

Image Geometrical Manipulation and Warping

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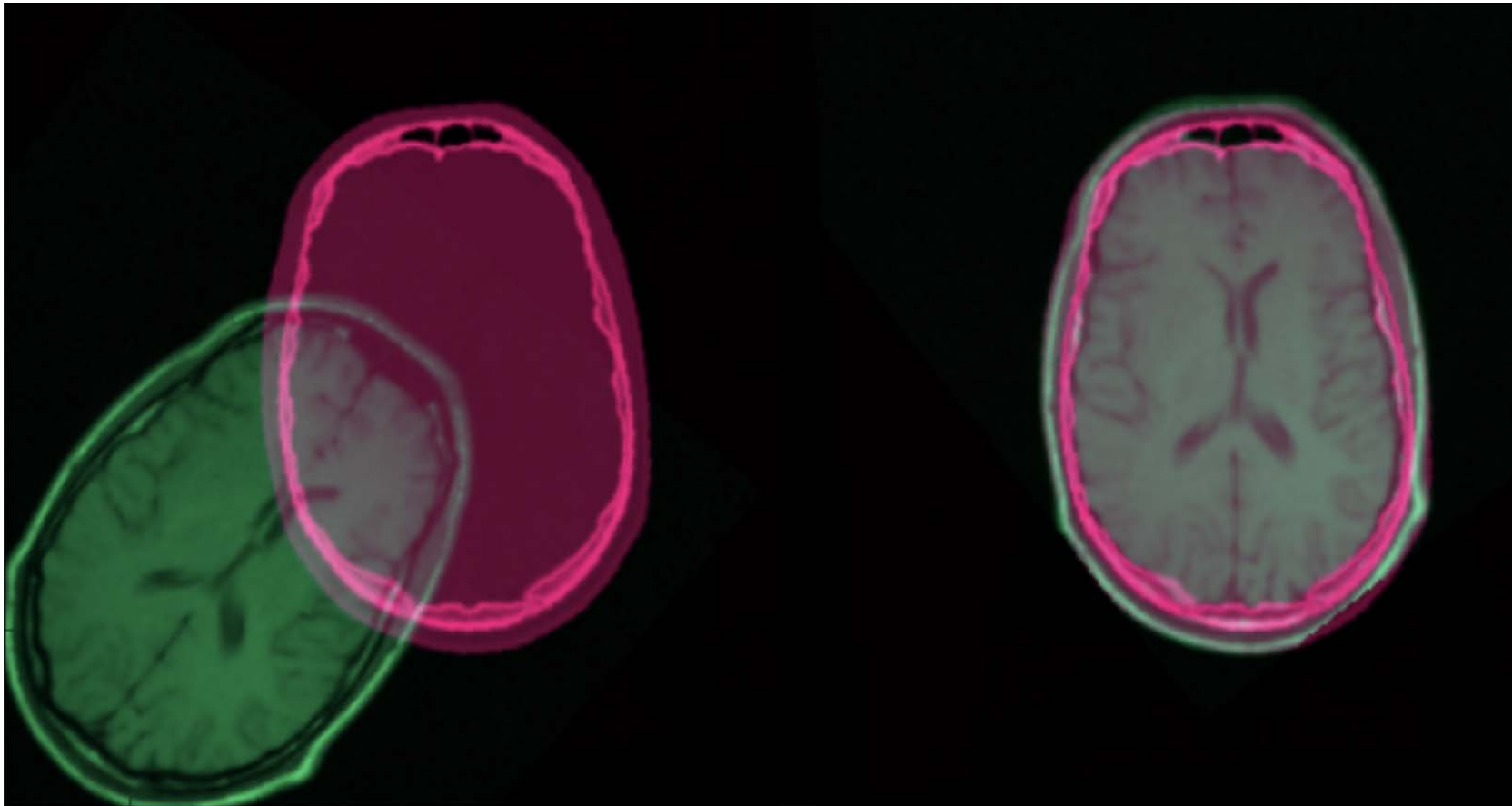
University of Southern California

Introduction

- Geometrical Manipulation
 - Translation
 - Scaling (zoom-in and zoom-out)
 - Rotation
 - Affine transformation
- Advanced Manipulation
 - Image warping
 - 3D object warping
 - Computer graphic rendering (from 3D world coordinates to 2D image coordinates)

