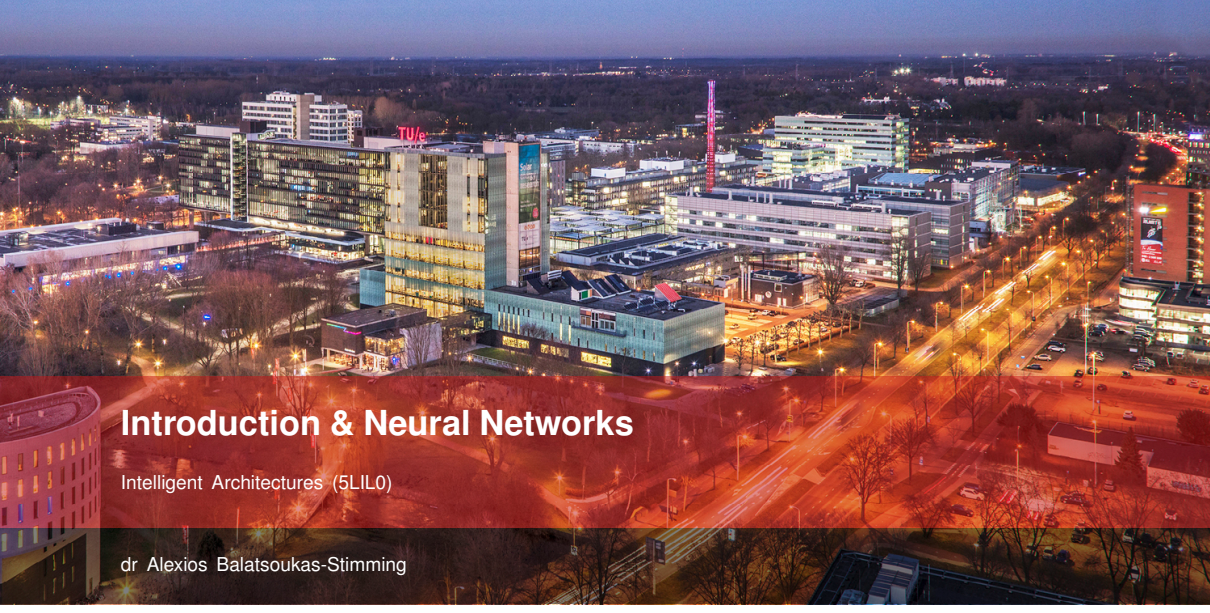

Introduction & Neural Networks

Intelligent Architectures (5LIL0)

dr Alexios Balatsoukas-Stimming

Department of Electrical Engineering, Electronic Systems Group

TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Intelligent Architectures (5LIL0)

Efficient deep learning algorithms and hardware architectures.

“Efficient...”

Efficiency

“Efficient...”

Efficiency

The ratio of useful output O to total input I:

$$E = \frac{O}{I}$$

“Efficient...”

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Efficiency examples:

“Efficient...”

Efficiency

The ratio of useful output O to total input I:

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Efficiency examples:

1. Automotive: km/L
2. Lighting: lm/W
3. Communications: bps/Hz
4. Computing: pJ/operation

Is Improved Efficiency Always Beneficial?

Jevons Paradox (William S. Jevons, 1865)

Increases in efficiency with which a resource is used tend to increase the rate of consumption of that resource due to increasing demand.

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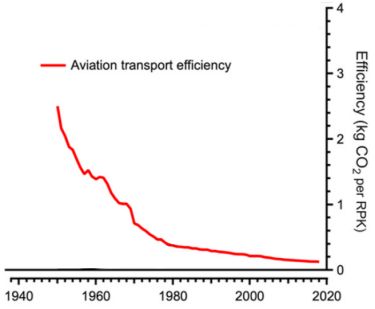

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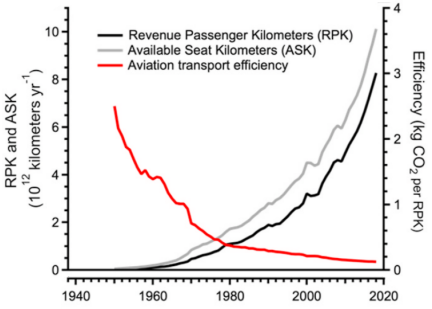

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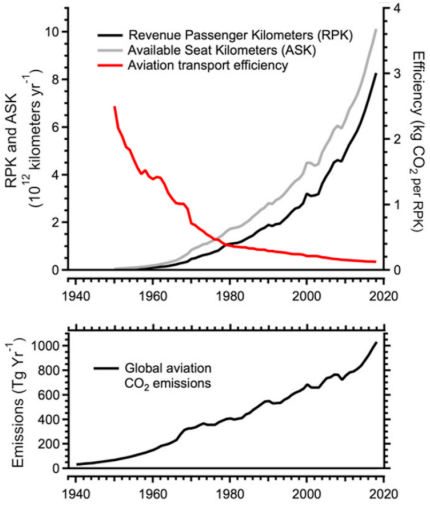

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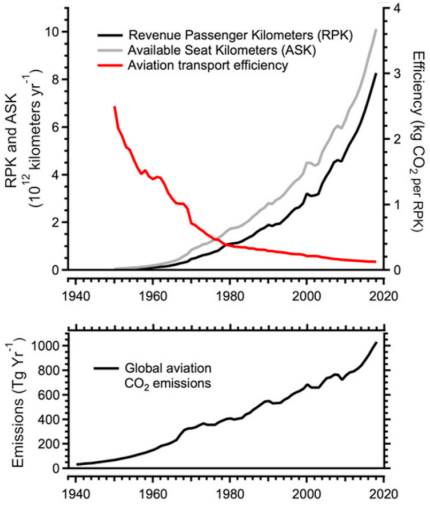
So: Don't look at energy efficiency in isolation!



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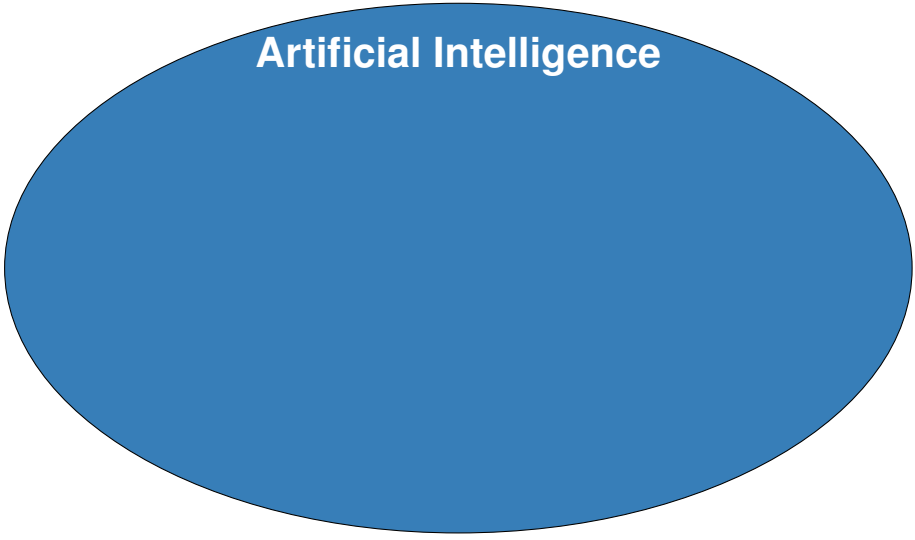

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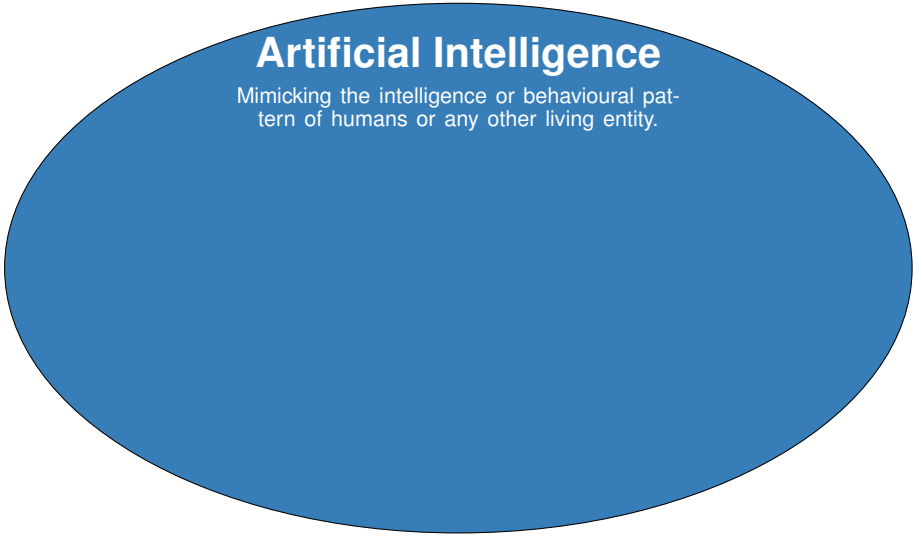

