Morphological Image Processing

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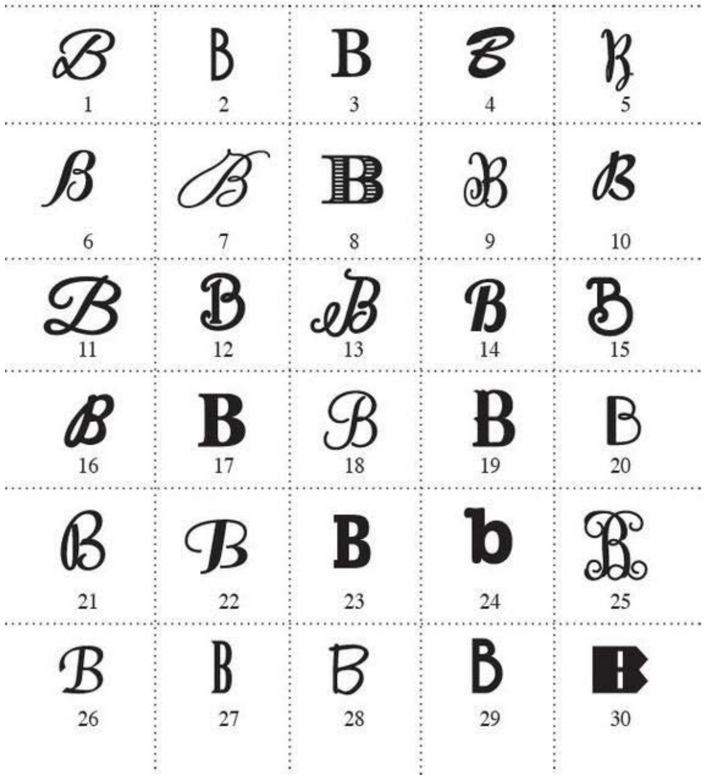
Introduction

What Is Morphology?

- Morph: shape
- Morphology: study of shapes
- In the context of image processing
  - Input: binary images
  - Output: processed binary images
    - Denoising
    - Thinning
    - Etc.

Example

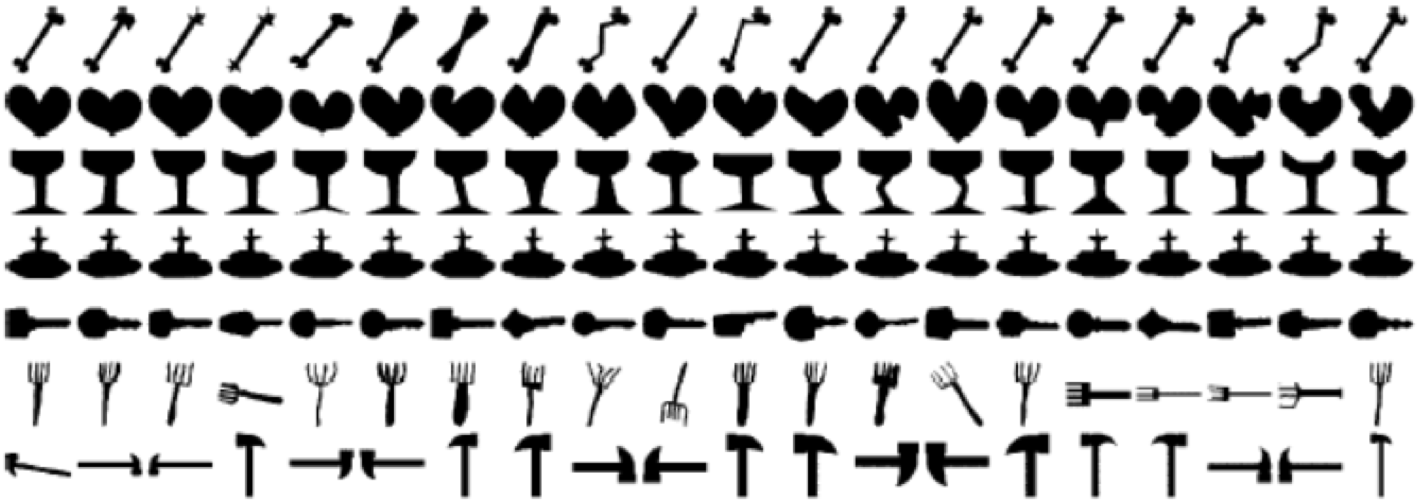
- Essential shape of an image
  - It has nothing to do with the stroke width



Morphological Processing

- Some objects contain shapes formed by line segments, arcs and curves
- Applications
  - Optical character recognition (OCR)
  - Fingerprint recognition
  - Shape retrieval
  - Etc.

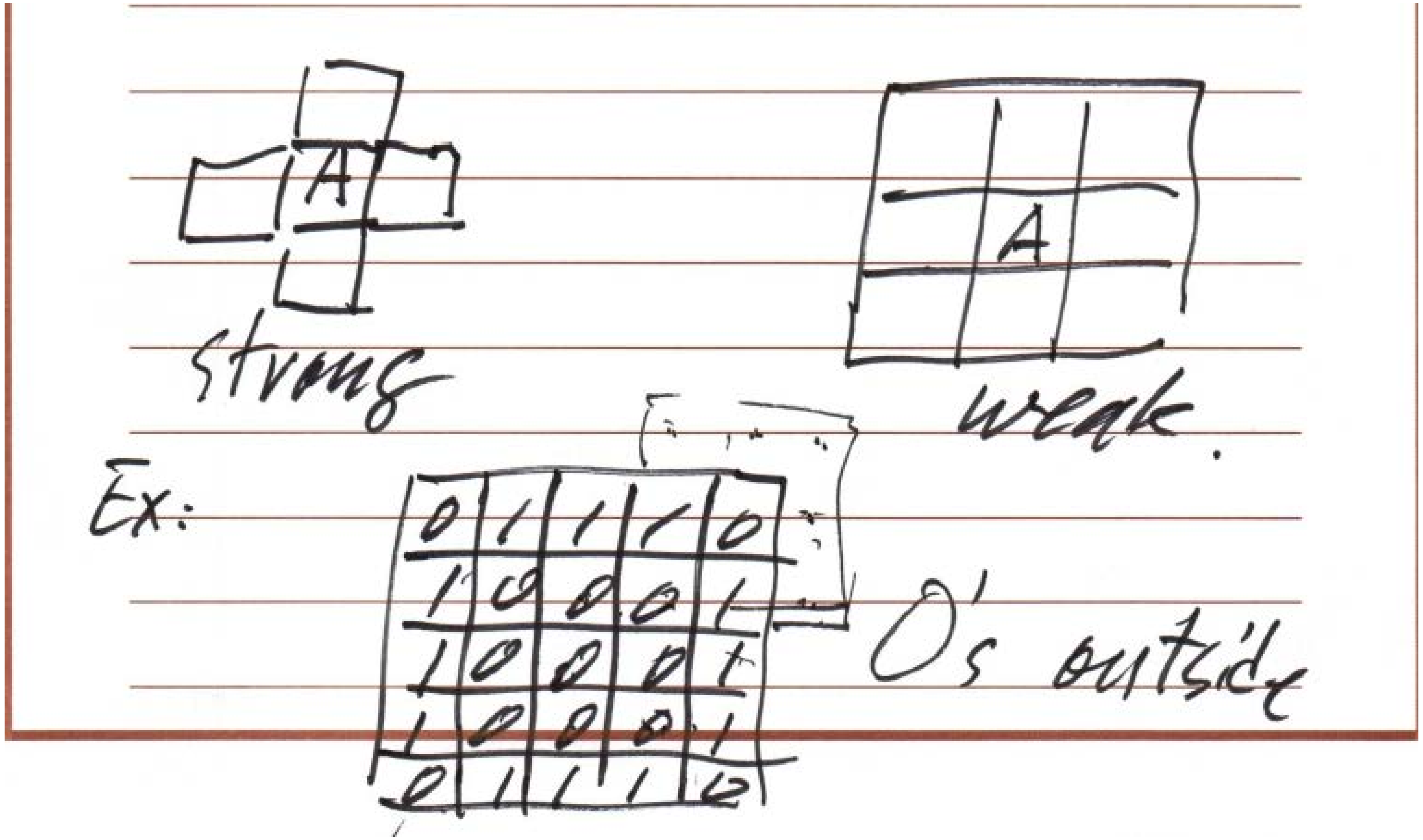
MPEG-7 Shape Dataset



Binary Image Connectivity

- 1: object pixel (black)
- 0: background pixel (white)
- 4-connectivity:
  - A pixel is 4-connected if its value is the same as one (or more) of its four nearest neighbors
- 8-connectivity:
  - A pixel is 8-connected if its value is the same as one (or more) of its eight nearest neighbors

Example



Object Counting

- How many objects in the last example?
  - 4-connectivity rule
    - No. of objects: 4
    - No. of background regions: 2
  - 8-connectivity rule
    - No. of objects: 1
    - No. of background regions: 1
  - Hybrid connectivity rule
    - 8-connectivity for objects and 4-connectivity for background
    - No. of objects: 1
    - No. of background regions: 2

Another Connectivity Measure: Bond

- Side connectivity: 2 pts.
- Corner connectivity: 1 pt.
- Bond= 2 x (no. of the same side neighbors)

  + 1 x (no. of the same corner neighbors)
- Example:

0	0	0		0	1	1		0	0	0
1	1	0		1	1	1		0	1	0
0	1	1		1	1	1		0	0	0

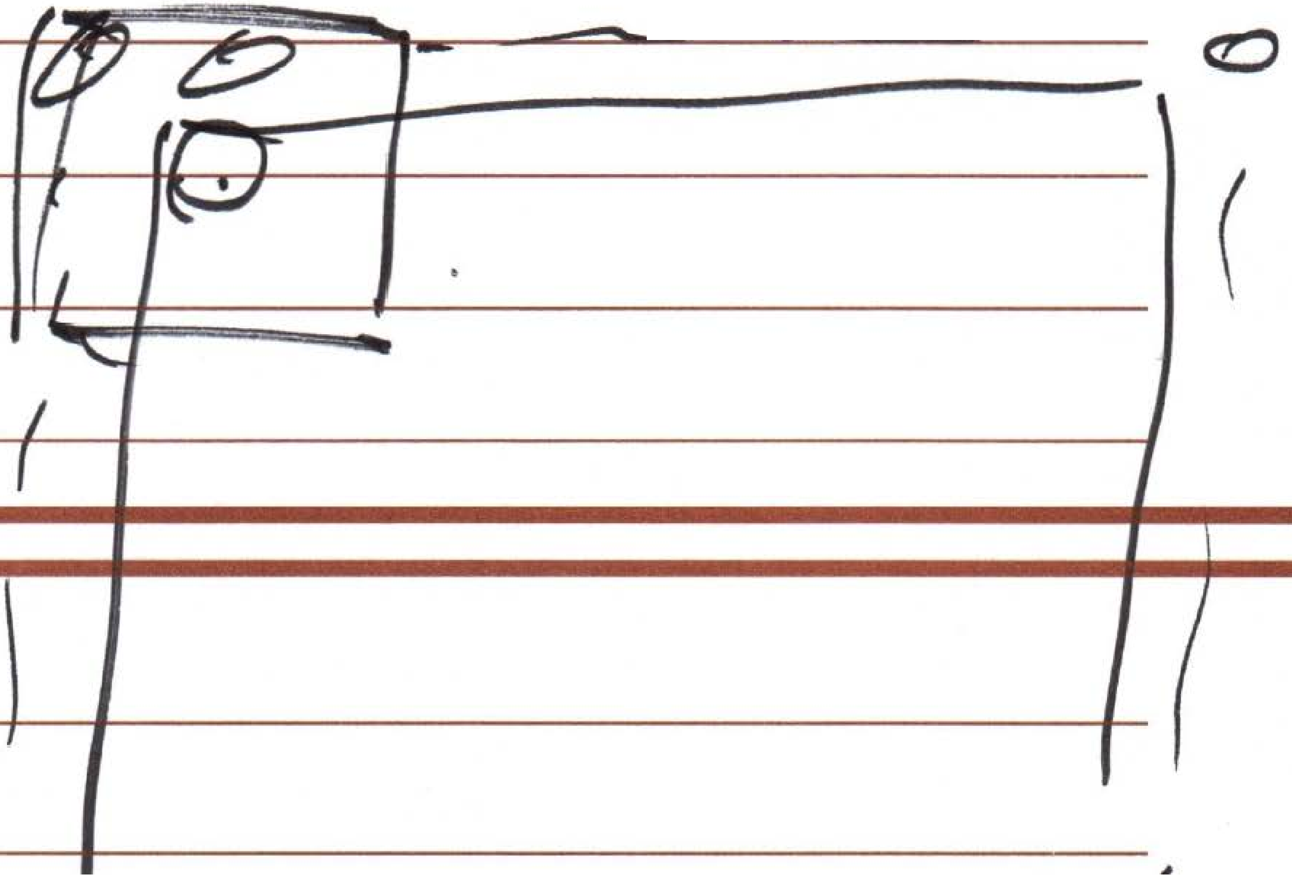
$B=5$ $B=11$ $B=0$

Basic Morphological Filters

Hit or Miss Morphological Filters

- Use an odd-size mask (typically 3x3) to scan a binary image
- Pre-define a set of hit masks
- If the underlying patch pattern matches one of the hit masks, it is called a “hit”. Otherwise, it is called a “miss”
- Action:
    - Hit -> take action on the central pixel (usually, change 0 to 1, change 1 to 0)
    - Miss -> no action on the central pixel (copy the central pixel value to the same location of the output image)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Hit

If hit. take action

* immediate action

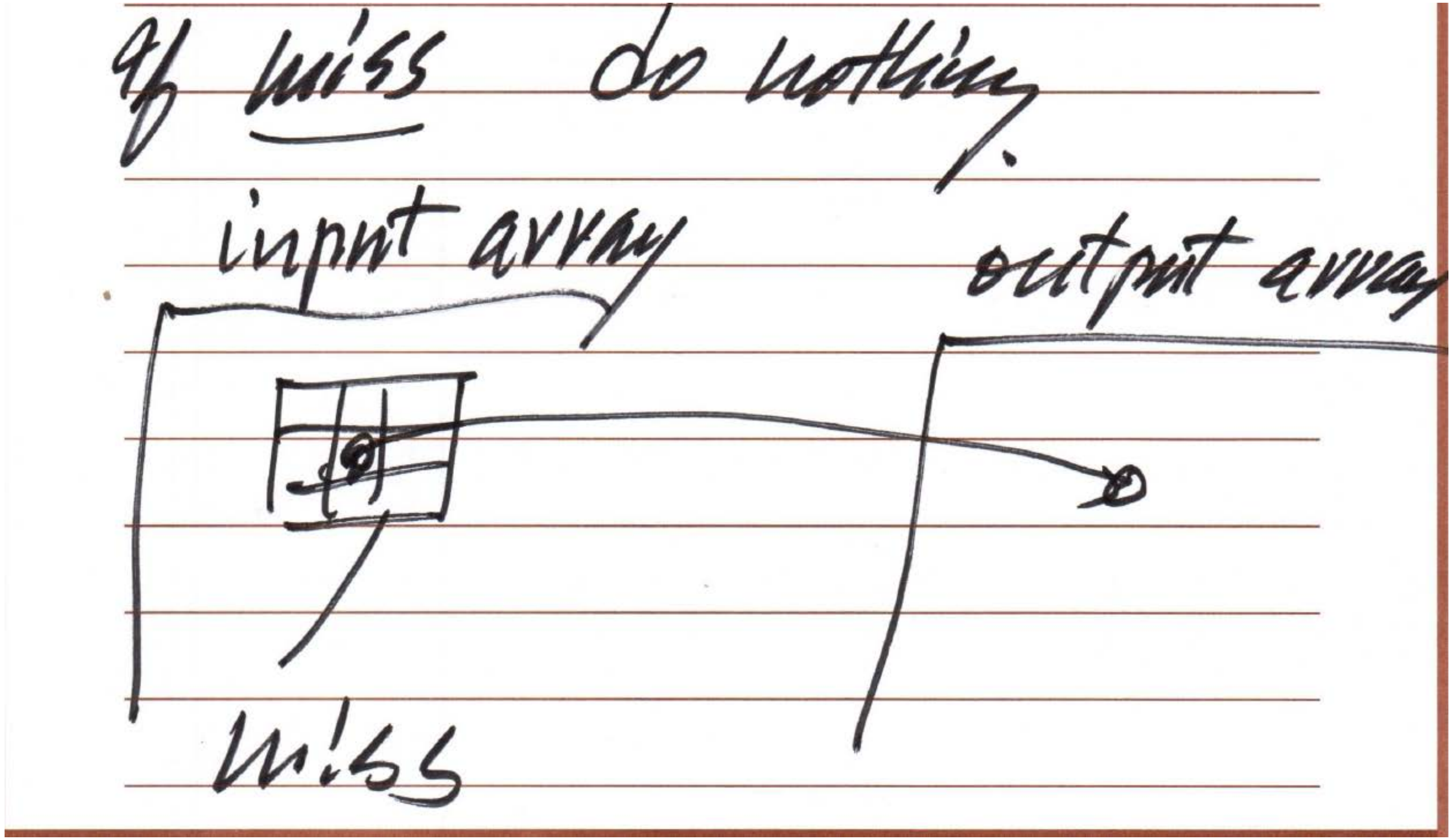
$1 \rightarrow 0$
$0 \rightarrow 1$

* label it and consider whether to take action later

Simple Filter

Advanced Filter

Miss



Example: Isolated Dots Removal

- Isolated black dots can be viewed as noise in black/white images

Mask Design

~~hit~~ mask. (a morphological filter).

$$
\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \rightarrow \text{hit} \quad (\text{center pixel } 1 \rightarrow 0)
$$

otherwise $\longrightarrow$ miss

Mathematical Representation of Morphological Filters

$F(j,k)$

input

$$\begin{matrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{matrix}$$

$X, X_i, i = 0, \cdots, 7$

logical variables

0, 1

Write a logical express. to represent the filter.

Logical Expression of Noise Removal Filter

$$G(j,k) = X \cap (X_0 \cup X_1 \cup \cdots \cup X_7)$$

output

logical "OR"

logical "AND"

Simple Morphological Filters

- Additive Filters
  - Action: Converting “0” (white, background) in the input image to “1” (black, foreground) in the output image
- Subtractive Filters
  - Action: Converting “1” (black, foreground) in the input image to “0” (white, background) in the output image

Example of Additive Filters (1)

- Interior Fill

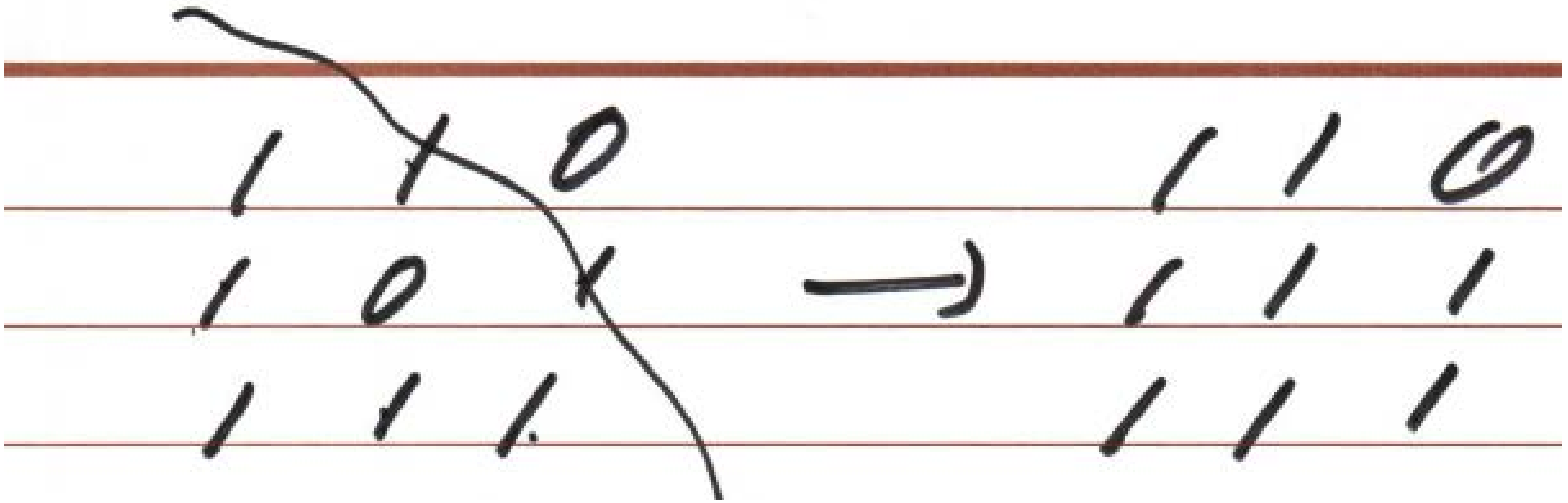
$$\begin{bmatrix} x & 1 & x \\ 1 & 0 & 1 \\ x & 1 & x \end{bmatrix}$$

x: Don't care term.

hit masks $2^4 = 16$.