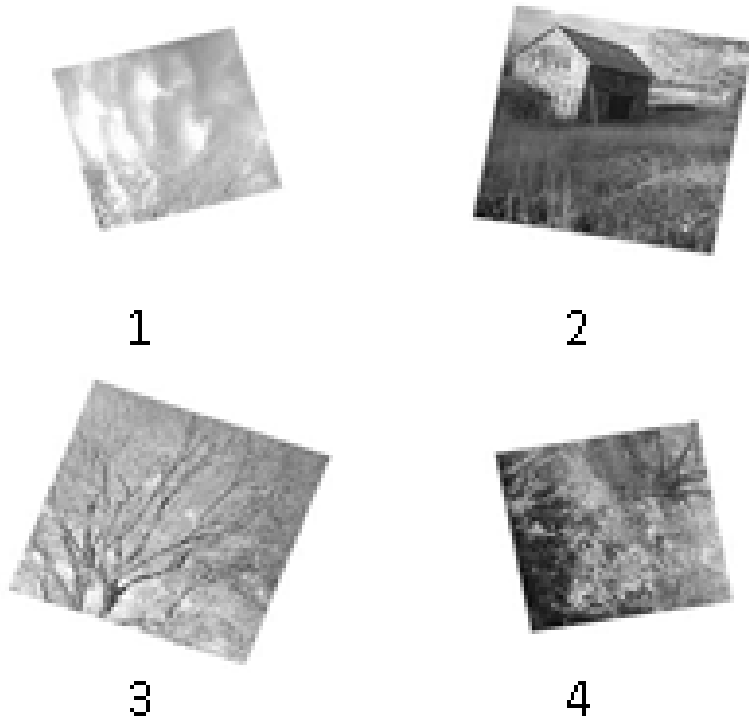
Applications

- Image registration



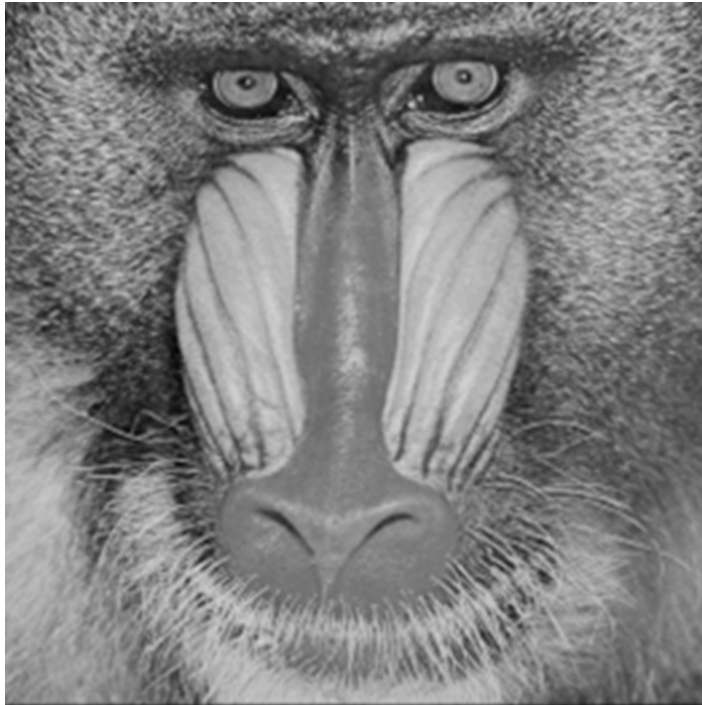
Applications

- Image registration



Applications

- Image warping



Applications

- Image warping

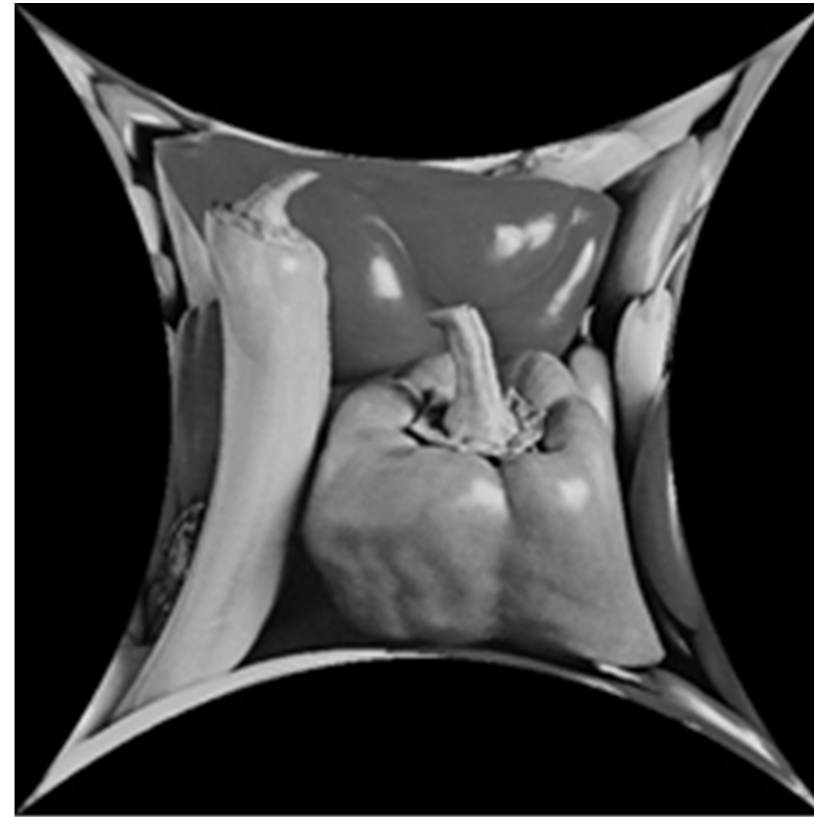
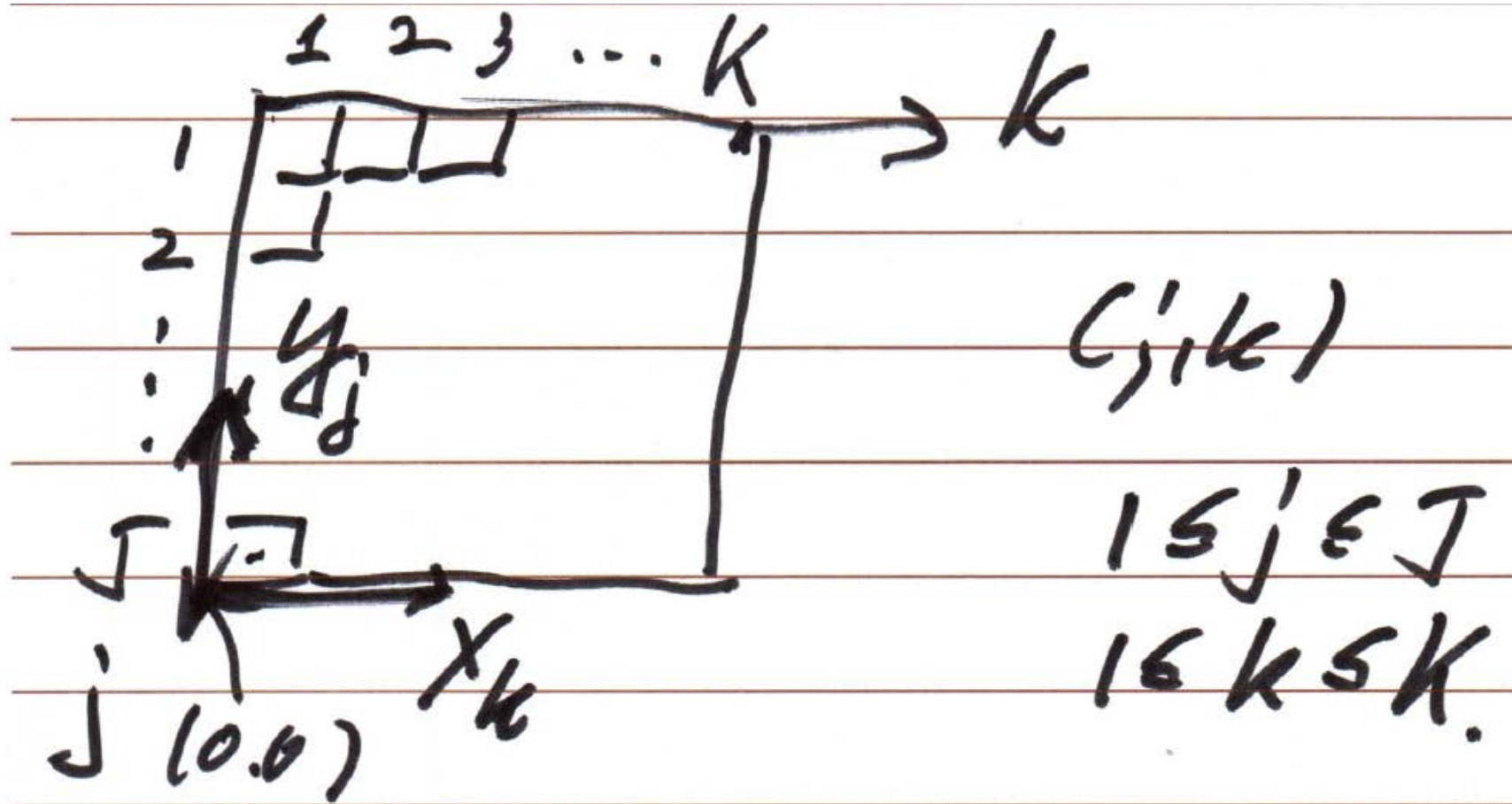