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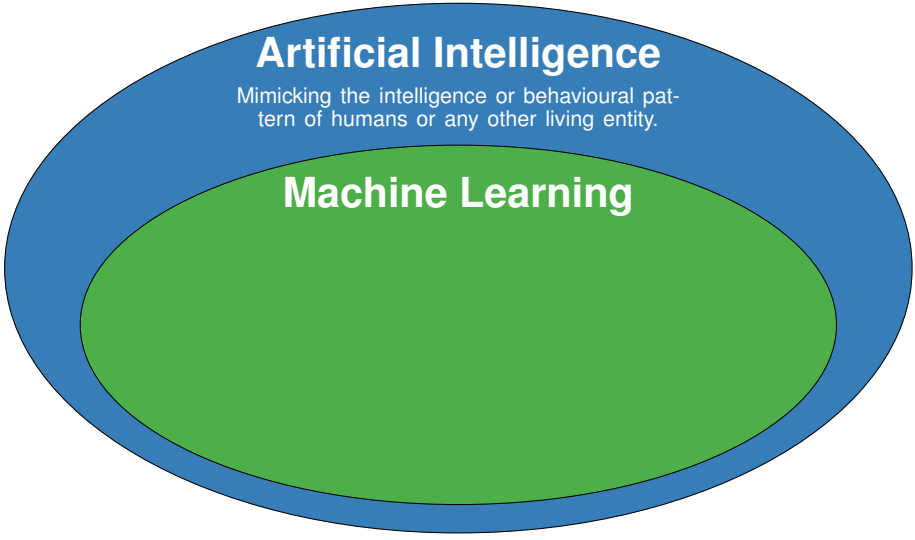

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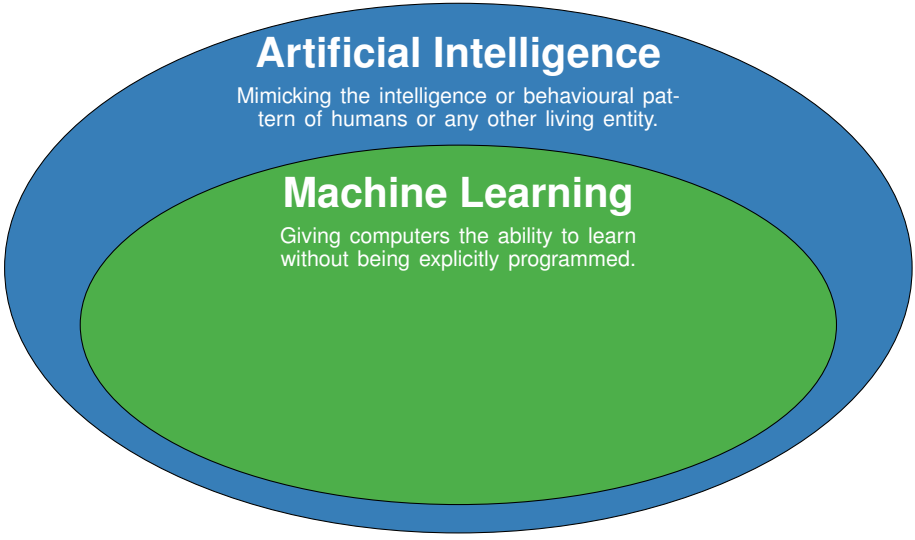

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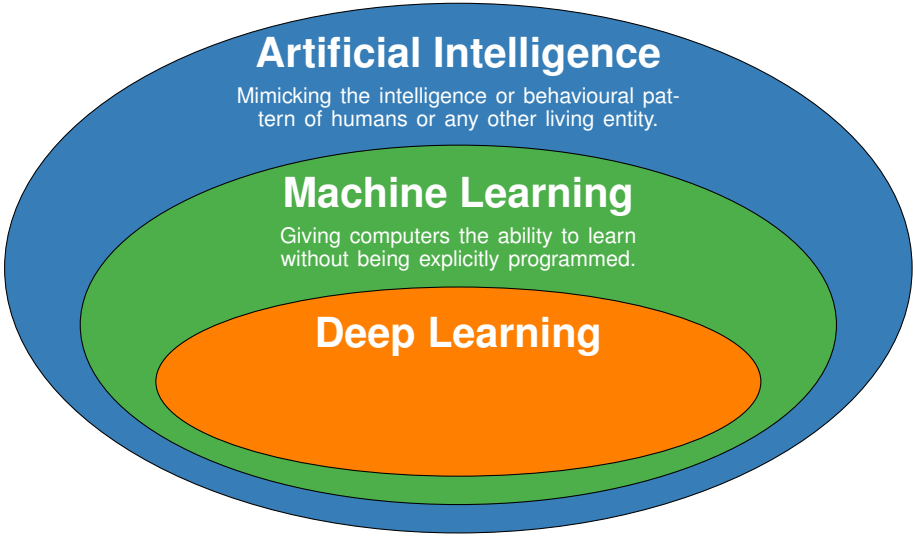

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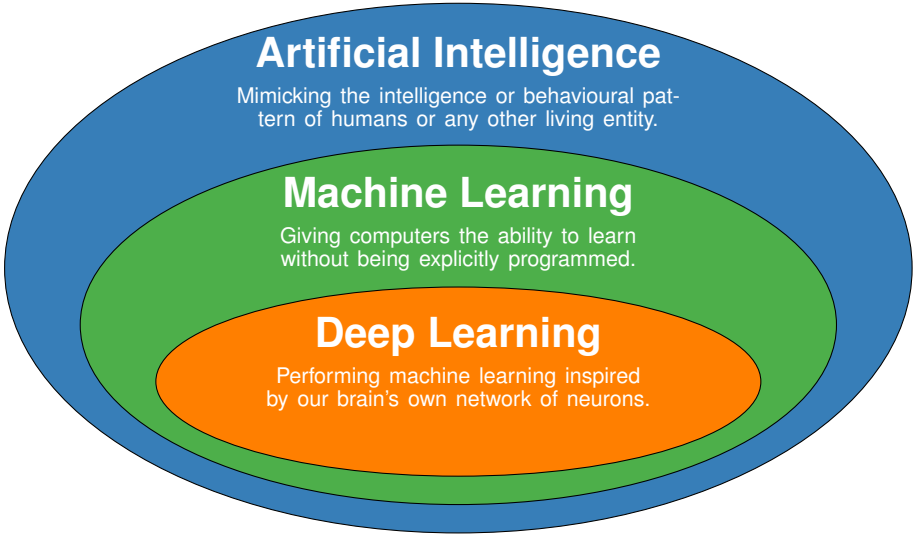

Types of Learning

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In practice, a combination of approaches is often used.

For example, ChatGPT uses both supervised learning and reinforcement learning.

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Basic components of a (supervised) learning system:

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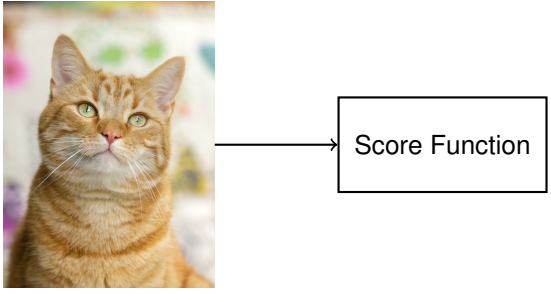

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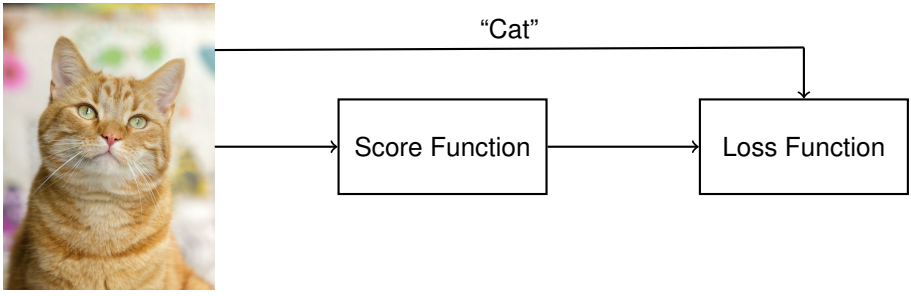

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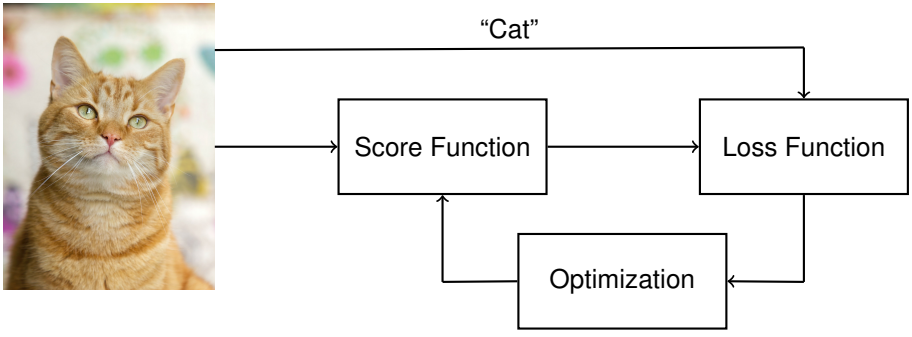

