Neural Network Controllers in Control Systems: A Survey on Stability, Robustness, and Adaptive Control

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Abstract

Neural network (NN) controllers have become integral to modern control systems, offering enhanced performance in managing complex, nonlinear dynamics. This survey explores the adaptability, robustness, and stability of NN controllers across various applications, emphasizing their superiority over traditional methods. Key findings demonstrate that NN controllers effectively address challenges like time delays and disturbances, significantly improving system performance. The integration of advanced learning frameworks, such as imitation learning, ensures stability and safety, meeting stringent performance requirements. Experiments with OS-net highlight the potential of NN controllers in modeling chaotic systems, while ReachNN* showcases advancements in verification processes. Practical applications, including quadcopter control and mixed autonomy strategies, illustrate the effectiveness of NN controllers in managing uncertainties and protecting sensitive parameters. The survey underscores the transformative potential of NN controllers in advancing control system technology, balancing safety and performance, and paving the way for future innovations. As research evolves, NN controllers are poised to address emerging challenges and expand their applicability across diverse and dynamic environments, driven by foundational elements like adjoint methods in optimization and stability analysis. Overall, NN controllers represent a significant advancement in control methodologies, offering robust solutions for complex system dynamics.

1 Introduction

1.1 Importance of Neural Network Controllers

Neural network (NN) controllers are integral to contemporary control systems, adept at managing complex, nonlinear dynamics with high precision. Unlike traditional controllers, NN controllers leverage their universal approximation capabilities to model a wide array of intricate systems, effectively addressing challenges such as unmodeled dynamics, nonlinearities, and time delays. This adaptability enhances performance and ensures safety through rigorous validation methods, including the Keep-Close approach, which maintains system outputs within safe bounds relative to robust reference models. Advanced techniques like stochastic barrier functions and reachability analysis tools, exemplified by ReachNN*, further bolster safety guarantees and performance optimizations, establishing NN controllers as a robust choice for modern applications [1, 2, 3]. This capability is especially vital in environments characterized by uncertain or partially observable dynamics, where traditional model-based controllers often underperform.

The robustness of NN controllers is enhanced through methodologies such as imitation learning, which fills gaps in robustness analysis and improves safety guarantees. These controllers effectively manage parametric variability, ensuring operational safety and stability across dynamic systems [4]. The integration of advanced strategies, such as Memory-Augmented Model Predictive Control (MAMPC)—which combines linear quadratic regulators (LQR) with neural networks—further augments performance without requiring extensive pre-computation [5].

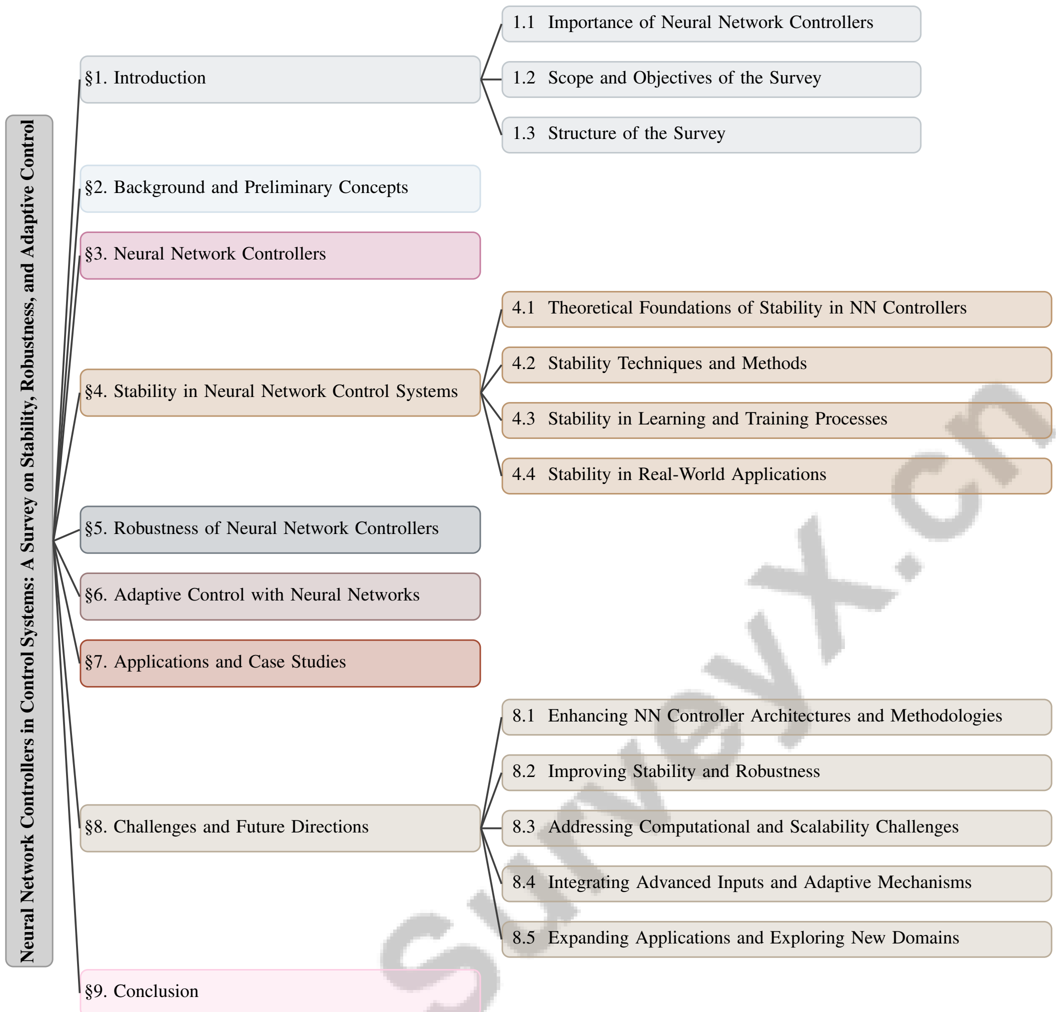

Figure 1: chapter structure

In scenarios involving time delays and disturbances, NN controllers demonstrate superior adaptability and robustness, outperforming traditional model-free adaptive control methods [6]. They also effectively navigate the stability-plasticity dilemma in continual learning, balancing the retention of past performance with the acquisition of new tasks [7]. Their application in vehicle dynamics control, particularly in in-wheel motor-driven vehicles, underscores their superiority over conventional physics-based modeling methods [8].

NN controllers play a crucial role in machine learning-based optimal feedback control for microgrids, enhancing stability and adaptability beyond traditional methods [9]. Despite the limitations of existing data-driven deep learning frameworks in modeling physical systems, NN controllers offer a physics-based approach that ensures robustness and error control [10]. In adaptive control scenarios, NN controllers exhibit improved tracking performance and robustness against disturbances compared to traditional control methods [11].

Additionally, NN controllers enhance usability and efficiency in complex systems, including partial differential equations (PDEs) with input delays, and excel in dynamic traffic signal control, surpassing traditional handcrafted methods. They also facilitate the learning of complex locomotion skills in soft agents, addressing the need for improved generalization and robustness [12].

The integration of NN controllers in control systems signifies a substantial advancement, providing enhanced performance, reliability, and adaptability across diverse applications. Their compactness and improved resilience against adversarial attacks render them suitable for deployment in resource-

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constrained environments [13]. However, the phenomenon of Neural Collapse, which can impact the robustness of neural networks both positively and negatively, necessitates careful consideration [14].

1.2 Scope and Objectives of the Survey

This survey offers a comprehensive examination of neural network (NN) controllers within control systems, emphasizing their adaptability, robustness, and stability across various real-time applications. Key objectives include exploring the adaptability of NN controllers in dynamic scenarios, such as aiding children with disabilities in toy manipulation, thereby illustrating their potential to enhance quality of life through real-time adaptability [15]. Another critical aspect is the integration of neural networks in managing uncertainties and improving performance in real-time applications, highlighting their significance in modern control systems [16].

The survey addresses the limitations of current NN controller methodologies by proposing a flexible control synthesis approach capable of accommodating multiple Signal Temporal Logic (STL) specifications without the need for retraining [17]. This approach is vital for developing versatile and resilient control systems. Additionally, it investigates the stability of time-delay systems governed by functional differential equations, focusing on stability concepts, Lyapunov methods, and the impact of inputs on system stability [18].

A significant emphasis is placed on synthesizing NN controllers with guaranteed stability and safety, particularly in nonlinear dynamics, which present unique challenges [19]. The survey also aims to rectify performance inadequacies in model-free adaptive control (MFAC) systems by modifying the equivalent-dynamic-linearization model (EDLM) to better manage time delays and disturbances [6].

