A Survey of Continuous Normalizing Flows and Knowledge Graph Embeddings

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Abstract

Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs) represent significant advancements in generative modeling and knowledge representation, respectively. CNFs utilize Neural Ordinary Differential Equations to transform simple probability distributions into complex ones, offering precise density estimation and sampling efficiency, crucial for high-dimensional data applications. KGEs, on the other hand, embed entities and relationships from knowledge graphs into continuous vector spaces, enhancing tasks such as link prediction and entity classification. This survey highlights the integration of geometric and probabilistic approaches in CNFs and KGEs, underscoring their role in improving model robustness and scalability. Despite their potential, challenges remain, including optimizing computational frameworks and integrating domain-specific knowledge. Future directions emphasize the development of hybrid models and novel architectures to enhance computational efficiency and adaptability. The synergy between CNFs, KGEs, and other generative models offers promising avenues for advancing the expressiveness and flexibility of generative frameworks, driving innovation in machine learning and data science. As the field evolves, the continued refinement of CNFs and KGEs will be essential for unlocking new potentials and applications across various domains.

1 Introduction

1.1 Overview of Continuous Normalizing Flows and Knowledge Graph Embeddings

Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs) are pivotal advancements in machine learning, particularly in generative modeling and representation learning. CNFs employ Neural Ordinary Differential Equations to transform simple probability distributions into complex ones, addressing challenges in density estimation and sampling inefficiencies [1, 2]. This transformation enhances generative models' flexibility and scalability, facilitating efficient data representation in unsupervised learning [3]. Furthermore, CNFs significantly contribute to manifold learning, essential for generative modeling in complex, non-Euclidean spaces [4].

Conversely, KGEs focus on the semantic representation of entities and relations within knowledge graphs, embedding them into continuous vector spaces for tasks such as link prediction and entity classification [5]. This embedding captures intricate semantic relationships while addressing data sparsity and computational complexity. Advanced frameworks, including graphically structured diffusion models, enhance KGEs by integrating algorithmic reasoning capabilities [6], crucial for improving robustness in domain-specific applications [7].

The convergence of CNFs and KGEs is evident in innovations utilizing stochastic differential equations and diffusion-based models, which enhance forecasting accuracy in spatiotemporal data [8]. These methodologies underscore the potential of CNFs and KGEs to collectively advance generative models and improve multivariate time series forecasting [9]. Subsequent sections will explore the methodologies, applications, and innovations associated with CNFs and KGEs, providing a

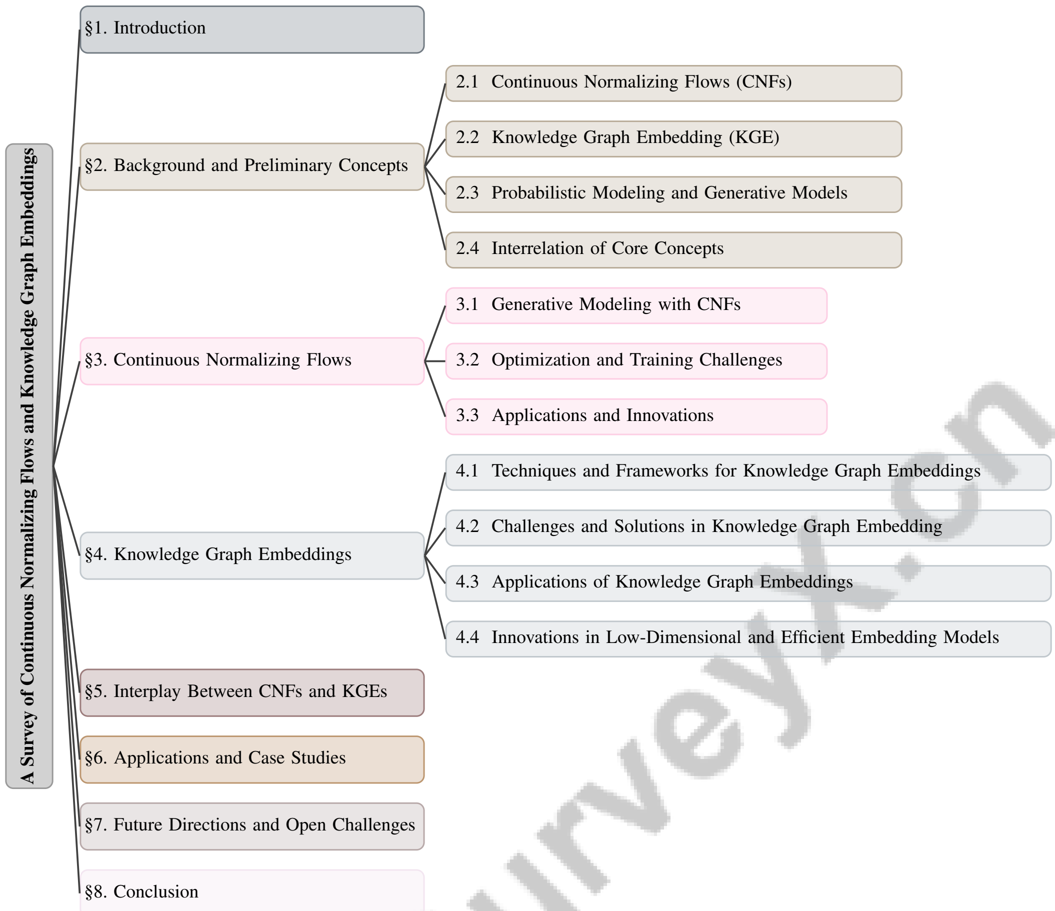

Figure 1: chapter structure

comprehensive framework for understanding their roles in contemporary data science and machine learning.

1.2 Significance in Generative Models and Probabilistic Modeling

CNFs and KGEs significantly enhance generative models and probabilistic modeling capabilities. CNFs utilize invertible transformations to map simple probability distributions into complex ones, enabling the modeling of intricate data patterns and overcoming the limitations of fixed priors like the Normal distribution [10, 11]. This adaptability is crucial for precise density estimation and generative modeling, capturing complex data dependencies [12].

Integrating CNFs with stochastic differential equations (SDEs) broadens their applicability, providing robust frameworks for maximum likelihood estimation and variational inference [6]. This is particularly beneficial for high-resolution image generation and video prediction, where traditional Gaussian variational distributions fall short [13]. CNFs also utilize manifold matching frameworks to generate samples that accurately represent the geometric structure of real data distributions, a feature often lacking in conventional methods [14]. By leveraging Riemannian geometric structures, CNFs effectively represent and sample from discrete joint distributions, addressing complex data dependencies [15].

Simultaneously, KGEs play a vital role in probabilistic modeling by embedding entities and relations from knowledge graphs into continuous vector spaces, facilitating tasks like link prediction and entity classification [5]. This embedding process is essential for integrating logical constraints and enhancing the robustness and accuracy of probabilistic models. For instance, the NS-KGE framework improves efficiency and accuracy by considering all negative instances during model learning [16]. The ability of KGEs to enhance data quality and trustworthiness is particularly significant for applications reliant on social media data [17].