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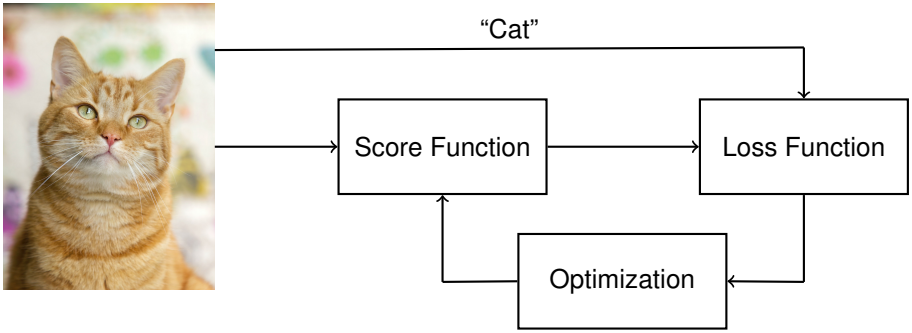

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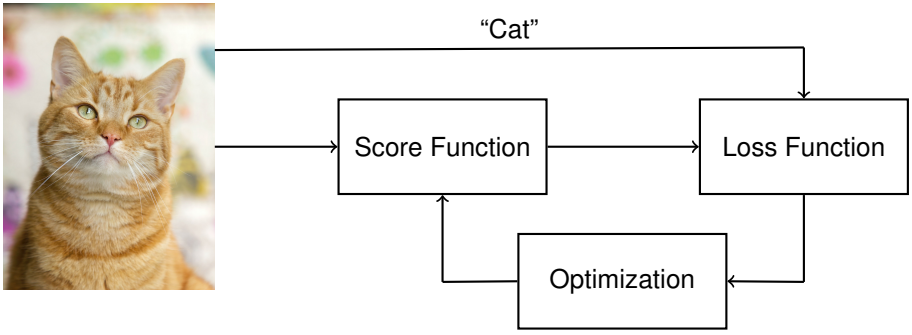

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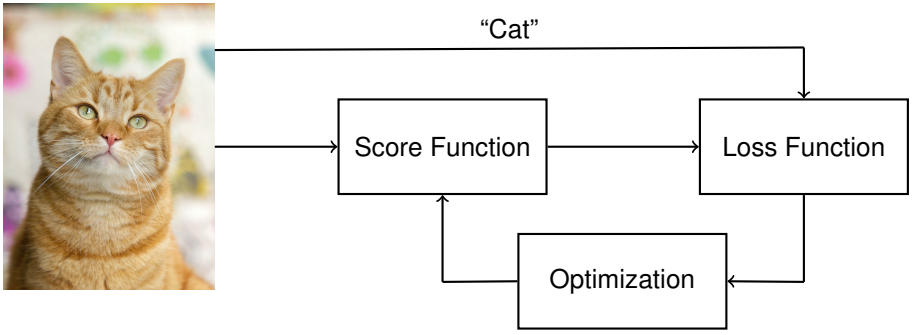

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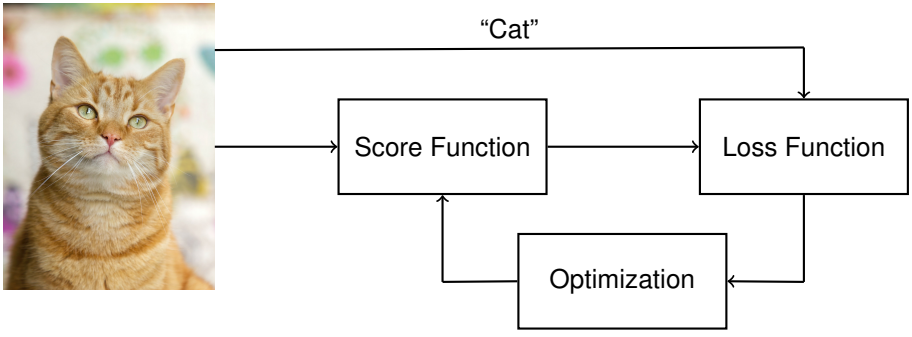
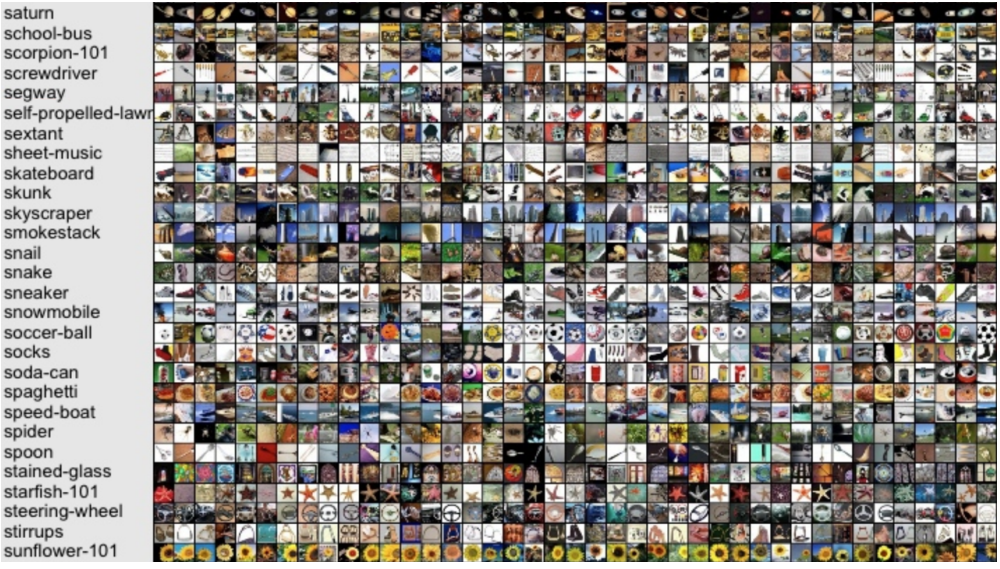


The ImageNet Challenge



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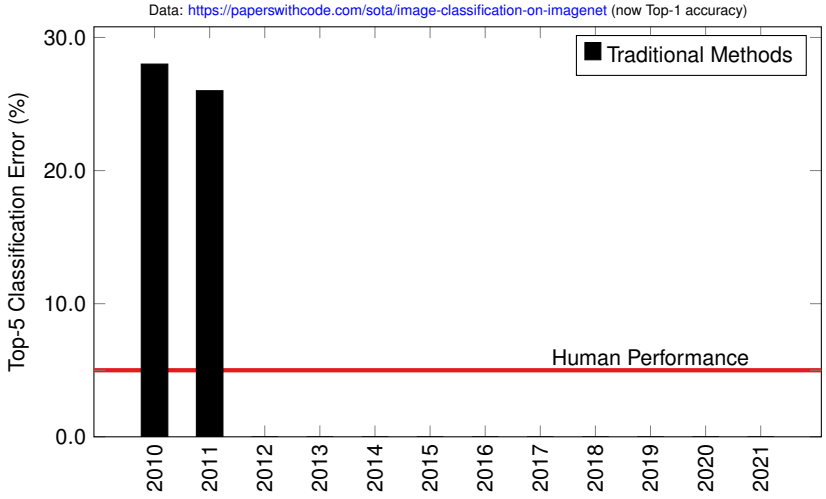

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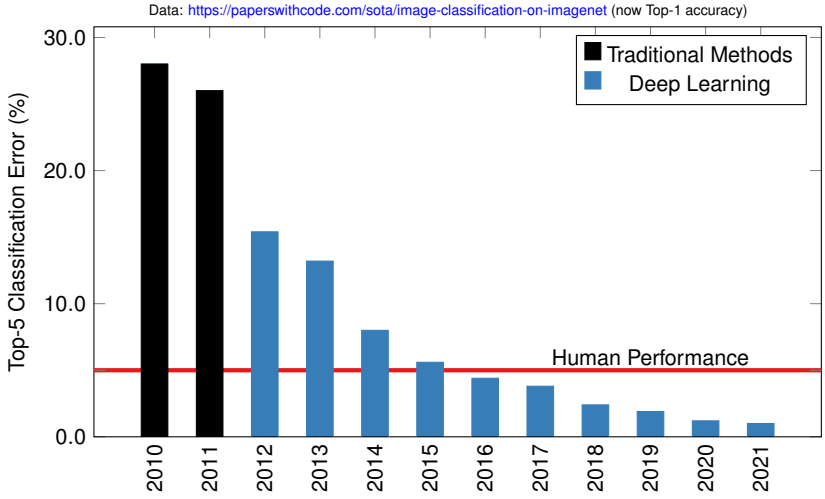

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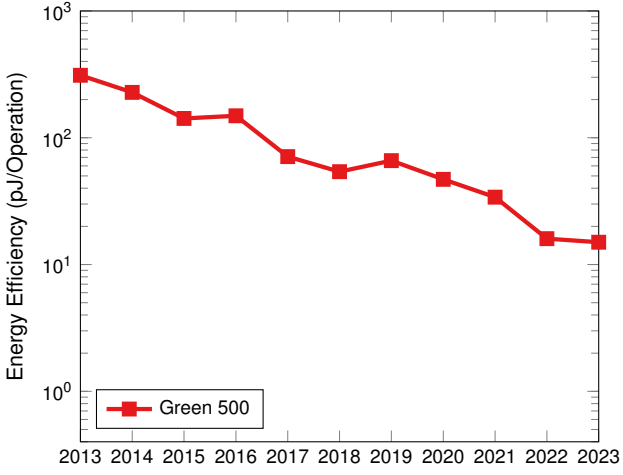
Data: https://top500.org/lists/green500/, https://en.wikipedia.org/wiki/Tensor_Processing_Unit



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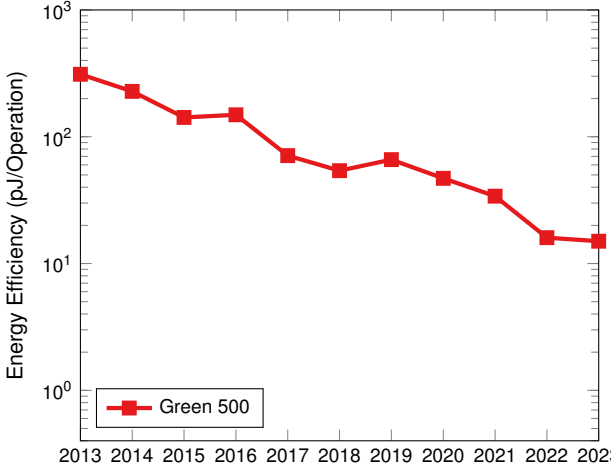

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Energy efficiency of brain: 10^{-8} pJ/Operation!!!

