 5 m wide, but infinitely long, the test mass would hit the side of the shaft at 21.9 seconds. Since the time for the test mass to hit the side of the wall is less than the time for the test mass to hit the bottom of the shaft, it means the test mass, in a shaft that is 4 km deep and 5 m wide the test mass will hit the side of the shaft before it ever reaches the bottom. It was also determined that the test mass would hit the side of the shaft at a depth of about 2.1 km from the start of the shaft.

In the second case, if we include drag, again if the mine was infinitely wide, but only had a depth of 4 km, the mass would hit the bottom of the shaft at 29.7 seconds. However, if the shaft was 5 m wide, but infinitely long, the test mass would hit the side of the shaft at 84.3 seconds. Therefore, again since the time for the test mass to hit the side of the wall is less than the time for the test mass to hit the bottom of the shaft, it means the test mass, in a shaft that is 4 km deep and 5 m wide the test mass will hit the side of the shaft before it ever reaches the bottom. It was also determined that the test mass would hit the side of the shaft at a depth of about 1.2 km from the start of the shaft.

Thus, incorporating drag will make the mass hit the side of the shaft about 7.8 seconds later, at about 0.9 km higher in the shaft than without drag. However it does not change the fact that the test mass would hit the side of the wall before it ever reaches the bottom of the shaft. Therefore, when including the effects from the Coriolis force and the depth dependent force due to gravity, and regardless if drag is included or not, it is not advisable to continue this depth measurement technique. Since the mass will never hit the bottom of the shaft before it hits the side of the shaft the time that is measured won’t be accurate in determining the depth of the shaft without knowing a lot more information.

IV Calculation of Crossing Times for Homogeneous and Non-Homogeneous Earth

In this section, we consider both a homogeneous and a non-homogeneous earth, meaning respectively that the density is either constant or not constant at different radii from the center of the earth. We also consider a trans-planetary tunnel, which is a shaft that goes all the way through the center of the earth and through to the other side. Here we don’t need to include the drag force, or the Coriolis force. We define the crossing time as the time when the mass travels all the way through the earth, turns around and reaches back where it started. With a constant density earth, the crossing time is about 4952.2 seconds.

For a non-homogeneous earth, the density of the earth can be modeled by the equation $p(r) = p_n(1 - \frac{r^2}{R^2})^n$ where R is the radius of the earth, and p_n is some density constant. Thus for different values of n, the density, and thus the way the mass falls through the earth changes. If n = 0, then it is back to a homogeneous earth. For the case that n = 0 the crossing time is about 4955.2 seconds. This time is slightly different from the time above, possibly due to the different methods of calculations, however the difference is too small to be a cause for concern. If we, however, set n = 9, we get a crossing time of 3661.2 seconds. This shows that the crossing time for a uniform density earth is about 1.4 times larger than for a non-uniform density earth with n=9. If we choose n to be any other number the relationship holds that as n increases the crossing time decreases.

If we think instead, think about a homogeneous moon and a trans-lunar tunnel, meaning a shaft extending through the center of the moon and through to the other side, the crossing time is different as the density of the moon is different. The crossing time for the moon is about 6354.0 seconds. This is about 1.3 times larger than the crossing time of a homogeneous earth. We can also look at how long it takes for the test mass to reach the center of different objects with different densities. The larger the density the smaller the time it takes for the test mass to reach the center of the object. But more than that there appears to be an exponential decay relationship between the time it takes for the test mass to reach the center of the object and the density of the object.

V Discussion and Future Work

In doing these calculations there were some assumptions made. One assumption made was in trying to find the crossing time. We looked for where the object reached 99 percent of its original position because the object never fully reaches back to its original position. While looking for 99 percent of the original position gets close to the actual crossing time, since we don't know what the actual largest radius that the object returns to is, we can't get the exact answer. Another assumption we made was that the earth is perfectly spherical, even though it isn't. In order to improve further calculations representing the earth as having a slightly more elliptical shape, as well as explicitly looking for what the maximum radius the test mass returns to could help to make these calculations better simulate what is really physically happening.

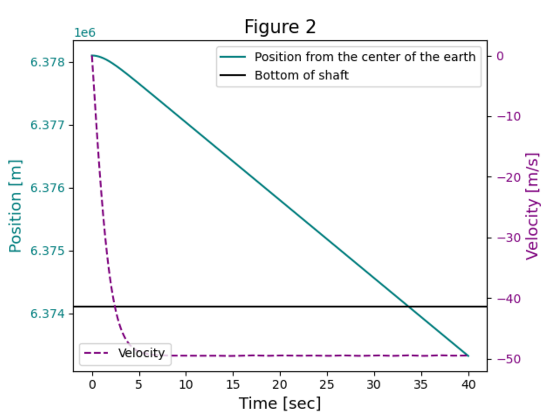

Figure 2: Position and velocity of the test mass with a depth dependent force due to gravity and drag

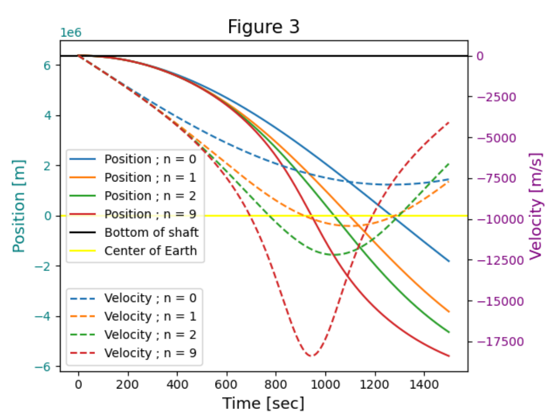

Figure 3: Position and velocity of the test mass for different density concentrations