Furthermore, the survey examines the capabilities of NN controllers in managing complex locomotion tasks involving multiple skills and higher degrees of freedom, essential for advancing robotic applications [12]. It also evaluates the robustness and stability of NN controllers in applications such as traffic signal control, striving to overcome the limitations of existing methods [20]. By addressing these objectives, the survey seeks to enhance the understanding and implementation of NN controllers in diverse and challenging environments, including applications of adjoint in linear algebra, ordinary differential equations, partial differential equations, neural networks, least squares and inverse problems, and PDE-constrained optimization [21].

1.3 Structure of the Survey

This survey is systematically structured to provide an in-depth exploration of neural network (NN) controllers in control systems, emphasizing stability, robustness, and adaptability. It tackles critical challenges such as the validation and certification of NN controllers against uncertainties, including unmodeled dynamics and time delays. The survey incorporates advanced methodologies, such as Lyapunov-based stability certificates and data-driven approaches, to ensure operational safety and performance. Additionally, it highlights innovative techniques for synthesizing stochastic barrier functions to enhance safety guarantees in complex dynamical systems, offering comprehensive insights into the practical applications and theoretical foundations of NN controllers in dynamic environments [22, 2, 23, 1, 4]. The paper commences with an **Introduction**, underscoring the critical importance of NN controllers and outlining the survey's scope and objectives. Following this, **Background and Preliminary Concepts** provides foundational insights into neural networks, control systems, stability, robustness, and adaptive control, setting the stage for subsequent discussions.

The core of the survey is divided into several sections, beginning with **Neural Network Controllers**, which examines the design and implementation of NN controllers, discussing innovative architectures, methodologies, and integration with control techniques, as well as exploring biologically-inspired and lightweight designs. The section on **Stability in Neural Network Control Systems** addresses theoretical foundations, techniques, and methods to ensure stability, alongside real-world applications and challenges.

In **Robustness of Neural Network Controllers**, the survey analyzes the robustness of NN controllers against uncertainties and external disturbances, exploring strategies to enhance robustness and the role of formal verification. The section on **Adaptive Control with Neural Networks** focuses on adaptive control principles and methodologies, its application in dynamic environments, and integration with traditional control methods.

The survey provides a comprehensive overview of , showcasing successful real-world implementations of NN controllers. It includes detailed case studies on quadcopter control, where NN controllers are utilized to manage flight dynamics, and motion planning for autonomous systems, such as an autonomous racing car using LiDAR data for steering decisions. Moreover, it discusses the verification of NN controllers in complex scenarios, including the formal verification of vision-based autonomous landing systems and reachability analysis of NN-controlled systems, demonstrating their robustness in uncertain environments and real-world applications [24, 1, 25, 3]. Finally, **Challenges and Future Directions** identifies current challenges and potential research directions, culminating in a **Conclusion** that summarizes key findings and reflects on the significance of NN controllers in advancing control systems.The following sections are organized as shown in Figure 1.

2 Background and Preliminary Concepts

2.1 Fundamental Concepts of Neural Networks

Neural networks (NNs) are advanced computational frameworks inspired by biological neural systems, designed to learn complex patterns through layered neuron interconnections. Each neuron processes inputs through linear and nonlinear transformations, crucial for handling intricate, nonlinear dynamics in control systems. The backpropagation algorithm, a cornerstone of NNs, refines performance by iteratively updating weights via gradient descent. However, its efficiency and scalability diminish in high-dimensional systems due to reliance on explicit derivative calculations [12].

To address these limitations, architectures like DeepONet have been developed to approximate nonlinear operators, enhancing NNs' capability to manage complex system dynamics [26]. Additionally, frameworks such as deep reinforcement learning and Bayesian meta-learning, exemplified by BM-DQN, underpin NN controllers' adaptability in dynamic settings [20]. In control systems, NNs are often integrated with other techniques to boost performance and safety. Differentiable physical simulators, for instance, provide precise analytical gradients for efficient NN training [12]. Despite their empirical success, learning-based controllers, including NNs, frequently lack formal safety assurances, a critical deployment consideration [27].

The synergy between NNs and classical control techniques extends their applicability across diverse fields. The integration of deep convolutional recurrent autoencoders offers a non-intrusive, data-driven approach to model reduction, effectively learning low-dimensional representations of complex systems [28]. This integration is vital for optimizing control strategies in systems with high degrees of freedom and complex dynamics.

NNs are pivotal in modern control systems, providing robust solutions for managing complex, nonlinear dynamics. Their adaptability to unforeseen tasks while ensuring safety and stability is essential in dynamic and uncertain environments. The ongoing evolution of NN architectures and methodologies holds significant promise for enhancing their effectiveness in control systems. Addressing challenges such as validating and certifying NN controllers against uncertainties, including unmodeled dynamics and time delays, is particularly important. Innovations like the Keep-Close approach for robustness analysis and tools such as ReachNN* for reachability analysis are paving the way for improved performance and reliability. These developments expand the applicability of NN controllers across various domains, including robotics and autonomous systems, facilitating formal verification of their safety and effectiveness, ensuring their integration into complex real-world applications [2, 17, 25, 3, 1].

2.2 Control Systems and Nonlinear Dynamics

Control systems are fundamental for regulating dynamic systems, ensuring desired performance and stability across various applications. They rely on feedback mechanisms to adjust inputs based on real-time observations, maintaining behavior within predefined operational parameters, particularly in complex scenarios like data-driven control and reinforcement learning. Model-free optimal control approaches, utilizing estimated cost functions and gradient information from physical system experiments, enable robust stability and performance amid uncertainties. Machine learning advancements have further refined feedback controller designs, allowing adaptation to varying conditions while optimizing overall performance [4, 22, 29, 9].

Nonlinear dynamics introduce complexities that challenge conventional control methodologies. Ensuring stability and safety in systems governed by nonlinear interactions is demanding, as traditional techniques often lack the requisite robustness. Recent advancements, including algorithms for learning stability certificates from trajectory data, address these challenges, enabling stability guarantees without precise system models. Additionally, NN-based algorithms enhance learning accuracy while maintaining global stability, underscoring the need for innovative strategies to manage nonlinear dynamics effectively [30, 31, 29, 4, 32]. Unlike linear systems, nonlinear systems exhibit behaviors such as bifurcations, chaos, and limit cycles, necessitating sophisticated modeling and control strategies.

Learning nonlinear dynamical systems from data is crucial for developing effective control strategies. [32] highlights the challenges of learning from a single trajectory, a common scenario in control systems, time series analysis, and reinforcement learning. Accurate capture of underlying dynamics from limited observations is vital for designing adaptive controllers capable of predicting complex behaviors.

Incorporating NNs into control systems provides robust solutions to challenges posed by nonlinear dynamics. NNs are adept at modeling nonlinear systems due to their ability to approximate complex functions and dynamic behaviors. This capability facilitates advanced controller designs that effectively address real-world application challenges. Recent advancements in stability certification, including Lyapunov-based stability certificates and stability-guaranteed training algorithms, enhance NN-controlled systems' robustness and safety. These innovations tackle critical concerns related to parametric variability and uncertainties, ensuring controllers maintain performance despite unmodeled dynamics and time delays. Techniques like the Keep-Close approach and the integration of Lyapunov theory with Integral Quadratic Constraints have validated NN controllers' effectiveness in various scenarios, including robotic arm control and spacecraft guidance, demonstrating their practical utility in uncertain environments [1, 4].

Understanding and managing nonlinear dynamics is crucial for the successful implementation of control systems across various domains. The ongoing development of advanced methodologies for learning and controlling dynamic systems is set to significantly enhance control system performance and applicability. This progress addresses the complexities of operating in nonlinear environments by integrating robust approaches, such as NN-based algorithms that ensure stability while learning from demonstration data. The application of model-free optimal control techniques and the construction of certificate functions from trajectory data provide formal safety and stability guarantees, essential for deploying reinforcement learning policies in real-world robotic applications. These innovations collectively contribute to more effective control solutions capable of adapting to the intricacies of real-time dynamics [30, 29, 33, 31].

3 Neural Network Controllers

Category	Feature	Method
Innovative Architectures and Methodologies	Knowledge and Learning Optimization	ROCLF[7]
	Mathematical and Bayesian Integration	NS[10], BM-DQN[20]
	Safety and Risk Management	CBF-NN[34], RVF[35]
	Delay and Stability Enhancements	DDB[26], DPLF[12]
Integration with Control Techniques	Hybrid Neural Networks	RN*[3], NLV[25]
Biologically-Inspired and Lightweight Designs	Biologically-Inspired Mechanisms	NNL-LES[36]

Table 1: This table provides a comprehensive overview of recent advancements in neural network controllers, categorizing them into innovative architectures and methodologies, integration with control techniques, and biologically-inspired and lightweight designs. Each category highlights specific features and methods, along with relevant references, showcasing the diverse approaches employed to enhance control system performance, stability, and adaptability in dynamic environments.