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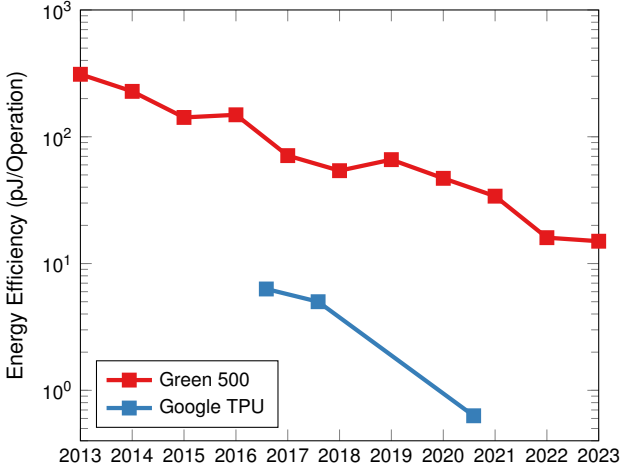

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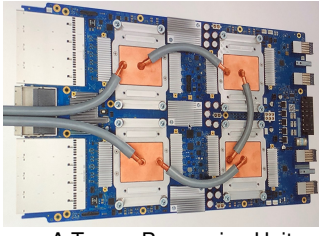

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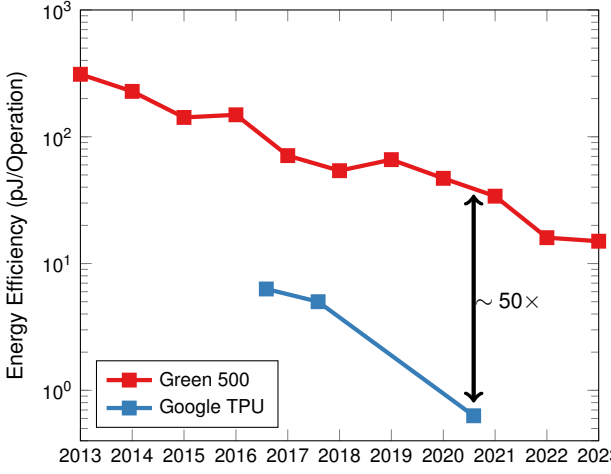

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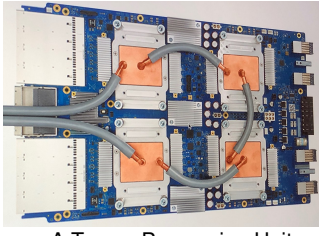

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A Tensor Processing Unit.

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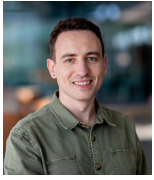
Course Team

Core Team

dr Alexios Balatsoukas-Stimming
Responsible lecturer



dr Federico Corradi
Co-lecturer

Zhanbo Shen
Teaching assistant

Lorenzo Pes
Teaching assistant

Guest Lecturers

Prof. Henk Corporaal
TU/e Emeritus

dr Maurice Peemen
Thermo Fischer Scientific

dr Willem Sandberg
NXP

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Prerequisites:

1. Basic linear algebra.
2. Basic differential calculus.
3. Very good programming skills (ideally in Python).

Course Organization

- **Lectures:** Tuesdays 10:45-12:30 & Fridays 15:30-17:15
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- **Course material:** Canvas.

Use of Generative AI

Not allowed

Using AI to **create content** like images, source code, video, text or any other kind.

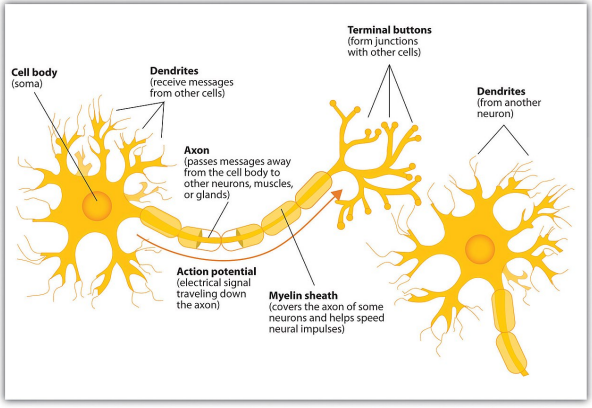
Allowed

Use of AI-powered tools to improve **your own** content that operate at the level of at most a single sentence (e.g. spelling checkers) or a single line of code (as provided by many IDEs).

Time For A Break

> "*Our civilization is first and foremost a civilization of means; in the reality of modern life, the means, it would seem, are more important than the ends.*"
>
> – Jacques Ellul, "The Technological Society," 1954.

Our Brain



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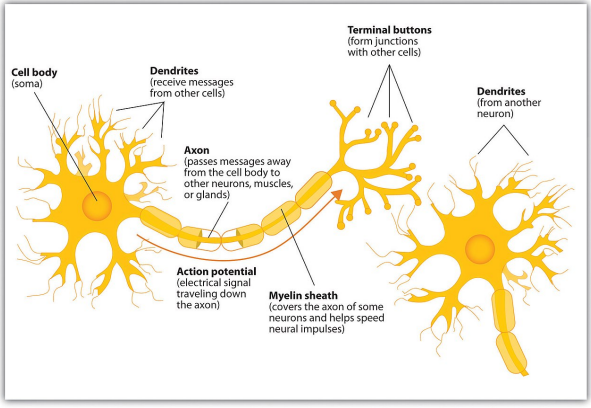
- Neurons receive input signals from **dendrites** and produce output signals along their **axon**, which interact with the dendrites of other neurons via **synaptic weights**.

Our Brain



Jennifer Walinga, CC BY-SA 4.0, via Wikimedia Commons

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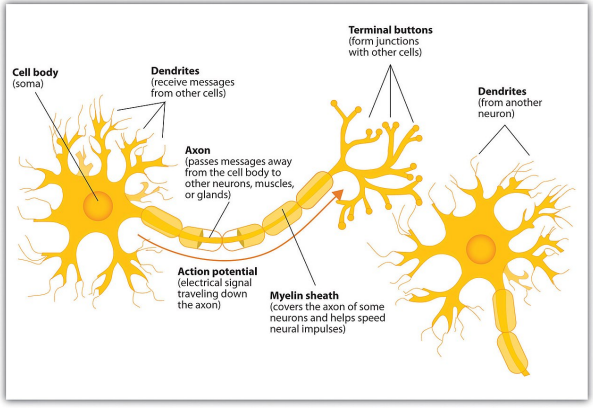
- About 10^{14} to 10^{15} synapses.

Our Brain



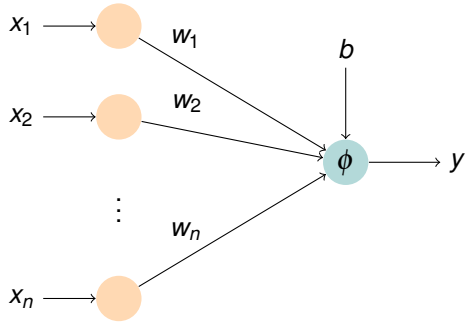
Jennifer Walinga, CC BY-SA 4.0, via Wikimedia Commons

The basic computational unit of the brain is a **neuron**.

- Neurons receive input signals from **dendrites** and produce output signals along their **axon**, which interact with the dendrites of other neurons via **synaptic weights**.
- About 80 billion neurons.
- About 10^{14} to 10^{15} synapses.

The synaptic weights are **learnable** and they control the influence between neurons.