The synergy between CNFs and KGEs is exemplified in applications utilizing conditional generative models, which incorporate classifier-free guidance to enhance performance in complex generative tasks [3]. As generative models evolve, integrating CNFs and KGEs with advanced probabilistic techniques remains a critical research area, promising significant advancements in both theoretical understanding and practical applications. Addressing the challenge of accurately identifying out-of-distribution inputs is crucial for applications like anomaly detection and ensuring safety in real-world deployments. Recent methods, such as G-Residual Flows and G-Coupling Flows, which incorporate equivariant maps into classical normalizing flow architectures, show promise in enhancing generative models [17]. Additionally, the potential of self-supervised learning to leverage unlabeled data through probabilistic embeddings offers promising avenues for future research [18]. Probabilistic generative models of graphs are also essential for enabling representation and sampling, contributing to the broader field of probabilistic modeling [19].

1.3 Key Concepts: Jacobian Matrix and Manifold Learning

The Jacobian Matrix and manifold learning are integral to understanding and advancing CNFs and KGEs. The Jacobian Matrix, comprising all first-order partial derivatives of a vector-valued function, is crucial in the transformation processes of CNFs. It provides insights into how input variable changes affect output changes, influencing the stability and scalability of CNF training [20]. Computing the trace of the Jacobian in closed form enhances computational efficiency, facilitating CNFs' application to larger datasets.

Manifold learning aims to uncover low-dimensional structures within high-dimensional data. In CNFs, it is essential for defining normalizing flows on complex manifold structures, exemplified by the Continuous Normalizing Flows on Manifolds (CNF) method, which employs vector fields on smooth manifolds [21]. This approach broadens the applicability and effectiveness of CNFs in generative modeling.

Incorporating manifold learning into CNFs enhances their capability to model complex data distributions and improves out-of-distribution (OOD) detection, critical for ensuring machine learning models' reliability and safety in real-world applications [22]. This significance is amplified in scenarios where models operate under uncertainty and handle novel inputs effectively.

Moreover, integrating manifold learning and the Jacobian matrix in probabilistic modeling frameworks opens new possibilities for representation learning. Recent research highlights the incorporation of stochasticity, which enhances performance, facilitates information compression, and improves OOD detection [18]. This synergy underscores the potential of CNFs and KGEs to address complex data modeling challenges.

Additionally, the dynamics of generalization in neural networks, particularly those trained via gradient descent, are influenced by the Jacobian matrix structure [23]. Understanding these dynamics is essential for optimizing CNFs and KGEs' learning processes, enabling them to capture complex data distributions more effectively. By leveraging autoregressive modeling principles, composable generative models (CGMs) can further enhance these frameworks' representation capabilities [24].

The interaction between the Jacobian matrix and manifold learning plays a crucial role in enhancing the theoretical foundations and practical implementations of CNFs and KGEs. This interaction facilitates effective CNF training on manifolds through probability path divergence minimization while enabling more precise predictions in KGEs by expanding knowledge representation from discrete points to continuous manifolds [25, 26, 23, 27, 28]. Their integration into existing and novel frameworks promises to enhance generative models' robustness, scalability, and versatility, paving the way for sophisticated and reliable machine learning solutions.

1.4 Structure of the Survey

This survey is structured to provide a comprehensive exploration of CNFs and KGEs, highlighting their significance and interconnections within the broader context of machine learning. The introduction outlines foundational concepts and the significance of CNFs and KGEs in generative models and probabilistic modeling. Section 2 delves into the background and preliminary concepts,

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offering detailed definitions and explanations of core topics such as CNFs, KGEs, the Jacobian Matrix, probabilistic modeling, generative models, and manifold learning, setting the stage for understanding the intricate relationships and collective importance of these concepts in machine learning and data science.

Section 3 focuses on CNFs, discussing their role as generative models, the use of differential equations for transforming probability distributions, and the critical function of the Jacobian Matrix in these processes. This section also addresses optimization and training challenges, along with applications and innovations in CNF usage. Section 4 explores KGEs, detailing employed techniques and frameworks, challenges and solutions in embedding high-dimensional data, and various applications of KGEs in real-world scenarios, highlighting innovations aimed at creating low-dimensional and efficient embedding models.

Section 5 analyzes the interplay between CNFs and KGEs, examining synergies that leverage manifold learning and probabilistic modeling to enhance both fields. This section explores existing research and potential directions for integrating CNFs and KGEs with advanced techniques to improve model robustness and efficiency. Section 6 provides applications and case studies, showcasing real-world implementations of CNFs and KGEs and their impact on industries and research areas.

The survey concludes with Section 7, identifying open challenges and future research directions in CNFs and KGEs, considering technological, theoretical, and practical aspects. This section emphasizes the need for interdisciplinary approaches and standardization, along with theoretical advancements and novel architectures. Finally, Section 8 summarizes the key points discussed, reflects on the current state of research, and reiterates the importance of continued exploration in these dynamic fields.The following sections are organized as shown in Figure 1.

2 Background and Preliminary Concepts

2.1 Continuous Normalizing Flows (CNFs)

Continuous Normalizing Flows (CNFs) are advanced generative models that leverage continuous-time dynamics via Neural Ordinary Differential Equations (Neural ODEs) to transform simple probability distributions into complex target distributions. Unlike traditional generative models with discrete layers, CNFs employ neural network-parameterized vector fields for transformations, allowing precise density estimation and statistical inference, thereby effectively capturing intricate data dependencies [29]. The computation of the Jacobian determinant is central to CNFs, ensuring invertibility and facilitating exact likelihood computation, which is crucial for efficient density estimation, especially in high-dimensional settings where traditional methods may struggle [4]. Techniques like Neural Autoregressive Flows (NAF) further enhance CNFs by generating pseudo-parameters for invertible transformations, improving the modeling of complex distributions [30].

Despite their advantages, CNFs face computational challenges, particularly in the discretization of ODEs during training, complicating optimization. Methods such as JKO-iFlow address these inefficiencies by unfolding the Wasserstein gradient flow, promoting efficient training and accurate density estimation [31]. CNFs have demonstrated significant potential in generating high-resolution images, surpassing traditional methods like GANs and VAEs in efficiency [29]. Their versatility extends to applications such as molecular data generation, using E(n) equivariant graph neural networks to produce molecular structures through continuous-time normalizing flows. CNFs are also pivotal in out-of-distribution detection, overcoming limitations of conventional normalizing flows reliant on high-dimensional likelihood estimations [32]. They enable meaningful latent variable manipulation in generative models, crucial for robust and scalable solutions [33]. The Conditional Euler Generator (CEGEN) exemplifies CNFs' potential in conditional generative modeling by minimizing the distance between transition probability distributions of generated and real time series [34].