Table 3 presents a comparative overview of cutting-edge neural network controller methodologies, emphasizing their distinct features and contributions to advancing control system performance and safety. The exploration of neural network controllers has witnessed transformative advancements that significantly enhance control system performance and adaptability, while also introducing innovative methodologies to tackle challenges in dynamic environments. As illustrated in **??**, the hierarchical structure of neural network controllers is pivotal in understanding these advancements. This figure highlights various innovative architectures and methodologies, along with their integration with control techniques, and showcases biologically-inspired and lightweight designs. Each section of the figure elaborates on specific approaches and their contributions to enhancing control system performance, safety, and adaptability. Table 1 presents a detailed summary of the latest methods and approaches in neural network controllers, emphasizing their significance in advancing control system capabilities. This subsection further underscores the importance of these developments in the evolution of neural network controllers.

3.1 Innovative Architectures and Methodologies

Method Name	Control Performance	Safety and Risk Assessment	Learning Techniques
CBF-NN[34]	Optimal Control Synthesis	Risk-aware Verification	Train A Neural
DDB[26]	Exponential Stability	Stability Guarantees	Deeponet Framework
BM-DQN[20]	Adaptation Speed	-	Bayesian Learning
DPLF[12]	Better Results	Safety Constraints	Periodic Activation Functions
RN*[3]	Efficiency Improvements	Verify Nncss	Knowledge Distillation
RVF[35]	System Performance	Risk Verification Framework	-
ROCLF[7]	Superior Performance	-	Bayesian Neural Network
NS[10]	Prediction Accuracy	-	Regression Problem

Table 2: This table presents a comparative analysis of various innovative neural network (NN) controller architectures and methodologies, highlighting their control performance, safety and risk assessment capabilities, and employed learning techniques. The methodologies include approaches such as optimal control synthesis, exponential stability, and Bayesian learning, each contributing to advancements in stability, adaptability, and performance in NN-controlled systems. References to the original studies are provided to facilitate further exploration of these cutting-edge techniques.

Innovative architectures and methodologies have substantially advanced neural network (NN) controllers, improving control performance, stability, and adaptability. One notable architecture is the Control Barrier Function-based Neural Network Controller (CBF-NN), which synthesizes controllers that satisfy Signal Temporal Logic (STL)-based safety constraints while optimizing performance rewards [34]. This is crucial for ensuring safety in dynamic environments.

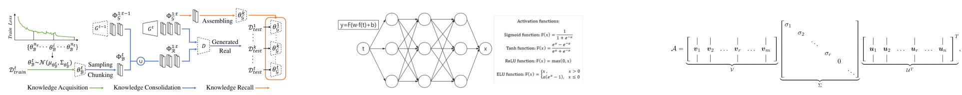
The DeepONet-based Delay-Compensated Backstepping method simplifies control gain computation for reaction-diffusion partial differential equations (PDEs), enhancing the efficiency of NN controllers in complex systems [26]. Similarly, the BM-DQN framework integrates Bayesian learning with deep Q-learning to bolster adaptation and robustness in traffic signal control, demonstrating the application of advanced learning techniques to real-world challenges [20].

The Differentiable Physics-Based Learning Framework (DPLF) enhances training robustness and efficiency of NN controllers in complex locomotion scenarios by leveraging differentiable physics for accurate gradient computation [12]. Additionally, the ReachNN* method employs knowledge distillation to reduce the Lipschitz constant of neural networks, improving performance while maintaining the original network's capabilities [3].

In risk verification, the approach by [35] collects trajectories from a stochastic control system with an NN controller, computing risk metrics based on robustness values against specified safety constraints. This emphasizes the necessity of rigorous risk assessment in NN controller deployment.

These innovative architectures and methodologies illustrate the transformative potential of neural networks in control systems, providing robust solutions that enhance stability, adaptability, and performance across various applications. For instance, the Autonomous Dynamic System (ADS) algorithm effectively balances learning precision and system stability during motions, utilizing a neural Lyapunov function for convergence to stable limit cycles. Advances in stability certification for NN-controlled nonlinear systems establish a Lyapunov-based stability certificate and a stability-guaranteed training algorithm, maximizing long-term utility while ensuring robust performance amid parametric variability. Furthermore, a data-driven approach for designing feedforward NN controllers guarantees stability for systems with unknown dynamics, outperforming traditional model-based methods [22, 30, 4]. The continuous evolution of NN controller designs aims to address existing challenges and expand their applicability in increasingly complex environments.

Table 2 provides a comprehensive overview of the key methodologies and architectures that have significantly advanced the development of neural network controllers, detailing their control performance, safety assessments, and learning techniques.



(a) Knowledge-based generative adversarial network (KG-GAN) for multi-task learning[7]

(b) A neural network with activation functions[10]

(c) The image depicts a mathematical matrix equation.[21]

Figure 2: Examples of Innovative Architectures and Methodologies

As illustrated in Figure 2, innovative architectures and methodologies in neural network controllers continue to evolve, enhancing the versatility and efficiency of machine learning models. The Knowledge-based Generative Adversarial Network (KG-GAN) for multi-task learning optimally consolidates knowledge across tasks, enhancing the learning process. Additionally, neural network structures incorporating various activation functions like sigmoid, tanh, ReLU, and ELU showcase the complexity of these systems in processing input data to achieve desired outputs. The application of mathematical matrix equations within these architectures underscores the foundational role of linear algebra in structuring and solving complex computational problems, highlighting the innovative strides in neural network controllers [7, 10, 21].

3.2 Integration with Control Techniques

Integrating neural network (NN) controllers with traditional and modern control techniques is crucial for enhancing performance, safety, and adaptability. The NNLander-VeriF framework exemplifies this integration by combining a perception NN with an NN controller to create an augmented neural network that captures the intricate relationship between aircraft states and control actions, demonstrating the potential of NN controllers to improve decision-making in aviation control systems [25]. Figure 3 illustrates the integration of neural network controllers with various control techniques, highlighting the NNLander-VeriF framework, reachability analysis using ReachNN*, and hybrid control methods incorporating traditional strategies like Model Predictive Control and Linear Quadratic Regulators.

Reachability analysis is vital for ensuring the safety and reliability of NN controllers. The ReachNN* method approximates the neural network's output and provides guaranteed error bounds using Bernstein polynomials, facilitating rigorous verification processes to ensure NN controllers operate within predefined safety margins while maintaining high performance [3].

Moreover, integrating NN controllers with traditional techniques such as Model Predictive Control (MPC) and Linear Quadratic Regulators (LQR) has yielded promising results. These hybrid approaches leverage the predictive capabilities of NNs to enhance adaptability and robustness, particularly in uncertain and dynamic environments. By combining the approximation capabilities of data-driven NN models with the analytical rigor of classical control strategies, these systems improve trajectory tracking performance and ensure closed-loop stability against external disturbances, even amidst model approximation errors. Structured designs optimize NN parameters and incorporate data-driven stability conditions into the training process, allowing for explicit tracking error bounds and effective controller parameter selection. The effectiveness of these integrated approaches is validated through simulations and numerical examples, demonstrating significant improvements over traditional methods [23, 22].

The integration of NN controllers with traditional and modern control techniques marks a significant advancement in control engineering. By harnessing the complementary strengths of these methodologies, control systems can be developed that are more efficient, reliable, and capable of adapting to real-world complexities. The ongoing enhancement of integrated approaches in NN controller design is expected to broaden their applications across various fields, addressing critical challenges such as validating NN controllers for uncertain systems, ensuring robustness against unmodeled dynamics and time delays, and leveraging tools like ReachNN* for reachability analysis. Data-driven methods are also being developed to guarantee stability in feedback loops and synthesize controllers that can adapt to new specifications without extensive retraining, promising to enhance the reliability and efficiency of NN controllers and drive progress in control system engineering [22, 17, 23, 3, 1].

