Neural Network Training

Intelligent Architectures (5LIL0)

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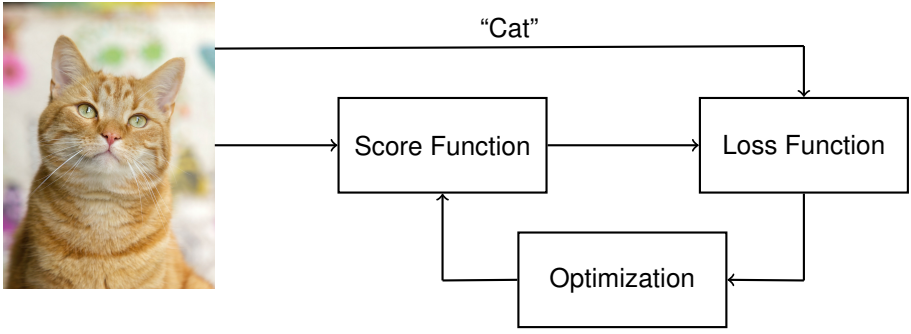
TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Last Time



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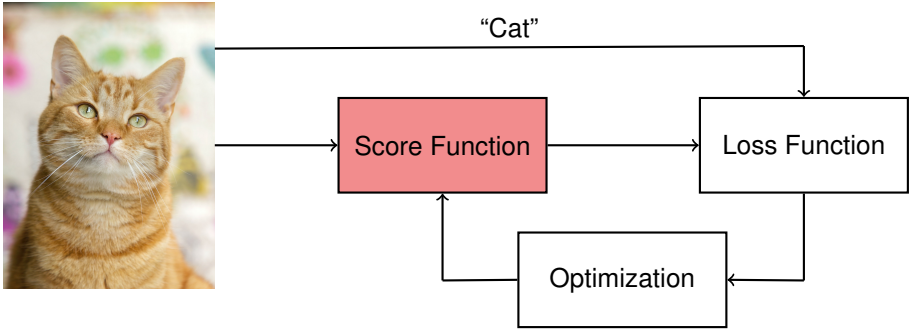

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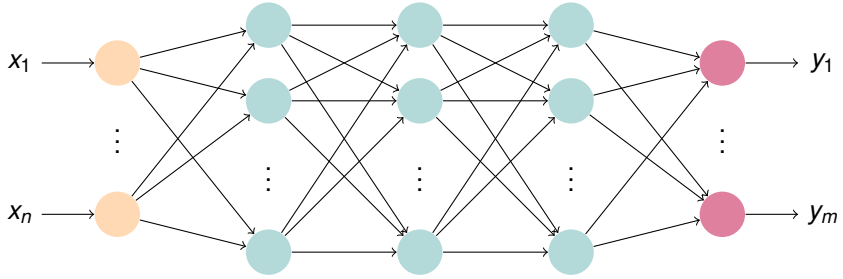
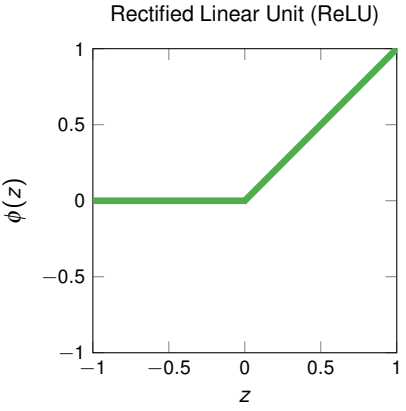


Neural network score function ingredients:

- Input layer
- Hidden layers
- Output layer
- Weights, biases, and activation functions

Last Time

- **Activation functions:**



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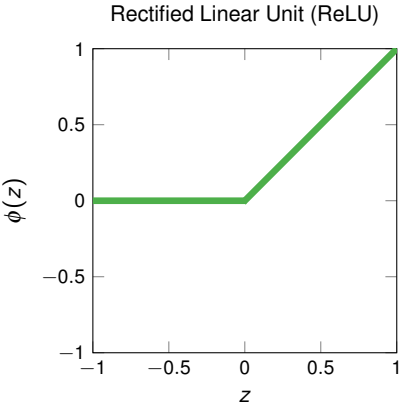
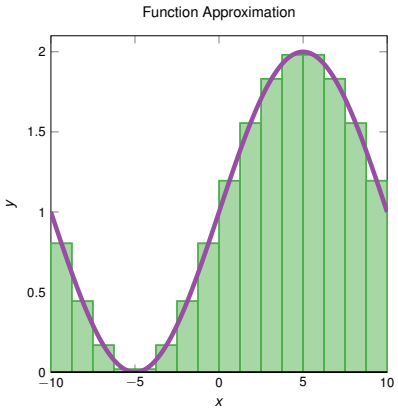