2.2 Knowledge Graph Embedding (KGE)

Knowledge Graph Embedding (KGE) is essential for knowledge representation, transforming entities and relationships in Knowledge Graphs (KGs) into continuous vector spaces, enhancing computational efficiency and predictive accuracy in tasks such as link prediction and entity classification [25]. KGs, structured as directed relational graphs, often face incompleteness, necessitating sophisticated embedding techniques to predict missing links and improve graph quality [35]. Traditional KGE methods use geometric transformations to map entities and relations into vector spaces, essential for link prediction and node classification [36]. However, these methods often struggle to uniformly capture diverse relational patterns, including hierarchical and logical structures inherent in KGs [37]. Recent advancements emphasize improving embedding interpretability and effectiveness, with models like TransG capturing multiple relation semantics, enhancing KGEs' representational capacity [35]. Despite progress, KGE models face challenges in providing understandable explanations for link predictions, crucial for user trust and compliance with legal frameworks [37]. Addressing these challenges necessitates developing models that achieve high predictive accuracy while offering transparency in decision-making processes.

KGEs are vital for enabling machines to comprehend and reason over relational data. Continuous advancements in KGE methodologies are expected to significantly enhance the accuracy and efficiency of knowledge-driven applications, particularly in link prediction tasks. These improvements aim to foster the development of more sophisticated systems by addressing complex mapping properties of relations within KGs, mitigating biases associated with KG structure, and facilitating multimodal information integration. Reinterpreting KGE models as generative circuits allows for efficient maximum-likelihood estimation and sampling, ensuring compliance with logical constraints and enhancing scalability in large-scale applications [38, 39, 40]. Developing scalable and adaptable KGE models capable of capturing intricate relational patterns remains a critical area of exploration.

2.3 Probabilistic Modeling and Generative Models

Probabilistic modeling and generative models are crucial in advancing Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs) by providing structured frameworks that encapsulate uncertainty and variability in complex datasets. CNFs use Neural Ordinary Differential Equations (Neural ODEs) to transform simple probability distributions into complex ones, enhancing computational efficiency in high-dimensional spaces [10]. This transformation is essential for applications requiring precise likelihood estimation and sample generation, as CNFs can explicitly estimate likelihoods across diverse distributions [16].

In KGEs, generative models embed entities and relations from knowledge graphs into continuous vector spaces, crucial for link prediction and entity classification tasks, where probabilistic modeling techniques address challenges in capturing conditional distributions of structured data [16]. Integrating probabilistic approaches in KGEs enhances model robustness, enabling effective capture of complex relational patterns and logical constraints. A significant challenge in probabilistic modeling is the inability to explicitly compute the joint density of generator outputs in differentiable generative models, complicating inference tasks requiring conditioning on observed data [41]. This limitation highlights the necessity of developing approaches capable of managing complex dependencies between data points. Integrating probabilistic embeddings within self-supervised learning (SSL) frameworks presents challenges in adhering to stochasticity assumptions while maintaining model performance [18].

The complexity of modeling associations between node attributes and graph structures in probabilistic graphs necessitates models that can simultaneously capture structure and attributes [19]. This dual capture is vital for ensuring accuracy and expressiveness in probabilistic models, particularly in applications like healthcare, where data privacy and the need for interpretable machine learning models are paramount [40]. Generative word embedding models address limitations in probabilistic modeling by capturing the distribution of words and their contexts, enhancing semantic understanding of data [42]. Adaptive learning frameworks like LWGAN illustrate probabilistic modeling's role in generative models by adjusting latent distributions to learn data manifolds' intrinsic dimensions, improving model generalization and performance [43].

Despite advancements, existing generative methods, particularly GANs, struggle with accurately representing temporal dynamics of time series due to data's complex structure [34]. Addressing these challenges is crucial for developing generative models capable of accurately capturing and predicting temporal patterns. The integration of probabilistic modeling with CNFs and KGEs promises to enhance their robustness and effectiveness, driving innovation in machine learning and data science. Creating scalable and adaptable probabilistic models that effectively capture intricate relational patterns is emerging as a vital area of investigation, with potential for significant theoretical advancements in understanding complex data structures and practical applications across various domains, including self-supervised learning, generative modeling, and statistical inference [18, 44, 45, 46].

2.4 Interrelation of Core Concepts

The interrelation of Continuous Normalizing Flows (CNFs), Knowledge Graph Embeddings (KGEs), the Jacobian Matrix, probabilistic modeling, generative models, and manifold learning forms a foundational framework for advancing machine learning and data science. CNFs utilize bijective transformations to model complex target distributions, addressing challenges posed by lower-dimensional manifolds that can lead to large density values and complicate optimization. This capability is essential for robust and accurate generative modeling, allowing efficient sampling and density evaluation critical for applications like density estimation and outlier detection. Leveraging advanced techniques such as Normalizing Flows provides a coherent framework for distribution learning, enhancing generative models' usability while addressing challenges like memorization and privacy concerns [47, 48].

In KGEs, capturing complex relational characteristics within Knowledge Graphs (KGs) is vital for enhancing KGE model performance [49]. Existing KGE models often overlook numeric values associated with edges, impacting their predictive power in link prediction tasks [49]. Techniques like RotatE model relations as rotations in complex vector spaces, enhancing KGE expressiveness by effectively capturing symmetry, inversion, and composition patterns [50]. Methods like JEL-EKG integrate structural and literal information, addressing traditional KG embedding models' limitations [51].

Probabilistic modeling enriches CNFs and KGEs by encapsulating uncertainty and variability within datasets. Theoretical perspectives emphasize latent variables' probabilistic nature and their role in capturing dependencies [46]. This foundation is critical for tasks requiring precise likelihood estimation and sample generation, enabling models to estimate likelihoods across diverse distributions [52]. Integrating probabilistic approaches in KGEs enhances model robustness, effectively capturing complex relational patterns and logical constraints [52].

Manifold learning, particularly through innovations like Manifold Continuous Normalizing Flows (MCNFs), leverages local geometry for probability computations, overcoming limitations of methods requiring global diffeomorphisms [12]. This approach extends CNFs' applicability to complex manifold structures, broadening their generative modeling utility.

The synergy between these core concepts is exemplified in addressing real-world challenges such as predicting missing links in knowledge graphs, vital for knowledge completion and validation [53]. Methods integrating multiple KGE models using an attention mechanism enhance prediction accuracy by focusing on the most suitable representations for each query [53]. Integrating hyperbolic geometry in lightweight models demonstrates potential for improved efficiency and performance through simplified calculations [54].

Theoretical contributions related to generative adversarial networks (GANs) provide insights into generative models, particularly in image generation, discussing frameworks inspired by game theory, statistical theory, and dynamical systems [55]. This highlights the broader applicability and influence of generative models within the machine learning landscape.

The interplay between CNFs, KGEs, and related mathematical constructs such as the Jacobian Matrix and manifold learning is essential for developing advanced models that capture and represent complex relational patterns effectively. This integration allows for a nuanced understanding of relation properties, such as symmetry and composition, while addressing challenges posed by sparse and dynamic knowledge graphs, thereby enhancing performance and interpretability of knowledge-augmented applications [25, 39, 40, 56, 57]. Refining these concepts through innovative methodologies promises significant advancements in both theoretical understanding and practical applications, enhancing the robustness and effectiveness of machine learning solutions.