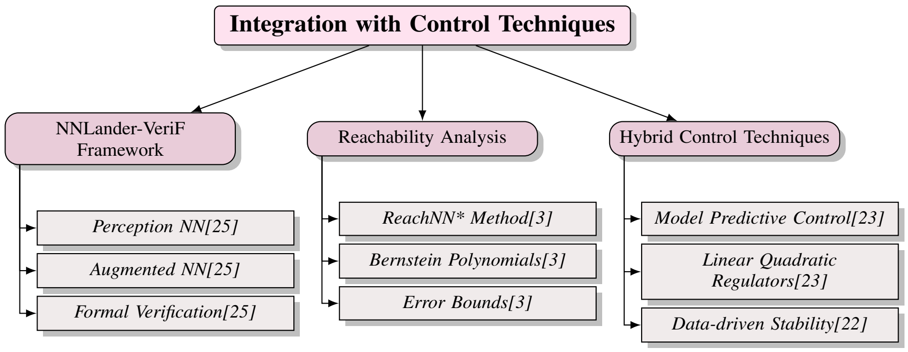


Figure 3: This figure illustrates the integration of neural network controllers with various control techniques, highlighting the NNLander-VeriF framework, reachability analysis using ReachNN*, and hybrid control methods incorporating traditional strategies like Model Predictive Control and Linear Quadratic Regulators.

3.3 Biologically-Inspired and Lightweight Designs

The advancement of biologically-inspired and lightweight neural network (NN) architectures is crucial for enhancing the efficiency and adaptability of NN controllers in control systems. These designs draw inspiration from biological neural processes, incorporating mechanisms such as local learning and neuromodulation to improve learning capabilities in dynamic environments. The method proposed by [36] exemplifies this approach, utilizing a lightweight architecture that integrates local learning and neuromodulation to tackle challenges in online continual learning.

This biologically-inspired architecture mimics the adaptive and efficient learning processes observed in natural neural systems, enabling NN controllers to perform effectively in real-time applications with limited computational resources. Local learning mechanisms facilitate decentralized processing, reducing computational burdens and enhancing scalability. Neuromodulation dynamically adjusts learning rates and synaptic strengths in response to environmental changes, allowing for rapid adaptations to new tasks while minimizing the risk of catastrophic forgetting. By integrating local learning rules and leveraging biologically-inspired architectures, neuromodulation supports robust performance across diverse tasks, evidenced by advancements in continual learning and stability in memory-augmented networks [30, 12, 36, 37, 38].

The lightweight nature of these architectures is particularly advantageous in resource-constrained scenarios, such as embedded systems and edge computing environments, where performance and efficiency are critical for real-time data processing. Recent developments in compact neural networks demonstrate that architectures utilizing structured matrices, like Toeplitz, can achieve high accuracy while significantly reducing the number of parameters, enhancing deployability in such applications. The integration of biologically-inspired techniques and local learning mechanisms further optimizes performance in dynamic environments with limited computational power [37, 36, 13, 10]. By minimizing computational overhead while maintaining high performance, these designs enable broader deployment of NN controllers in applications with stringent energy and processing constraints.

The integration of biologically-inspired and lightweight designs in NN controllers signifies a substantial advancement in control system technology, enhancing adaptability and efficiency by meeting diverse signal temporal logic (STL) specifications without retraining, while also improving robustness and safety verification in uncertain environments through innovative validation techniques [39, 1, 17, 3]. By leveraging principles from biological neural systems, these architectures offer enhanced adaptability, efficiency, and scalability, paving the way for versatile and robust control solutions across various domains. The ongoing development of these innovative designs promises to extend the capabilities and applications of NN controllers, addressing existing challenges and expanding their impact in control engineering.

Feature	Control Barrier Function-based Neural Network Controller (CBF-NN)	DeepONet-based Delay-Compensated Backstepping	BM-DQN framework
Control Performance	Optimizes Performance Rewards	Enhances Gain Computation	Bolsters Adaptation, Robustness
Safety Measures	Stl-based Safety Constraints	Not Specified	Not Specified
Learning Technique	Signal Temporal Logic	Delay Compensation	Bayesian Deep Q-learning

Table 3: This table provides a comparative analysis of three advanced neural network controller architectures: the Control Barrier Function-based Neural Network Controller (CBF-NN), the DeepONet-based Delay-Compensated Backstepping method, and the BM-DQN framework. It highlights key features such as control performance, safety measures, and learning techniques, illustrating their respective contributions to enhancing system adaptability, robustness, and safety in dynamic environments.

4 Stability in Neural Network Control Systems

4.1 Theoretical Foundations of Stability in NN Controllers

Ensuring stability in neural network (NN) controllers is critical for their performance in dynamic and uncertain environments. Lyapunov functions are central to embedding stability guarantees within learning algorithms, enabling estimation of attraction regions across system modes when integrated with differentiable planners [39]. Neural Contraction Metrics (NCMs) enhance contraction theory, providing a framework to design controllers that ensure stability amid system variability [40]. Exponential stability in closed-loop systems further underlines the theoretical assurances essential to NN controller design [26].

Challenges such as vanishing and exploding gradients in recurrent neural networks necessitate robust training methodologies to maintain stability [41]. The impact of time-delay effects is evident in global asymptotical stability studies of quaternion-valued neural networks, which consider leakage and time-varying delays [42]. Research on analytic neural networks with event-triggered synaptic feedbacks provides insights into stability under discrete feedback conditions [43]. Verifying safety in NN outputs for high-dimensional inputs, such as LiDAR measurements, is crucial for stability in autonomous systems [24].

The one-shot reachability analysis framework is vital for verifying unrolled neural network dynamical systems, enhancing stability understanding in NN controllers [44]. Probabilistic modeling in the BM-DQN framework mitigates training instability, enhancing robustness and adaptability [20]. The DPLF framework supports stable gradient propagation, crucial for NN controller stability [12].

Adjoint methods are significant in stability analysis for differential equations, highlighting their mathematical importance [21]. Viewing dynamic systems through operators in Hilbert spaces clarifies system properties, offering a comprehensive stability perspective [45].

Theoretical stability foundations in NN controllers are reinforced by methodologies addressing challenges posed by dynamic environments. These include validating NN controllers against uncertainties such as unmodeled dynamics and nonlinearities. The Keep-Close approach maintains output consistency in closed-loop systems, while Lyapunov-based certificates establish safe operating regions for NN-controlled nonlinear systems. Stochastic barrier functions provide safety guarantees by optimizing safety probabilities in neural network dynamic models. Data-driven techniques enhance stability assurance for feedback loops in plants with unknown dynamics, illustrating diverse strategies for reliable NN controller deployment [22, 1, 2, 4].

4.2 Stability Techniques and Methods

Various techniques ensure stability in neural network (NN) controllers, addressing challenges in dynamic control systems. Integrating Lyapunov theory with Integral Quadratic Constraints (IQCs) provides a robust framework for stability analysis, enhancing transient response guarantees and offering robustly invariant ellipsoids through finite-horizon conditions [46]. Lyapunov-based certificates are crucial for robust stability, defining maximal Lipschitz bounds for NN controllers [4].

The Critic Lyapunov Function (CALF) method embeds Lyapunov-like constraints into the learning process, ensuring NN controller stability [47]. Multiple ISS-Lyapunov functions address complexities in randomly switched systems, offering sufficient conditions for input-to-state stability [48].

Research categorizes stability concepts, including input-to-state stability (ISS) and input-to-output stability, offering a comprehensive framework for understanding interrelations [18]. Graph-theoretical considerations and multiple operators analyze synchronization stability in neural networks, critical for coherent system behavior [49]. The Master Stability Function (MSF) examines synchronized states' stability under varying coupling strengths [50].

As illustrated in Figure 4, the categorization of stability techniques and methods in neural network controllers highlights Lyapunov-based methods, synchronization stability, and advanced stability techniques. Each category includes specific methods and frameworks that address various challenges in ensuring stability and robustness in dynamic control systems.

Practical applications, such as iterative optimization of vehicle dynamics, demonstrate stability methods in NN controllers to enhance real-world performance [8]. The Recall-Oriented Continual Learning (ROCL) framework exemplifies stability techniques in learning environments, balancing new task acquisition with past knowledge retention [7].

Advanced methods, including the Lyapunov-Krasovskii functional method, provide stability criteria for quaternion-valued neural networks (QVNNs) with time-varying delays [42]. The event-triggered feedback mechanism reduces computational demands, enhancing stability in analytic neural networks [43]. The One-Shot Reachability Analysis (OSRA) method minimizes errors during reachable set analysis, essential for NN control system stability [44]. Parameterizing Neural Contraction Metrics (NCMs) ensures positive definiteness, contributing to exponential stabilization [40].

The method by [51] augments gradients with sampled data, ensuring uniform global asymptotic stability, highlighting data-driven techniques' importance in NN controller stability.

These stability techniques emphasize a multifaceted approach to dynamic environment challenges. Integrating diverse strategies—such as Lyapunov's methods, neural network algorithms, and dissipation theory—enhances system robustness and reliability. This integration addresses non-stationary trajectory challenges, ensuring adaptability and stability under various conditions, ultimately improving performance in applications like robotics and control systems [30, 46, 52, 1, 21].