- **Universal approximation theorem:**



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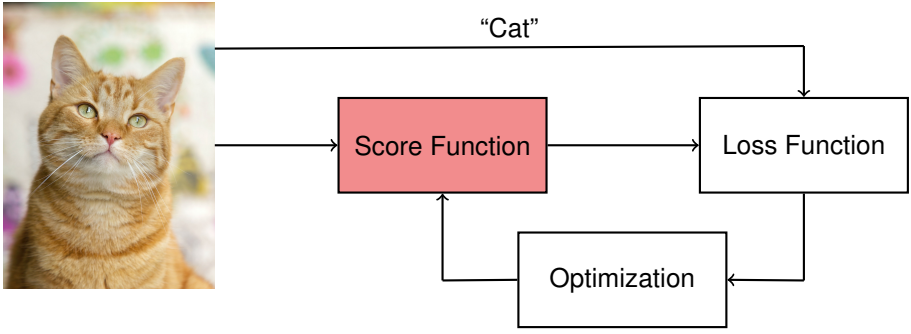

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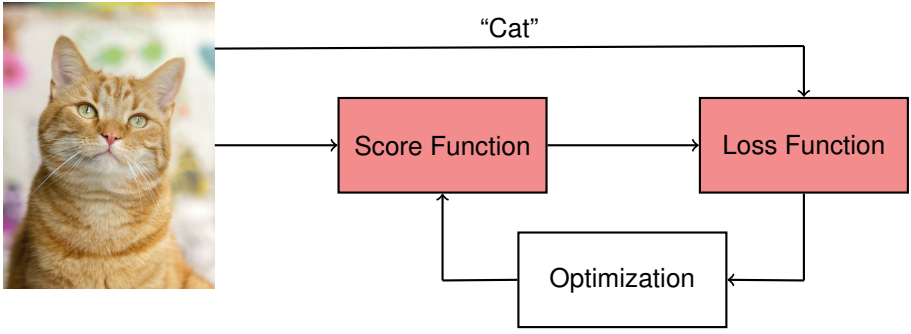

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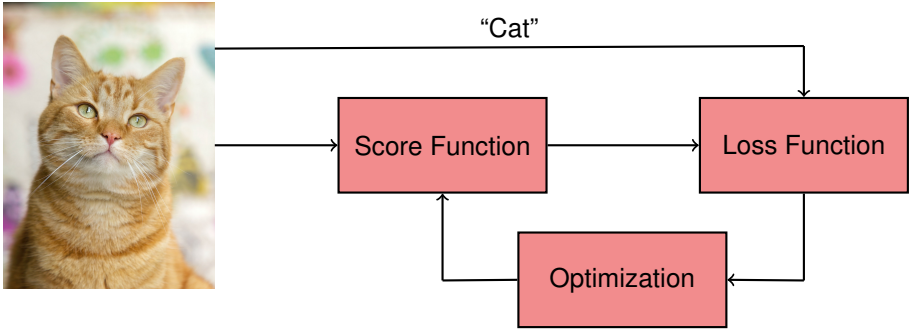

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Tune the weights and biases so that the neural network implements a desired function.

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2. Evaluate the loss function using the score function output and labels.
3. Update neural network weights/biases to reduce the loss function (**backward pass**).
4. Repeat.

Loss Functions

Intuitive definition

A loss function is a measure of “wrongness”: it represents the “cost” of getting the neural network output $\hat{\mathbf{y}}$ when the desired output was $\mathbf{y}$. We denote it by $L(\mathbf{y}, \hat{\mathbf{y}})$.

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How?

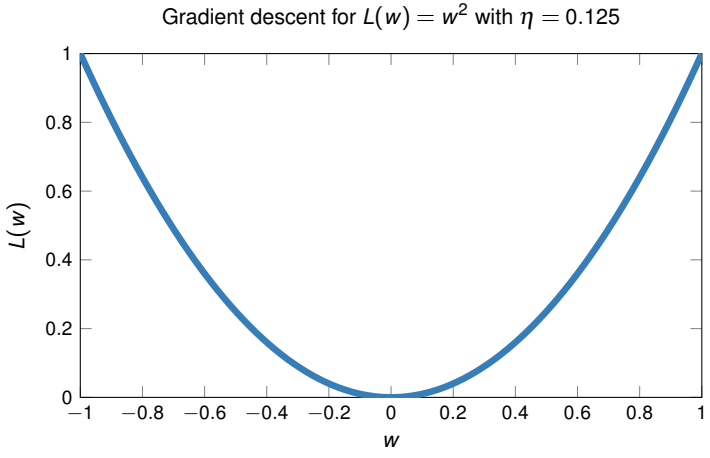
We use an iterative minimum finding technique called **gradient descent**:

$$\mathbf{w}_k = \mathbf{w}_{k-1} - \eta \cdot \frac{\partial L}{\partial \mathbf{w}_{k-1}},$$

$$\mathbf{b}_k = \mathbf{b}_{k-1} - \eta \cdot \frac{\partial L}{\partial \mathbf{b}_{k-1}},$$

where $k = 1, \ldots$ is the iteration and η is called the **learning rate**.

Gradient Descent



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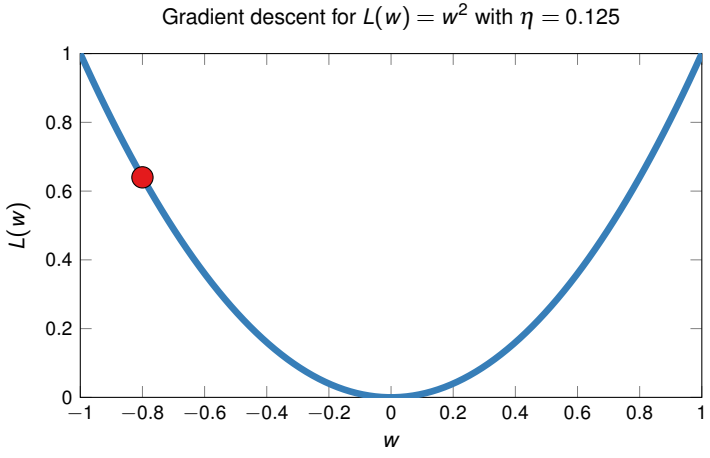

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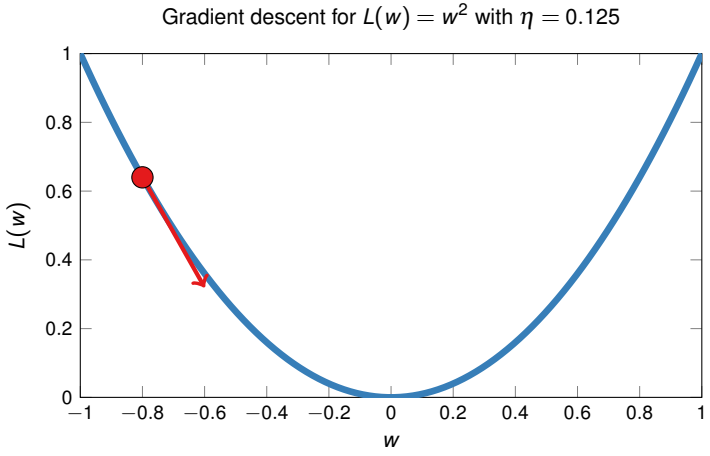

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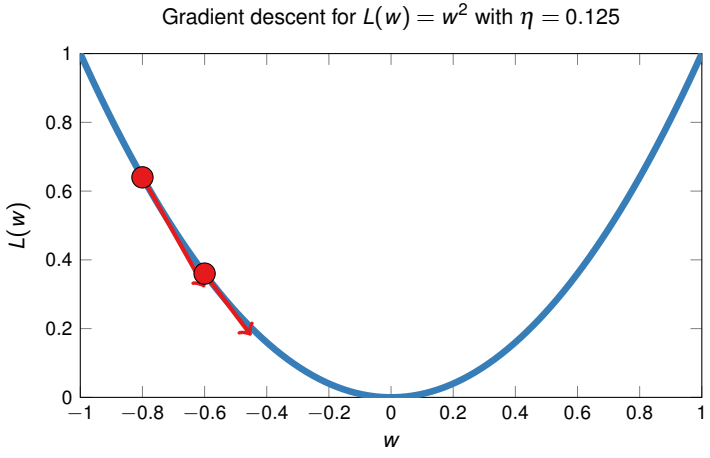