3 Continuous Normalizing Flows

Continuous Normalizing Flows (CNFs) play a pivotal role in generative modeling by transforming simple probability distributions into complex target distributions through continuous-time dynamics and Neural Ordinary Differential Equations (Neural ODEs). This section explores the intricacies of CNFs, their optimization challenges, and their innovative applications across various domains.

3.1 Generative Modeling with CNFs

CNFs revolutionize generative modeling by employing Neural ODEs to parameterize vector fields, enabling precise modeling of high-dimensional data dependencies [29]. These transformations maintain manifold structures through diffeomorphic mappings, enhancing model expressiveness [4]. CNFs offer exact density evaluation and efficient sampling, crucial for capturing complex patterns. Techniques like Flow Matching allow CNFs to transform distributions efficiently, enhancing model versatility in applications such as multivariate time series analysis [41, 19].

Integrating CNFs with probabilistic frameworks, such as the Relational Variational Autoencoder (RVAE), enhances generative capabilities over graph-structured data [40]. Equivariant Graph Neural Networks further extend CNFs for generating 3D graphs, respecting symmetries [55]. In molecular generation, CNFs transform latent representations into complex molecular structures using techniques like Sinkhorn loss for optimal transport [36, 42]. Recursive construction using simpler flows ensures numerical stability and expressiveness [43].

CNFs' innovation is evident in methods like OT-Flow, which reformulates CNFs as an optimal transport problem, offering efficient trace computation [34]. CNF Matching (CNFM) and Entropy-Kantorovich potentials optimize CNF training, avoiding ODE evaluations and enhancing computational efficiency [4]. These advancements allow CNFs to outperform traditional diffusion-based methods, offering superior sample quality and generation speed [58, 59, 60].

3.2 Optimization and Training Challenges

The optimization of CNFs involves challenges due to the computational burden of integrating ODEs, necessitating numerous function evaluations (NFEs) [29]. Balancing model complexity with output quality demands significant computational resources [2]. Non-uniqueness of flow transport and numerical instability in non-Euclidean projections further complicate training [31, 61].

Efficient Jacobian trace computation is critical for stable training and scalability [41]. The quadratic scaling of edge reasoning in large graphs poses additional challenges [62]. Methods like Markovian Flow Matching (MFM) improve exploration and generalization, addressing these optimization hurdles [26, 63]. Semi-Equivariant Conditional Normalizing Flows (SECNF) demonstrate high validity and improved predictions, highlighting CNFs' optimization successes [64].

As illustrated in Figure 2, the primary challenges and solutions in optimizing and training Continuous Normalizing Flows (CNFs) are summarized, emphasizing computational challenges, efficient methods, and future directions for improvement. Addressing these challenges is crucial for enhancing CNF scalability and applicability. Integrating sophisticated mathematical frameworks and methodologies is expected to bridge theoretical gaps and develop robust generative models capable of high-dimensional data generation [65, 1, 66].

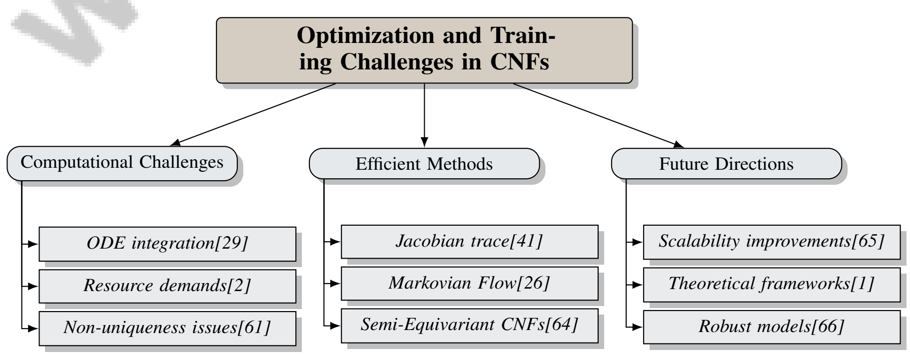


Figure 2: This figure illustrates the primary challenges and solutions in optimizing and training Continuous Normalizing Flows (CNFs), highlighting computational challenges, efficient methods, and future directions for improvement.

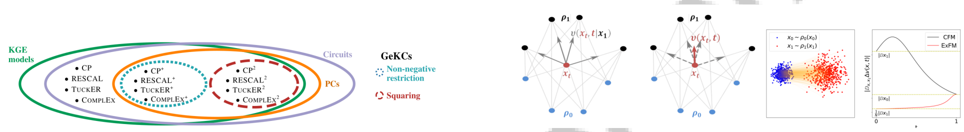
3.3 Applications and Innovations

CNFs have become instrumental in manifold learning, enhancing sampling efficiency and modeling data on arbitrary smooth manifolds, essential for tasks like 3D object reconstruction [21]. The FFJORD method excels in density estimation and variational inference, crucial for applications requiring precise probabilistic modeling [67]. Temporal optimization techniques, such as TO-FLOW, enhance CNF training speed for real-time applications [68].

Sinkhorn divergence expands CNFs' applicability to large-scale settings, benefiting image synthesis and generative art [69]. CNFs also offer privacy-preserving solutions, suitable for secure data generation [47]. E-Geodesic Flow Matching exemplifies CNFs' ability to learn complex joint distributions, relevant for natural language processing [70].

Innovations in training, like the ExFM method, improve flow-based model scalability and stability [71]. Markovian Flow Matching (MFM) achieves computational efficiency, reducing costs while maintaining performance [72]. SparseFlow enhances CNF generalization with fewer parameters, advantageous for embedded systems [73].

The LWGAN framework addresses traditional GAN and VAE limitations through adaptive learning, showcasing CNFs' potential in generative modeling [43]. The JKO-Flow method consistently improves CNF performance, essential for scientific computing applications [74]. These innovations underscore CNFs' versatility and effectiveness in addressing complex data modeling challenges, positioning them as transformative tools in deep generative modeling [58, 60, 46, 42].



(a) Geometric Kernels for Graph Neural Networks: A Comprehensive Overview[38]

(b) Exponential Flow Model (ExFM) for Graph Neural Networks: A Comparative Study[71]

Figure 3: Examples of Applications and Innovations

As illustrated in Figure 3, CNFs provide a powerful framework for modeling complex data distributions through continuous transformations. The studies "Geometric Kernels for Graph Neural Networks: A Comprehensive Overview" and "Exponential Flow Model (ExFM) for Graph Neural Networks: A Comparative Study" exemplify CNFs' innovative applications in enhancing graph neural networks (GNNs). The first study explores geometric kernels within GNNs, detailing models such as CP, RESCAL, TUCKER, and COMPLEX, crucial for graph data representation. The second study compares ExFM and Continuous Flow Model (CFM) applications in GNNs, highlighting their impact on graph structures. These studies showcase CNFs' cutting-edge innovations in advancing GNN capabilities [38, 71].