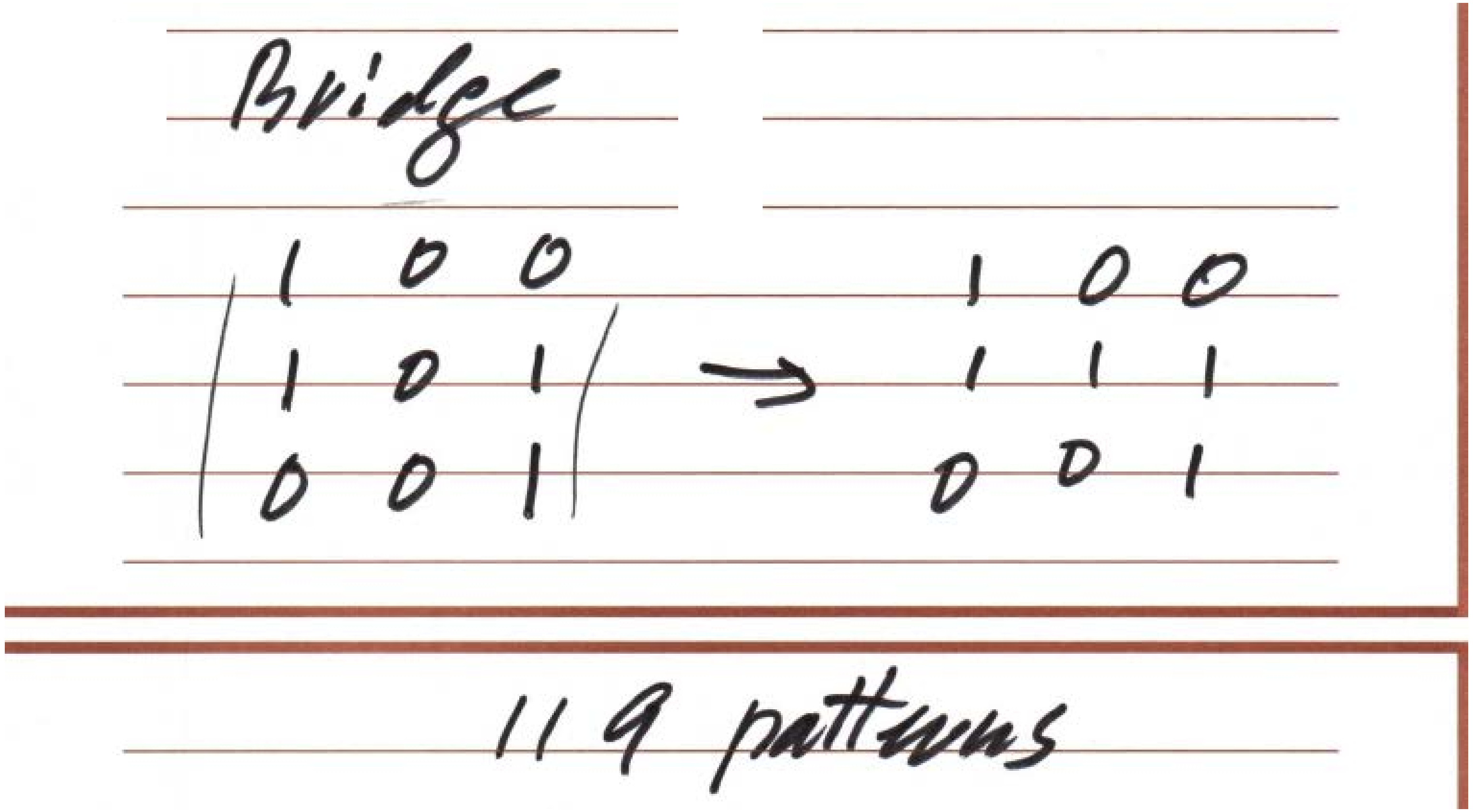
Example of Additive Filters (2)

- Diagonal Fill



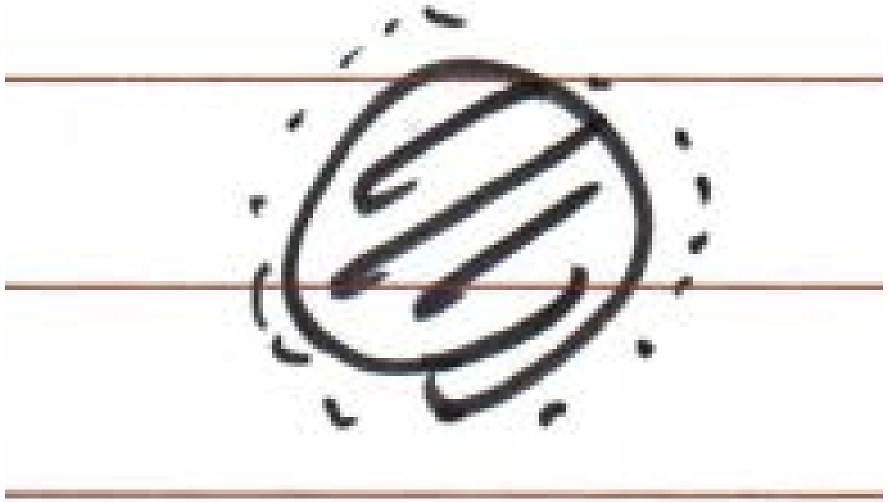
Example of Additive Filters (3)

- Bridge



Example of Additive Filters (4)

- Eight-Neighbor Dilation
  - Goal: grow the size of an object

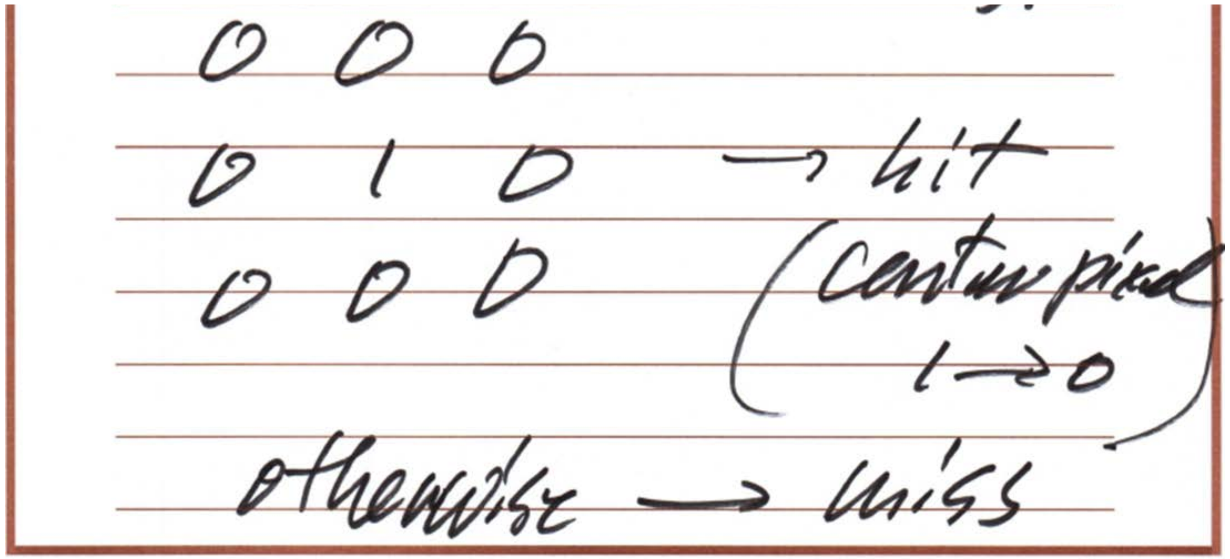


$$\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \longrightarrow \begin{array}{ccc} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}$$

any of 8 neighbors is "me"

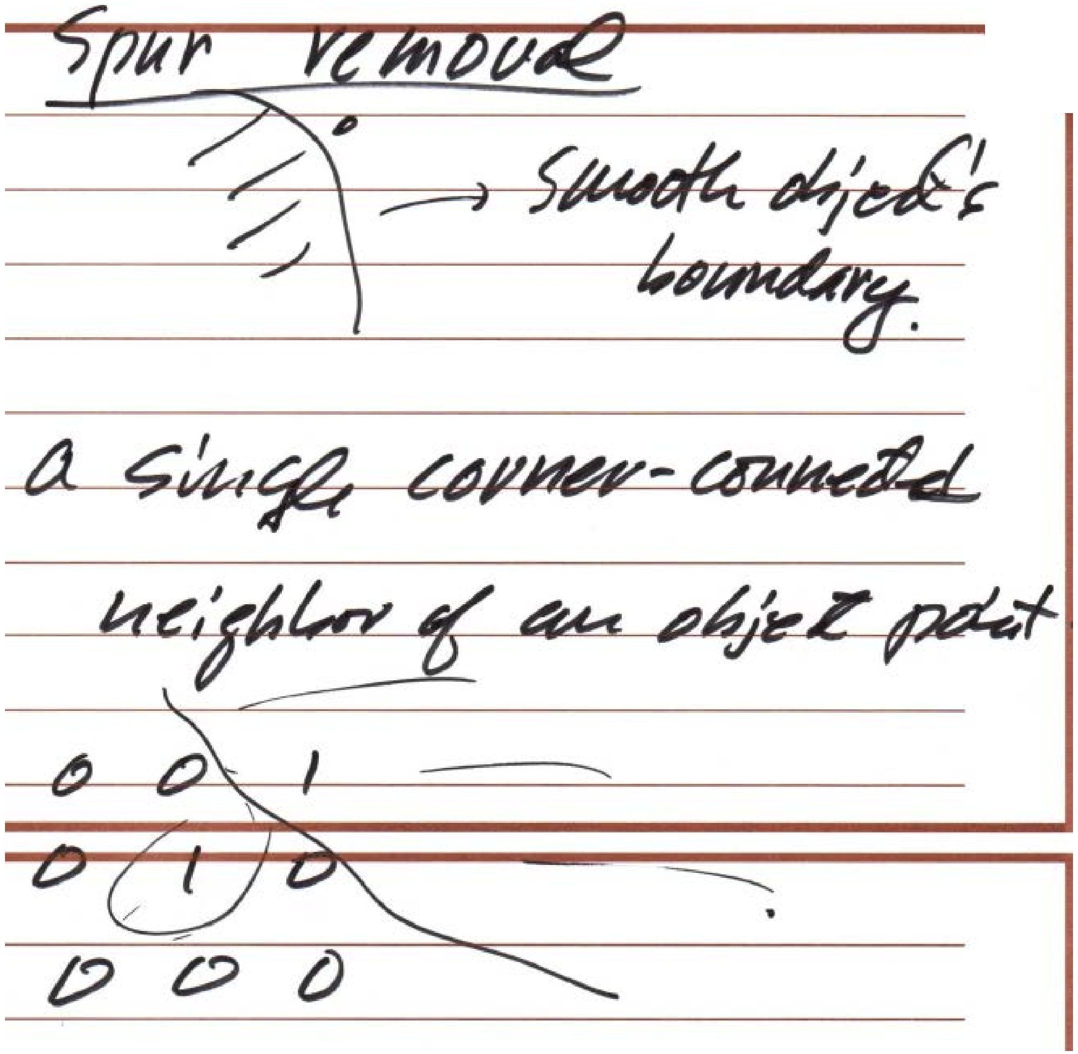
Example of Subtractive Filters (1)

- Isolated pixel removal



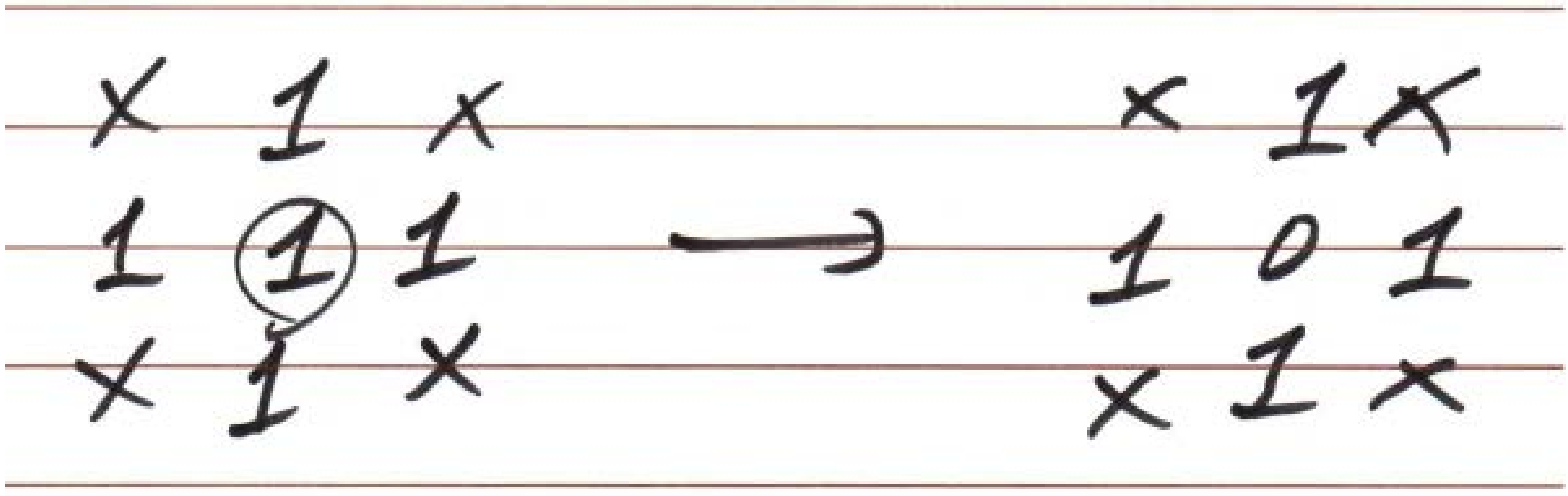
Example of Subtractive Filters (2)

- Spur removal

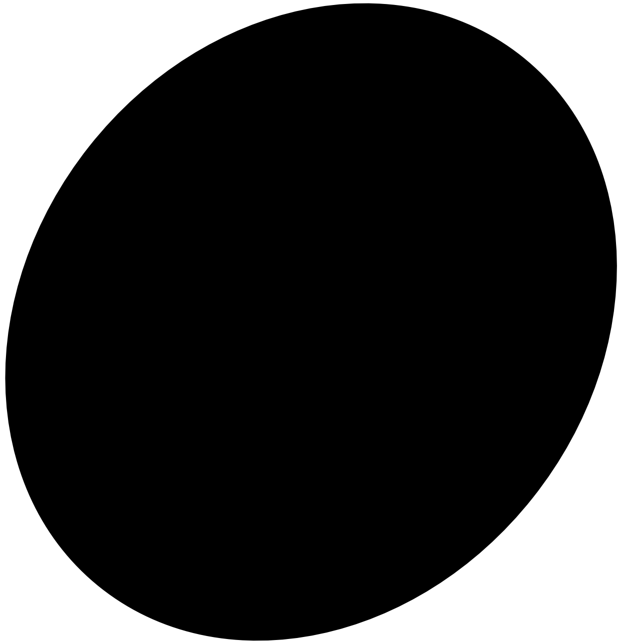


Example of Subtractive Filters (3)