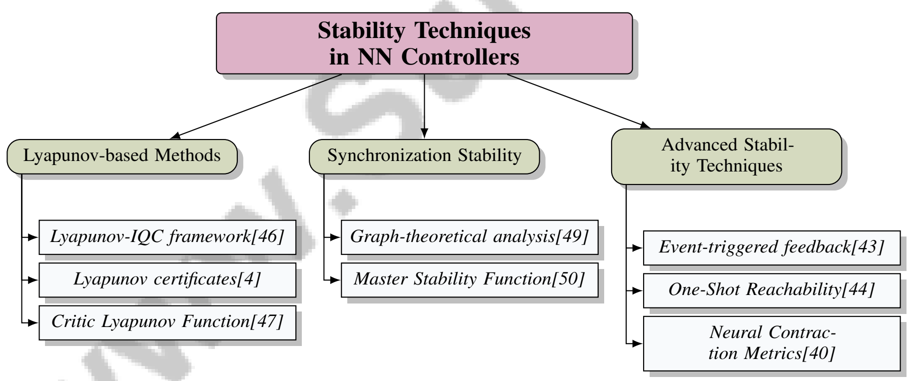


Figure 4: This figure illustrates the categorization of stability techniques and methods in neural network controllers, highlighting Lyapunov-based methods, synchronization stability, and advanced stability techniques. Each category includes specific methods and frameworks that address various challenges in ensuring stability and robustness in dynamic control systems.

4.3 Stability in Learning and Training Processes

Stability during learning and training phases is vital for neural network (NN) controllers to perform reliably in dynamic environments. Lyapunov functions are instrumental in verifying nonlinear systems' stability, simplifying the verification process through their relationship with finite-time Lyapunov functions [53]. Embedding stability guarantees directly into learning algorithms enhances NN controllers' robustness.

The Stability-Guided Training (SGT) method applies Lyapunov stability theory in training NN controllers, ensuring stability under bounded parametric variations [4]. Multiple ISS-Lyapunov Functions provide a framework for robust stability analysis in systems with stochastic switching signals [48].

Feedback mechanisms in analytic neural networks avoid Zeno behaviors by maintaining minimum update intervals, preserving stability during learning [43]. Continuity of solution maps and open map properties contribute to maintaining stability, ensuring effective learning [52].

Multi-operator treatments in neural networks provide eigenvalue bounds, offering a complete analysis of asymptotic stability for strongly connected networks [49]. Embedding strengths in the perceptron model, validated by Kuhn-Tucker conditions, reinforce learning process stability [54]. Addressing time-delay effects during training is underscored by sufficient conditions for global asymptotic stability of QVNNs with leakage and delays [42].

Innovative methodologies reinforce NN controllers' stability during learning, tackling dynamic environment challenges. These include validating NN controllers against uncertainties like unmodeled dynamics through the Keep-Close method, maintaining closed-loop system outputs near reference models. Lyapunov-based certificates certify robustness under parametric variability, while neural Lyapunov functions guarantee stability during learning from demonstration. Safety guarantees for NN dynamic models use stochastic barrier functions to maximize safety probabilities. Data-driven NN controller design incorporates stability conditions into training, ensuring feedback loops remain stable amid unknown dynamics [22, 30, 2, 1, 4].

4.4 Stability in Real-World Applications

Real-world applications of neural network (NN) controllers present unique stability challenges requiring robust solutions for reliable performance in diverse environments. In visual navigation, the DECISION architecture significantly improves stability, emphasizing advanced NN architectures' role in overcoming practical challenges [55].

Stability is crucial in high-precision assembly tasks, as demonstrated by the RL-OSFC method, highlighting reinforcement learning techniques' importance in adaptively managing stability [56]. Numerical simulations evaluating stability margins provide a comprehensive method for assessing stability conditions, ensuring NN controllers maintain desired performance [57].

Reachability analysis using TLL NNs offers practical solutions for stability challenges in dynamical systems. The L-TLLBox framework performs reachability analysis faster than existing methods, facilitating real-time NN controller application, crucial for dynamic systems requiring rapid responses [58].

In mixed-autonomy scenarios, maintaining stability while protecting sensitive parameters is challenging. The parameter privacy-preserving strategy effectively maintains control performance and stability, vital for safe NN controller operation in systems with varying autonomy levels [59].

Safety guarantees are critical for stability in real-world applications. The method by [2] enhances safety probability in complex systems modeled by neural networks, essential for maintaining stability and reliability in safety-critical environments.

In physical human-robot interaction (pHRI), proposed controllers demonstrate effective compliance and safety, tracking reference trajectories without exceeding constraints, underscoring the importance of designing stable NN controllers for human-centric applications [60].

Risk verification frameworks assess NN controllers' robustness in stochastic systems, estimating risk effectively and providing insights into robustness under various conditions [35]. This assessment is vital for ensuring stability in uncertain and variable environments.

Extending stability concepts to non-stationary trajectories through open maps preserves stability across dynamical systems [52]. This approach ensures NN controllers adapt to changing conditions while preserving performance.

These examples highlight the diverse stability challenges in real-world NN controller applications and innovative solutions addressing these challenges. Advanced stability techniques and methodologies promise significant improvements in NN controller reliability and effectiveness. These advancements tackle critical challenges in validating and certifying NN controllers, particularly concerning performance under uncertainties like unmodeled dynamics, nonlinearities, and time delays. Integrating Lyapunov theory with Integral Quadratic Constraints (IQCs) allows robustness analysis against bounded parametric variations and establishes safety certifications. This comprehensive framework enhances operational stability, applicable across diverse scenarios, as demonstrated in case studies involving complex systems like the Single-Link Robot Arm and Apollo Lander control [1, 4].

5 Robustness of Neural Network Controllers

The robustness of neural network (NN) controllers is crucial for ensuring reliable performance in complex environments. Addressing the intricate challenges in establishing robust control mechanisms is essential, particularly in dynamic contexts.

5.1 Challenges in Ensuring Robustness

Achieving robustness in NN controllers is challenging due to inherent complexities and nonlinearities. A significant issue is balancing safety margins with optimal performance; conservative measures may limit performance, while aggressive optimization can compromise safety [34]. Verification in real-world applications is complicated by uncertainties affecting system performance [35]. The lack of precise mathematical specifications complicates behavior analysis in continuous dynamics, posing challenges in unpredictable environments [61].

In traffic signal control, existing methods often struggle with inefficiency and instability in new scenarios, necessitating adaptive strategies that maintain stability [20]. Control Barrier Function (CBF)-based safety filters may lead to suboptimal long-term performance by neglecting cumulative control impacts [27]. The requirement for persistent excitation limits current methods' applicability [51]. Adversarial attacks exploit NN vulnerabilities, threatening robustness, with some methods enhancing resilience yet lacking universal applicability [14, 13].

These challenges highlight the need for innovative methodologies to manage complexities from nonlinearity, computational demands, and environmental variability. As illustrated in Figure 5, the key challenges in ensuring robustness in neural network controllers can be categorized into three main areas: balancing safety and performance, verification and uncertainties, and adversarial and adaptation issues. Each category underscores specific aspects such as safety margins, real-world verification, and adversarial attacks, based on the cited literature. Addressing validation and certification challenges is crucial for ensuring reliable NN controller performance, including managing uncertainties like unmodeled dynamics and time delays. Techniques such as stochastic barrier functions and Lyapunov-based stability certificates can establish robust stability and safety protocols, enhancing operational reliability across scenarios [35, 1, 2, 4].

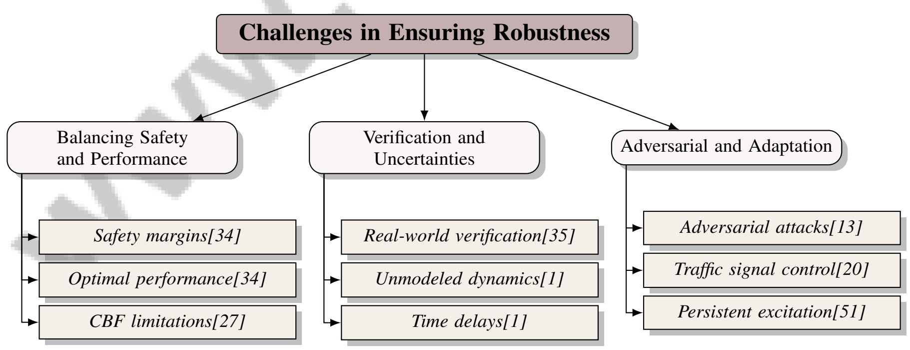


Figure 5: This figure illustrates the key challenges in ensuring robustness in neural network controllers, categorized into balancing safety and performance, verification and uncertainties, and adversarial and adaptation issues. Each category highlights specific aspects such as safety margins, real-world verification, and adversarial attacks, based on the cited literature.