4 Knowledge Graph Embeddings

In Knowledge Graphs (KGs), transforming discrete structures into continuous vector representations is crucial for computational efficiency and diverse applications. Numerous techniques and frameworks optimize Knowledge Graph Embeddings (KGEs), focusing on foundational methods, innovative adaptations, and the interplay of geometric, probabilistic, and neural network-based techniques to enhance expressiveness and scalability. Table 1 presents a detailed classification of methodologies and advancements in Knowledge Graph Embeddings, underscoring the diverse strategies employed to address challenges and expand applications in this domain. Additionally, Table 4 offers a comparative overview of various Knowledge Graph Embedding methods, detailing their optimization techniques, dimensionality, and application domains to illustrate the diversity in approaches and their relevance to different tasks.

?? illustrates the hierarchical structure of Knowledge Graph Embeddings, categorizing techniques and frameworks, challenges and solutions, and applications. This figure highlights foundational and innovative methods, as well as domain-specific solutions and scalable approaches under the umbrella of techniques and frameworks. Furthermore, it addresses challenges such as high-dimensional

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Category	Feature	Method
Techniques and Frameworks for Knowledge Graph Embeddings	Complex Relation Modeling	DRE[75], TransG[35]
	Geometric Embedding Spaces	ManifoldE[25]
	Uncertainty and Probabilistic Methods	PrTransX[5]
Challenges and Solutions in Knowledge Graph Embedding	Confidence and Attention Techniques	KANE[76], CSD[77]
	Scalability and Efficiency Approaches	NS-KGE[78], FHRE[79], ATTH[80]
	Optimization and Tuning Strategies	SSA[81], HCF[82]
	Iterative and Linear Methods	LR[83]
Applications of Knowledge Graph Embeddings	Geometric and Spatial Representation	TorusE[84], WPM[28]
	Dimensional Adaptability	3H-TH[85], MREL[86]
	Visual Quality Improvement	SGN[2]
Innovations in Low-Dimensional and Efficient Embedding Models	Space Efficiency	PYKE[87]

Table 1: This table provides a comprehensive summary of various methodologies and innovations in the field of Knowledge Graph Embeddings (KGEs), categorized into techniques and frameworks, challenges and solutions, applications, and low-dimensional models. It highlights specific features and methods such as geometric embedding spaces, probabilistic approaches, and scalability techniques, illustrating the breadth of research and development in enhancing KGE expressiveness and efficiency.

embeddings and presents solutions like the KANE framework. Finally, the figure categorizes applications into link prediction and efficiency and scalability, showcasing models such as TorusE and PYKE. This comprehensive depiction not only complements the textual analysis but also enhances the understanding of the complexities involved in KGE methodologies.

4.1 Techniques and Frameworks for Knowledge Graph Embeddings

Knowledge Graph Embeddings (KGEs) transform discrete knowledge graph structures into continuous vector spaces, improving computational efficiency and enabling applications like link prediction and entity classification. Techniques for enhancing KGE expressiveness and scalability include relation-aware mapping models capturing complex relational properties, tensor decomposition, and neural network-based approaches. Innovations integrate auxiliary information, hyperbolic spaces, and polar coordinates to capture intricate relation patterns such as symmetry, asymmetry, and composition. Future research focuses on multimodal information integration and modeling dynamic, sparse Knowledge Graphs to address structural biases impacting KGE performance [39, 40].

Translation-based models are foundational in KGE methodologies. TransE, for instance, represents relations as translations in embedding space, effectively capturing relational patterns [5]. Extensions like RotatE define relations as rotations in complex vector space, capturing symmetry and inversion, while TorusE uses a torus embedding space to maintain TransE principles without regularization [84].

Geometric approaches expand KGE capabilities through manifold-based models. ManifoldE measures triple distances from a manifold, offering a novel embedding perspective [25]. TransG enhances representational capacity by using a mixture of relation component vectors to embed various latent meanings [35].

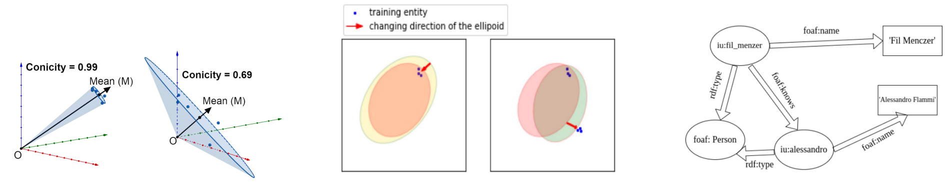
Neural network-based approaches advance KGEs by integrating Graph Neural Networks (GNNs) to handle knowledge graph complexities, capturing high-order structures and facilitating embedding propagation. Techniques like Kronecker decomposition reduce redundancy and encourage feature reuse, improving predictive performance and resilience against noise [88, 89, 75, 90, 91].

Probabilistic approaches enrich KGE frameworks by incorporating uncertainty. The PrTransX algorithm, for example, incorporates triplet uncertainty, outperforming corresponding TransX algorithms in link prediction tasks [5].

Domain-specific knowledge graphs enhance KGE model accuracy and effectiveness by leveraging unique domain characteristics and relationships. Methods like TransC represent concepts as spheres and instances as vectors, effectively modeling relationships [38, 92, 39, 40, 91].

Hardware-agnostic frameworks allow large knowledge graph embedding computations without code modification, enhancing scalability and accessibility. Performance benchmarks provide insights into practical applications, particularly in link prediction, where understanding graph structure and embedding effectiveness is crucial [91, 38, 40].

The diverse techniques and frameworks for developing KGEs highlight the evolving nature of the field, driven by the increasing application of KGs and the need to address challenges related to knowledge representation and data quality. Recent research emphasizes these factors to enhance link prediction tasks and understanding of interactions between Knowledge Graphs and embeddings [38, 92, 40].



(a) Conicity Analysis in Machine Learning: Understanding the Impact on Data Representation[93]

(b) Changing Direction of the Ellipsoid[75]

(c) The image depicts a simple graph representation of a knowledge graph, showing relationships between entities.[91]

Figure 4: Examples of Techniques and Frameworks for Knowledge Graph Embeddings

As shown in Figure 4, various techniques and frameworks enhance the representation of complex data structures in knowledge graph embeddings. The first image, "Conicity Analysis in Machine Learning," shows how conicity impacts data representation. The second image, "Changing Direction of the Ellipsoid," highlights the dynamic nature of training entities within geometric structures. The third image illustrates a simple graph representation of a knowledge graph, depicting relationships crucial for understanding graph semantics. These examples showcase the methodologies underpinning knowledge graph embedding development and application [93, 75, 91].