An Artificial Neuron



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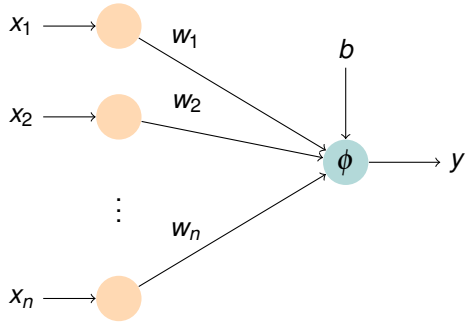

$$y = \phi\left(\sum_{i=1}^{n} w_i x_i + b\right) = \phi\left(\mathbf{w}^\top \mathbf{x} + b\right)$$

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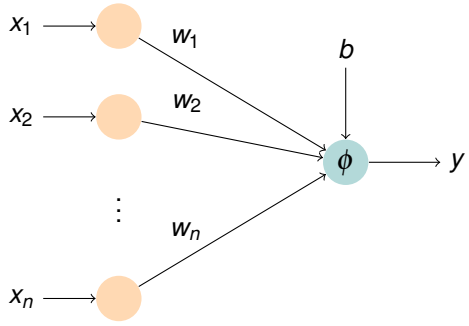

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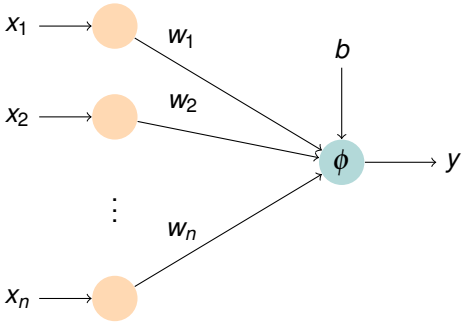

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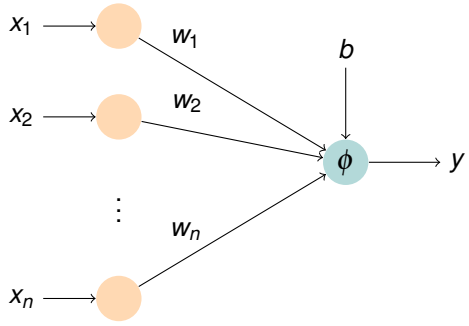

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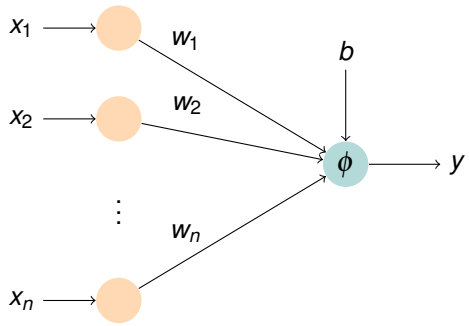

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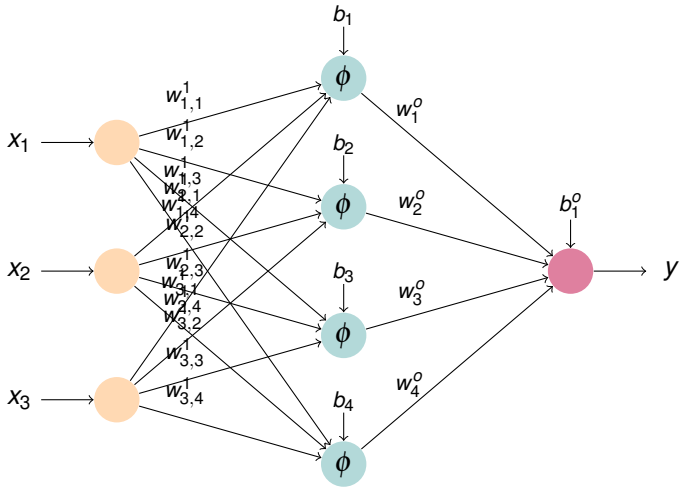
- **Output** y corresponds to action potential on axon.

A Single-Layer Single-Output Neural Network



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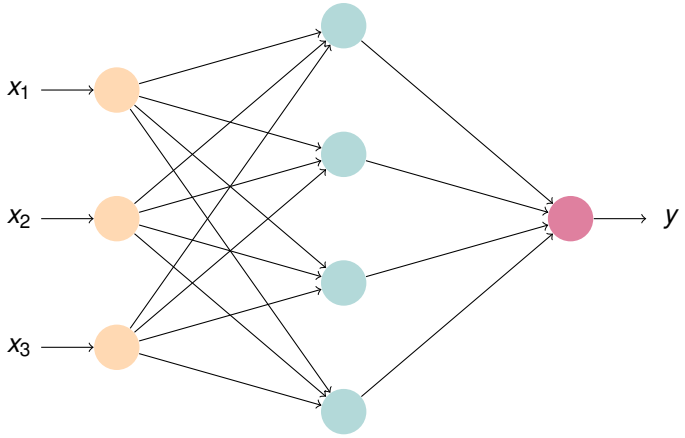

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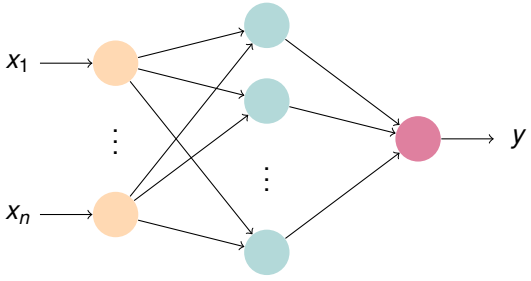

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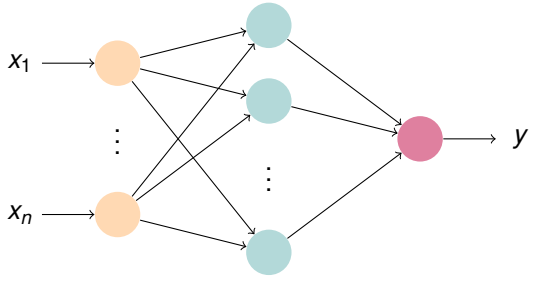
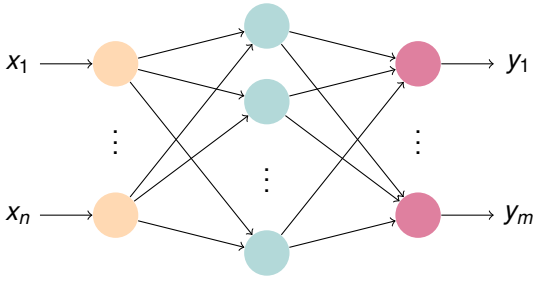


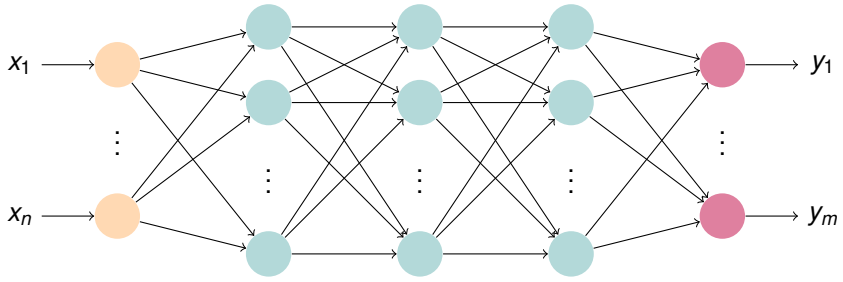
$$y = \left(\mathbf{w}^{[2]}\right)^{\top} \phi\left(\mathbf{W}^{[1]}\mathbf{x} + \mathbf{b}^{[1]}\right) + b^{[2]}$$

A Single-Layer Multi-Output Neural Network



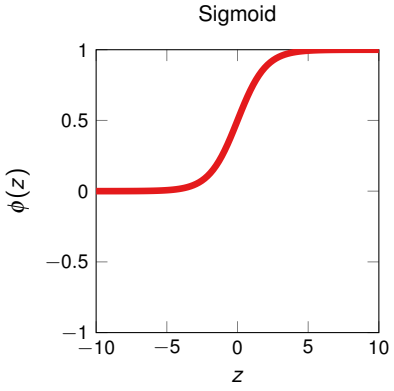
$$\mathbf{y} = \mathbf{W}^{[2]}\phi\left(\mathbf{W}^{[1]}\mathbf{x} + \mathbf{b}^{[1]}\right) + \mathbf{b}^{[2]}$$

A Multi-Layer Multi-Output Neural Network



Let $f^{[i]}(\mathbf{z}) = \phi\left(\mathbf{W}^{[i]}\mathbf{z} + \mathbf{b}^{[i]}\right)$. Then, we have $\mathbf{y} = f^{[4]}\left(f^{[3]}\left(f^{[2]}\left(f^{[1]}(\mathbf{x})\right)\right)\right)$

Traditional Activation Functions



$$\phi(z) = \frac{1}{1+e^{-z}} = \sigma(z)$$

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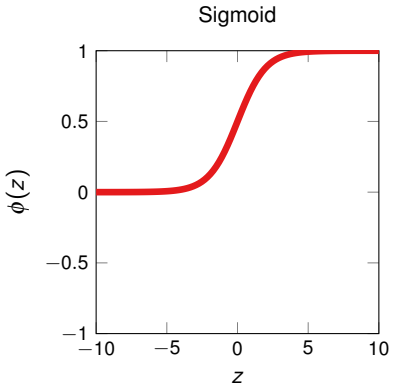