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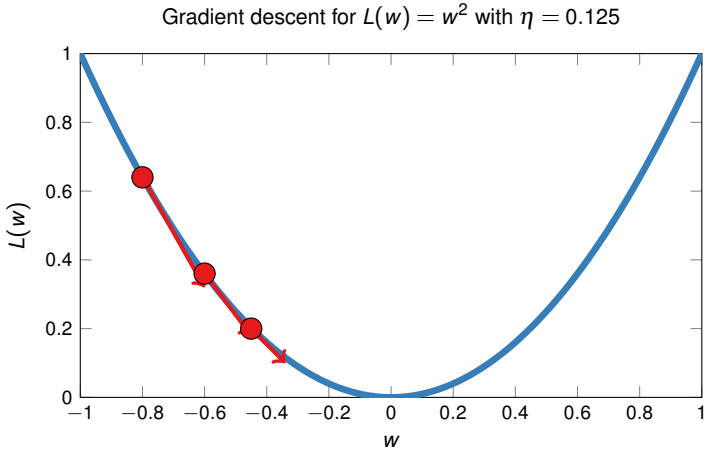

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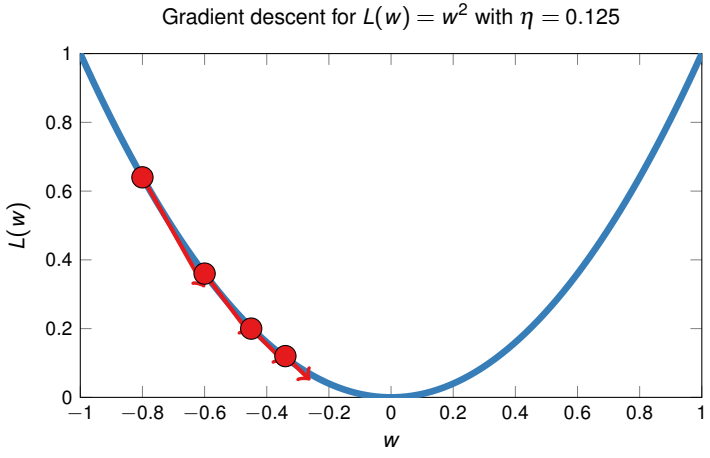

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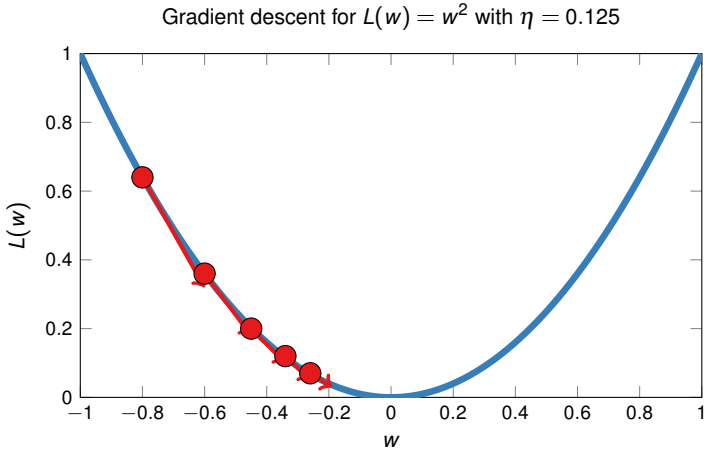

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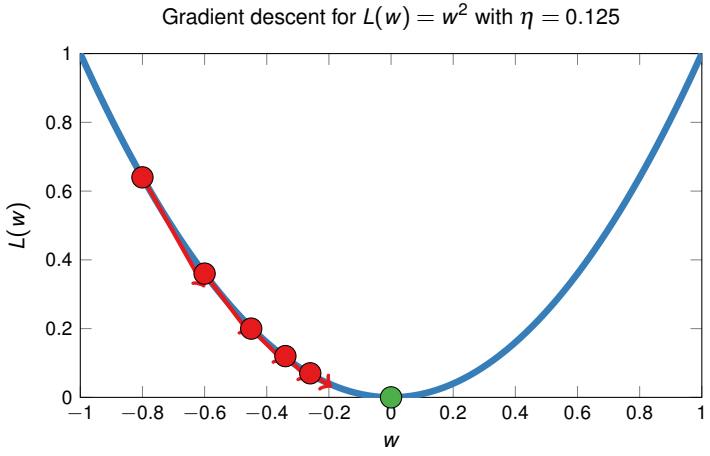

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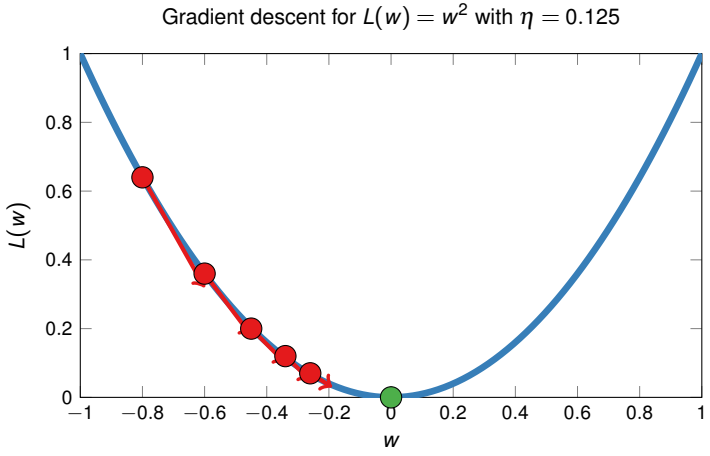

Challenge: a good choice of the learning rate η is crucial!