4.2 Challenges and Solutions in Knowledge Graph Embedding

Method Name	Scalability Challenges	Modeling Limitations	Optimization Techniques
CSD[77]	Computation And Memory	Hierarchical Structures	Hyperparameter Tuning
SSA[81]	Computational Time Constraints	Hierarchical Structures Capturing	Sobol Sensitivity Analysis
PYKE[87]	Quadratic Time Complexity	Ppmi Similarity Measure	Simulated Annealing
WPM[28]	-	Complex Graph Structures	Hyperparameter Tuning
KANE[76]	Computational Complexity Impact	Hierarchical Structures Capturing	Hyperparameter Tuning Leveraging
NS-KGE[78]	Higher Computational Costs	-	Efficient Computations
MREL[86]	High Training Costs	Complex Graph Patterns	Low-dimensional Embeddings
LR[83]	High Computational Complexity	Linear Regression Assumptions	Optimize The Model
HCF[82]	Computational And Time	Complex Relationships Capturing	Dynamic Configuration Adjustments
FHRE[79]	-	Hierarchical Structures	Hyperparameter Tuning
ATTH[80]	Computational Complexity Involved	Inadequate Representations Fail	Hyperparameter Settings Learning

Table 2: This table presents a comprehensive comparison of various Knowledge Graph Embedding (KGE) methods, highlighting their scalability challenges, modeling limitations, and optimization techniques. The table serves to underscore the diverse approaches and specific issues encountered by each method, providing insights into their performance dynamics and potential areas for improvement.

Knowledge Graph Embeddings (KGEs) face challenges impacting their performance and scalability, including high-dimensional embeddings leading to increased computational demands and deployment challenges in resource-constrained environments. While low-dimensional strategies offer promise, they often require extensive pre-training or involve complex non-Euclidean operations [77].

Table 2 provides a detailed overview of the scalability challenges, modeling limitations, and optimization techniques associated with different Knowledge Graph Embedding methods, emphasizing the intricacies and solutions pertinent to advancing KGE frameworks. Hyperparameter tuning inefficiencies and redundancies present significant challenges, testing many task-irrelevant hyperparameters and hindering rapid KGE deployment and adaptation [81]. Scalability remains a concern due to quadratic time complexity in processing large knowledge graphs, limiting large-scale applicability [87].

Modeling hierarchical relations is challenging, as current KGE models struggle to capture these structures without excessive auxiliary information reliance, limiting applicability [39]. The noisy, heterogeneous nature of real-world knowledge graphs complicates embedding processes, with methods like WEIGHTED-PM affected by input graph noise or inaccuracies [28].

Innovative solutions address these challenges. Techniques like KANE incorporate high-order structural relationships and attribute triples, enhancing representation and overcoming existing method limitations [76]. The NS-KGE framework improves accuracy and efficiency by leveraging all instances, reducing negative sampling reliance and enhancing robustness [78].

Ensemble learning approaches enhance scalability and reduce computational demands but depend on low-dimensional embedding size and model number choices, indicating a complexity-performance trade-off [86]. Linear regression assumptions in some models restrict performance in complex relational scenarios, suggesting the need for flexible modeling techniques [83].

Lack of support for pseudo-labeling and Auto-ML techniques limits framework capabilities [82]. Addressing this could enhance KGE adaptability and efficiency. Existing hyperbolic rotation models face obstacles due to reliance on logarithmic and exponential mappings, complicating training [79]. The ATT H model improves representation accuracy by modeling hierarchical and logical relationships but involves potential computational complexity [80].

Addressing KGE challenges requires integrating advanced geometric, probabilistic, and computational techniques. By advancing scalable, efficient, and flexible embedding techniques, future KGE frameworks can enhance accuracy and robustness, modeling complex relationships and patterns within Knowledge Graphs for applications in deep learning and social media analytics [38, 39, 40, 91, 57].

4.3 Applications of Knowledge Graph Embeddings

Method Name	Application Domains	Methodological Innovations	Performance Metrics
TorusE[84]	Knowledge Graph Completion	Embedding ON Torus	Link Prediction Accuracy
PYKE[87]	Drugbank Dbpedia	Physical Model	Cluster Purity
3H-TH[85]	Link Prediction Tasks	3D Rotation Translation	Ranking-based Metrics
MREL[86]	Link Prediction	Multiple Run Ensemble	Link Prediction Accuracy
WPM[28]	Recommendation Systems	Scoring Mechanism	Average Distortion
SGN[2]	Latent Spaces	Spherical Interpolation	Visual Quality

Table 3: This table summarizes various knowledge graph embedding methods, their application domains, methodological innovations, and corresponding performance metrics. It highlights the diverse applications of these methods across different fields, demonstrating their effectiveness in enhancing knowledge graph utility and performance.

Knowledge Graph Embeddings (KGEs) have extensive applications across various domains, transforming complex graph structures into continuous vector spaces. A prominent application is in link prediction, where KGEs excel in predicting missing links, enhancing knowledge graph completeness and utility. The TorusE model improves link prediction accuracy by embedding knowledge graphs onto a torus, capturing intricate relational patterns without regularization [84].

In biomedicine, KGEs predict links within biomedical knowledge graphs, potentially reducing costs and accelerating research by identifying novel biological relationships, crucial for drug discovery and personalized medicine [94].

KGE applications extend to scenarios requiring efficient, scalable solutions for large knowledge graphs. The PYKE model offers significant runtime efficiency and embedding quality improvements for large-scale applications with computational constraints [87]. The 3H-TH model outperforms state-of-the-art models in low-dimensional space while maintaining competitive high-dimensional performance, showcasing adaptability across applications [85].

Ensemble learning approaches enhance KGE applications by leveraging multiple low-dimensional models, improving link prediction accuracy and training efficiency while reducing computational overhead, suitable for real-time applications [86].

The WEIGHTED-PM method improves graph embedding quality by capturing complex geometric structures in real-world data, enhancing performance on various downstream tasks, beneficial in heterogeneous graph settings [28].

KGEs also improve generative model visual quality. Sampling generative networks (SGN) insights highlight KGEs' practical implications in enhancing visual realism, vital for virtual reality and computer graphics [2].

Overall, KGE applications underscore their versatility and effectiveness in transforming knowledge graphs into actionable insights. As research advances, innovative KGE methodologies will enhance applicability, improve link prediction performance, and address biases related to Knowledge Graph structures, contributing to a deeper understanding of knowledge representation and practical implications [38, 95, 92, 39, 40]. Table 3 provides a comprehensive overview of selected knowledge graph embedding methods, detailing their application domains, innovative methodologies, and performance metrics, thereby illustrating their diverse applications and contributions to the field.

4.4 Innovations in Low-Dimensional and Efficient Embedding Models

Innovations in low-dimensional and efficient embedding models significantly advance Knowledge Graph Embeddings (KGEs), addressing scalability and computational efficiency challenges. The PYKE model integrates a physical model with simulated annealing, achieving linear space and near-linear time complexity, enabling effective scaling for large knowledge graphs in resource-constrained environments [87].

Low-dimensional embeddings reduce computational and memory demands while maintaining expressiveness for capturing complex relational patterns. Ensemble learning combines multiple low-dimensional models, enhancing predictive accuracy and training efficiency compared to single high-dimensional models. This approach enhances generative model performance, enabling efficient sampling and density evaluation, minimizing computational overhead through Kronecker decomposition, suited for real-time applications and large-scale deployments [88, 82, 89, 48].