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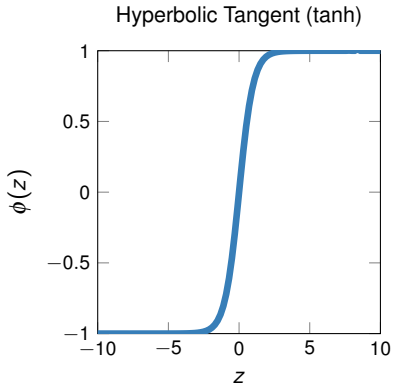
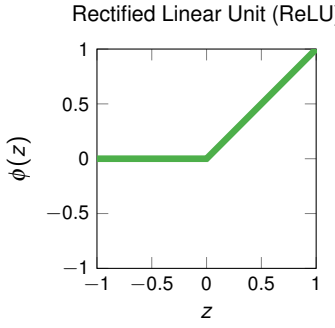


$$\phi(z) = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$$

Modern Activation Functions



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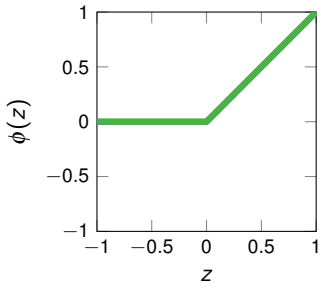
Modern Activation Functions

Rectified Linear Unit (ReLU)



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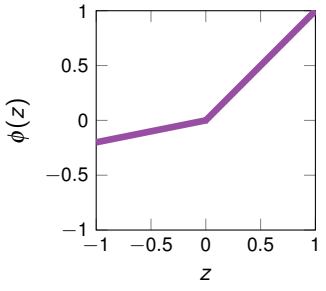

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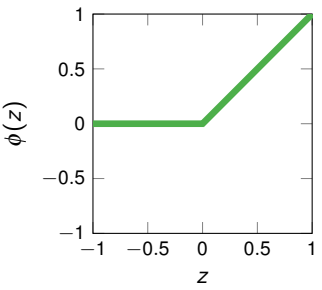

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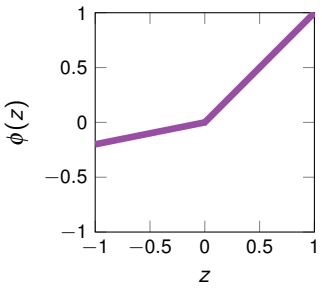

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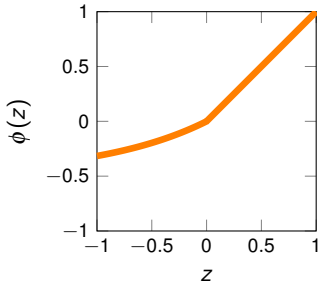
Exponential Linear Unit (ELU, $\alpha = 0.5$)



$$\phi(z) = \begin{cases} \alpha(e^z - 1), & z < 0, \\ z, & z \geq 0. \end{cases}$$

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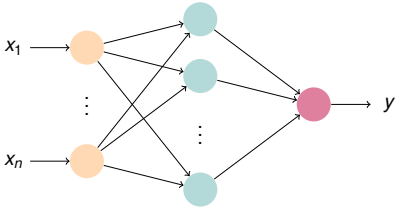

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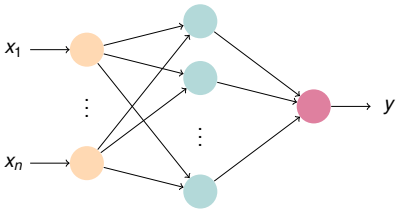

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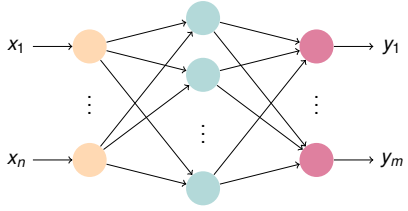
Multi-class classification



$$y_i = \text{softmax}(\mathbf{z}) = \frac{e^{z_i}}{\sum_{k=1}^{m} e^{z_k}}$$

Universal Approximation Theorem

Theorem 1. *Let σ be any continuous discriminatory function. Then finite sums of the form*

$$G(x) = \sum_{j=1}^{N} \alpha_j \sigma(y_j^T x + \theta_j) \tag{2}$$

are dense in $C(I_n)$. In other words, given any $f \in C(I_n)$ and $\varepsilon > 0$, there is a sum, $G(x)$, of the above form, for which

$$|G(x) - f(x)| < \varepsilon \qquad \text{for all} \quad x \in I_n.$$

G. Cybenko "Approximation by superpositions of a sigmoidal function," Mathematics of Control, Signals and Systems, 1989.

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Huh?!?

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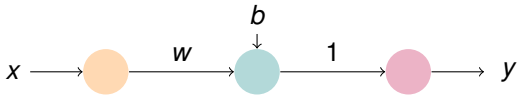
Neural Network as a Step Function



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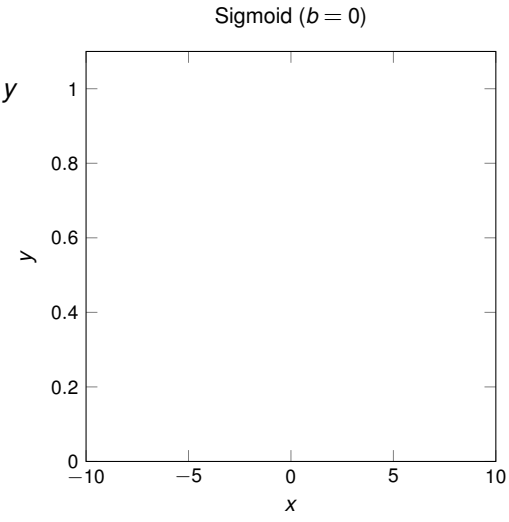

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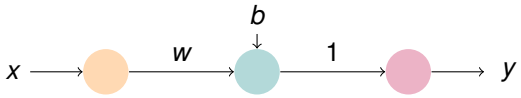

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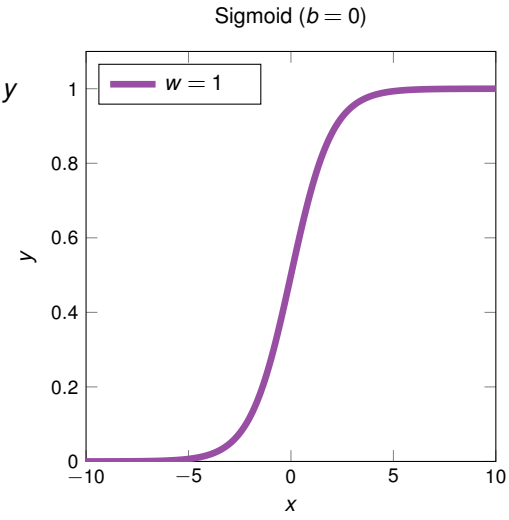
1. Assume $b = 0$ and vary w



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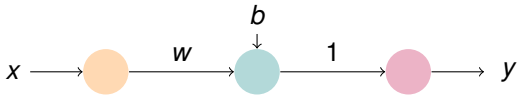

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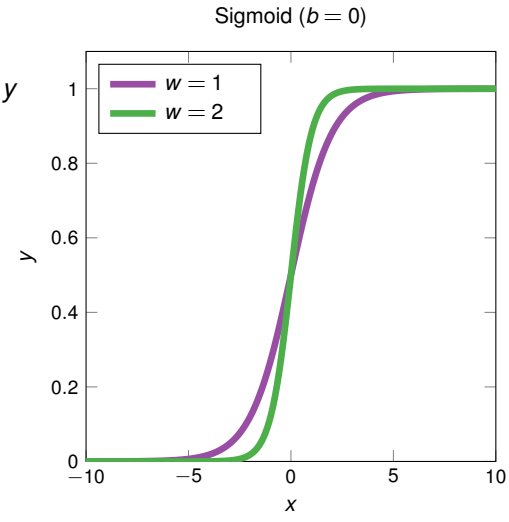

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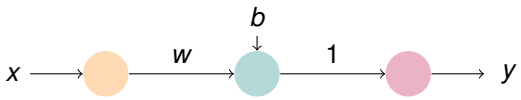

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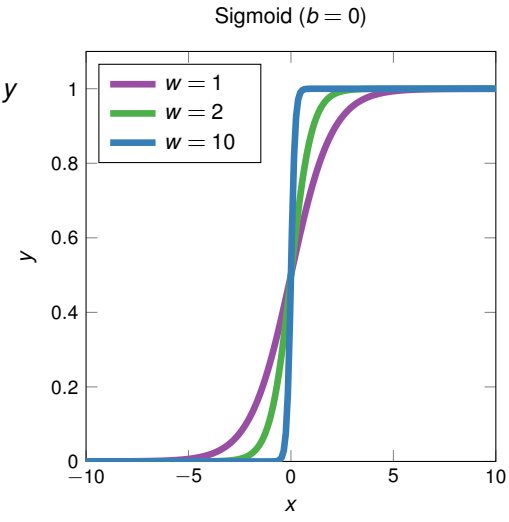

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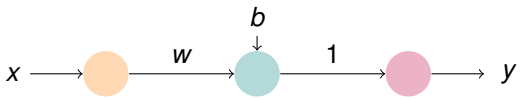