Why Gradient Descent?

- For $L(w) = w^2$, we could have found the minimum analytically: $\frac{\partial L(w)}{\partial w} = 0 \Leftrightarrow w = 0$

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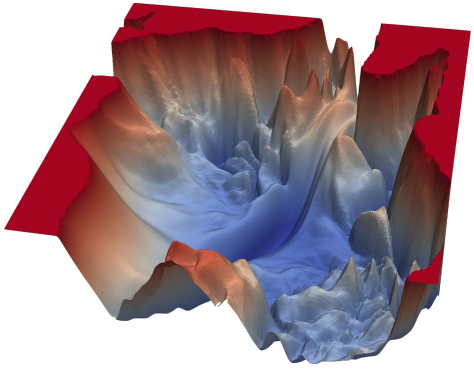
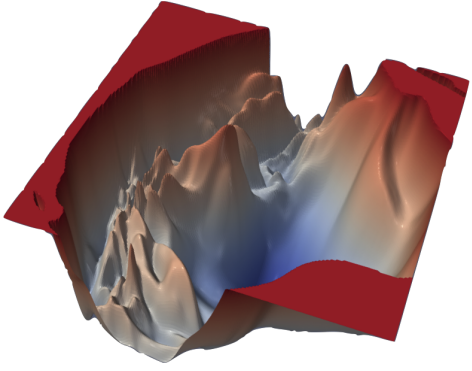
- For $L(w) = w^2$, we could have found the minimum analytically: $\frac{\partial L(w)}{\partial w} = 0 \Leftrightarrow w = 0$
- In reality, the loss functions look more like this [1]:





[1] H. Li, Z. Xu, G. Taylor, C. Studer, T. Goldstein, "Visualizing the loss landscape of neural nets," 2018.

Backpropagation

Backpropagation is an efficient way to calculate $\frac{\partial L}{\partial \mathbf{w}}$ and $\frac{\partial L}{\partial \mathbf{b}}$ required for gradient descent.

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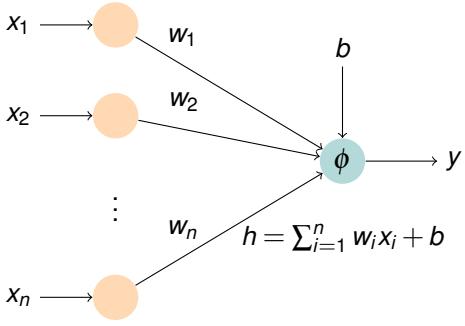

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- Single-neuron neural network.
- Squared error loss function.
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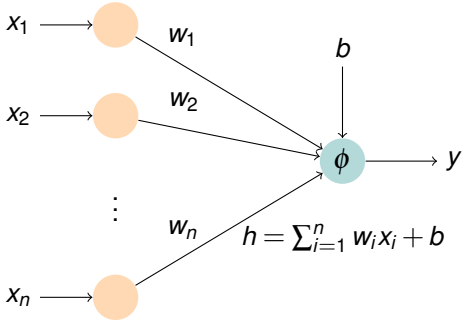

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Loss function: $L(y, \hat{y}) = (y - \hat{y})^2$

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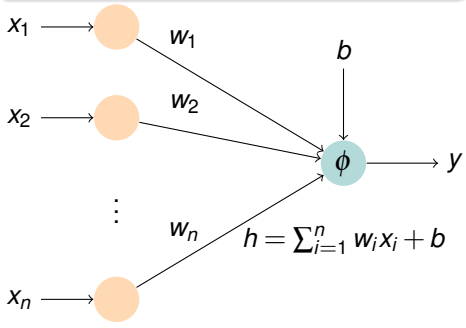
Chain Rule

Use helper variable z: $\frac{\partial x}{\partial y} = \frac{\partial x}{\partial z} \cdot \frac{\partial z}{\partial y}$