Recent advancements in geometric approaches, particularly lightweight Euclidean-based models and fully hyperbolic rotation techniques, enhance KGE model efficiency. While traditional hyperbolic models like RotH showed potential in low-dimensional spaces, newer models like RotL and Rot2L simplify hyperbolic operations, improving representation capabilities with reduced computational costs. The novel fully hyperbolic model leveraging the Lorentz model addresses limitations in existing hyperbolic rotation methods, achieving state-of-the-art performance on challenging datasets with fewer parameters, underscoring geometric understanding's importance in optimizing KGE methods [79, 54, 93].

Overall, innovations in low-dimensional and efficient embedding models emphasize balancing computational efficiency with accurate complex data structure representation. As research in Knowledge Graphs (KGs) and Knowledge Graph Embedding Models (KGEMs) progresses, advancements will improve model scalability and versatility across domains, enhancing link prediction tasks and addressing data sparsity, computational complexity, and biases introduced by KG structure. These developments will facilitate sophisticated, resource-efficient knowledge representation solutions leveraging deep learning techniques and social media data [91, 40].

Feature	TransE	RotatE	TorusE
Optimization Technique	Translation-based	Rotation-based	Torus Embedding
Dimensionality	Low-dimensional	Complex Space	Low-dimensional
Application Domain	General	Symmetry	Link Prediction

Table 4: This table provides a comparative analysis of three different Knowledge Graph Embedding (KGE) methods—TransE, RotatE, and TorusE—highlighting their distinct optimization techniques, dimensionality characteristics, and application domains. The table underscores the diversity in approaches, ranging from translation-based to rotation-based and torus embedding strategies, each tailored to specific relational properties and prediction tasks within knowledge graphs.

5 Interplay Between CNFs and KGEs

5.1 Knowledge Transfer and Generative Modeling

The integration of knowledge transfer and generative modeling significantly enhances Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs), improving adaptability and accuracy in complex environments. In CNFs, generative modeling techniques, particularly those leveraging Neural Ordinary Differential Equations (Neural ODEs), transform simple distributions into complex ones, crucial for tasks like novelty detection by modeling intricate distributions and evaluating unseen data [41]. For KGEs, generative models such as TransG enhance representational capacity by capturing multiple latent meanings of relations, thus improving predictive accuracy [35]. The integration of probabilistic embeddings introduces robustness, enhancing performance in uncertain environments by decorrelating features [18].

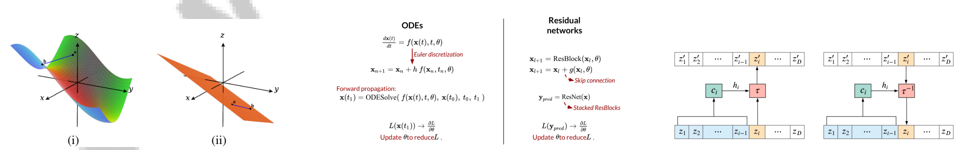
Models like LGGM and generative word embedding models contribute significantly to cross-domain knowledge transfer, enhancing the performance of graph generative models by learning transferable patterns [36, 42]. Tunable models in graph generation, such as those used in drug discovery, add flexibility and precision by allowing tailored generative outputs [96]. Innovations in adaptive learning, exemplified by LWGAN, optimize generative processes by learning intrinsic data manifold dimensions [43].

These advancements are pivotal for developing Knowledge Graphs (KGs) and Knowledge Graph Embedding Models (KGEMs), enhancing their ability to predict new facts while addressing challenges such as multiple relation semantics and biases in KG structures. The reinterpretation of KGE score functions as generative circuits facilitates maximum-likelihood estimation and efficient sampling, thus improving scalability and performance in scenarios involving millions of entities [38, 39, 35, 40]. By integrating advanced mathematical frameworks and domain-specific knowledge, these models achieve greater robustness and accuracy, paving the way for sophisticated applications in machine learning and data science.

5.2 Geometric and Probabilistic Enhancements

Geometric and probabilistic approaches are vital for advancing CNFs and KGEs, providing robust frameworks for modeling complex data distributions and capturing intricate relational patterns. Integrating geometric principles with normalizing flows enables tractable computations of complex distributions, effectively managing uncertainty, particularly in high-dimensional settings [57]. Decomposing the Jacobian matrix into information and nuisance spaces enhances the training and generalization processes by focusing on informative features [23]. Probabilistic enhancements in KGEs, as seen in frameworks like MetaEU, leverage meta-learning to generate embeddings adaptively, addressing the forgetting challenge in dynamic knowledge graphs [97].

The integration of geometric and probabilistic approaches enhances the expressiveness and scalability of CNFs and KGEs, laying the foundation for sophisticated models capable of capturing the nuances of complex data landscapes. Recent advancements in knowledge graph embeddings, generative word embedding models, and self-supervised learning techniques promise to significantly enhance machine learning and data science, yielding more accurate models by addressing challenges related to model architecture and training approaches [18, 89, 42].



(a) Graphical Representation of a Function in Three Dimensions[98]

(b) Residual Networks and Ordinary Differential Equations (ODEs)[99]

(c) Sequential Representation Learning with Temporal Context[10]

Figure 5: Examples of Geometric and Probabilistic Enhancements

The exploration of CNFs and KGEs is enriched by geometric and probabilistic enhancements, as illustrated in Figure 5. The first example presents a three-dimensional graphical representation of a function, where a gradient color scheme illustrates the function's values across a surface grid, offering insights into spatial behavior. The second example juxtaposes Residual Networks with Ordinary Differential Equations (ODEs), highlighting forward propagation and prediction processes, underscoring advancements in solving differential equations within neural networks. Lastly, the depiction of sequential representation learning with temporal context showcases a model integrating a recurrent neural network (RNN) with a temporal context module, emphasizing the role of temporal dynamics in enhancing sequential data processing. Together, these examples underscore the potential of integrating geometric and probabilistic approaches to advance the understanding and application of CNFs and KGEs [98, 99, 10].

5.3 Synergies with Other Generative Models

The integration of CNFs, KGEs, and other generative models holds significant promise for advancing generative modeling through the synthesis of diverse methodologies. Frameworks like KE-GCN exemplify the potential to leverage strengths from multiple approaches for comprehensive multi-relational graph modeling [90]. KE-GCN integrates various Graph Convolutional Network (GCN) methods with knowledge embedding techniques, showcasing the synergies achievable by combining CNFs and KGEs with other generative models.

The synergies between CNFs and generative models such as Variational Autoencoders (VAEs) and Generative Adversarial Networks (GANs) enhance the expressiveness and flexibility of generative frameworks. CNFs provide precise likelihood estimation and efficient sampling, essential for improving the performance of VAEs and GANs, particularly in high-dimensional data spaces. Their ability to learn complex probability distributions through ordinary differential equations enables CNFs to achieve state-of-the-art results in applications like image synthesis and molecular generation. Techniques like Flow Matching improve training stability and speed, facilitating better generalization and faster sampling, positioning CNFs as powerful tools in generative modeling [58, 26, 100, 60]. Incorporating CNFs into these models can enhance data quality and facilitate more accurate density estimation.