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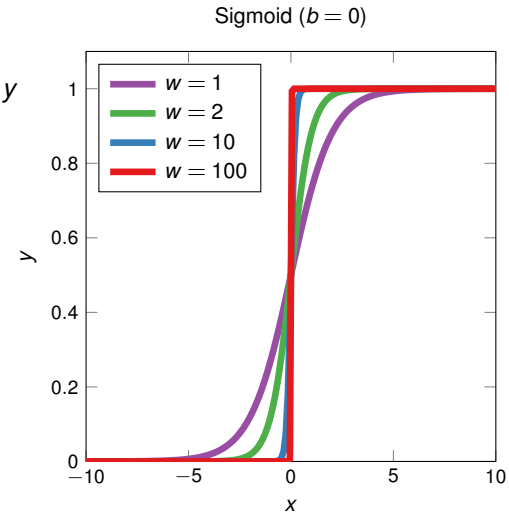

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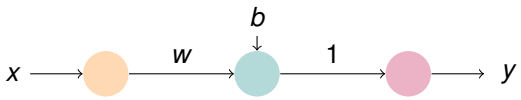

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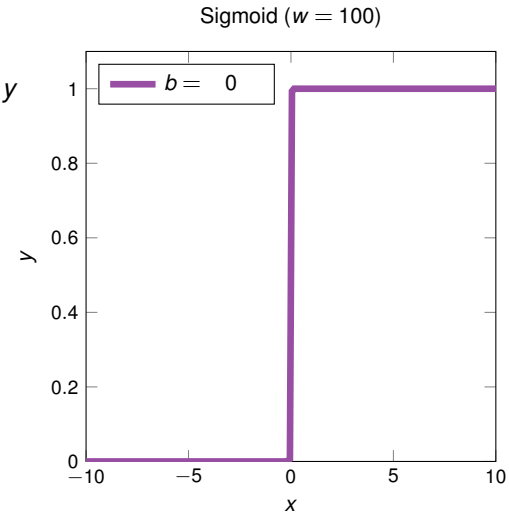

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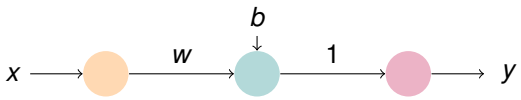

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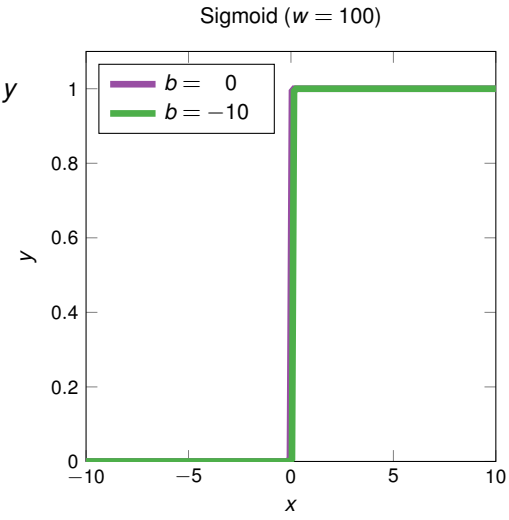
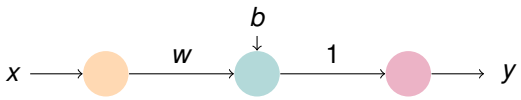
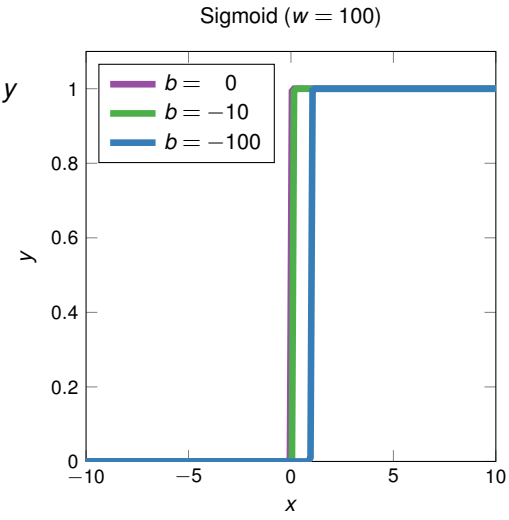


Neural Network as a Step Function

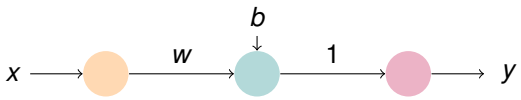


$$y = \sigma(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$

1. Assume $b = 0$ and vary w
2. Assume $w = 100$ and vary b



Neural Network as a Step Function



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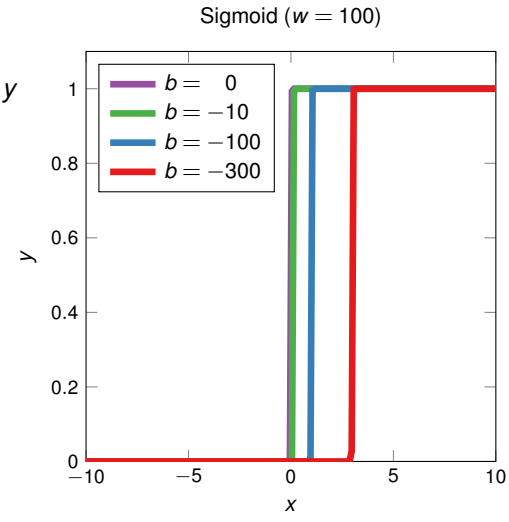
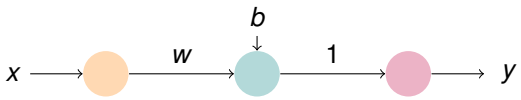


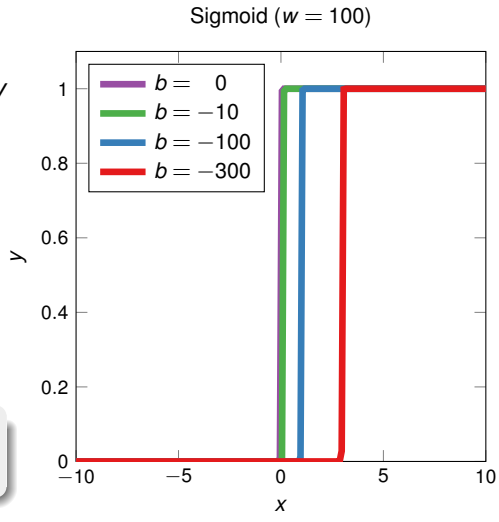
Neural Network as a Step Function



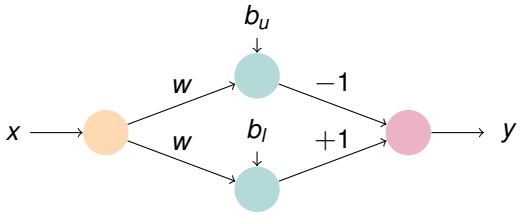
$$y = \sigma(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$

1. Assume $b = 0$ and vary w
2. Assume $w = 100$ and vary b

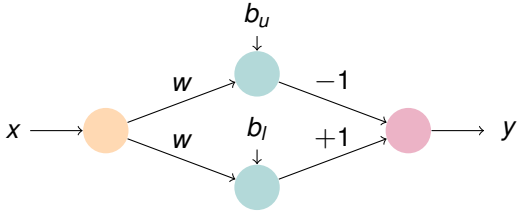
In general, $y = \sigma(wx + b)$ approximates a **step function** at $x = -\frac{b}{w}$.



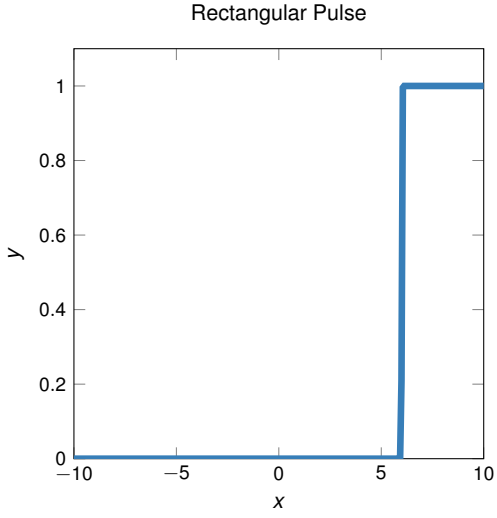
Neural Network as a Rectangular Function