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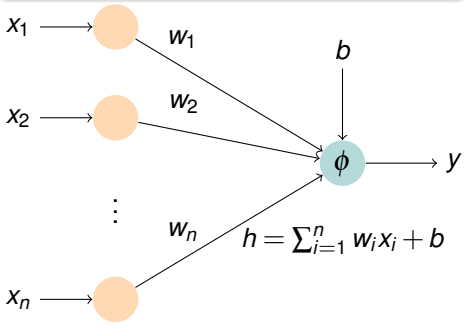

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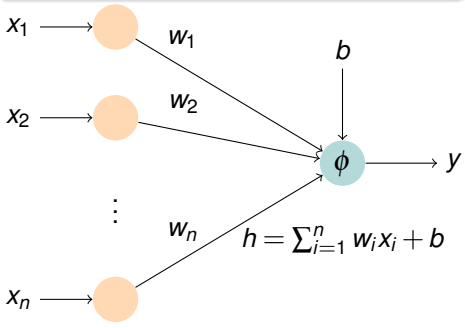

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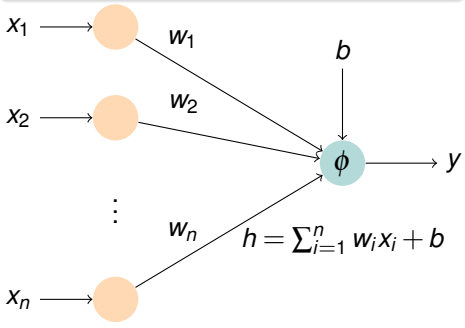

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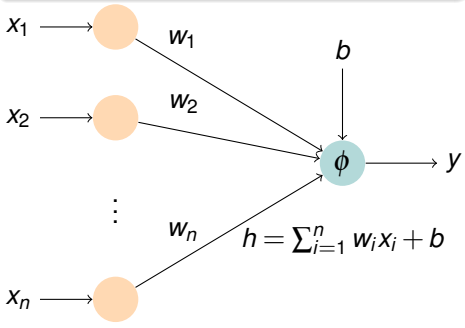

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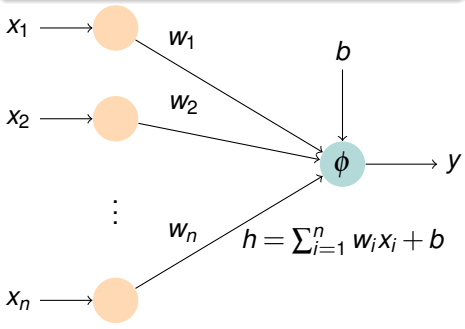

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$$\frac{\partial h}{\partial w_j} = \frac{\partial \sum_{i=1}^{n} w_i x_i + b}{\partial w_j} = x_j$$

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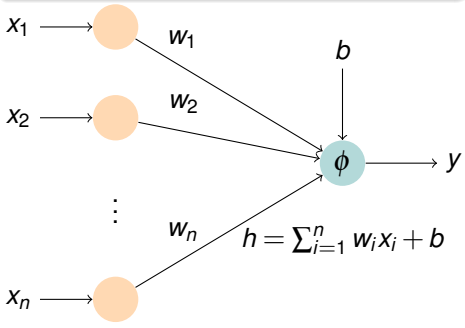

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$$\frac{\partial L}{\partial y} = \frac{\partial (y - \hat{y})^2}{\partial y} = 2(y - \hat{y})$$
$$\frac{\partial y}{\partial h} = \frac{\partial \phi(h)}{\partial h} = \phi'(h)$$
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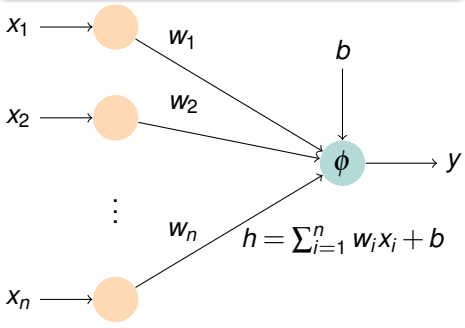

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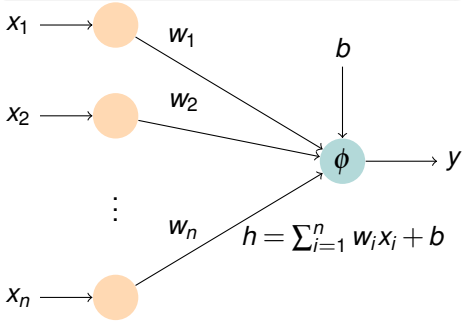