Image Coordinates versus Cartesian Coordinates



Transformation between Image and Cartesian Coordinates

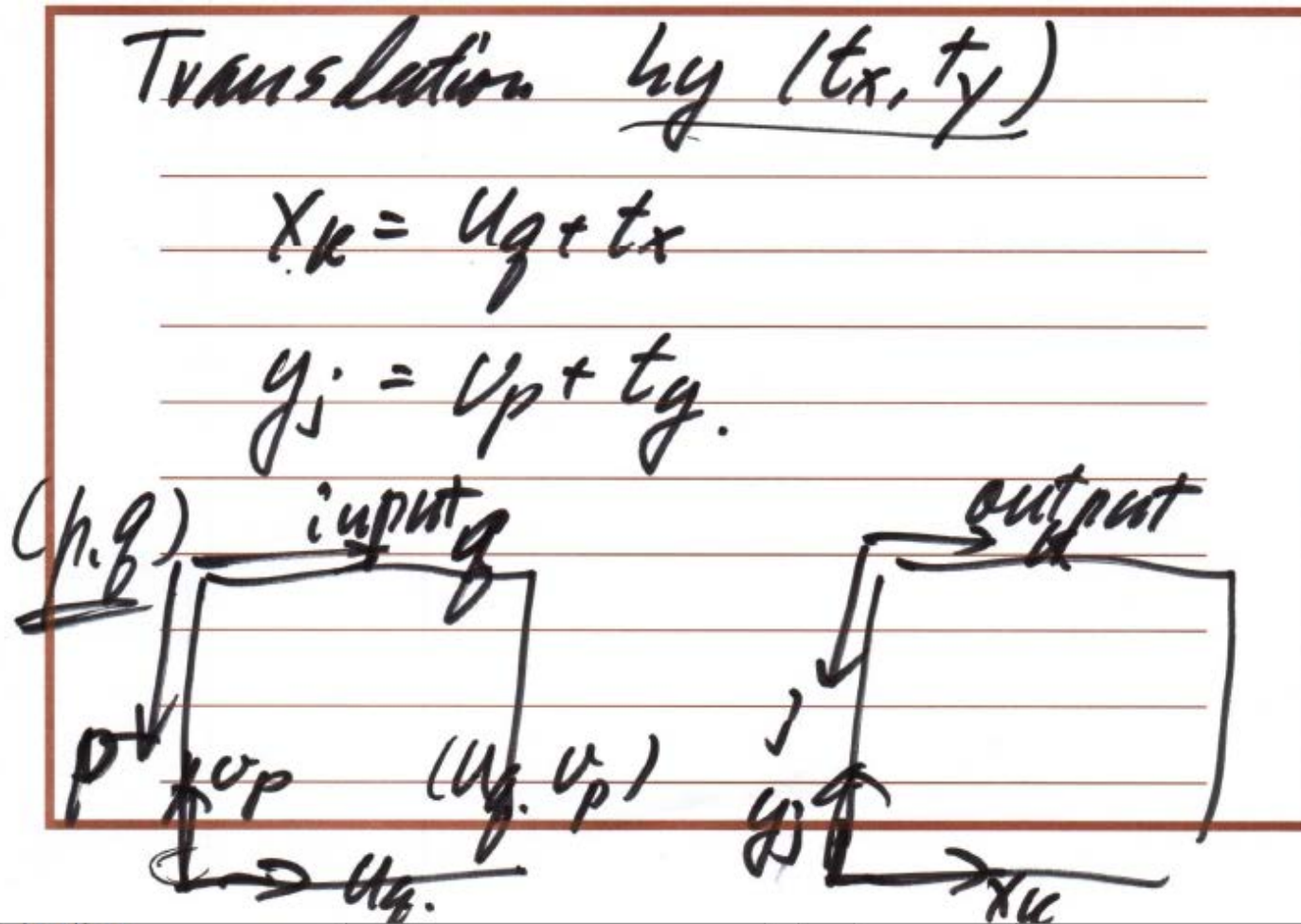
Coordinates conversion.

$$\begin{cases} x_k = k - \frac{1}{2} \\ y_j = J + \frac{1}{2} - j. \end{cases}$$

$k=1 \Rightarrow x_k = 0.5$

$j=J \quad y_j = \frac{1}{2}$

Translation



Translation

	Input	Output
Cartesian	$\begin{pmatrix} u \\ v \end{pmatrix}$	$\begin{pmatrix} x \\ y \end{pmatrix}$
image.	$\begin{pmatrix} p \\ q \end{pmatrix}$	$\begin{pmatrix} j \\ k \end{pmatrix}$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -\frac{L}{2} \\ -1 & 0 & p + \frac{L}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \\ 1 \end{pmatrix} \quad \text{Eq (2)}$$

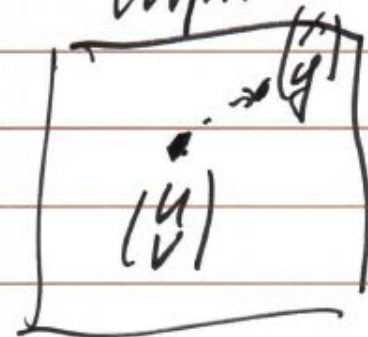
$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -\frac{L}{2} \\ -1 & 0 & j + \frac{L}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \\ 1 \end{pmatrix} \quad \text{Eq (1)}$$