5.2 Strategies for Enhancing Robustness

To enhance NN controller robustness, several strategies are employed. Formal verification techniques ensure stability and scalability in high-dimensional systems, enhancing robustness without explicit penalization [40]. Managing complex time delays is critical; criteria for handling these delays provide a practical framework for temporally variable systems [42]. Reducing computational overhead while

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maintaining robust performance is achieved by decreasing synaptic information exchange frequency [43].

Simplifying control gain computation through neural network approximations enhances robustness in systems with intricate dynamics [26]. Incorporating risk metrics derived from robustness values allows systematic assessments of NN controllers against safety constraints [35]. The NN-gauge controller emphasizes long-term effects in safety-critical applications, offering computational efficiency [27]. The concurrent learning approach achieves stability without stringent persistent excitation requirements, enhancing real-world applicability [51].

Diverse methodologies enhance NN controller robustness, including validating controllers against uncertainties, developing Lyapunov-based stability certificates, employing risk verification frameworks, and synthesizing stochastic barrier functions for safety guarantees. Each strategy addresses specific challenges in ensuring safety, stability, and performance, illustrating the multifaceted nature of robustness enhancement [17, 2, 35, 1**?**]. Integrating these techniques enables NN controllers to navigate diverse challenges, ensuring successful deployment across applications.

5.3 Formal Verification and Safety Guarantees

Formal verification is crucial for ensuring NN controllers' safety and robustness, particularly in dynamic and uncertain environments. The NNLander-VeriF framework exemplifies this in vision-based autonomous landing systems, demonstrating efficiency and effectiveness [25]. The Safe-by-Repair method systematically addresses unsafe behaviors in TLL NN controllers while preserving safe behaviors, ensuring reliability in safety-critical applications [62]. In reinforcement learning, stochastic barrier functions provide safety guarantees for NNDMs, facilitating minimally invasive controllers [2].

Finite state abstractions and reachability analysis enable formal verification by computing safe initial states [61]. One-shot reachability analysis offers a novel approach to safety verification, minimizing errors associated with existing recursive techniques [44]. The relationship between dissipation and Integral Quadratic Constraints (IQCs) enhances stability analysis methods, contributing to robust verification techniques [46].

Formal verification is essential for deploying NN controllers, ensuring necessary safety and robustness assurances for effective operation in complex environments. This process involves systematically analyzing NN behavior under uncertainties to guarantee adherence to safety specifications. Techniques such as reachability analysis, Satisfiability Modulo Convex encoding, and risk verification frameworks assess NN performance against benchmarks, ensuring stability and safety. Addressing challenges like mathematical modeling of sensor inputs and closed-loop dynamics complexities enhances NN controllers' reliability in safety-critical applications, facilitating integration into autonomous systems [24, 25, 35, 61, 1]. Continuous advancements in verification methodologies promise to further enhance NN controllers' reliability and effectiveness across domains.

6 Adaptive Control with Neural Networks

6.1 Principles and Methodologies of Adaptive Control

Adaptive control using neural networks (NNs) provides a dynamic framework for controllers to adjust to varying conditions, ensuring both stability and performance. A central principle involves integrating reinforcement learning to facilitate real-time adaptation in changing environments, as demonstrated by [56]. This adaptability is essential for developing NN controllers capable of optimal learning across diverse scenarios.

As illustrated in Figure 6, the hierarchical structure of adaptive control using neural networks categorizes key principles and techniques, applications and methods, as well as safety and performance considerations. This visual representation enhances our understanding of how these elements interconnect within the adaptive control framework.

The Safe Model-based Reinforcement Learning (SMBRL) method exemplifies embedding safety within policy optimization, ensuring stability in continuous state-action spaces [63]. This underscores the importance of incorporating safety in adaptive control strategies. The Composite Learning Backstepping Control (CLBC) method combines modular backstepping with composite learning, achieving parameter convergence and robust control even under relaxed excitation conditions [33]. This highlights the efficacy of integrating multiple control techniques for robust NN adaptive control.

The ML-OFC method demonstrates adaptability to disturbances in microgrid operations, showcasing the versatility of NN controllers [9]. Similarly, the Adaptive Neural Network Control (ANNC) enhances trajectory tracking for Autonomous Underwater Vehicles (AUVs), illustrating adaptive principles across various domains [11]. The L-TLLBox framework further supports adaptive control by combining exact and approximate reachability analyses, essential for managing real-world complexities [58].

Moreover, the hierarchical neural network-based control method (HNN-CM) stabilizes hybrid systems by integrating NNs for control and estimation [39]. The use of Control Barrier Functions (CBFs) alongside Signal Temporal Logic (STL) specifications optimizes performance while ensuring safety constraints [34].

Adaptive control principles with NNs are characterized by merging learning and control techniques, enabling effective adaptation to diverse tasks and environments. In AUV applications, adaptive designs leverage NNs to tackle disturbances, control nonlinearities, and model uncertainties. By employing a critic and action NN, these systems evaluate long-term performance and compensate for unknown dynamics, maintaining robust stability. Data-driven approaches further enhance stability guarantees, expanding applicability across uncertain systems and complex scenarios [22, 23, 11, 1, 4].

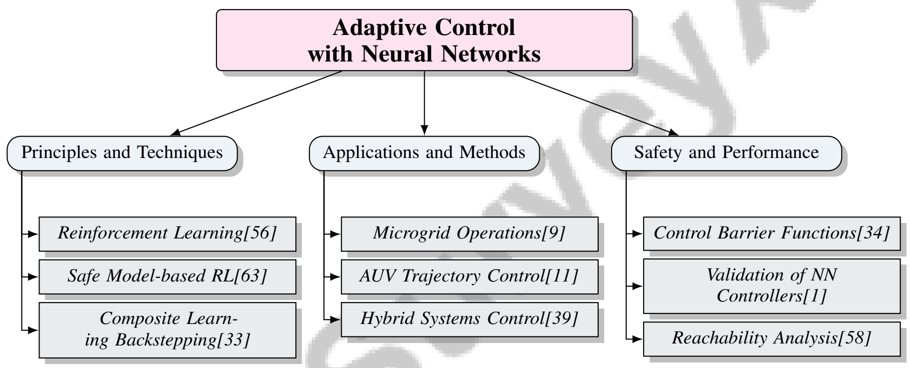

Figure 6: This figure illustrates the hierarchical structure of adaptive control using neural networks, categorizing key principles and techniques, applications and methods, and safety and performance considerations.

6.2 Adaptive Control in Dynamic Environments

Adaptive control in dynamic environments ensures NN controllers maintain performance amidst changing conditions. These methodologies enable dynamic adjustment of parameters and strategies in response to environmental variations, enhancing robustness. The RL-OSFC method employs reinforcement learning to optimize control policies through environmental interaction, effective in high-precision assembly tasks [56].

In environments with fluctuating system parameters and disturbances, adaptive control strategies are crucial for stability. The ML-OFC method manages disturbances in microgrid operations [9], while the ANNC method improves trajectory tracking for AUVs by adapting control parameters to environmental changes [11]. Such examples underscore the critical role of adaptive control in navigating dynamic environments.

Hierarchical control structures, like the HNN-CM method, provide a robust framework for stabilizing hybrid systems amid dynamic changes by integrating NNs for control and estimation [39]. Combining CBFs with STL specifications allows for performance optimization while satisfying safety constraints, further enhancing NN adaptability in dynamic settings [34].

Adaptive control in dynamic environments is defined by NN controllers' capability to adjust to changing conditions, ensuring stability and performance across various applications. Advancements in adaptive methodologies are expected to significantly enhance NN controller performance, facilitating application in complex environments. This is particularly relevant for systems like AUVs, where

NN controllers effectively tackle disturbances and model uncertainties. Recent research highlights integrating multiple NNs within adaptive designs to ensure stability and robustness, broadening operational capabilities in real-world scenarios [22, 1, 4, 11].

6.3 Integration with Traditional Control Methods

Integrating adaptive NN controllers with traditional control methods marks a significant advancement in control system technology, enhancing robustness, adaptability, and efficiency. This integration combines the learning capabilities of neural networks with the reliability of traditional techniques. Empirical findings emphasize the importance of robust memory architectures and stability certification in neural control systems, addressing stability and optimizing learning accuracy [30, 37, 1, 4].

A notable example is the fusion of Model Predictive Control (MPC) with neural networks to create Memory-Augmented Model Predictive Control (MAMPC). This approach utilizes MPC's predictive capabilities alongside NN adaptability to enhance performance without extensive pre-computation [5]. Similarly, integrating Linear Quadratic Regulators (LQR) with NN controllers improves tracking performance and resilience to disturbances, highlighting the complementary strengths of these methodologies.