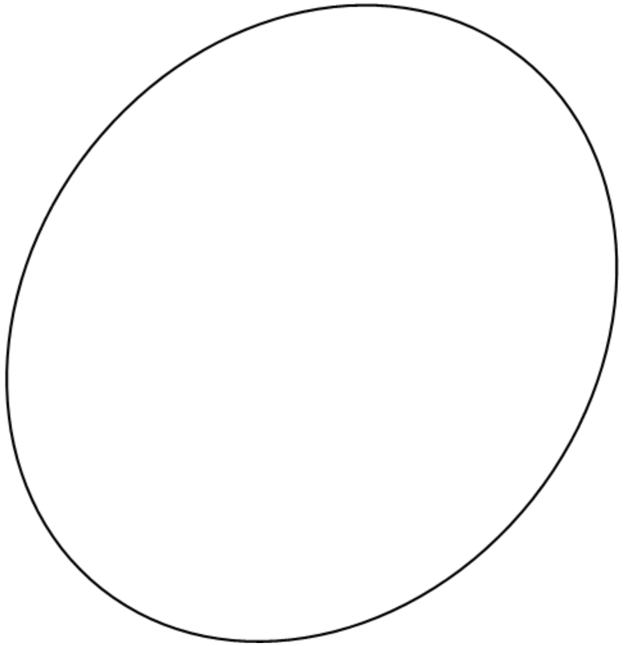
- Interior Pixel Removal



Overall Effect of Interior Pixel Removal



Input Image



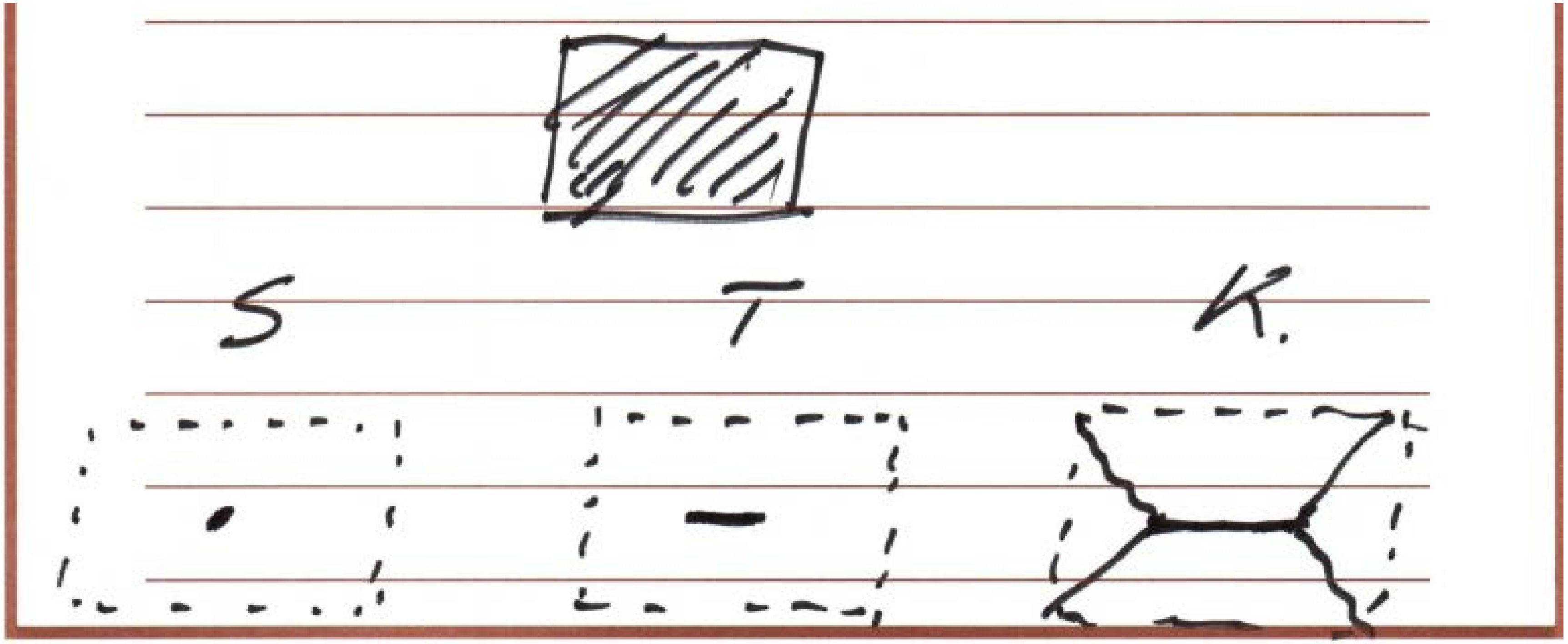
Output Image

Advanced Morphological Filters

Advanced Morphological Filters

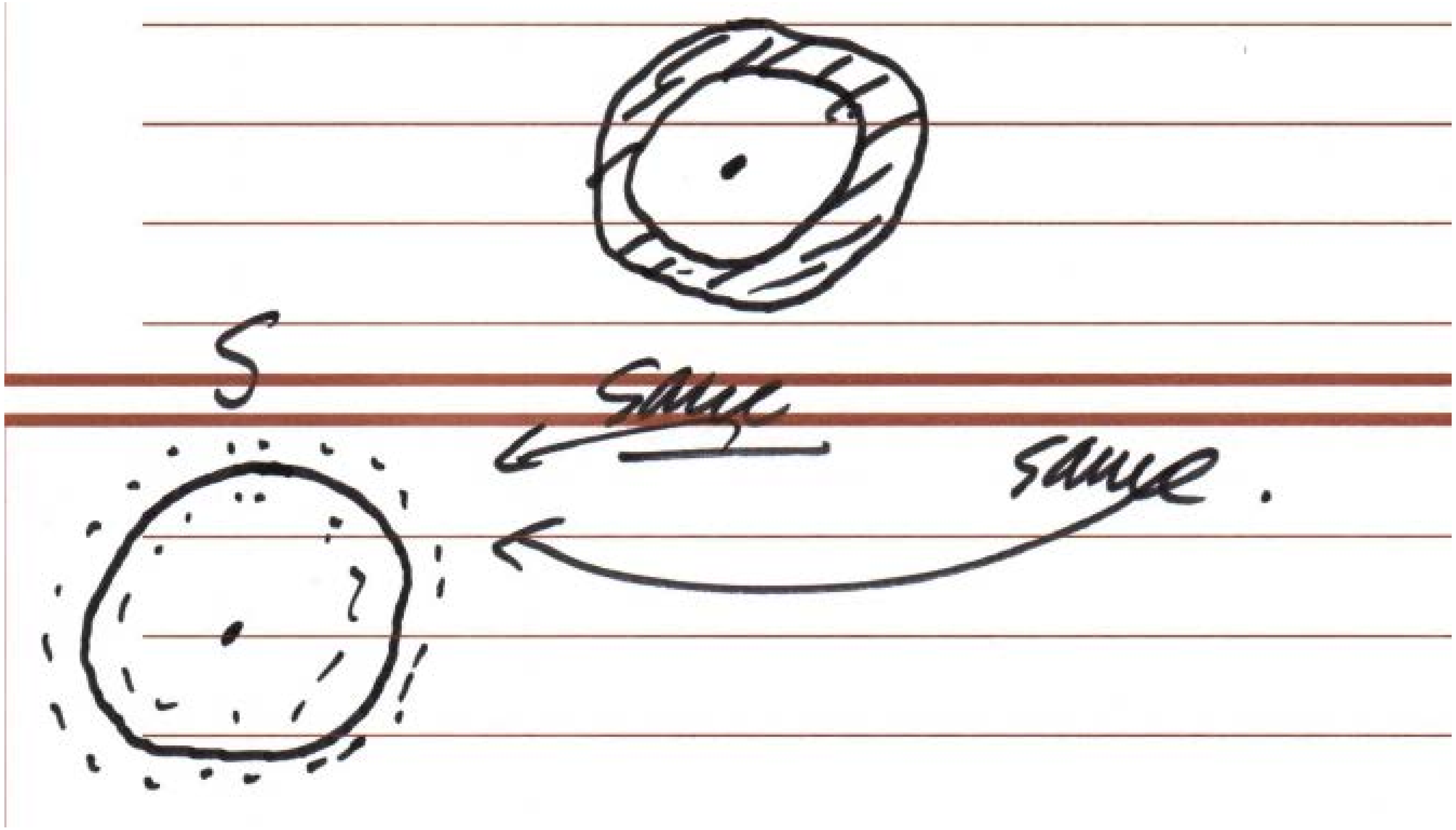
- Three subtractive filters
  - Shrinking
  - Thinning
  - Skeletonizing
- One additive filter
  - Thickening

Examples (1)

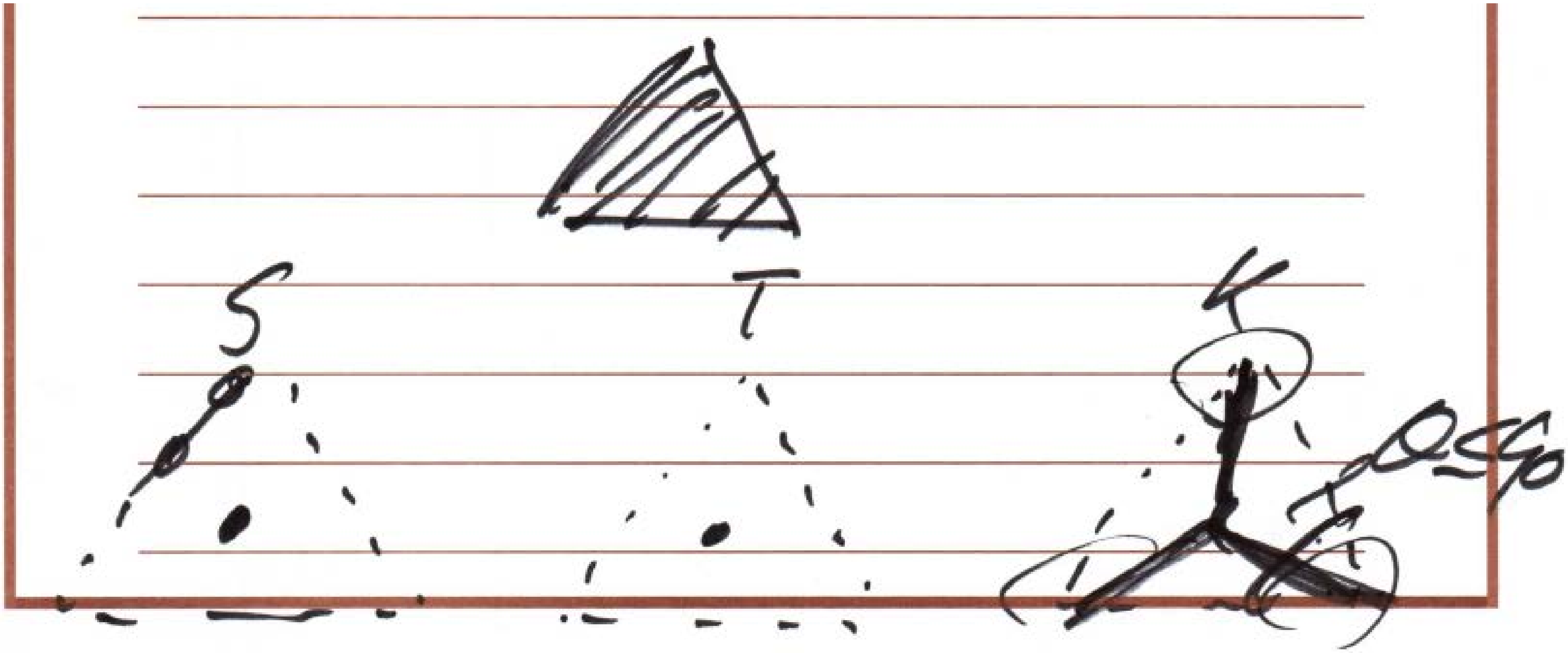


Examples (2)

Examples (3)



Examples (4)



One-Stage Filter Design

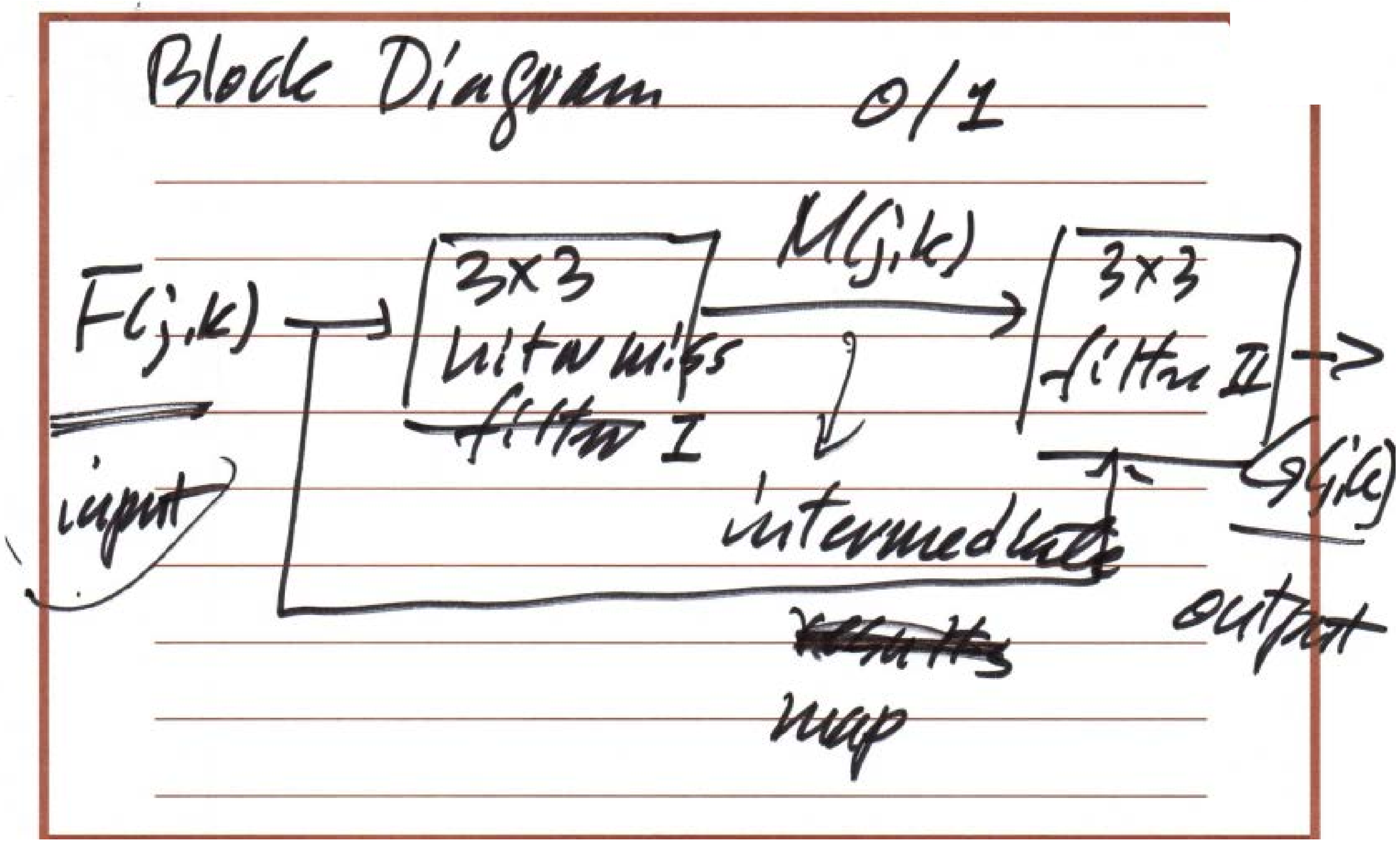
- If we adopt the single-stage hit-or-miss filter solution, the filter size has to be of 5x5=25

# of patterns (excluding center)

$$= \boxed{2^{24}}$$

Two-Stage Filter Design

- To simplify the design process, we decompose the one-stage 5x5 filter to two stages in cascade, where each stage consists of a 3x3 filter



Purpose of 1st Stage Design

1st stage

$$\begin{matrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{matrix} \longrightarrow \begin{matrix} M_3 & M_2 & M_1 \\ M_4 & M & M_0 \\ M_5 & M_6 & M_7 \end{matrix}$$

$F(j,k)$ $\quad$ $M(j,k)$

$M(j,k)=1$ X is an candidate for erasure

$M(j,k)=0$. X is not a candidate for erasure.

Purpose of 2nd Stage Design (1)

$G(j,k) = X \cap [\bar{M} \cup P(M, M_0 \cdots$

$\cdots M_7)]$

Consider $X=1$

$M=1.$

If $P(M, M_0 \cdots, M_7) = 1$

$G(j,k) = X$ Do nothing

(miss)

Purpose of 2nd Stage Design (2)

If $P(M, M_0 \cdots, M_7) = 0$

$G(j, k) = X \cap 0 = 0$

Erasure. (hit)

$P(M, M_0 \cdots M_7) = 1$

Erasure inhibiting logical variable.

First Stage (or M) Hit Masks (1)

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TABLE 14.3-1. Shrink, Thin and Skeletonize Conditional Mark Patterns [$M = 1$ if hit]

Table	Bond	Pattern							
		0 0 1	1 0 0	0 0 0	0 0 0				
S	1	0 1 0	0 1 0	0 1 0	0 1 0				
		0 0 0	0 0 0	1 0 0	0 0 1				
		0 0 0	0 1 0	0 0 0	0 0 0				
S	2	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 0	0 0 0	0 0 0	0 1 0				
		0 0 1	0 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0
S	3	0 1 1	0 1 0	0 1 0	1 1 0	1 1 0	0 1 0	0 1 0	0 1 1
		0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	1 1 0	0 1 1	0 0 1
		0 1 0	0 1 0	0 0 0	0 0 0				
TK	4	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 0	0 0 0	0 1 0	0 1 0				
		0 0 1	1 1 1	1 0 0	0 0 0				
STK	4	0 1 1	0 1 0	1 1 0	0 1 0				
		0 0 1	0 0 0	1 0 0	1 1 1				

(Continued)

First Stage (or M) Hit Masks (2)

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		1 1 0	0 1 0	0 1 1	0 0 1				
ST	5	0 1 1	0 1 1	1 1 0	0 1 1				
		0 0 0	0 0 1	0 0 0	0 1 0				
		0 1 1	1 1 0	0 0 0	0 0 0				
ST	5	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 0	0 0 0	1 1 0	0 1 1				
		1 1 0	0 1 1						
ST	6	0 1 1	1 1 0						
		0 0 1	1 0 0						
		1 1 1	0 1 1	1 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 1
STK	6	0 1 1	0 1 1	1 1 0	1 1 0	1 1 0	1 1 0	0 1 1	0 1 1
		0 0 0	0 0 1	0 0 0	1 0 0	1 1 0	1 1 1	1 1 1	0 1 1

(Continued)

First Stage (or M) Hit Masks (3)

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TABLE 14.3-1. (Continued)

Table	Bond	Pattern							
		1 1 1	1 1 1	1 0 0	0 0 1				
STK	7	0 1 1	1 1 0	1 1 0	0 1 1				
		0 0 1	1 0 0	1 1 1	1 1 1				
		0 1 1	1 1 1	1 1 0	0 0 0				
STK	8	0 1 1	1 1 1	1 1 0	1 1 1				
		0 1 1	0 0 0	1 1 0	1 1 1				
		1 1 1	0 1 1	1 1 1	1 1 1	1 1 1	1 1 0	1 0 0	0 0 1
STK	9	0 1 1	0 1 1	1 1 1	1 1 1	1 1 0	1 1 0	1 1 1	1 1 1
		0 1 1	1 1 1	1 0 0	0 0 1	1 1 0	1 1 1	1 1 1	1 1 1
		1 1 1	1 1 1	1 1 1	1 0 1				
STK	10	0 1 1	1 1 1	1 1 0	1 1 1				
		1 1 1	1 0 1	1 1 1	1 1 1				
		1 1 1	1 1 1	1 1 0	0 1 1				
K	11	1 1 1	1 1 1	1 1 1	1 1 1				
		0 1 1	1 1 0	1 1 1	1 1 1				

Second Stage (or P) Hit Masks for Shrinking and Thinning (1)

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TABLE 14.3-2. Shrink and Thin Unconditional Mark Patterns
[$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit][a]

Pattern							
Spur		Single 4-connection					
0 0 M	M 0 0	0 0 0	0 0 0				
0 M 0	0 M 0	0 M 0	0 M M				
0 0 0	0 0 0	0 M 0	0 0 0				
L Cluster							
0 0 M	0 M M	M M 0	M 0 0	0 0 0	0 0 0	0 0 0	0 0 0
0 M M	0 M 0	0 M 0	M M 0	M M 0	0 M 0	0 M 0	0 M M
0 0 0	0 0 0	0 0 0	0 0 0	M 0 0	M M 0	0 M M	0 0 M
4-Connected offset							
0 M M	M M 0	0 M 0	0 0 M				
M M 0	0 M M	0 M M	0 M M				
0 0 0	0 0 0	0 0 M	0 M 0				
Spur corner cluster							
0 A M	M B 0	0 0 M	M 0 0				
0 M B	A M 0	A M 0	0 M B				
M 0 0	0 0 M	M B 0	0 A M				

(Continued)

Second Stage (or P) Hit Masks for Shrinking and Thinning (2)

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Corner cluster

M M D
M M D
D D D

Tee branch

D M 0	0 M D	0 0 D	D 0 0	D M D	0 M 0	0 M 0	D M D
M M M	M M M	M M M	M M M	M M 0	M M 0	0 M M	0 M M
D 0 0	0 0 D	0 M D	D M 0	0 M 0	D M D	D M D	0 M 0

Vee branch

M D M	M D C	C B A	A D M
D M D	D M B	D M D	B M D
A B C	M D A	M D M	C D M

Diagonal branch

D M 0	0 M D	D 0 M	M 0 D
0 M M	M M 0	M M 0	0 M M
M 0 D	D 0 M	0 M D	D M 0

[a] $A \cup B \cup C = 1$ $D = 0 \cup 1$ $A \cup B = 1$.