coordinates
transformation

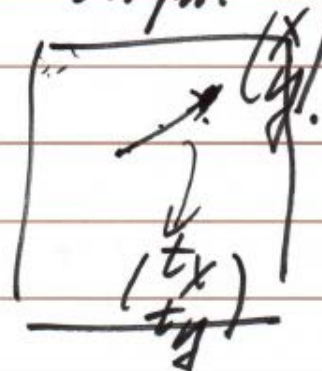
Non-Integer Translation

Translation

input



output



$$x = u + t_x$$

$$y = v + t_y$$

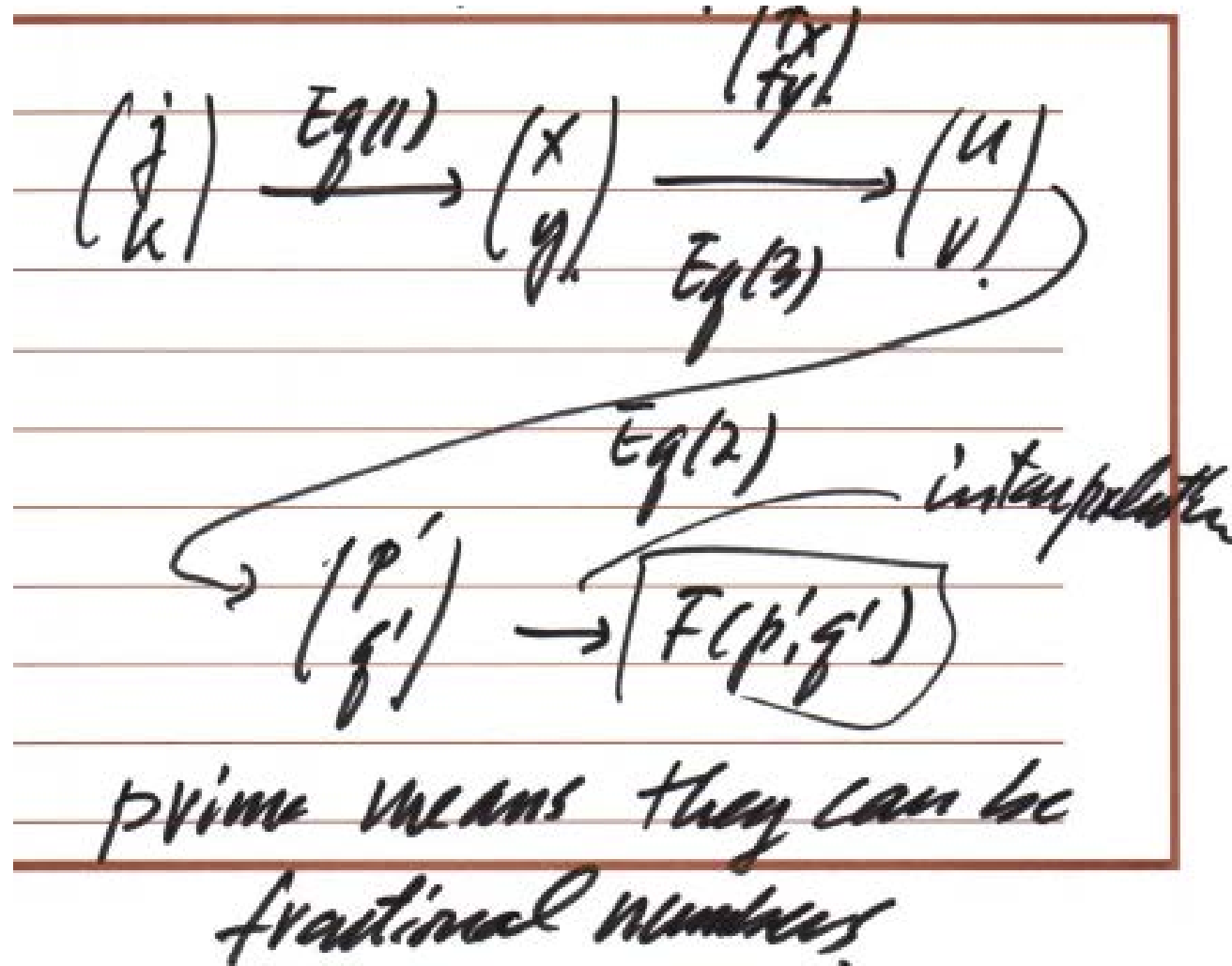
Eq (3)

What happens if t_x & t_y are not integers?

Inverse Address Mapping

- Render the output image (in integer pixel locations) by tracing back to the corresponding input pixel locations
- The input pixel locations can be fractional numbers
 - Use the interpolation technique to generate the corresponding values

Inverse Address Mapping



Rewrite Eq. (3): Affine to Linear System Conversion

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}. \quad \text{Eq (3)}$$

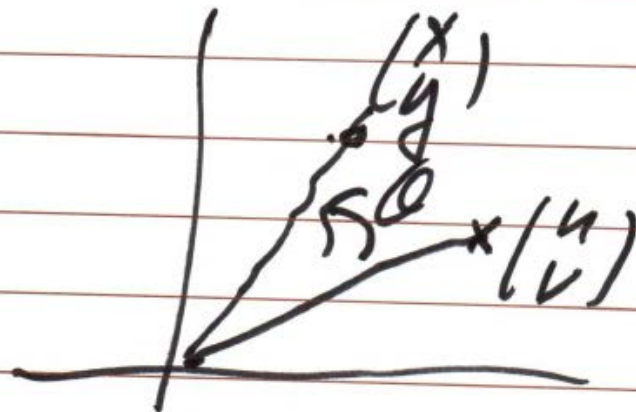
Scaling

$$x = s_x u$$
$$y = s_y v$$
$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$
$$\frac{\Delta x_1}{\Delta x_0} = s_x$$
$$\frac{\Delta y_1}{\Delta y_0} = s_y$$

Rotation

Rotation by θ .

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

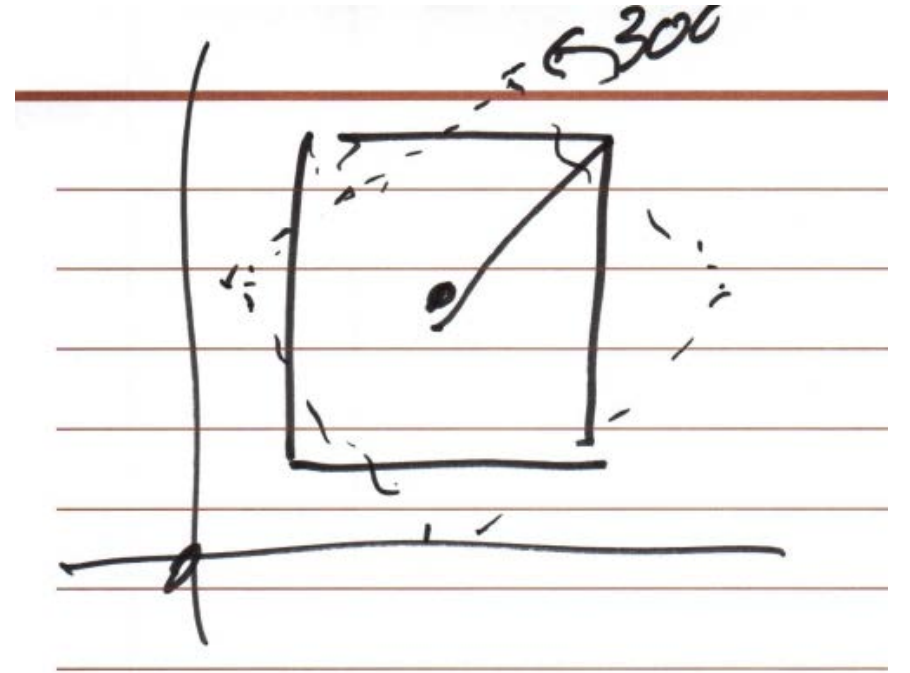


Compound Operation

1) translate by $\begin{pmatrix} t_x \\ t_y \end{pmatrix}$

2) rotate by θ .