The Composite Learning Backstepping Control (CLBC) method integrates traditional backstepping techniques with adaptive learning mechanisms, ensuring parameter convergence and control performance under relaxed excitation conditions [33]. This integration exemplifies synergy between classical control strategies and adaptive learning for robust control in dynamic environments.

Combining CBFs with STL specifications optimizes performance objectives while ensuring safety constraints [34]. This approach illustrates integrating optimization algorithms with adaptive control techniques to enhance NN controllers' robustness and safety.

The integration of adaptive NN controllers with traditional methods presents a promising path for advancing control system technology. By leveraging the strengths of various control methodologies, including neural networks and stability verification techniques, it becomes feasible to design systems that enhance efficiency and reliability while addressing real-world complexities. This integration allows for robust validation against uncertainties such as unmodeled dynamics and nonlinearities, ensuring safety and adaptability through learned stability certificates and region-of-attraction planning. Consequently, these advanced systems achieve superior performance across diverse scenarios, from robotic motion control to hybrid system stabilization [30, 31, 39, 35, 1]. The ongoing development of these integrated approaches promises to expand the applicability of NN controllers across various domains, driving innovation in control system design.

7 Applications and Case Studies

7.1 Case Studies and Applications

Neural network (NN) controllers exhibit remarkable versatility in managing complex system dynamics across various domains. Notable case studies include the control of a single-link robot arm and the guidance of the Apollo Lander. In the first case, NN controllers excel in precise manipulation and stability, outperforming traditional methods in managing nonlinear dynamics and optimizing control strategies [1]. The second case highlights the NN controller's role in the Apollo Lander's descent and landing, ensuring stability under uncertain environmental conditions, thus enhancing aerospace guidance and control systems' performance and reliability [1].

These examples demonstrate NN controllers' adaptability to diverse control challenges. They ensure robustness against unmodeled dynamics and time delays through methods like the Keep-Close approach. Tools such as ReachNN further enhance performance verification efficiency by facilitating reachability analysis of NN-controlled systems. Advancements in stability certification for NN-controlled nonlinear systems underscore their ability to maintain operational safety and robustness in dynamic environments [1, 4, 3]. The continuous evolution of NN controller designs promises broader applicability, driving innovation in control system technology.

7.2 Quadcopter Control with NN Controllers

NN controllers significantly enhance quadcopter control systems by navigating complex dynamics and improving system performance. They achieve stability and robustness through rigorous validation and certification methods, effectively handling uncertainties such as unmodeled dynamics and nonlinearities. Recent advancements, including robust, optimal, and safety-guaranteed training methods, empower NN controllers to maintain stability and minimize tracking errors, making them suitable for unmanned aerial vehicle applications [22, 23, 64, 1, 4].

Comparative studies in quadcopter simulations reveal NN controllers' superior robustness and adaptability over traditional nominal controllers, providing enhanced stability and optimal performance across varied scenarios [64]. NN controllers' ability to learn and adapt to real-time system dynamics presents a substantial advantage, particularly in ensuring robust performance amid uncertainties and nonlinearities. The Robust, Optimal, Safe and Stability Guaranteed Training (ROSS-GT) approach replaces traditional synthesis methods, establishing constraints that account for system nonlinearity and disturbances, minimizing control loss risks. These capabilities are crucial in mission-critical scenarios where safety and performance adherence are paramount [1, 34, 25, 64]. The integration of safety and stability constraints enhances reliability, providing a robust framework for managing quadcopter systems' complex dynamics.

7.3 Motion Planning with Complex Spatio-Temporal Tasks

NN controllers' integration in motion planning tasks involving complex spatio-temporal dynamics marks a significant advancement. These tasks require the fusion of high-dimensional sensory inputs with real-time decision-making processes, where NN controllers excel by managing nonlinearities and adaptively learning from data. Recent advancements in NN controller synthesis, such as encoder-decoder structured networks with attention mechanisms, enable encoding complex specifications like Signal Temporal Logic (STL) into control signals, enhancing training efficiency and flexibility. Robust stability frameworks provide operational safety through stability certificates and maximal Lipschitz bounds, while risk-aware learning approaches prioritize safety constraints and optimize performance in uncertain environments [1, 34, 17, 4].

In complex motion planning scenarios, NN controllers manage dynamic environments effectively. For example, in robotic motion planning, they seamlessly integrate sensory data such as LiDAR and visual inputs, facilitating real-time navigation and decision-making [55]. The Differentiable Physics-Based Learning Framework (DPLF) enhances NN controller robustness and efficiency by leveraging differentiable physics to provide accurate gradients for training, enabling effective learning of complex motion patterns [12].

Bayesian meta-learning techniques, exemplified by the BM-DQN framework, enable NN controllers to efficiently adapt to new environments, providing robust solutions for motion planning in dynamic settings [20]. This adaptability is essential for tasks requiring generalization across various scenarios, ensuring reliable performance.

NN controllers' integration in complex motion planning tasks signifies a pivotal advancement in control system technology, enhancing safety and robustness. Studies demonstrate NN controllers' effectiveness in managing uncertainties through innovative approaches like the Keep-Close method, ensuring outputs remain close to robust reference models despite input variations. Combining robust model predictive control (MPC) with NN function approximation allows reliable tracking of dynamic setpoints in robotic applications, effectively unifying planning and control processes. This synergy guarantees stability and compliance with constraints while improving computational efficiency, applicable to real-world scenarios, as evidenced by successful implementations in diverse problems, including robotic arm control and lunar lander guidance [1, 16]. By harnessing neural networks' learning capabilities, these controllers offer powerful tools for addressing challenges associated with spatio-temporal dynamics, paving the way for sophisticated motion planning solutions across various domains.

8 Challenges and Future Directions

8.1 Enhancing NN Controller Architectures and Methodologies

The advancement of neural network (NN) controller architectures is crucial for addressing modern control system complexities. Future research should emphasize refining theoretical frameworks and exploring numerical stability analysis methods to extend NN controllers' applicability across diverse domains [45]. Developing adaptive bounding polytopes can bridge performance gaps between one-shot and recursive approaches, enhancing efficiency and robustness [44]. The concurrent learning framework holds promise for complex real-world systems by refining learning algorithms to accommodate higher-order dynamics and integrating them with traditional strategies [51]. Simplifying NN architectures can improve scalability and efficiency, especially in real-time applications. Addressing vanishing or exploding gradients in recurrent neural networks is essential for stable training [34].

As illustrated in Figure 7, the enhancement of NN controller architectures encompasses critical aspects such as theoretical frameworks, learning and adaptation, and optimization and safety. This figure emphasizes the importance of refining theoretical frameworks, stability analysis methods, and adaptive bounding polytopes. Furthermore, it highlights the significance of concurrent learning, the simplification of architectures, and the resolution of gradient issues, all of which are vital for effective learning and adaptation.

Exploring event-triggered frameworks and extending them to self-triggered formulations can enhance NN controllers' applicability in dynamic optimization by reducing computational overhead while maintaining robust performance [43]. These research avenues promise improvements in robustness, adaptability, and efficiency, advancing control system technology. This progress will tackle challenges like safety validation and robustness against uncertainties, including unmodeled dynamics and time delays. Innovations in performance analysis, reachability, and stability certification will enable NN controllers to operate safely within specified parameters while optimizing performance. Techniques such as knowledge distillation and stochastic barrier functions will enhance reliability and safety, leading to efficiency gains and broader implementation capabilities in complex scenarios [2, 34, 3, 1, 4].

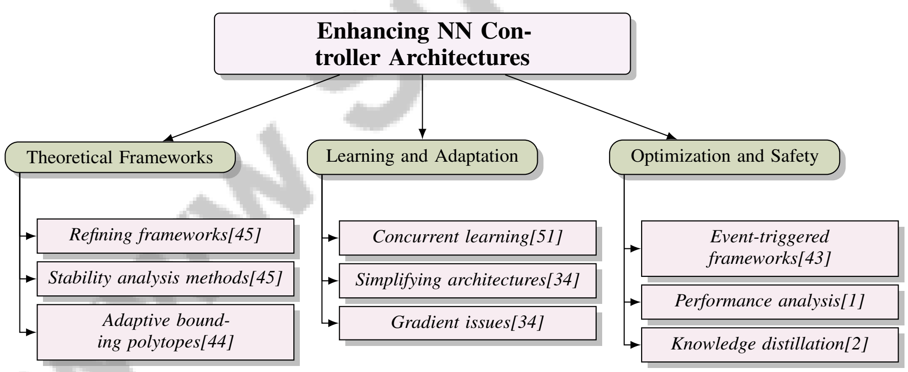

Figure 7: This figure illustrates the enhancement of neural network (NN) controller architectures, focusing on theoretical frameworks, learning and adaptation, and optimization and safety. It highlights the importance of refining theoretical frameworks, stability analysis methods, and adaptive bounding polytopes. Concurrent learning, simplifying architectures, and addressing gradient issues are emphasized for learning and adaptation. Optimization and safety are advanced through event-triggered frameworks, performance analysis, and knowledge distillation.