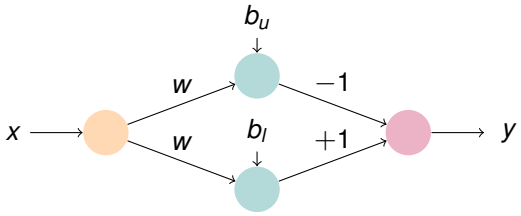
Neural Network as a Rectangular Function



$$y_u = \sigma(100x - 600)$$

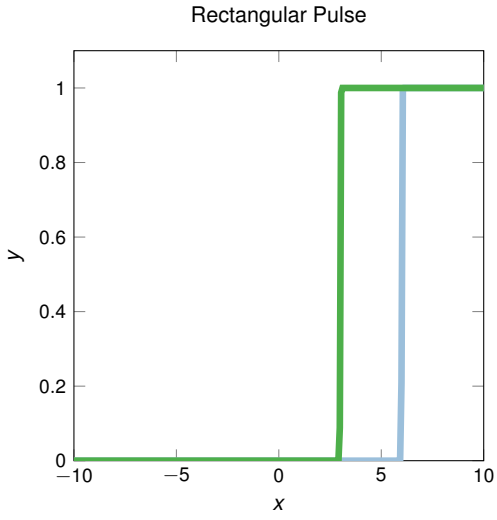


Neural Network as a Rectangular Function

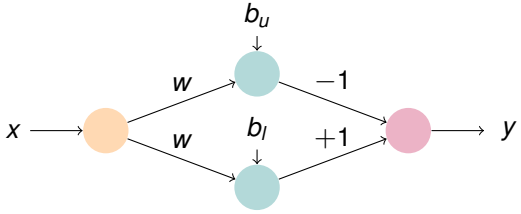


$$y_u = \sigma(100x - 600)$$

$$y_l = \sigma(100x - 300)$$



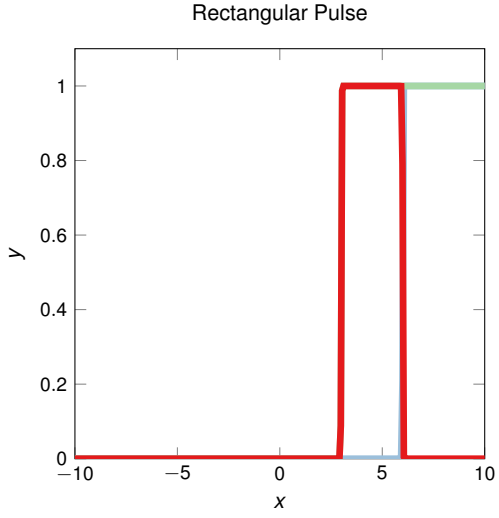
Neural Network as a Rectangular Function



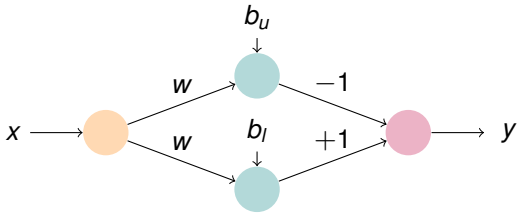
$$y_u = \sigma(100x - 600)$$

$$y_l = \sigma(100x - 300)$$

$$y = y_l - y_u = \sigma(100x - 300) - \sigma(100x - 600)$$



Neural Network as a Rectangular Function

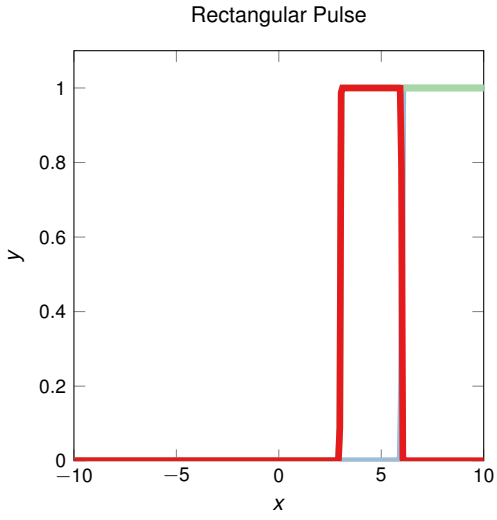


$$y_u = \sigma(100x - 600)$$

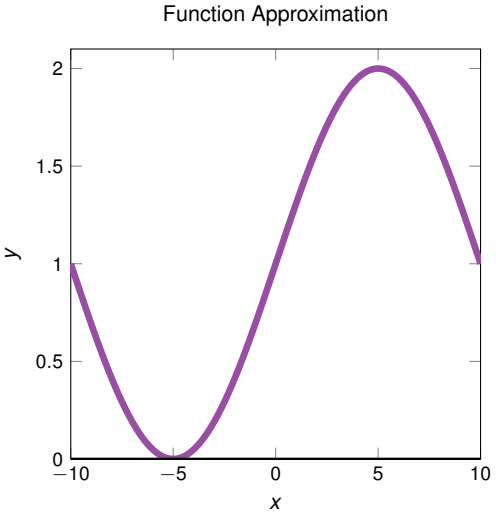
$$y_l = \sigma(100x - 300)$$

$$y = y_l - y_u = \sigma(100x - 300) - \sigma(100x - 600)$$

In general, $y = \sigma(wx + b_l) - \sigma(wx + b_u)$ approximates a **rectangular function** from $x_l = -\frac{b_l}{w}$ to $x_u = -\frac{b_u}{w}$.



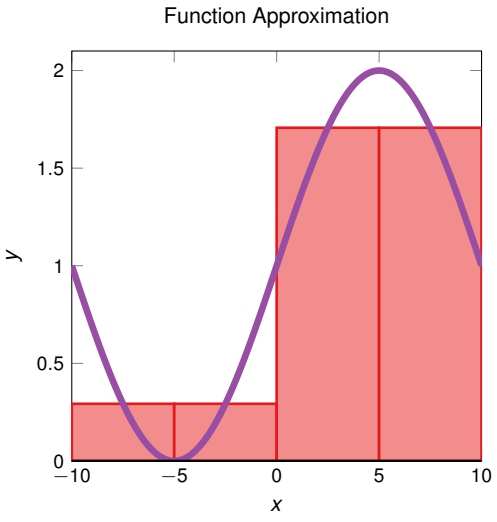
Neural Network as a Universal Approximator



Neural Network as a Universal Approximator

Generalization:

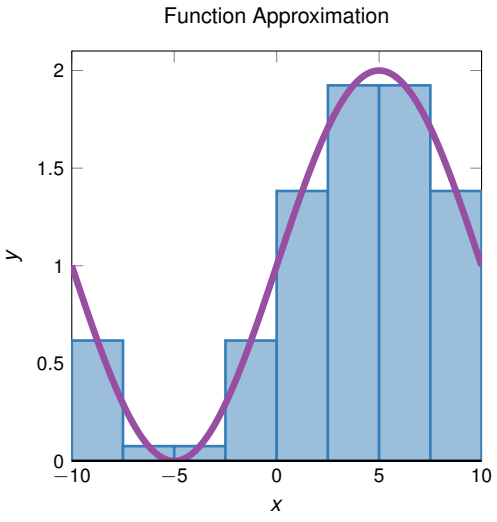
$$y = \sum_{j=1}^{4} \alpha_j \left(\sigma\left(wx - b_{l,j}\right) - \sigma\left(wx - b_{u,j}\right) \right)$$



Neural Network as a Universal Approximator

Generalization:

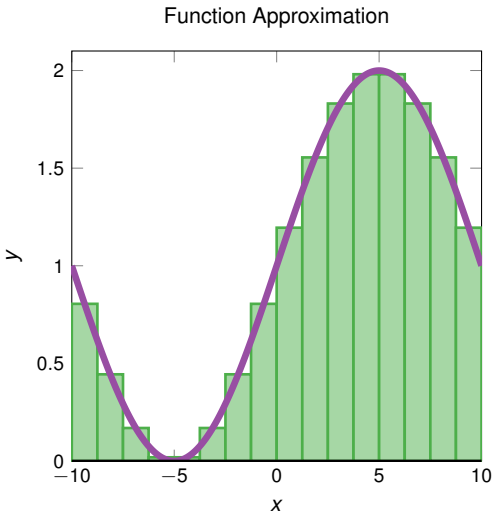
$$y = \sum_{j=1}^{8} \alpha_j \left(\sigma\left(wx - b_{l,j}\right) - \sigma\left(wx - b_{u,j}\right) \right)$$



Neural Network as a Universal Approximator

Generalization:

$$y = \sum_{j=1}^{16} \alpha_j \left(\sigma\left(wx - b_{l,j}\right) - \sigma\left(wx - b_{u,j}\right) \right)$$



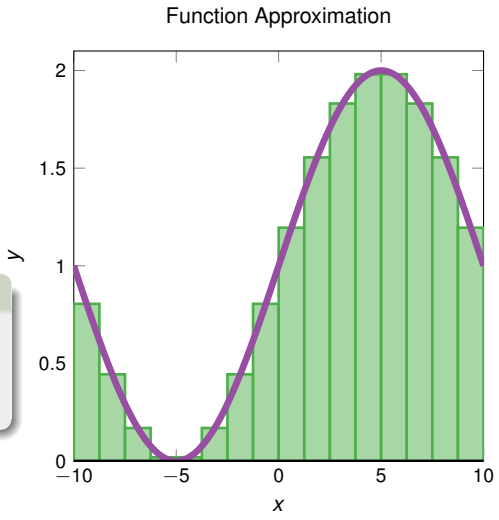
Neural Network as a Universal Approximator

Generalization:

$$y = \sum_{j=1}^{16} \alpha_j \left(\sigma\left(wx - b_{l,j}\right) - \sigma\left(wx - b_{u,j}\right) \right)$$

Result

We can use N rectangular functions constructed from $2N$ neurons to **approximate any function arbitrarily well**!



Conclusion

Summary:

1. Neural networks (NNs) have a long history, but recently became extremely popular.
2. NNs consist of artificial neurons that are brain-inspired.
3. NNs perform matrix-vector multiplications, vector additions, and activation functions.
4. NNs are universal function approximators.

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Summary:

1. Neural networks (NNs) have a long history, but recently became extremely popular.
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Next time:

- **NN training:** cost functions, (improved) gradient descent, backpropagation, weight initialization, data normalization.