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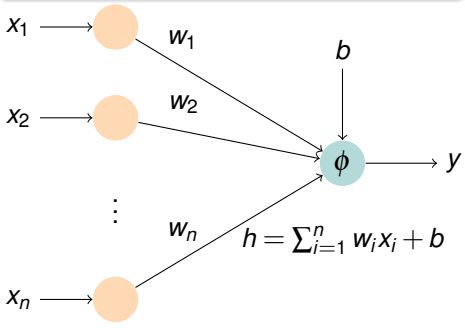

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Activation function derivatives

- **tanh**: $\phi'(h) = 1 - \tanh^2(h)$
- **sigmoid**: $\phi'(h) = \sigma(h)(1 - \sigma(h))$
- **ReLU**: $\phi'(h) = \begin{cases} 0, & h < 0, \\ 1, & h > 0. \end{cases}$

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One pass through the entire N-sample dataset is called an **epoch**.

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Time For A Break

> "*An ounce of practice is generally worth more than a ton of theory.*"
>
> – Ernst F. Schumacher, "Small Is Beautiful: A Study of Economics As If People Mattered," 1973.

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where n_{in} and n_{out} are the number of incoming and outgoing weights.

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- Works best for **ReLU-type** activations.

[1] X. Glorot and Y. Bengio, "Understanding the Difficulty of Training Deep Feedforward Neural Networks," 2010.
[2] K. He, X. Zhang, S. Ren, J. Sun, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification," 2015.

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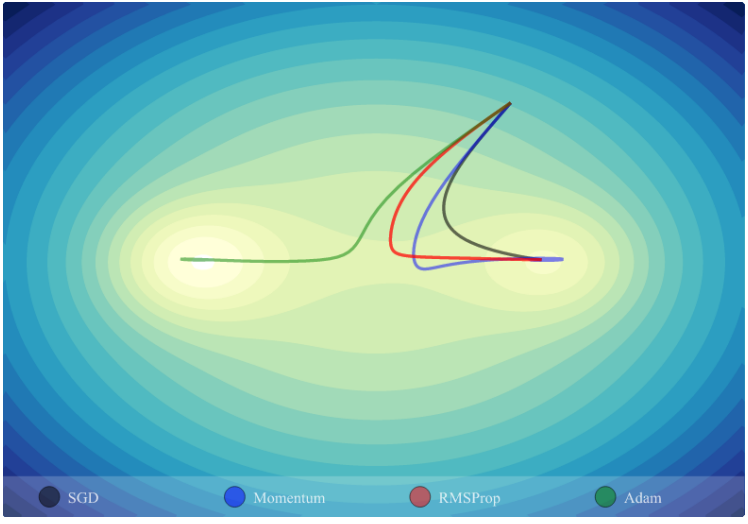
Improved SGD Methods Visualized



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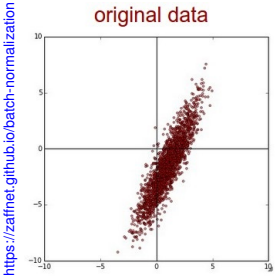

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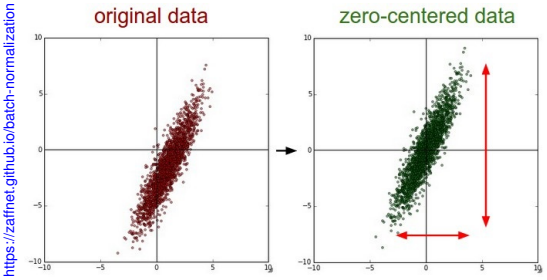

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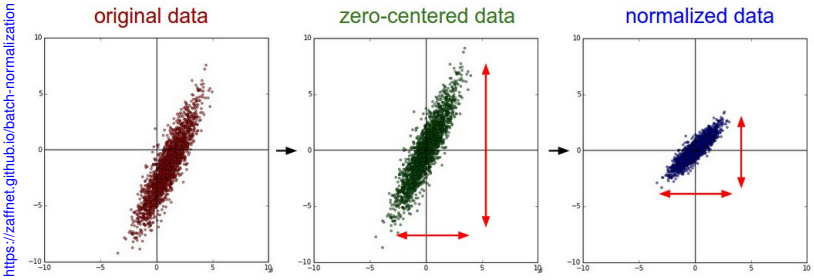