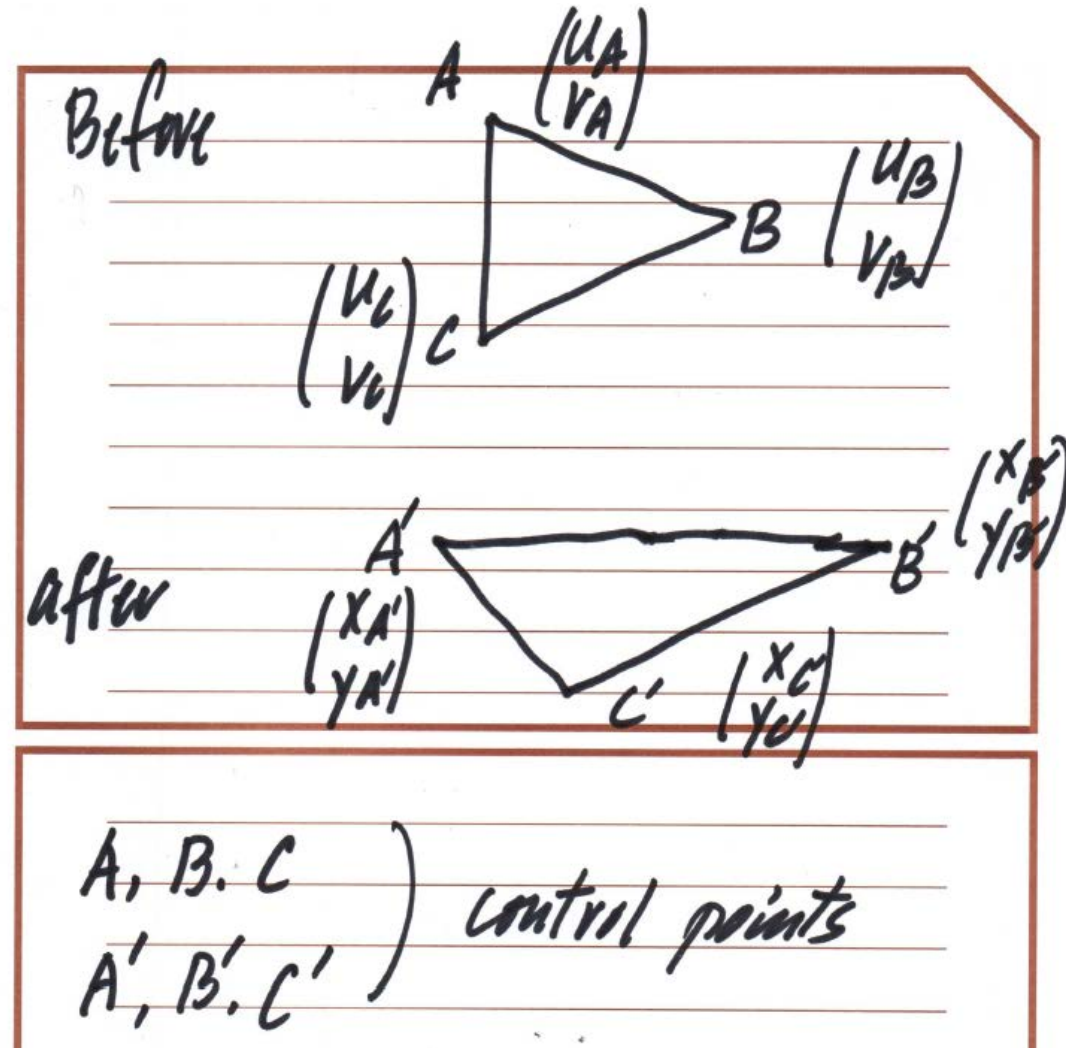
3) scale by $\begin{pmatrix} s_x \\ s_y \end{pmatrix}$



Compound Operation

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ 1 \end{pmatrix}$$

Manipulation of A Triangle



Manipulation of A Triangle

$$\bar{X}_{out} = T \bar{X}_{in}$$

$$\begin{pmatrix} X_A' & X_B' & X_C' \\ Y_A' & Y_B' & Y_C' \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} U_A & U_B & U_C \\ V_A & V_B & V_C \\ 1 & 1 & 1 \end{pmatrix}$$

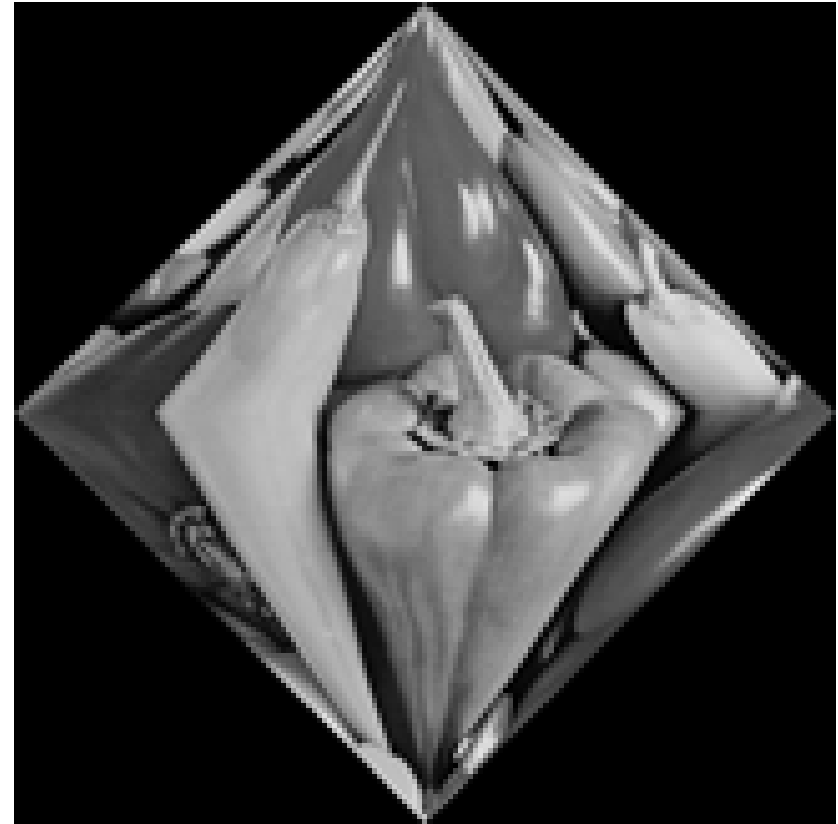
↑

A model

a system of
9 equations of 9 parameters

$$T = \bar{X}_{out} \bar{X}_{in}^{-1}$$

Polygon Manipulation Example (1)