$\{0,1\}$

P-Hit Masks for Skeletonizing (1)

TABLE 14.3-3. Skeletonize Unconditional Mark Patterns
[$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit][a]

Pattern											
Spur											
0	0	0	0	0	0	0	0	*M*	*M*	0	0
0	*M*	0	0	*M*	0	0	*M*	0	0	*M*	0
0	0	*M*	*M*	0	0	0	0	0	0	0	0
Single 4-connection											
0	0	0	0	0	0	0	0	0	0	*M*	0
0	*M*	0	0	*M*	*M*	*M*	*M*	0	0	*M*	0
0	*M*	0	0	0	0	0	0	0	0	0	0
L corner											
0	*M*	0	0	*M*	0	0	0	0	0	0	0
0	*M*	*M*	*M*	*M*	0	0	*M*	*M*	*M*	*M*	0
0	0	0	0	0	0	0	M	0	0	*M*	0
Corner cluster											
M	*M*	*D*	*D*	*D*	*D*						
M	*M*	*D*	*D*	*M*	*M*						
D	*D*	*D*	*D*	*M*	*M*						

(Continued)

P-Hit Masks for Skeletonizing (2)

Tee branch

D	M	D	D	M	D	D	D	D	D	M	D
M	M	M	M	M	D	M	M	M	D	M	M
D	D	D	D	M	D	D	M	D	D	M	D

Vee branch

M	D	M	M	D	C	C	B	A	A	D	M
D	M	D	D	M	B	D	M	D	B	M	D
A	B	C	M	D	A	M	D	M	C	D	M

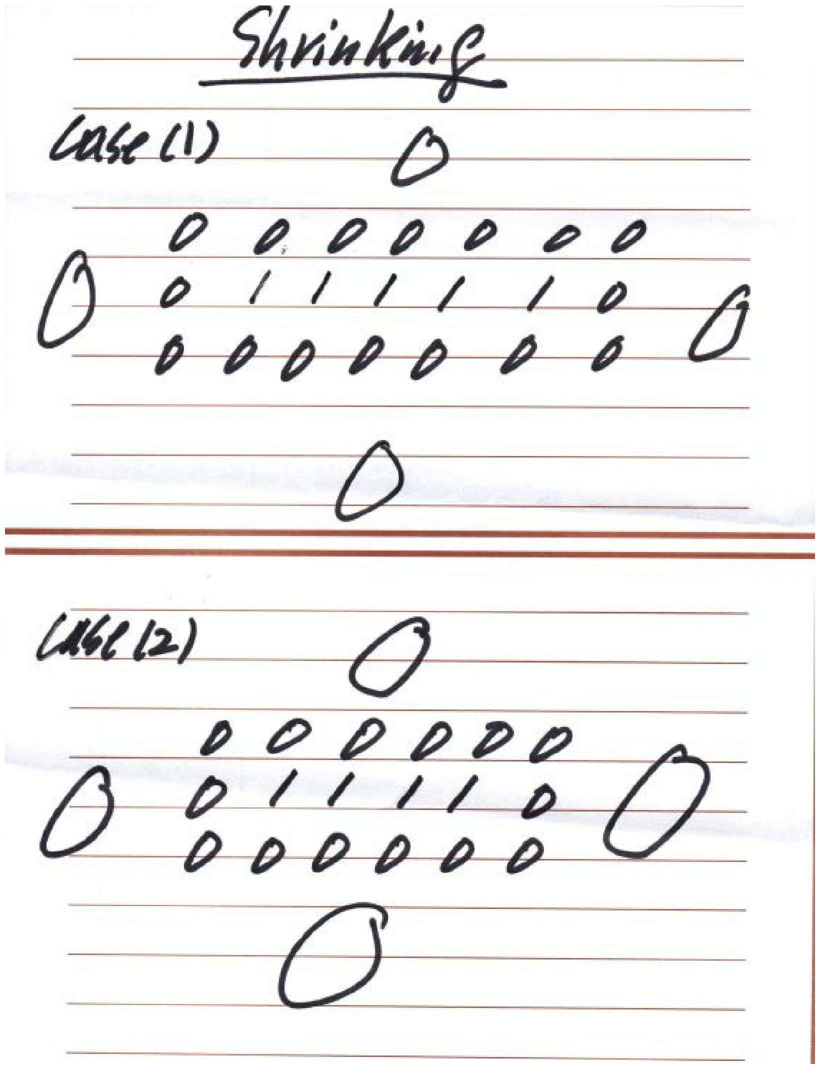
Diagonal branch

D	M	0	0	M	D	D	0	M	M	0	D
0	M	M	M	M	0	M	M	0	0	M	M
M	0	D	D	0	M	0	M	D	D	M	0

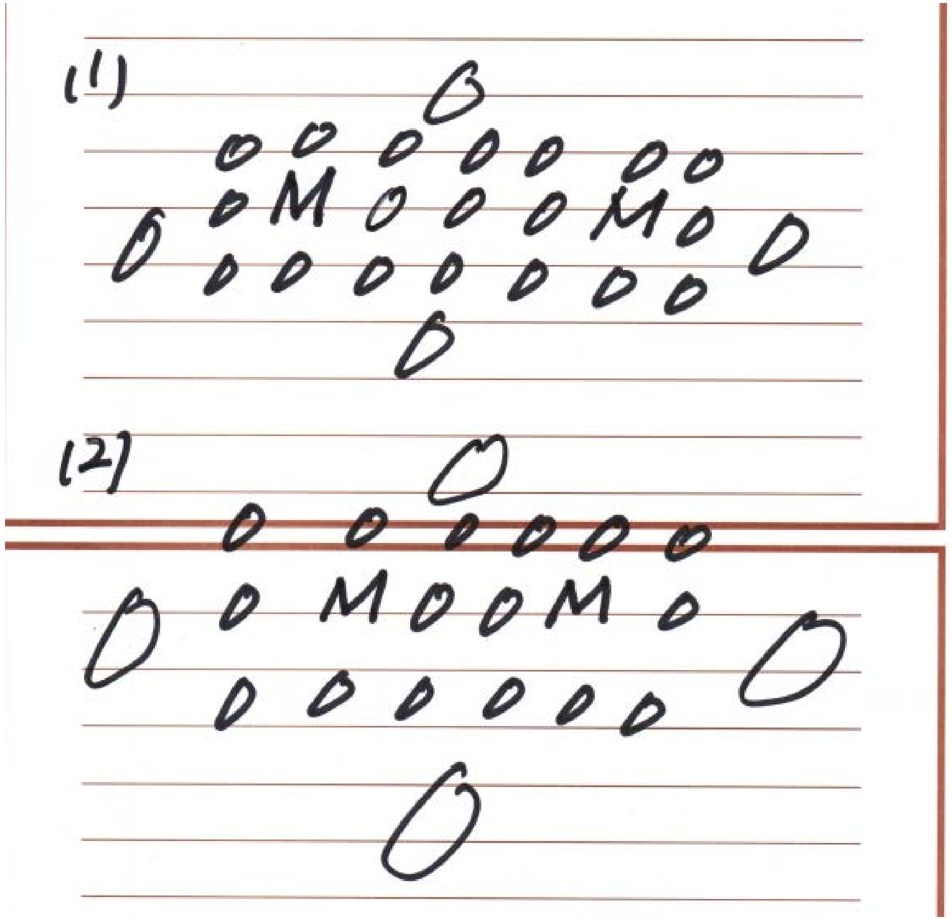
[a] $A \cup B \cup C = 1 \quad D = 0 \cup 1.$

Why Two-Stage Design?

- Consider the following two cases:

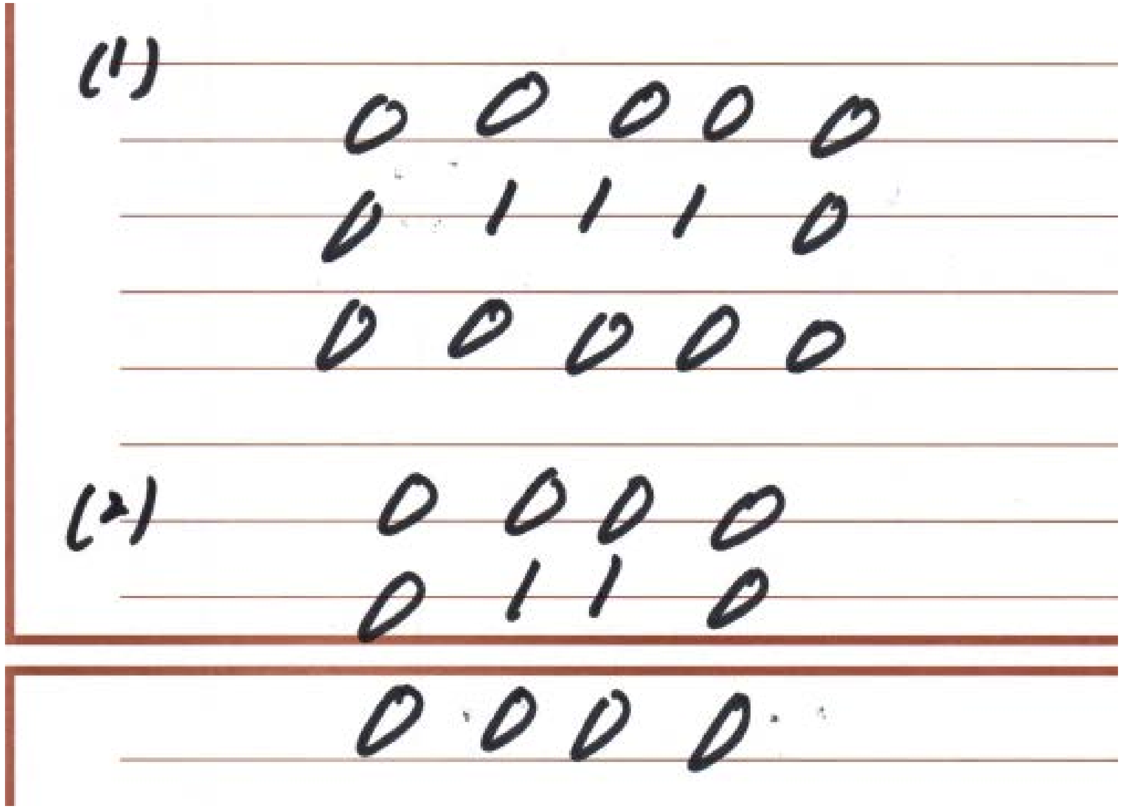


Iteration #1, M Filters

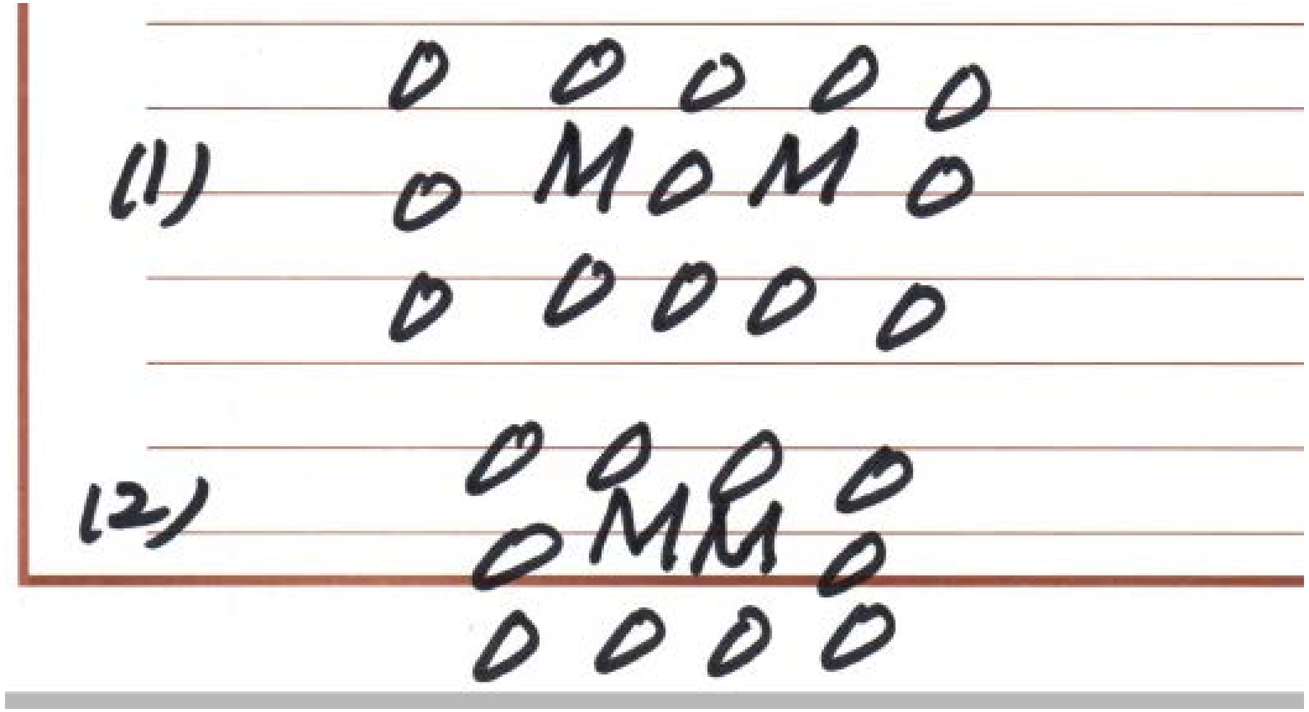


Iteration #1, P Filters

- No hit P filters - erasure is allowed
- Results:

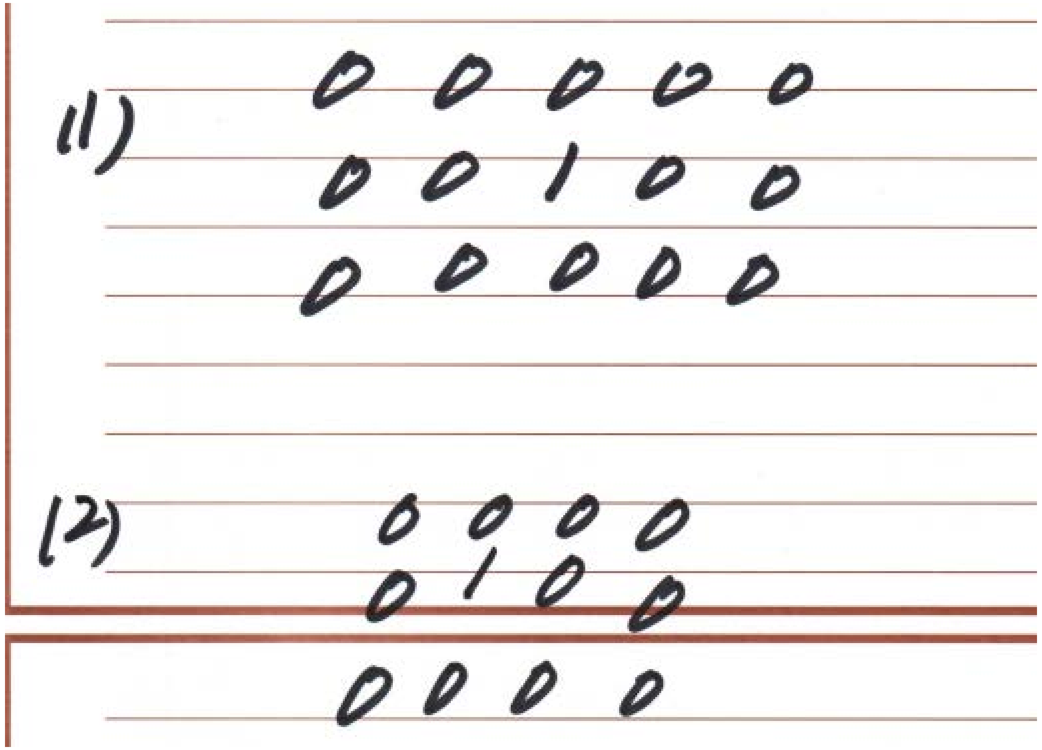


Iteration #2, M Filters



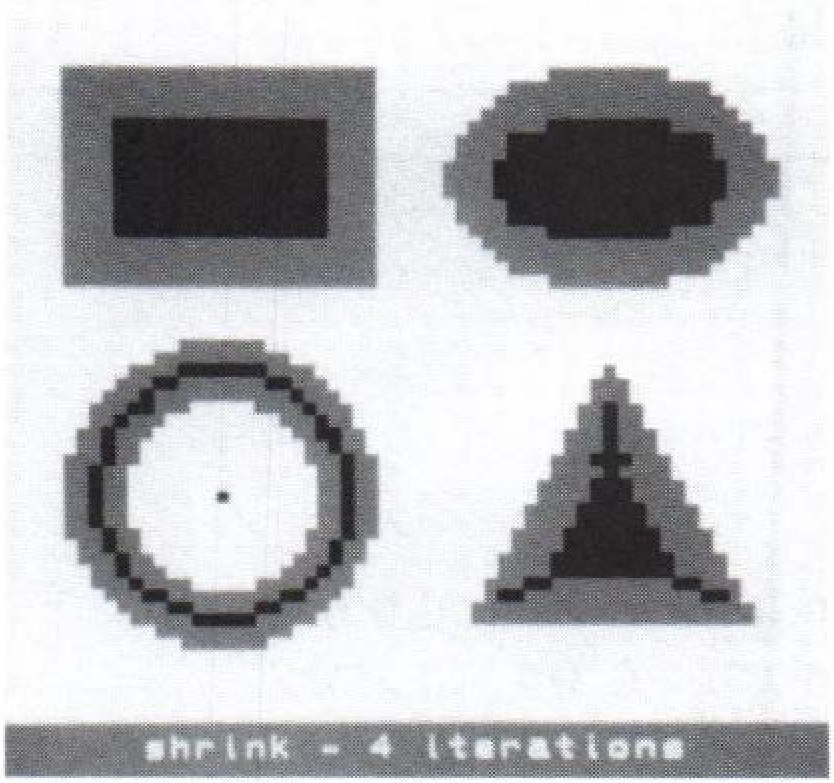
Iteration #2, P Filters

- No hit P filters in case (1) - erasure is allowed
- One hit P filter in case (2) – erasure in left M position is inhibited
- Results:

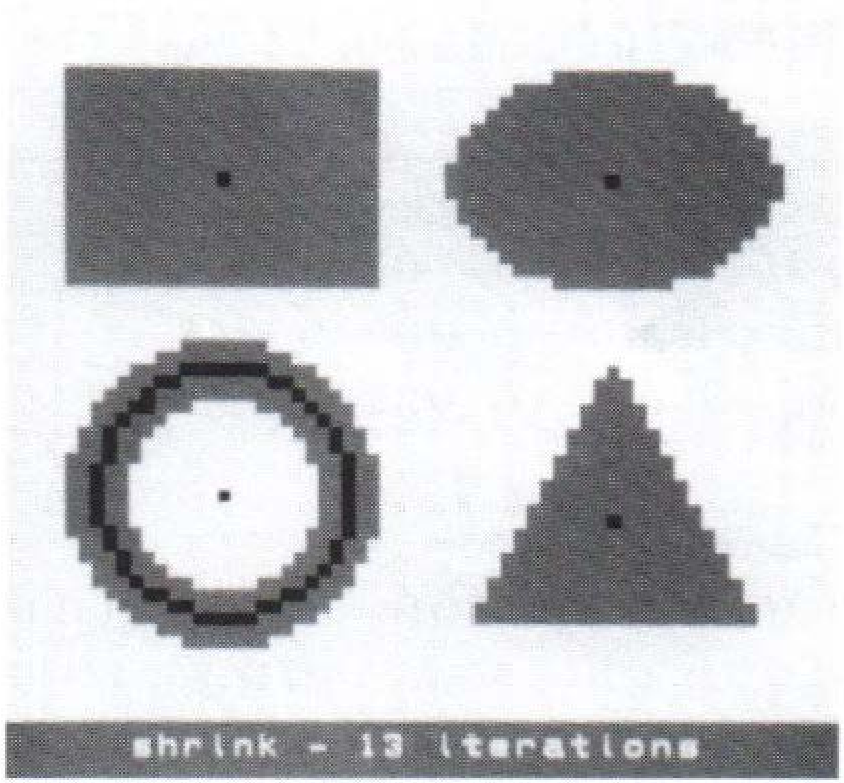


- No more change in future iterations

Iterative Application of Shrinking Filters Until Convergence

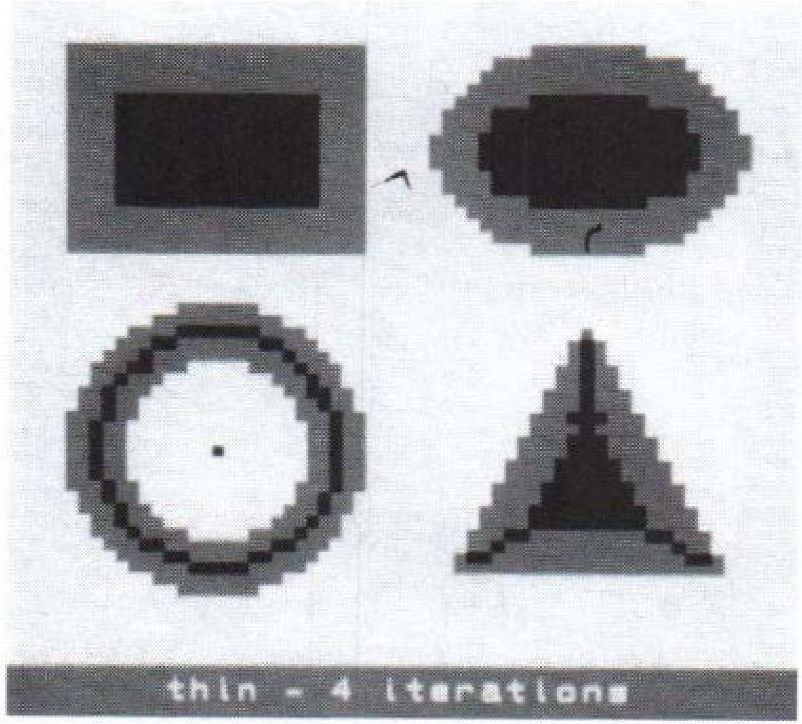


(*a*) Four iterations

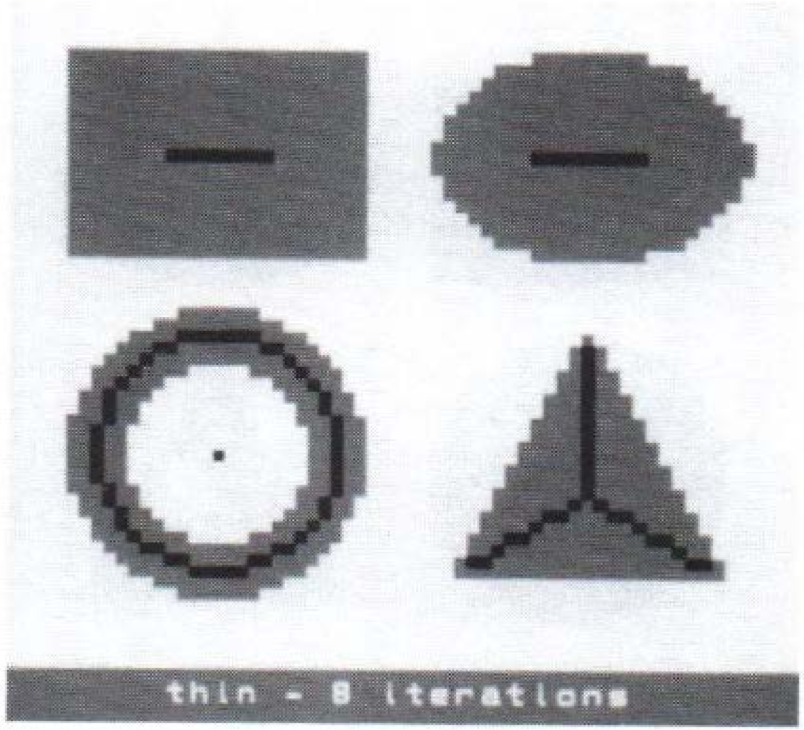


(*b*) Thirteen iterations

Iterative Application of Thinning Filters Until Convergence

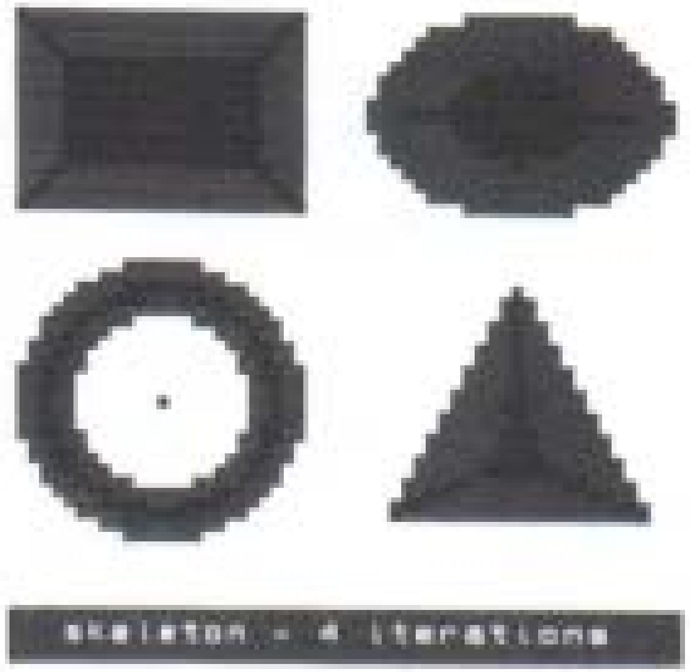


(a) Four iterations

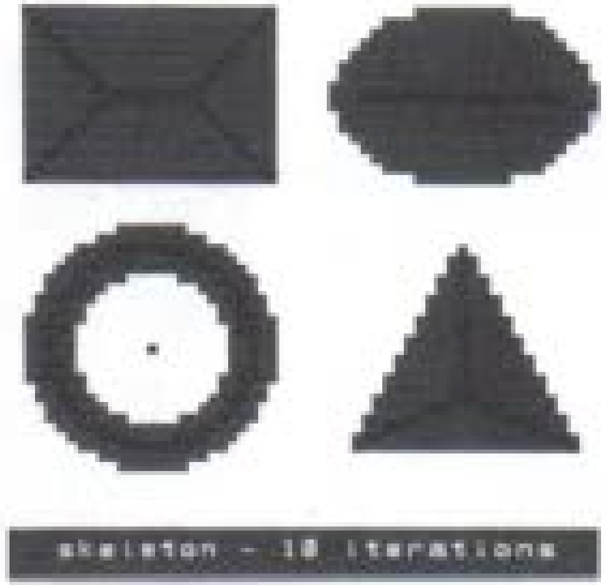


(b) Eight iterations

Iterative Application of Skeletonizing Filters Until Convergence



(*a*) Four iterations



(*b*) Ten iterations

Implementation of Morphological Filters

8 patterns

Shrinking op.

		0 0 1	0 1 1	1 1 0	1 0 0	0 0 0	0 0 0	0 0 0	0 0 0
S	3	0 1 1	0 1 0	0 1 0	1 1 0	1 1 0	0 1 0	0 1 0	0 1 1
		0 0 0	0 0 0	0 0 0	0 0 0	1 0 0	1 1 0	0 1 1	0 0 1

Bond number

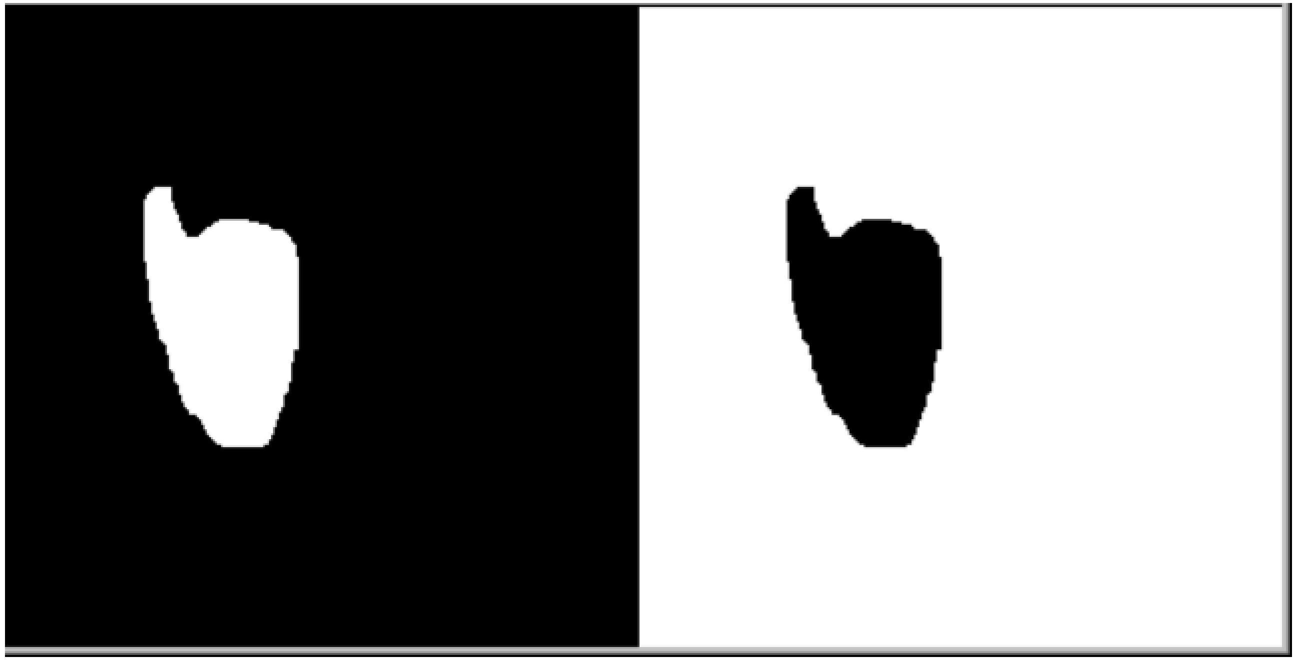
- Check filter type and bond number filter
- Center pixel always takes value “1” (if it is “0”, skip)
- Encode the eight neighbors with a binary sequence (bit-string)
- Begin with East, counter-clockwise
  - 11000000, 01100000, 00110000, 00011000
  - 00001100, 00000110, 00000011, 10000001

Image-Set-Based Morphology

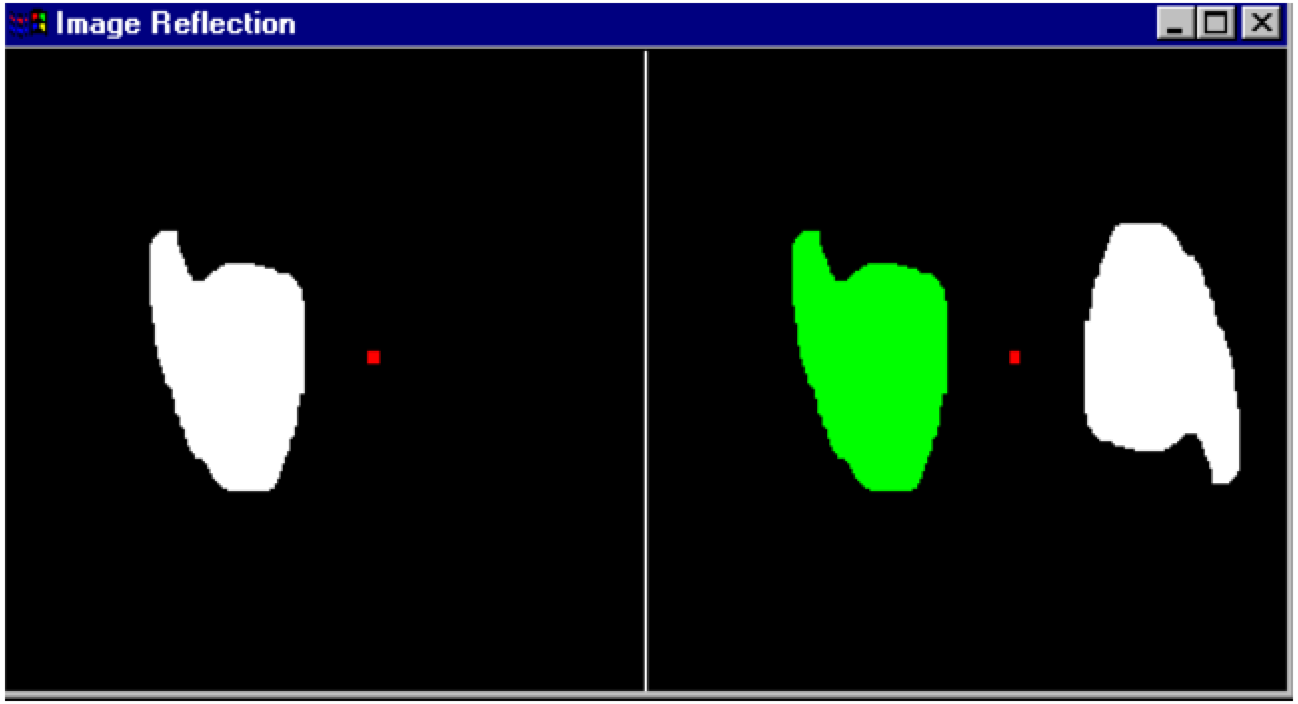
Example: Universal Set, Object Set and Complement Set

Object Set A

Complement of Object Set A

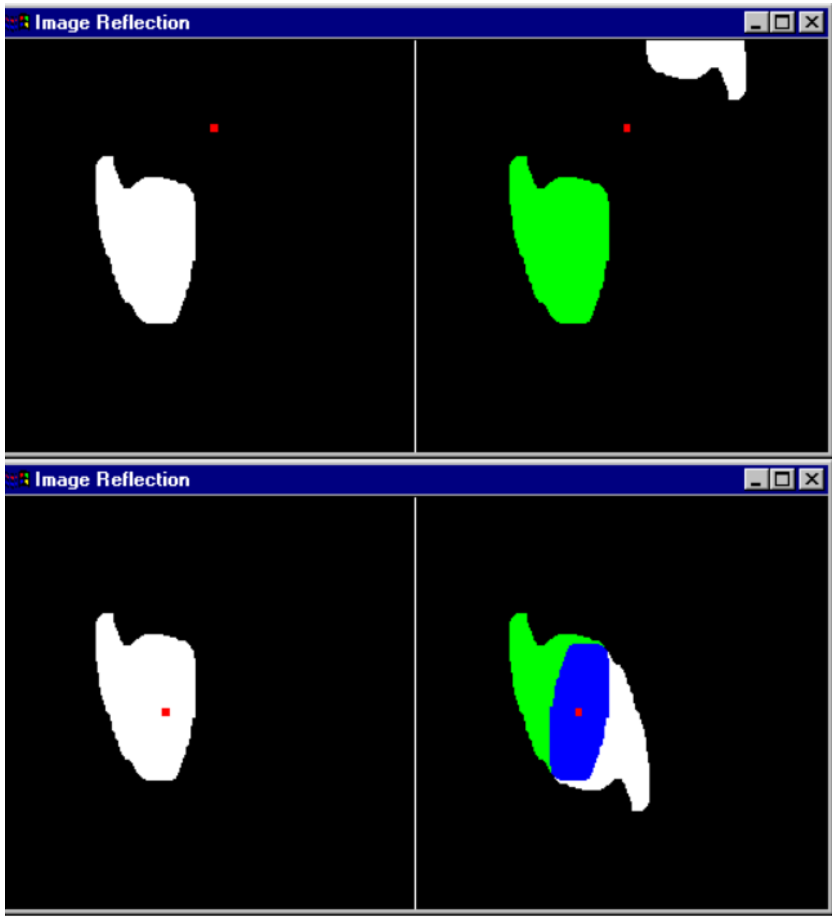


Example: Image Reflection



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

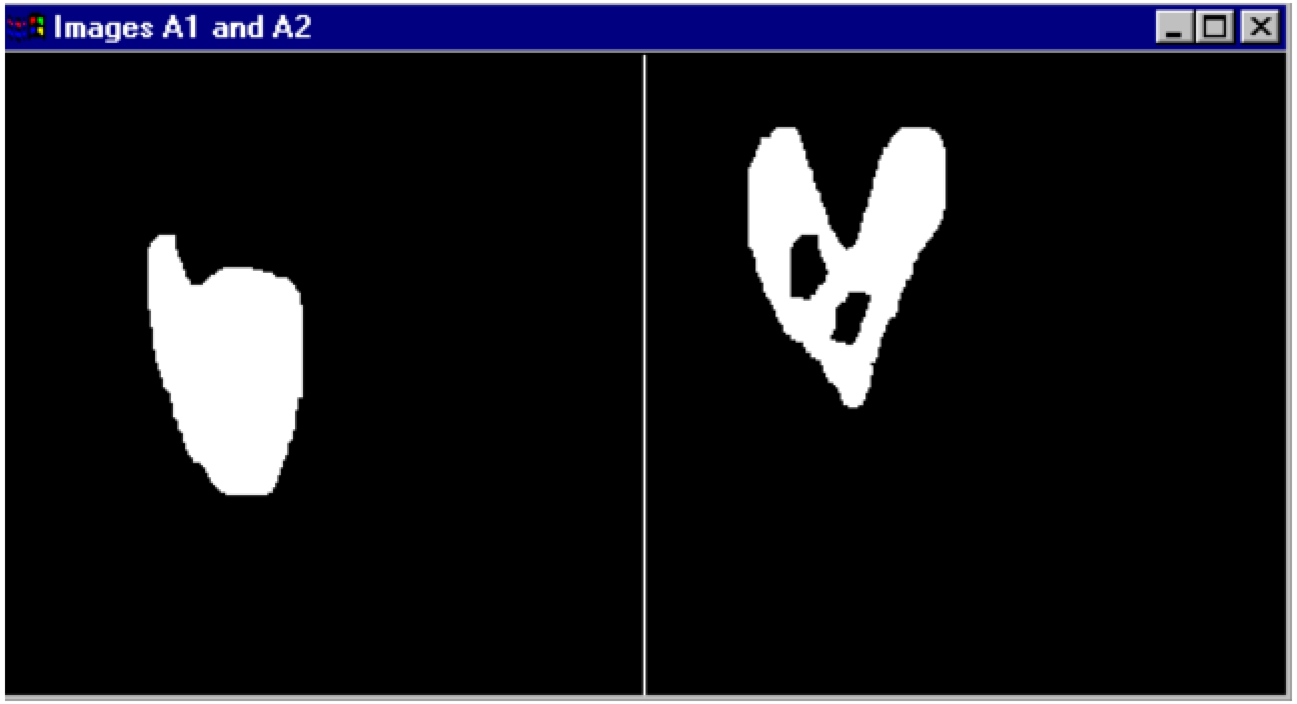
Example: Image Reflection, Union and Intersection