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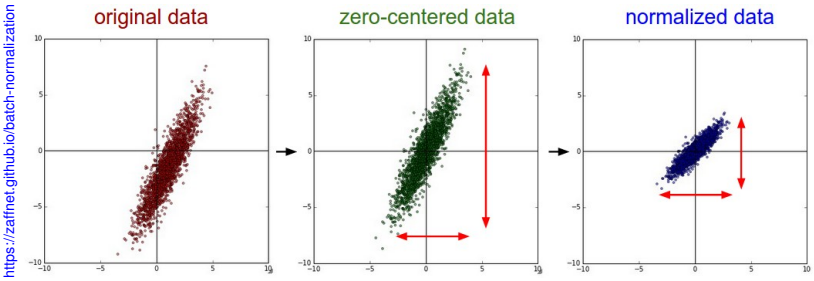
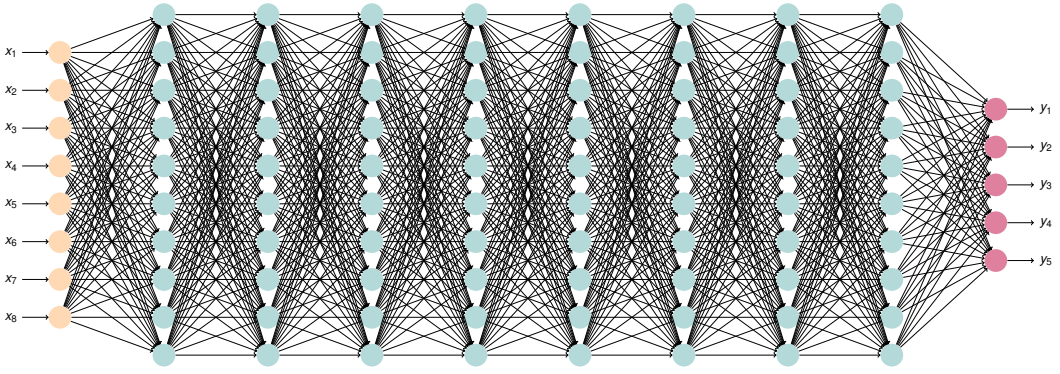


- **Batch normalization** extends the idea to internal layers [1].

[1] S. Ioffe, C. Szegedy, "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift," 2015.

Deep Learning Frameworks



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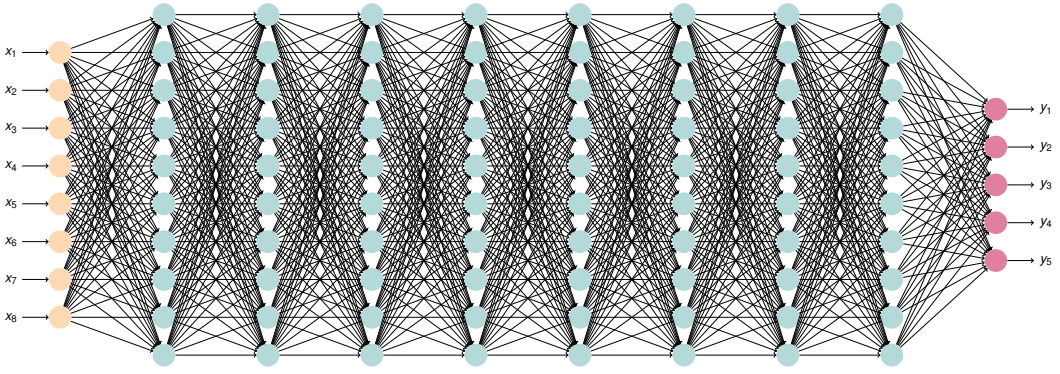

- Various deep learning frameworks automate the implementation and the training!

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   - Both high-level and low-level APIs
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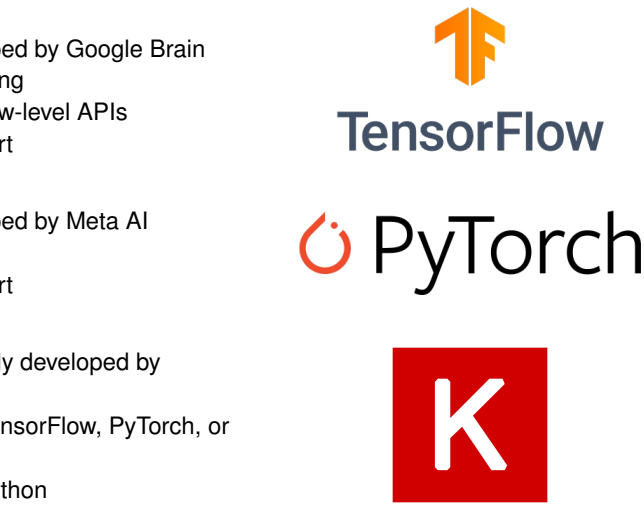

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Next time:

1. Overfitting and how to overcome it.
2. Hyperparameter tuning.
3. Convolutional neural networks.
4. Introduction to optimizations.