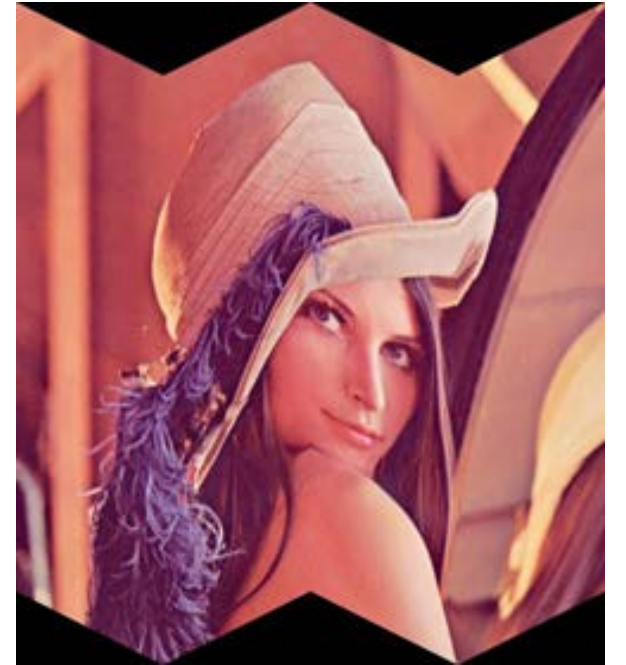
Polygon Manipulation Example (2)



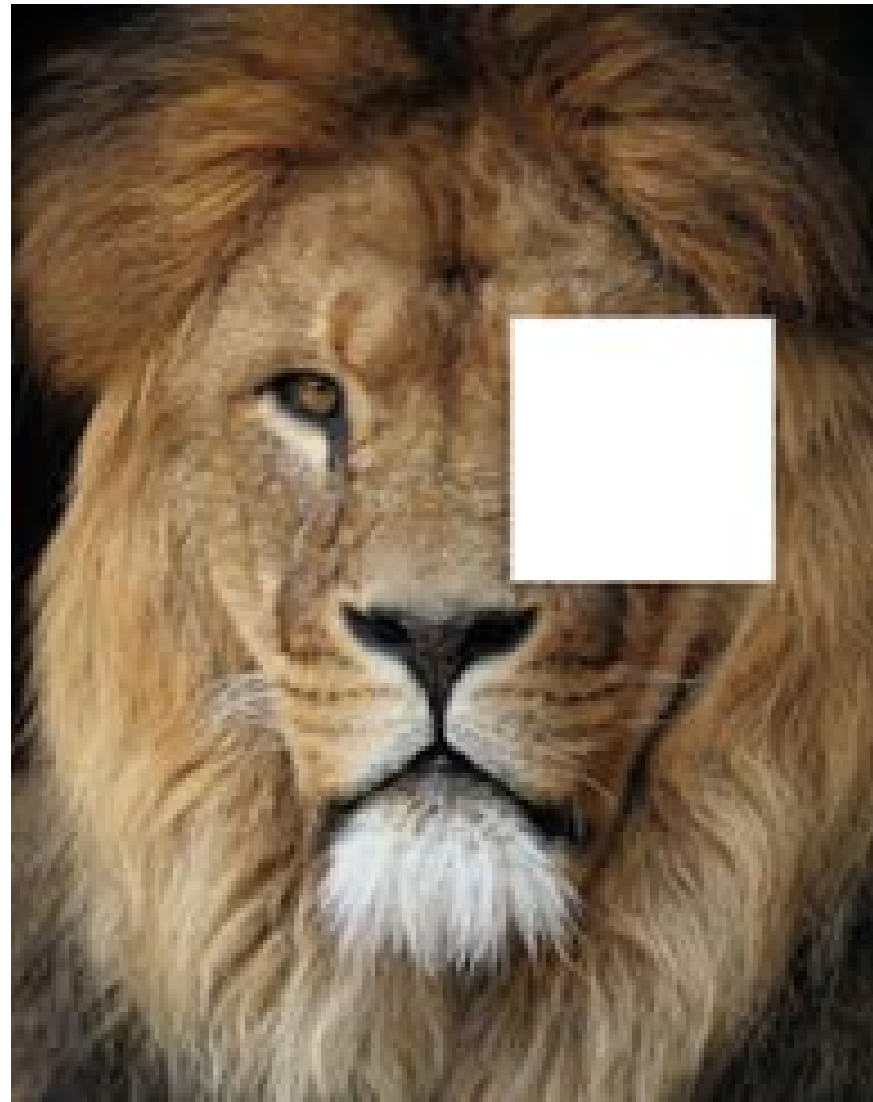
Polygon Manipulation Example (3)



Polygon Manipulation Example (4)