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

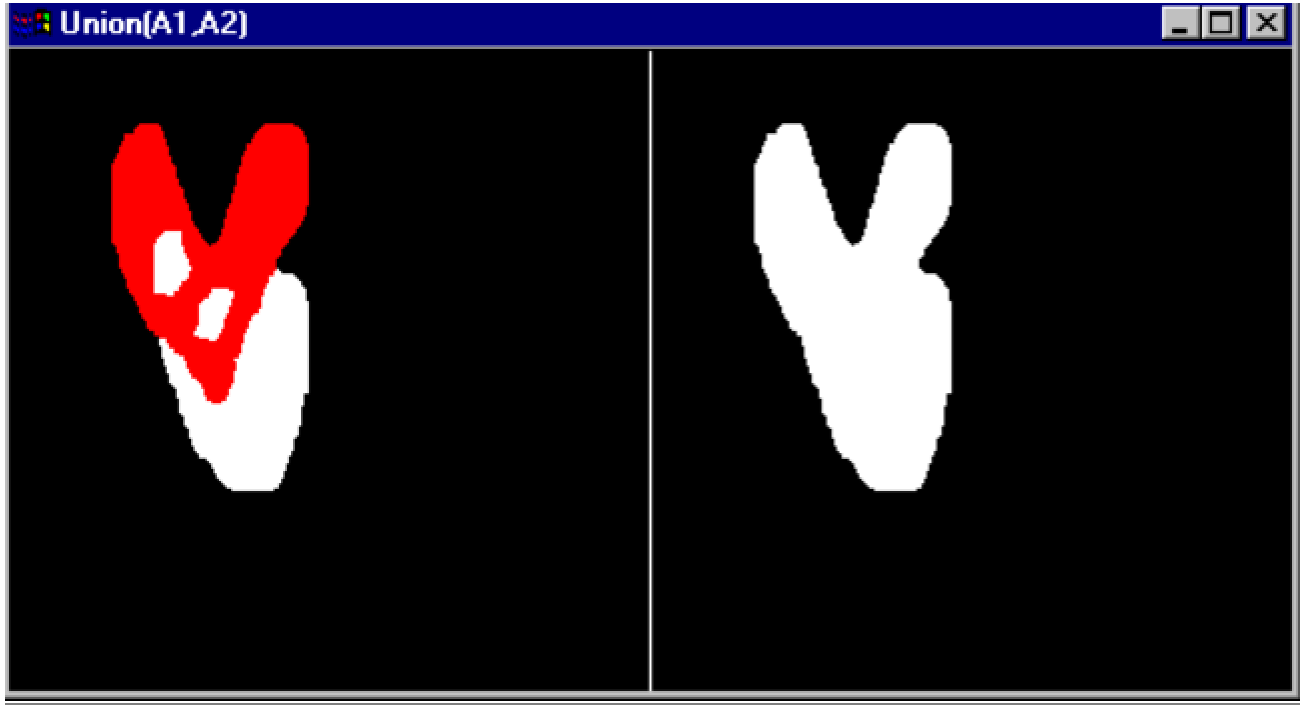
Example: Two Image Sets

A1 A2



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Union of Two Image Sets



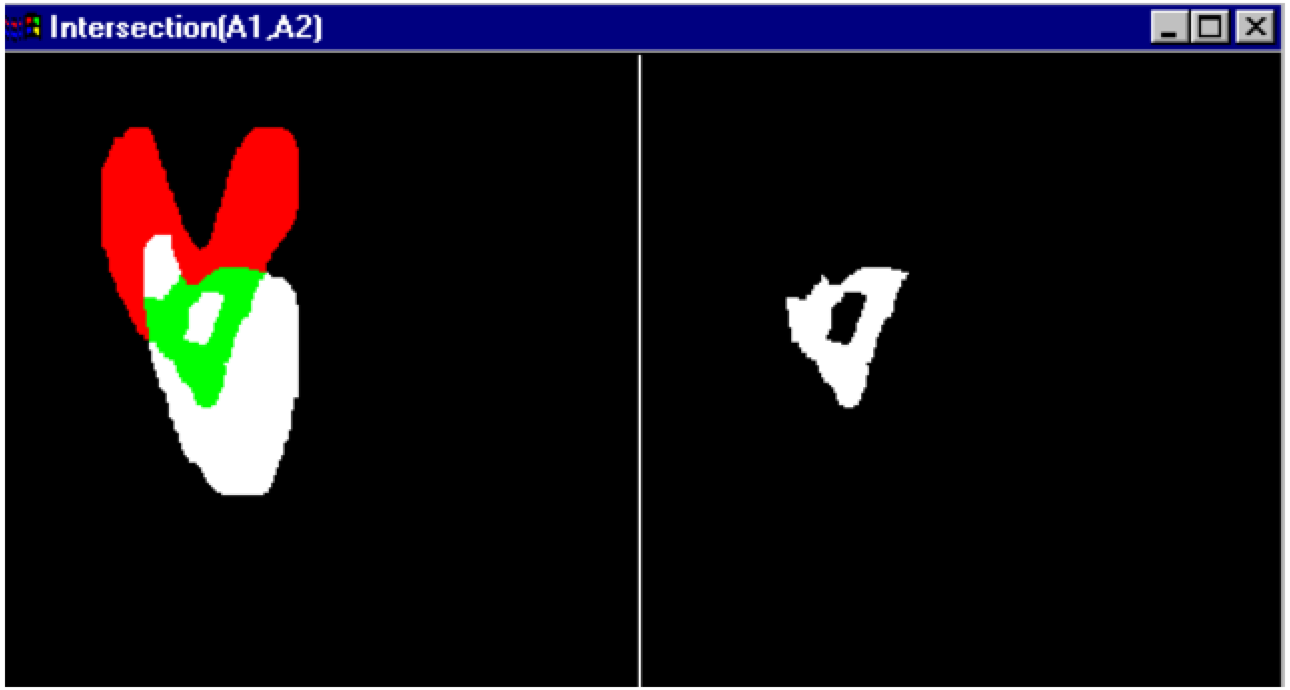
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Intersection and Differences of Two Image Sets

Green: Intersection of A1 and A2

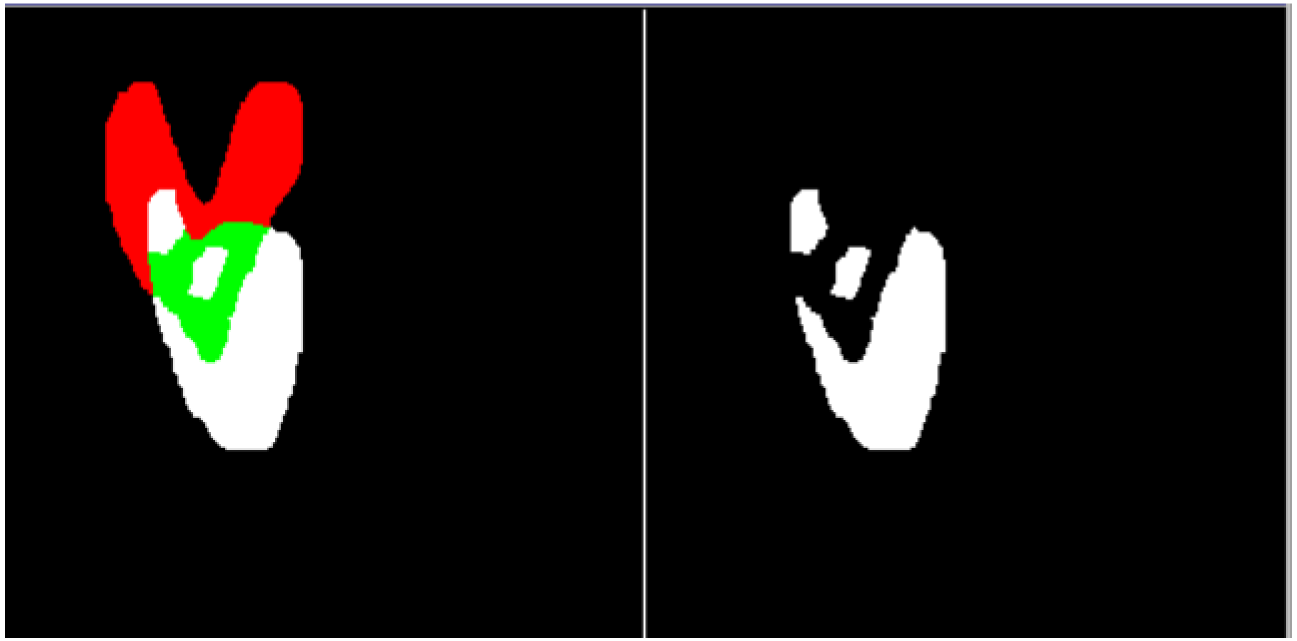
Red: A2-A1

White: A1-A2



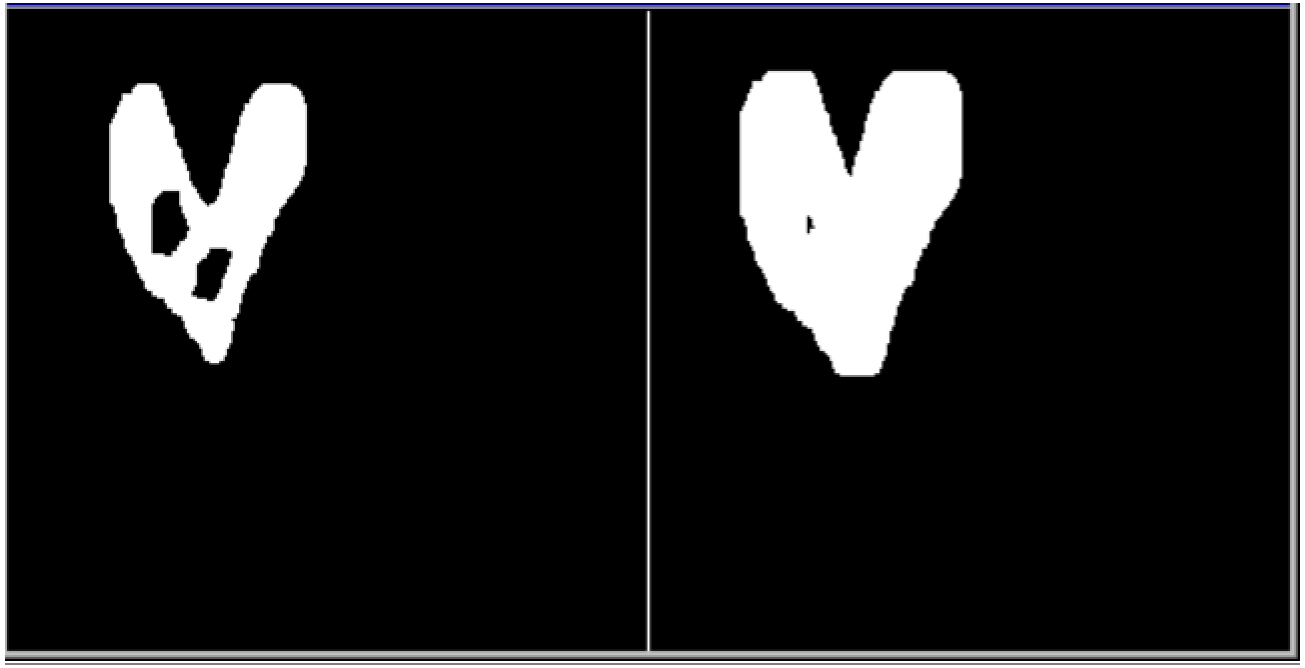
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: XOR



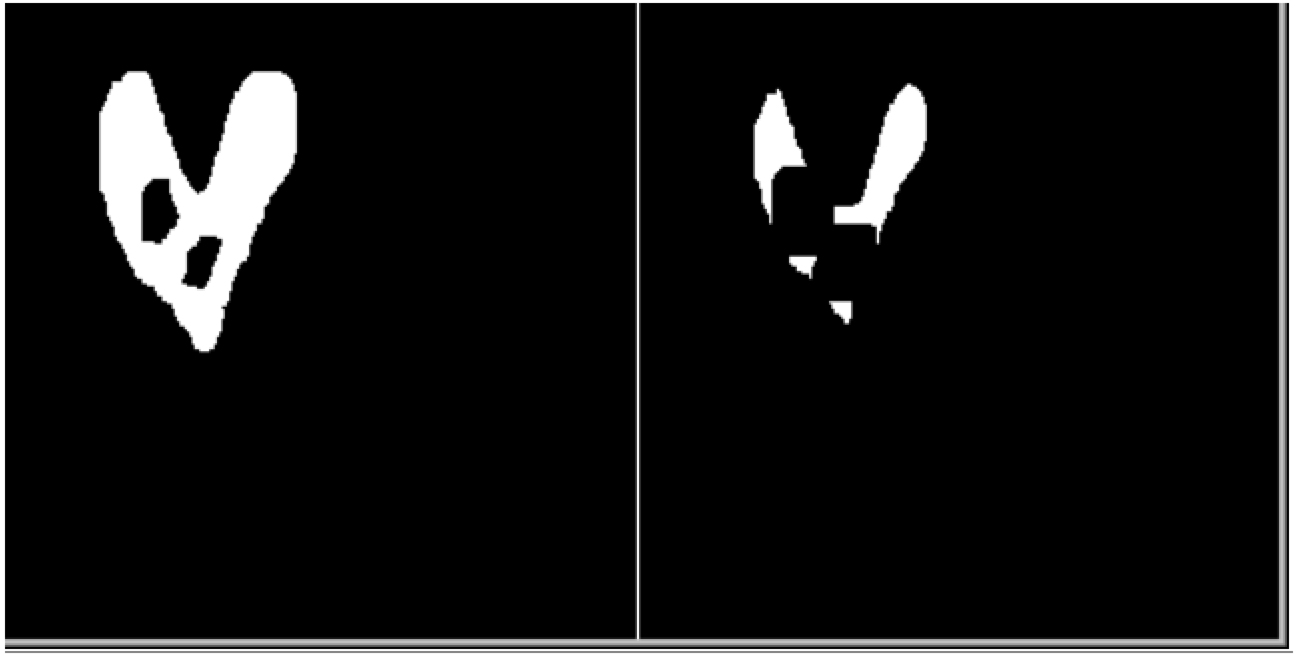
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Object Dilation



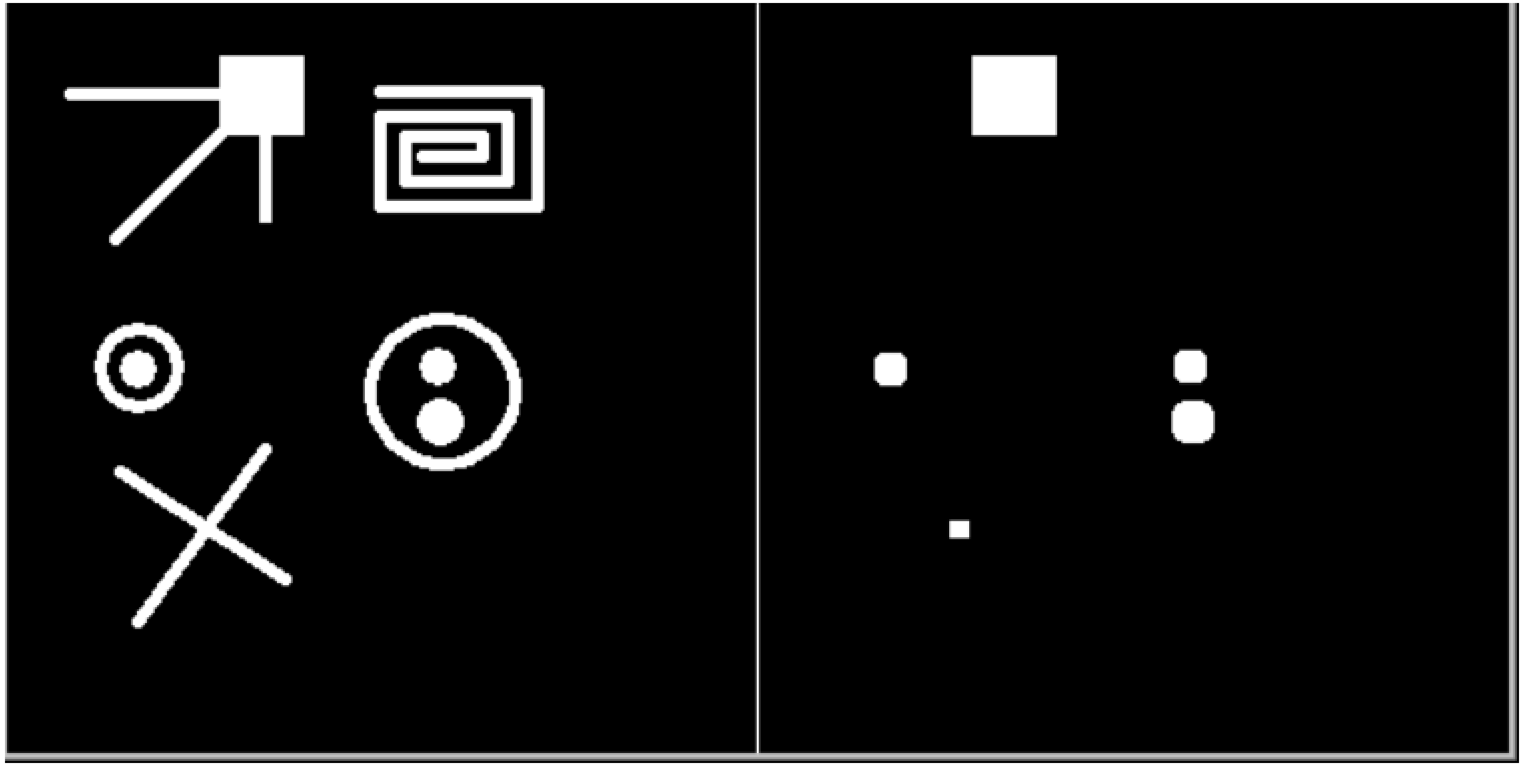
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Object Erosion



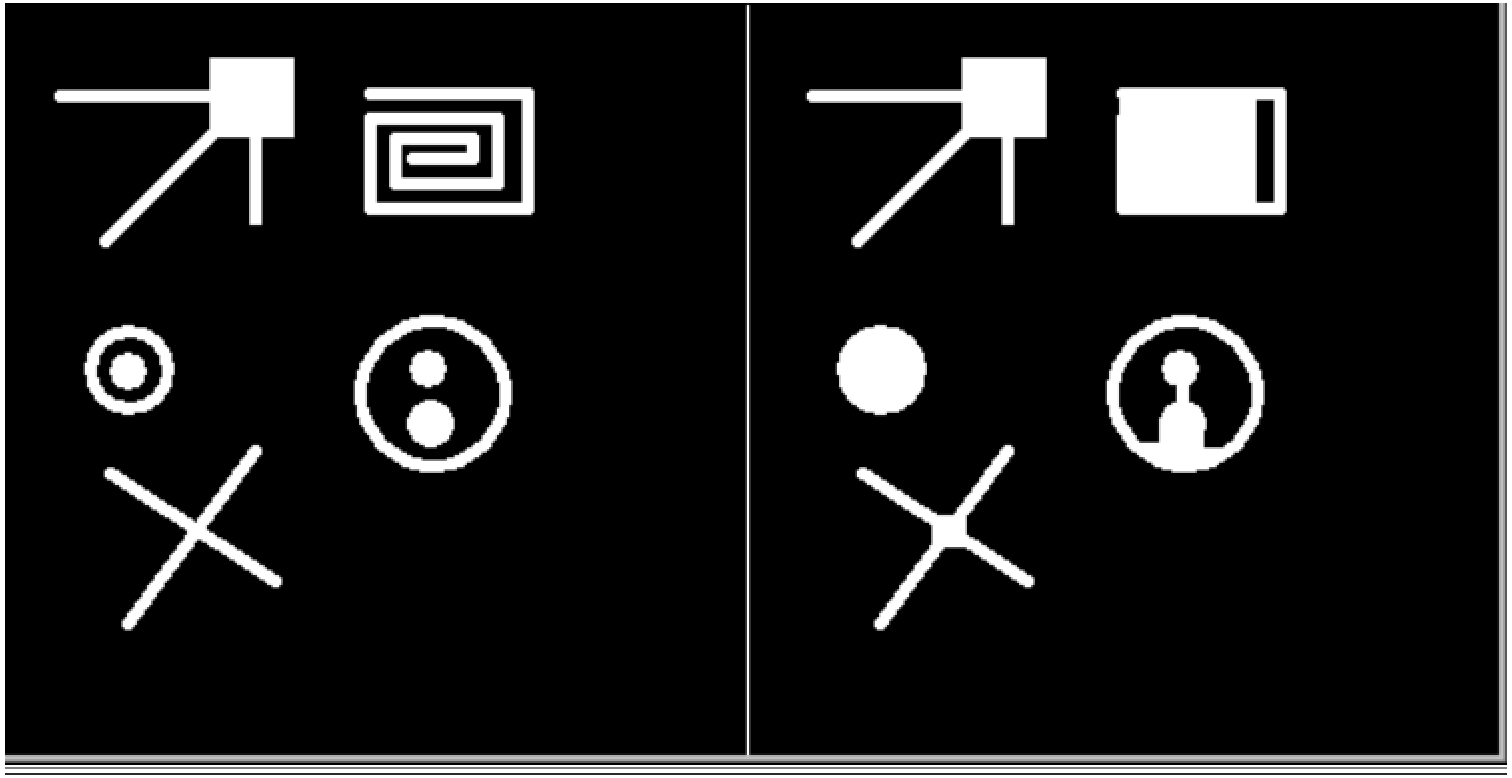
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Opening