8.2 Improving Stability and Robustness

Enhancing the stability and robustness of NN controllers is a critical research focus. Improving the scalability and efficiency of verification methods is essential to handle complexities from high-dimensional measurements and sensor data uncertainties, potentially leading to advanced reinforcement learning algorithms that incorporate safety measures and stability guarantees [65, 30, 56, 63]. Extending methods to accommodate complex NN architectures and various delay types is promising, exploring the stability implications of different delays [42]. Refining discrete update mechanisms in analytic neural networks could enhance convergence efficiency [43]. Scalable verification methods are crucial for robustness, especially in sim-to-real transfer scenarios [35].

Incorporating perception into control frameworks and effectively handling disturbances are vital for improving NN controllers' robustness in dynamic environments [39]. Future work should integrate these elements to create adaptable and resilient control systems. The one-shot reachability analysis framework (OSRA) shows promise in enhancing safety verification by providing tighter bounds than traditional methods [44]. Simplifying mathematical treatments in dynamic systems could facilitate practical applications of NN controllers across various contexts [45].

Pursuing these directions will enhance NN controllers' validation and certification, improving stability and robustness in uncertain environments with unmodeled dynamics, nonlinearities, and time delays. Innovative approaches like the Keep-Close method, Lyapunov-based stability certificates, and data-driven design techniques will ensure NN controllers maintain safety and reliability, driving innovation and broadening their applicability in complex systems like robotics and autonomous vehicles [22, 30, 3, 1, 4].

8.3 Addressing Computational and Scalability Challenges

Computational and scalability challenges in NN controllers hinder their widespread adoption, necessitating innovative solutions to enhance efficiency. High computational complexity in training and deploying NNs, especially in real-time applications, is a primary challenge. Recent advancements focus on developing lightweight NN architectures that reduce computational overhead without sacrificing performance [36]. Integrating event-triggered frameworks optimizes resources by updating control actions only when necessary, reducing the computational load and making NN controllers feasible for large-scale systems with limited processing capabilities [43]. Concurrent learning techniques enhance scalability by leveraging sampled data for gradient computations, ensuring uniform global asymptotic stability while reducing the need for persistent excitation [51].

Efficient verification methods, like one-shot reachability analysis, address computational challenges in ensuring safety and robustness by providing tighter bounds on reachable sets [44]. Differentiable physics-based learning frameworks facilitate training by providing accurate analytical gradients, improving computational efficiency in complex motion planning tasks [12]. Addressing computational and scalability challenges requires a multifaceted approach combining innovative architectural designs, efficient learning methods, and advanced verification techniques. Implementing these strategies will enhance operational efficiency and scalability, facilitating application across a range of complex control systems. Tools like ReachNN* for reachability analysis, leveraging Bernstein polynomials and GPU acceleration, exemplify this approach, improving performance significantly. Verification-aware knowledge distillation techniques optimize NN controller parameters, ensuring robust performance amid uncertainties and unmodeled dynamics, streamlining training and validation processes for diverse scenarios, from robotics to aerospace [3, 17, 1].

8.4 Integrating Advanced Inputs and Adaptive Mechanisms

Integrating advanced input mechanisms and adaptive features in NN controllers enhances flexibility and performance in complex environments. Incorporating sophisticated sensory inputs, like LiDAR and high-resolution visual data, enables NN controllers to perceive and interpret surroundings, crucial for applications like autonomous vehicles requiring real-time decision-making [55]. Differentiable physics-based learning frameworks facilitate the integration of advanced inputs by providing accurate analytical gradients, improving training efficiency [12]. Adaptive mechanisms such as reinforcement learning (RL) and Bayesian meta-learning enhance adaptability. The BM-DQN framework exemplifies this integration, allowing NN controllers to efficiently learn and generalize across different tasks and environments [20].

Integrating Control Barrier Functions (CBFs) with Signal Temporal Logic (STL) specifications optimizes performance while maintaining safety constraints, enhancing adaptability in real-time applications [34]. Incorporating advanced inputs and adaptive features promises to expand NN controllers' capabilities, enabling them to tackle complex tasks with efficiency and reliability. Ongoing advancements in integration strategies, like adaptive tracking control, hybrid systems neural control, and RL for robotic manipulation, will enhance control system technology. These innovations will facilitate sophisticated applications, especially in dynamic environments like robotics, requiring precise execution under varying conditions. For instance, adaptive tracking control manages load variations, hybrid control systems ensure stability during mode transitions, and RL with operational space force information allows high-precision autonomous tasks. Collectively, these advancements will enable intelligent robotic systems across diverse domains, including healthcare and manufacturing [39, 56, 15, 1].

8.5 Expanding Applications and Exploring New Domains

The potential for expanding applications and exploring new domains for NN controllers is vast, driven by their adaptability and robustness. In smart grid systems, NN controllers can optimize energy distribution and consumption in real-time, enhancing efficiency and reliability [9]. In autonomous vehicles, NN controllers can revolutionize navigation and control systems by offering enhanced adaptability to dynamic environments and unforeseen obstacles [55]. Their capability to process high-dimensional sensory inputs makes them ideal for real-time decision-making in complex traffic scenarios.

The healthcare sector presents a promising domain for NN controllers, especially in robotic-assisted surgeries and rehabilitation devices, where precise control and adaptability are crucial [60]. NN controllers can enhance accuracy and safety, providing personalized solutions tailored to individual needs. In aerospace applications, NN controllers can improve guidance systems' performance and safety, managing intricate dynamics associated with flight and space exploration [1]. Their real-time learning and adaptation are essential for navigating uncertainties in these environments.

NN controllers can also enhance advanced agricultural systems by optimizing autonomous farming equipment performance. Their adaptability to uncertainties improves operational efficiency and productivity while minimizing environmental impact. Techniques like control barrier functions ensure compliance with safety constraints articulated through Signal Temporal Logic, crucial for mission-critical objectives in agriculture [1, 34].

The continuous evolution of NN controller architectures promises to drive innovation across these domains, expanding applicability and impact. By harnessing their capabilities, NN controllers can tackle modern systems' challenges, such as unmodeled dynamics, nonlinearities, and time delays. This advancement enhances control systems' robustness and safety through methods like the Keep-Close approach and stochastic barrier functions while facilitating reachability analysis and formal verification in complex applications, from robotics to aerospace. Consequently, NN controllers pave the way for sophisticated and reliable solutions across various fields, evidenced by successful implementations in diverse scenarios, including single-link robot arm control and the deep guidance of the Apollo Lander [2, 61, 3, 1, 4].

9 Conclusion

Neural network (NN) controllers are pivotal in enhancing the capabilities of modern control systems, offering a sophisticated approach to managing intricate, nonlinear dynamics with precision and flexibility. This survey elucidates their substantial role in overcoming challenges such as time delays, disturbances, and uncertainties, thereby significantly boosting system performance. Techniques like ISTC and ROSS-GT exemplify the advancements in addressing these challenges across diverse applications. The incorporation of advanced learning paradigms, including imitation learning and innovative synthesis methods, ensures that NN controllers not only meet but exceed stringent performance and safety standards. The efficacy of NN controllers is further demonstrated through experiments with OS-net, which adeptly model chaotic systems and achieve stable periodic behavior, highlighting their proficiency in handling complex dynamical systems. Research into extreme value statistics also underscores their importance in maintaining network stability across varied conditions. Verification advancements are marked by ReachNN*, which offers substantial efficiency improvements, facilitating the verification of previously challenging NN controllers. In practical scenarios, such as quadcopter control and privacy-preserving strategies in mixed autonomy, NN controllers adeptly manage uncertainties while safeguarding sensitive information, maintaining robust control performance. These developments underscore the expanding influence of NN controllers across various sectors, propelling technological advancements in fields like autonomous vehicles and sophisticated locomotion tasks. The survey underscores NN controllers' crucial role in advancing control system technology, effectively balancing safety and performance in nonlinear systems, and setting the stage for future innovations. As research progresses, NN controllers are poised to address emerging challenges and broaden their applicability in complex, dynamic environments. Additionally, the significance of adjoint methods in establishing optimality conditions and their crucial role in the stability analysis of differential equations underscores the foundational principles underpinning the advancement of NN controllers.

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