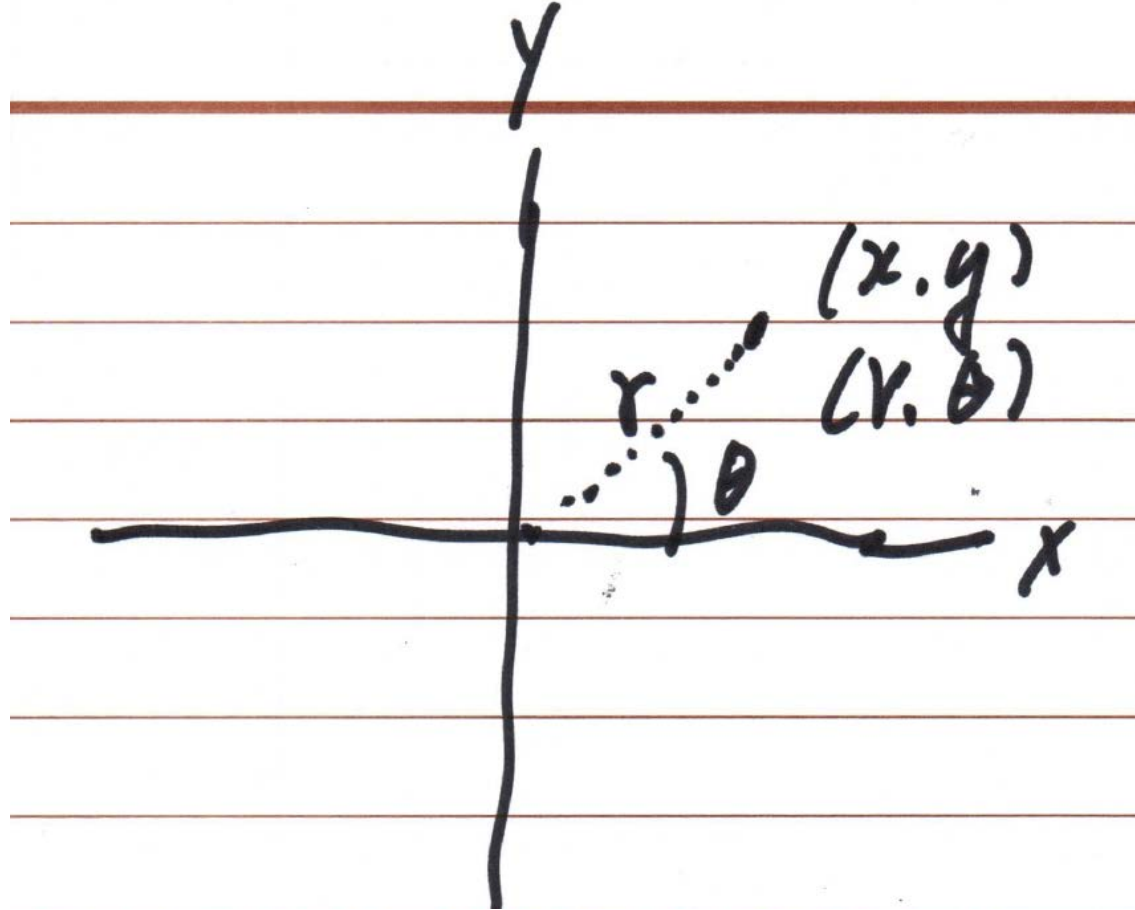


Polygon Manipulation Example (5)

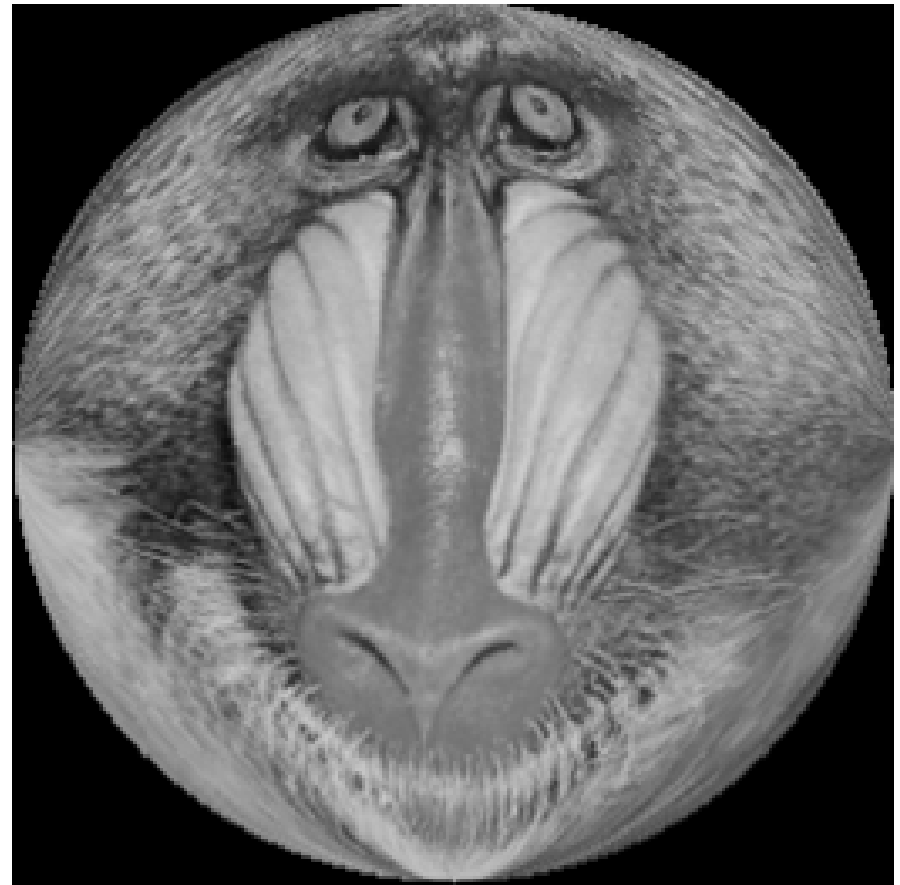
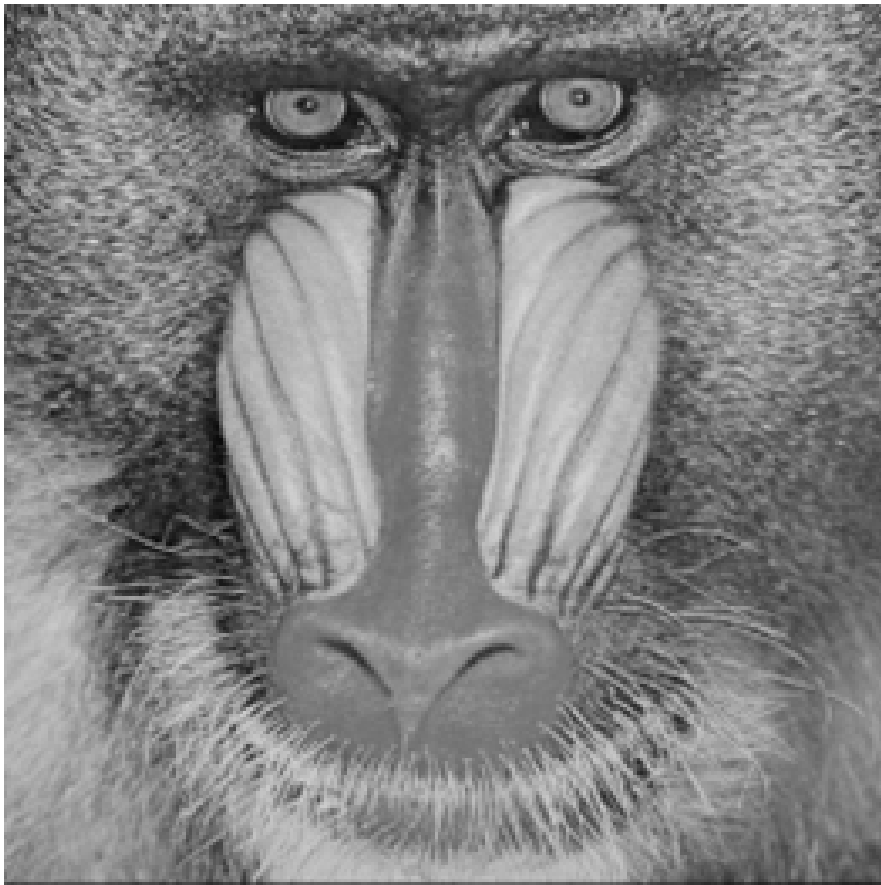