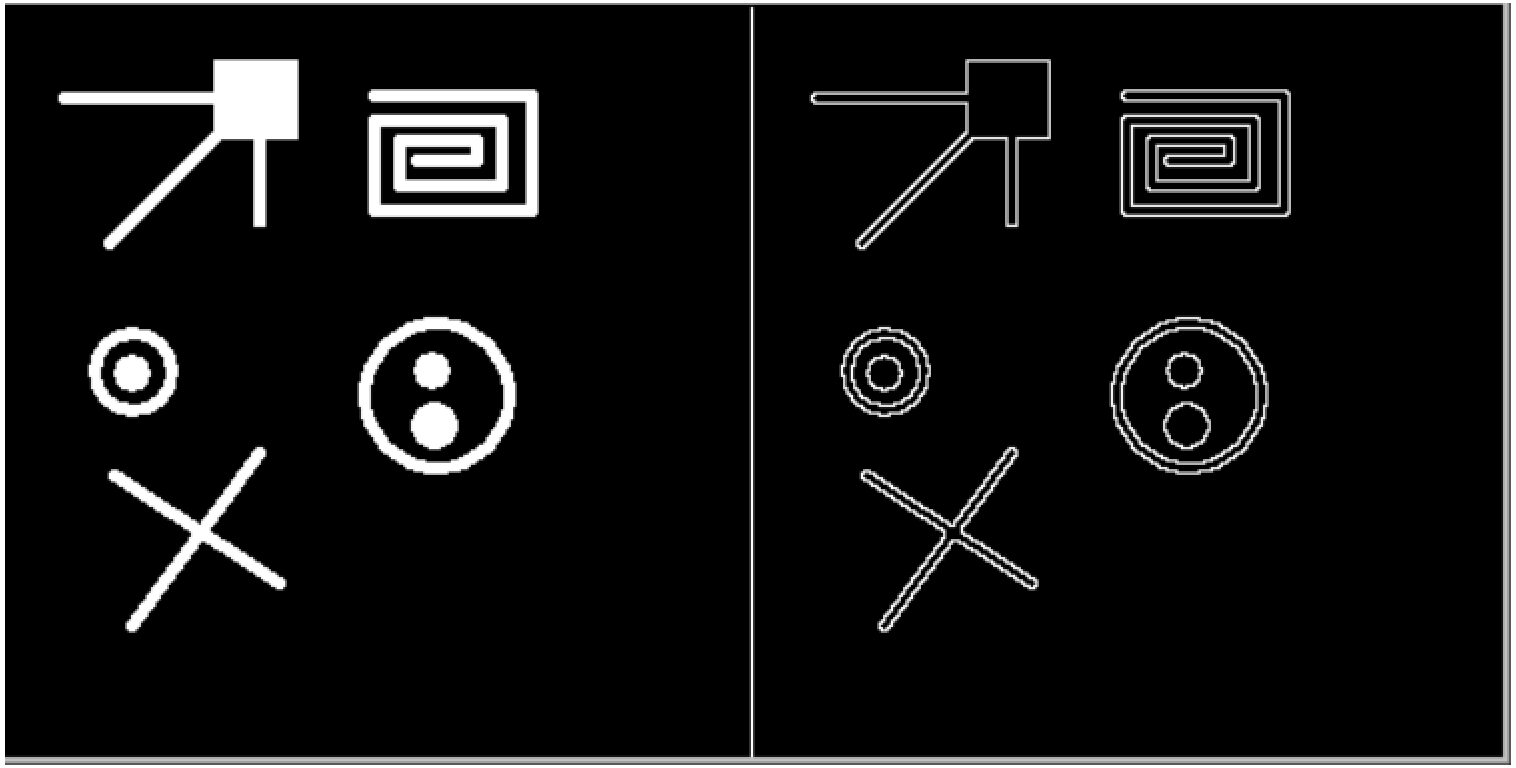
Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Closing



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Example: Boundary Extraction



Credit: https://www.cis.rit.edu/class/simg782.old/lec_morphology.html

Morphological Filter Design with Structuring Elements

Object: Set A, Structuring Element: Set B

Example of Structuring Elements

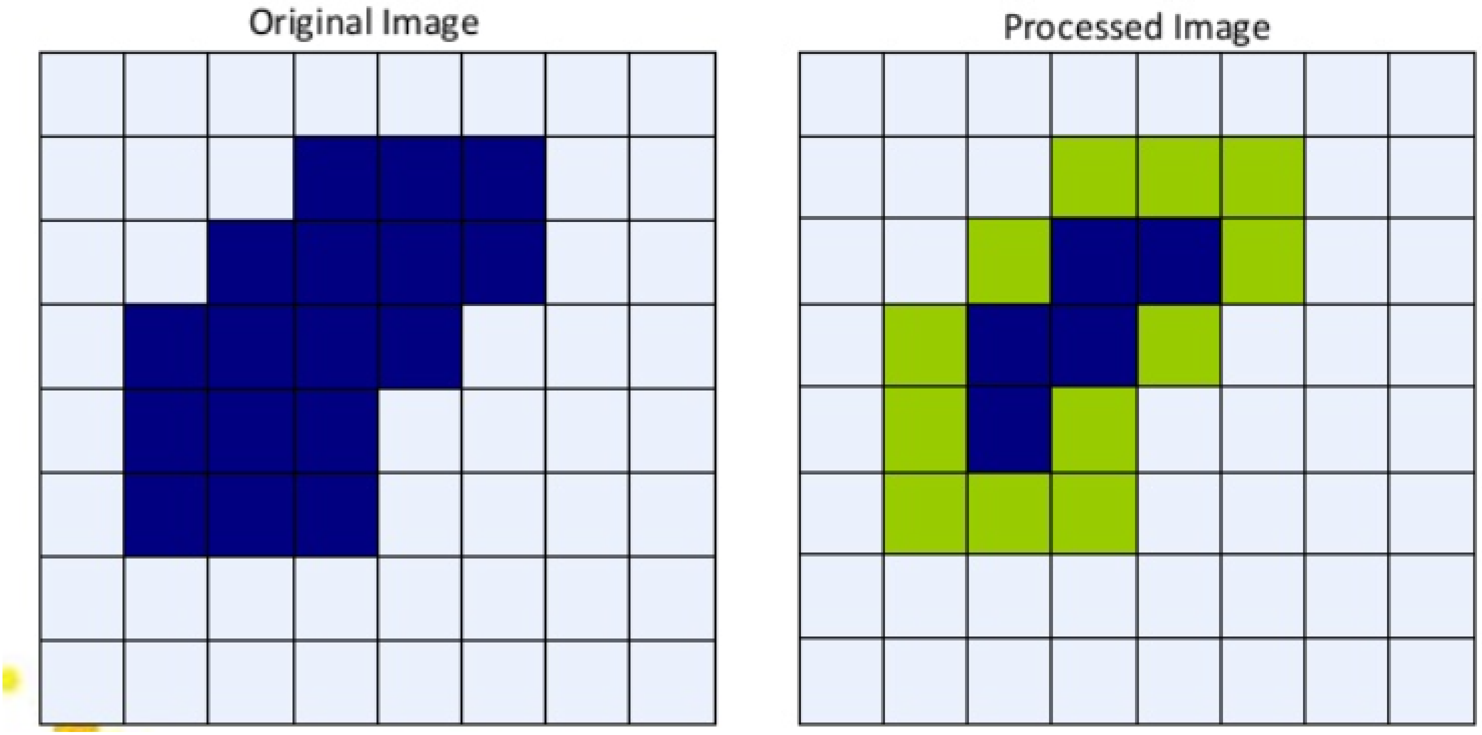
1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

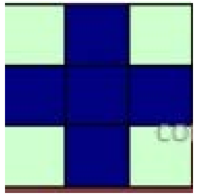
0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion with Structuring Element



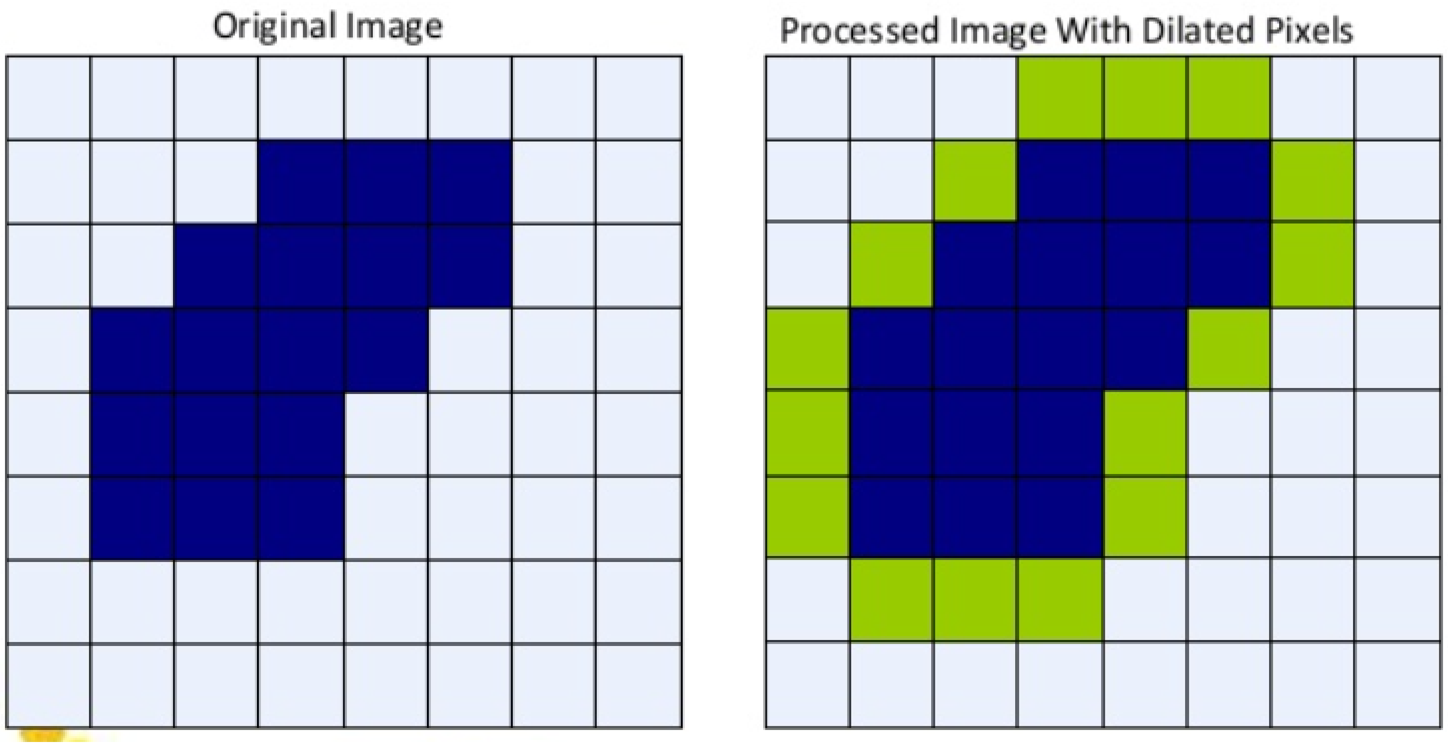
Structuring Element



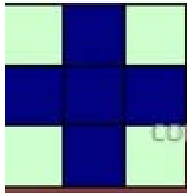
Use the center of SE to scan the object image
If hit, include the center pixel in the output image

Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Dilation with Structuring Element



Structuring Element



Use the center of SE to scan the object image
Include the union of the two in the output image

Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion and Dilation with Structuring Element

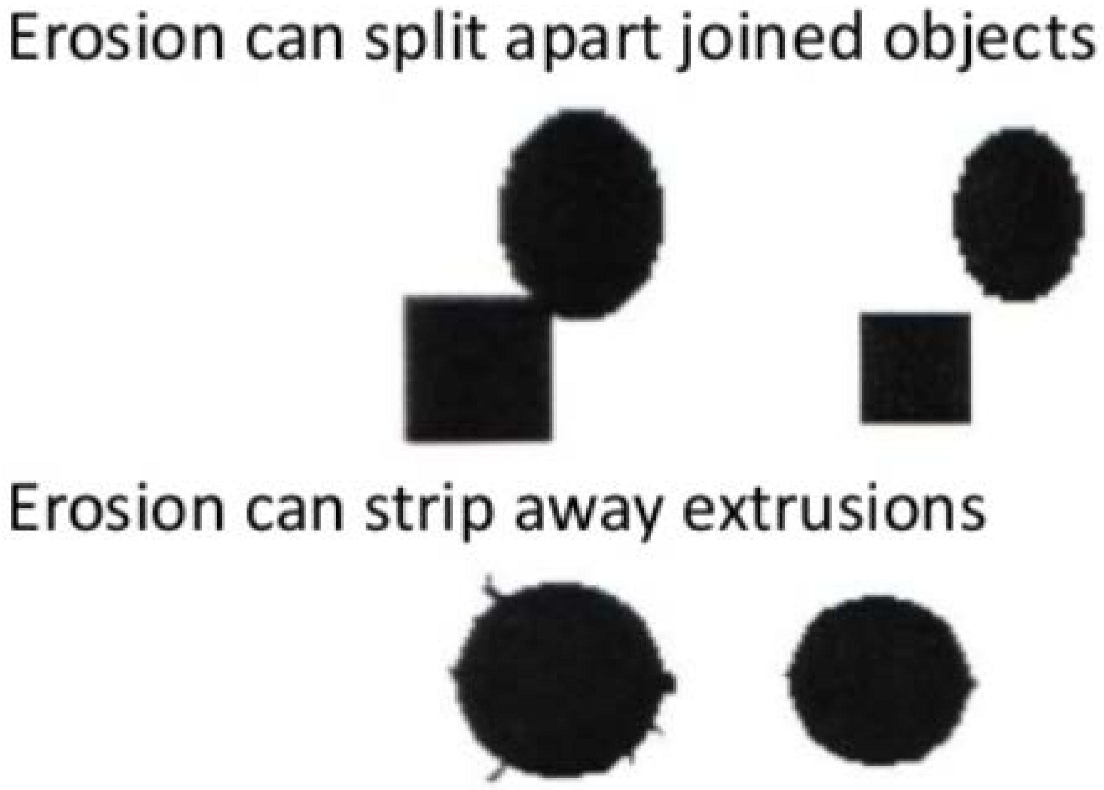
Original image

Erosion by 3*3 square structuring element

Erosion by 5*5 square structuring element

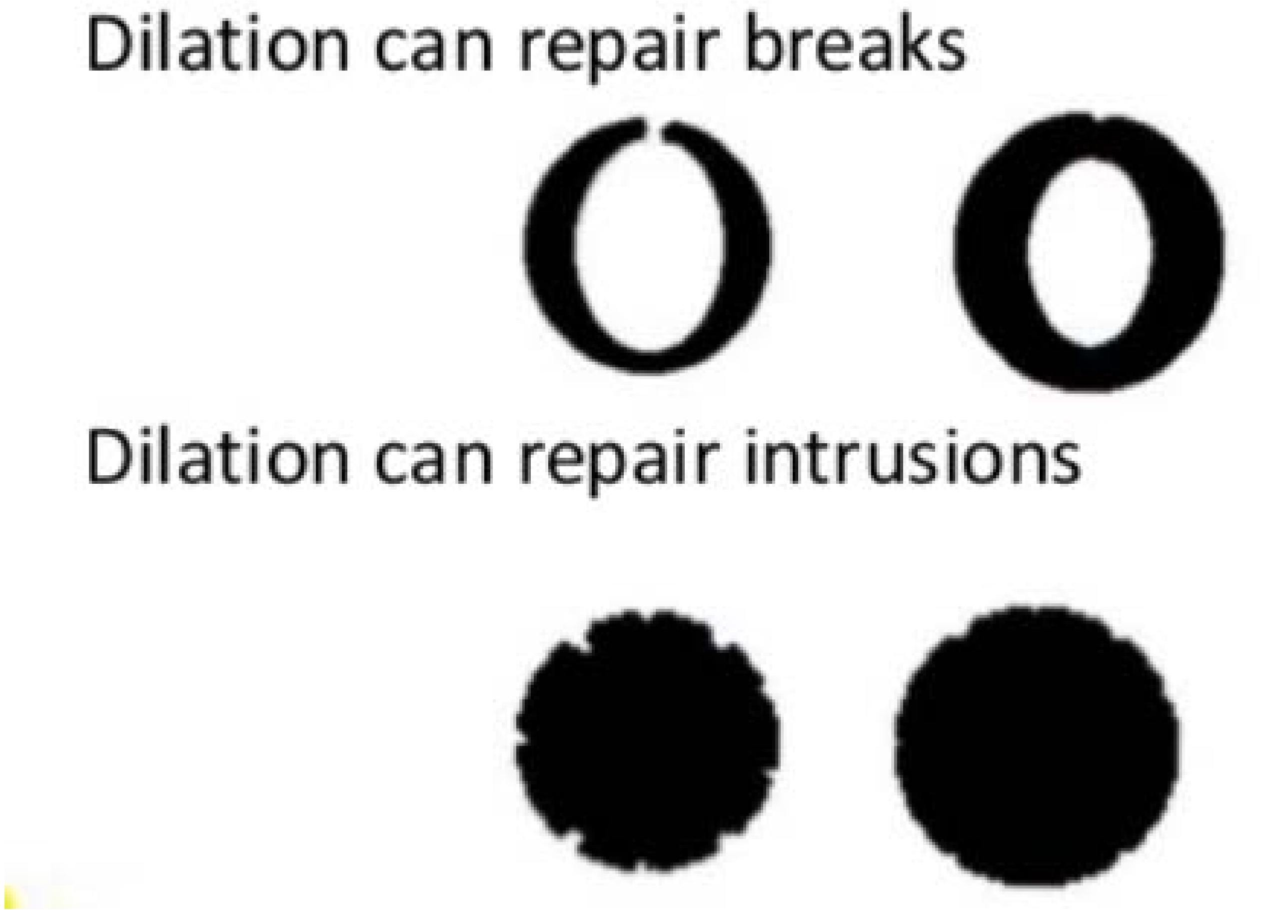
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Erosion Effect