Cartesian/Polar Coordinates Transformation

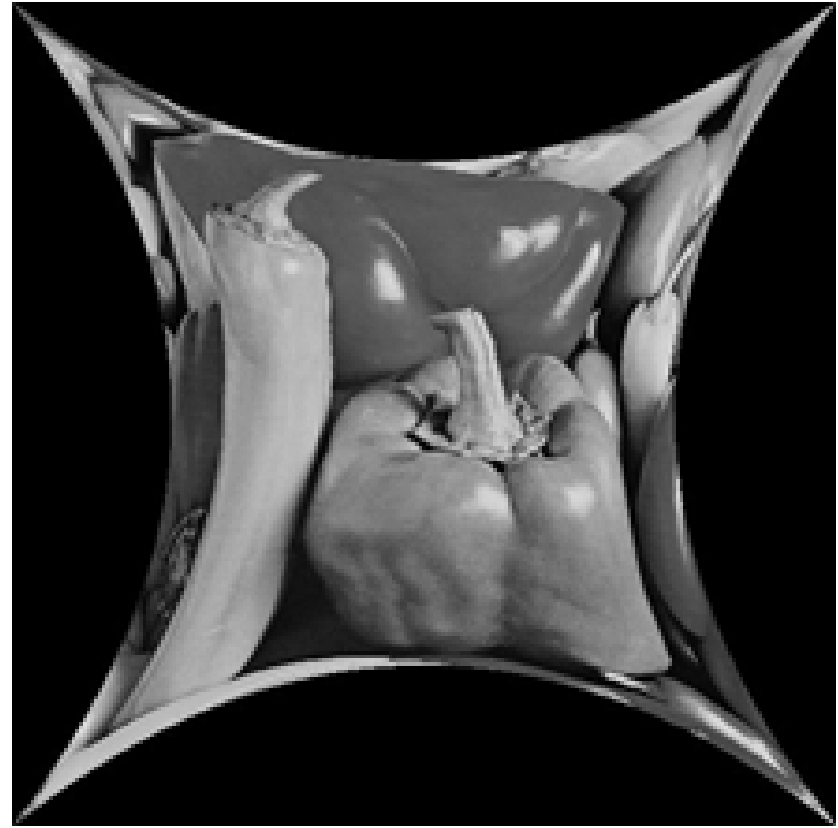
Some transformation
is easier to do in the
polar coordinates



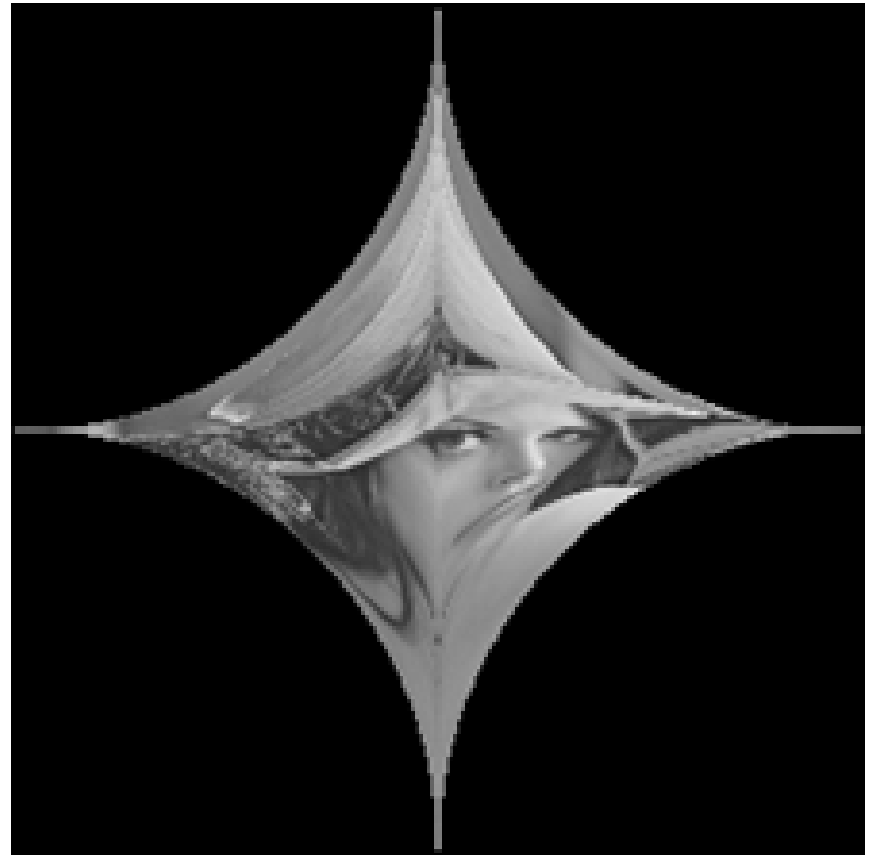
Example of Cartesian/Polar Coordinates Transformation (1)



Example of Cartesian/Polar Coordinates Transformation (2)



Example of Cartesian/Polar Coordinates Transformation (3)



Example of Cartesian/Polar Coordinates Transformation (4)



Example of Cartesian/Polar Coordinates Transformation (5)



Address Mapping + Intensity Mapping

- We have considered pixel address mapping without modifying its intensity (or color) value
- Can we do pixel address/intensity/color value mapping at the same time?
 - It is more challenging but feasible

Example: Facial Morphing



Trump



Biden

Face Image Morphing Video Clips

- Two sample projects on facial image morphing conducted by Prof. Kuo's graduate students in class EE569 (Introduction to Digital Image Processing) can be viewed from the following two websites:
- The best 14 results in the 2012 Fall class:
- The best 12 results in the 2011 Fall class:
<https://vimeo.com/user6200043>
- Each video clip was conducted by a single student.