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Dilation Effect



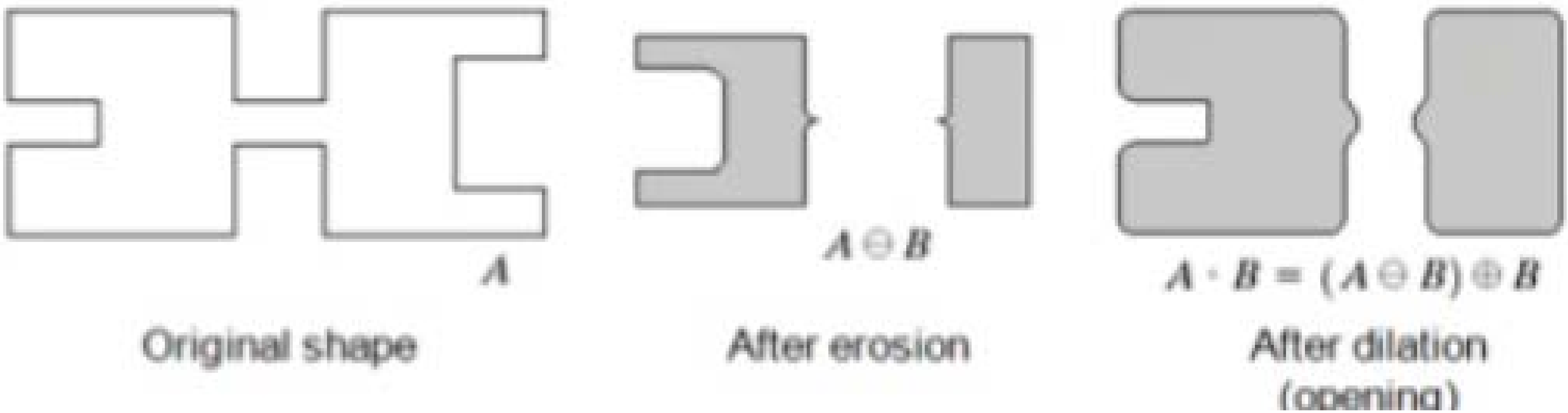
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Opening

– The opening of image f by structuring element s, denoted $f \circ s$ is simply an erosion followed by a dilation

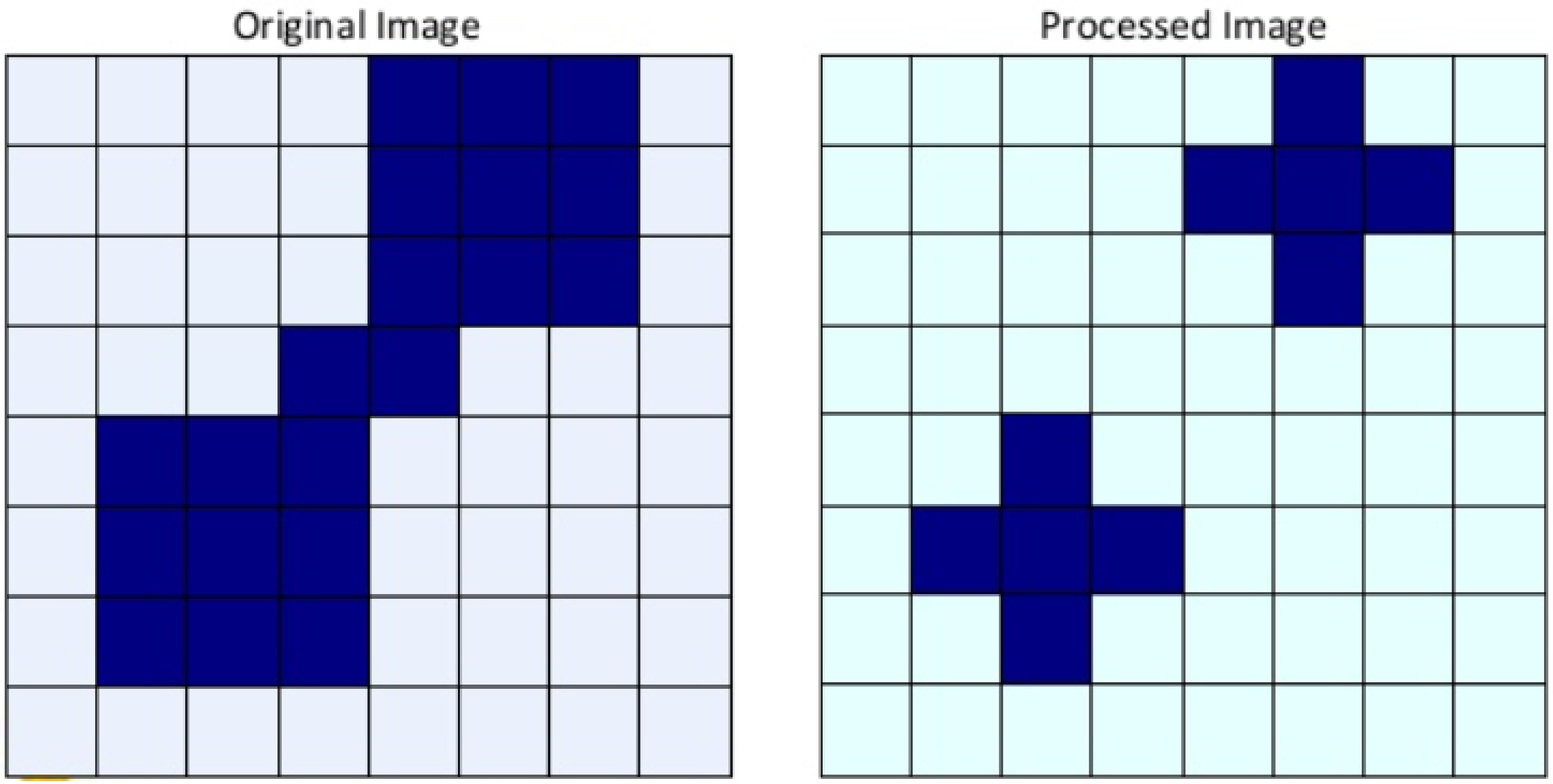
A disk-shaped SE is used

$$f \circ s = (f \ominus s) \oplus s$$

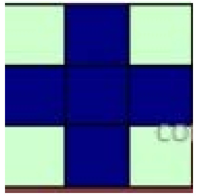


Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Opening (1)

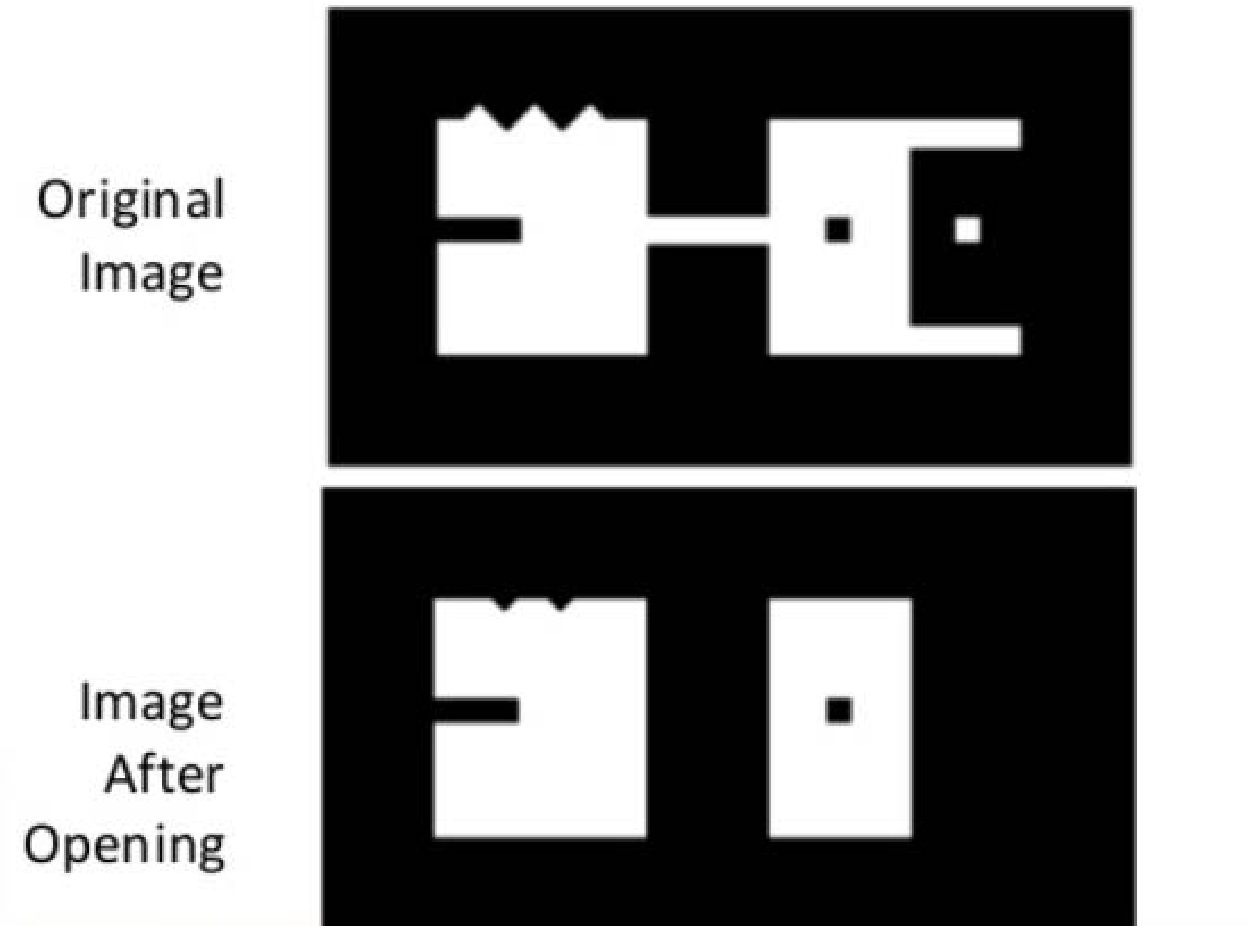


Structuring Element



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Opening (2)



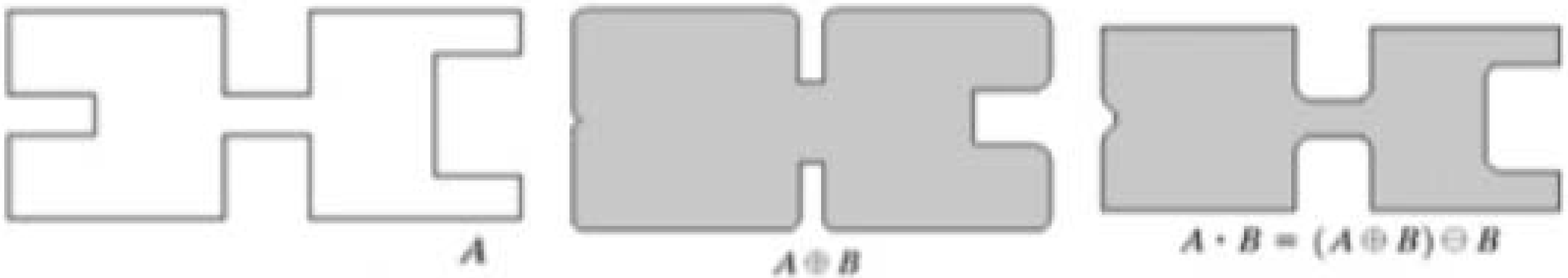
Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Closing

– The closing of image f by structuring element s, denoted $f \bullet s$ is simply a dilation followed by an erosion.

$$f \bullet s = (f \oplus s) \ominus s$$

A disk-shaped SE is used



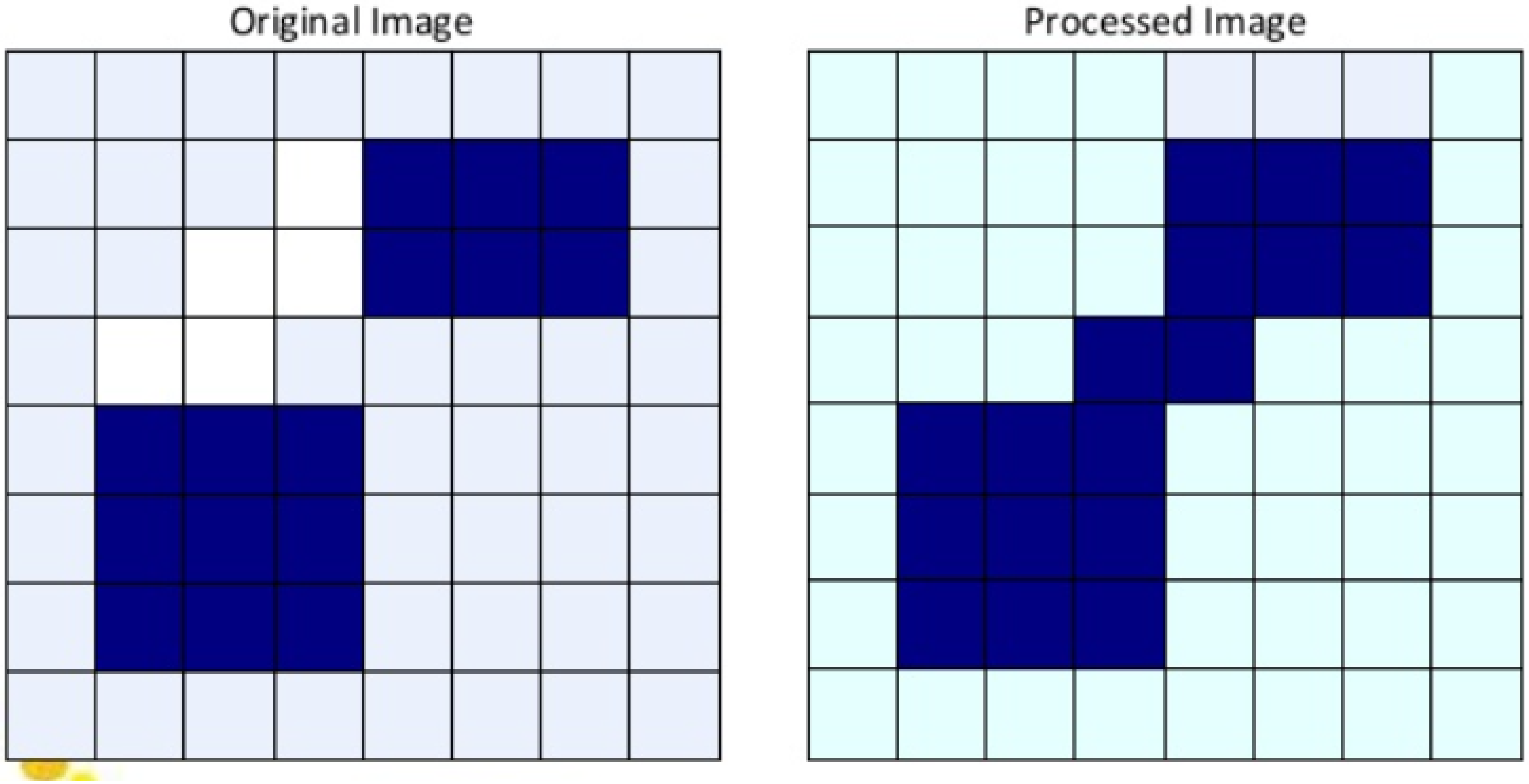
Original shape

After dilation

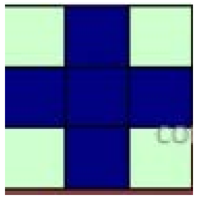
After erosion (closing)

Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Closing (1)

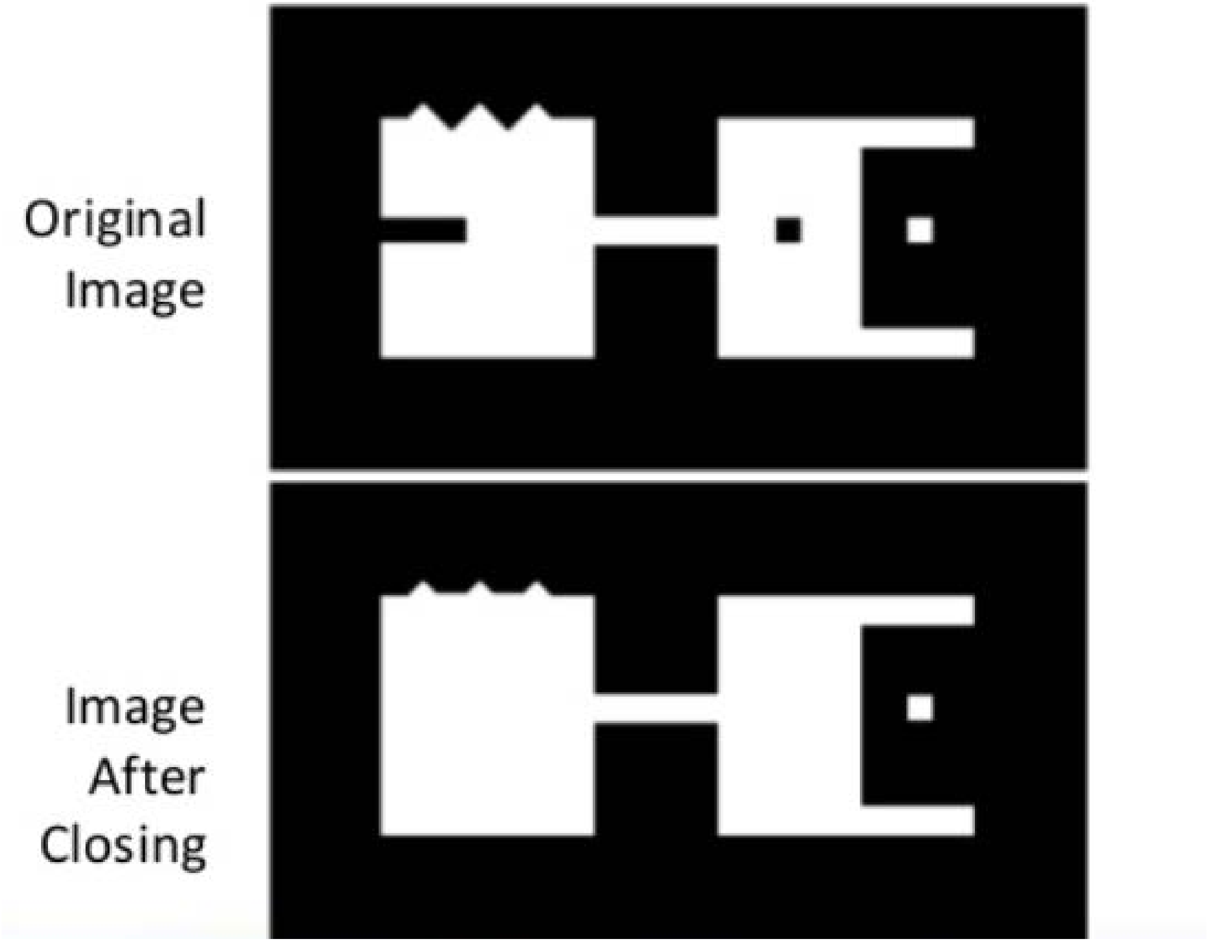


Structuring Element



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Examples of Closing (2)



Credit: https://www.slideshare.net/shkulathilake/morphological-image-processing-43465879

Qualitative Description of Opening and Closing

- Opening
  - Smooth the contour of an object, break narrow isthmuses and eliminate thin protrusions
- Closing
  - Fuse narrow breaks and long thin gulfs, eliminate small holes and fill gaps in the contours