KGEs also benefit from integration with other generative models, enhancing their ability to capture complex relational patterns and logical constraints within knowledge graphs. The probabilistic foundations of KGEs allow effective representation of uncertainty and variability in structured data, making them suitable for integration with probabilistic generative models. This integration improves tasks like link prediction and entity classification by capturing complex dependencies, as evidenced by advancements in knowledge graph embedding techniques that enhance parameter efficiency and robustness against noise while accommodating diverse relational patterns [88, 83, 89, 95, 53].

The synergies between CNFs, KGEs, and other generative models present a promising avenue for advancing generative modeling. By unifying diverse methodologies, these synergies can enhance robustness, scalability, and applicability across various domains. As research in generative modeling progresses, exploring synergies among methods—such as linear combinations of latent variables and innovative 3D representations—holds significant potential for fostering groundbreaking innovations. This exploration is expected to yield more sophisticated solutions that enhance data synthesis, improve control over generation processes, and address challenges in modeling complex high-dimensional data, ultimately shaping the future of applications in computer vision and beyond [99, 66, 47, 101, 1].

5.4 Integration of CNFs and KGEs with Advanced Techniques

Integrating CNFs and KGEs with advanced techniques is pivotal for enhancing model robustness and efficiency, expanding their applicability across diverse domains. This integration aims to leverage advanced methodologies in generative modeling and representation learning to address challenges such as the opacity of probabilistic interpretations, loss of corpus information in matrix factorization, and the need for controlled generation processes. By incorporating latent factors—such as topics, sentiments, or writing styles—into generative models and employing techniques like linear combinations of latent variables and embedding nudging, this integration seeks to improve interpretability, scalability, and the quality of generated outputs while addressing privacy and usability concerns [47, 66, 42].

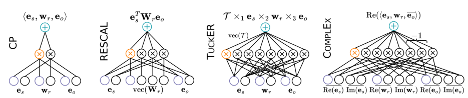
A primary avenue for this integration involves developing efficient algorithms for training and evaluating normalizing flows, critical for applying CNFs to large-scale datasets and complex domains [10]. Refining the algorithms that underpin CNFs enhances their expressiveness and flexibility, making them suitable for high-dimensional structured data [45].

Incorporating novel learning algorithms into CNFs and KGEs can further enhance performance, facilitating exploration in new domains, such as discrete data modeling and structured domains where traditional methods may struggle [10]. Additionally, integrating CNFs with other generative models, such as GANs, holds promise for improving the quality and diversity of generated data [44].

Exploring novel architectures is another critical aspect of integrating advanced techniques with CNFs and KGEs. Innovative architectural designs can enhance models' ability to capture complex dependencies and relational patterns [45]. This includes applying manifold learning techniques, such as the dynamic chart method in Manifold Continuous Normalizing Flows (MCNFs), which can improve stability and efficiency in modeling data on complex manifolds [12].

Extending the application of existing models, such as JKO-iFlow, to larger-scale applications and integrating them with other generative models can significantly enhance performance [31]. This approach underscores the importance of cross-pollination between different modeling frameworks to achieve more robust and efficient solutions.

The integration of CNFs and KGEs with advanced methodologies, including generative circuit models and enhanced relation pattern modeling, represents a promising avenue for future research, particularly in addressing the complexities of Knowledge Graph structures and improving link prediction tasks across diverse applications. This approach aims to mitigate biases inherent in KG structures while enhancing relational property modeling, contributing to a more comprehensive understanding of knowledge representation and its applications in machine learning [38, 39, 40]. By focusing on enhancing model efficiency, exploring novel learning algorithms, and applying these models to new domains, researchers can drive significant advancements in generative modeling and knowledge representation, ultimately leading to more sophisticated and reliable machine learning solutions.



(a) Four Neural Network Architectures for Estimating the Complex of a Matrix[38]

Model	Geometry of Orth.	Dimension of Orth.	Logical Patterns				Topological Structures	
			Symmetry	Antisymmetry	Inversion	Composition	Cyclicity	Hierarchy
RotatE (Sun et al., 2019)	$\mathbb{E}$	2	✔	✔	✔	✔	✔	✗
Rotate3D (Gao et al., 2020)	$\mathbb{E}$	3	✔	✔	✔	✔	✔	✗
DualE (Cao et al., 2021)	$\mathbb{E}$	3	✔	✔	✔	✗	✔	✗
QuatE (Zhang et al., 2019)	$\mathbb{E}$	4	✔	✔	✔	✗	✔	✗
HousE (Li et al., 2022)	$\mathbb{E}$	k	✔	✔	✔	✔	✔	✗
RefH (Chami et al., 2020)	$\mathbb{Q}$	2	✔	✔	✔	✔	✗	✔
RotH (Chami et al., 2020)	$\mathbb{Q}$	2	✔	✔	✔	✔	✗	✔
AttH (Chami et al., 2020)	$\mathbb{Q}$	2	✔	✔	✔	✔	✗	✔
GoldE	$\mathbb{P}(\mathbb{E}), \mathbb{Q}$	k	✔	✔	✔	✔	✔	✔

(b) The table compares various models for representing geometric structures in computer graphics.[102]

Figure 6: Examples of Integration of CNFs and KGEs with Advanced Techniques

The interplay between CNFs and KGEs is a burgeoning research area that seeks to integrate these techniques with advanced methodologies to enhance data representation and inference. As illustrated in Figure 6, the first example showcases "Four Neural Network Architectures for Estimating the Complex of a Matrix," highlighting diverse architectures—CP, RESCAL, TUCKER, and COMPLEX—designed to estimate matrix complexity through layered structures. This demonstrates the versatility of neural networks in processing complex data structures. The second example, a comparative table of models for representing geometric structures in computer graphics, underscores the diversity of approaches in capturing geometric and topological nuances, categorizing models like RotatE and Rotate3D based on orthogonality geometry, dimension, logical patterns, and topological structures. Together, these examples illustrate the potential of integrating CNFs and KGEs with advanced techniques to push the boundaries of data representation and analysis [38, 102].

6 Applications and Case Studies

The transformative impact of Knowledge Graph Embeddings (KGEs) is evident across numerous domains, significantly enhancing data-driven decision-making and operational efficiency. This section delves into real-world applications of KGEs, underscoring their importance in sectors such as smart cities and healthcare.

6.1 Knowledge Graph Embeddings in Real-World Scenarios

KGEs adeptly convert complex relational data into actionable insights across diverse domains. In smart cities, they optimize urban planning by accurately modeling entities and interactions, as demonstrated with datasets from smart city environments, where KGEs enhance urban systems and services [103]. In healthcare, KGEs improve predictive analytics by constructing medical knowledge graphs from over 10 million electronic medical records, uncovering hidden relationships that enhance patient outcomes through advanced link prediction [5]. This capability is crucial for drug discovery and personalized medicine, where understanding complex biological interactions is essential.

KGEs also excel in managing large-scale knowledge graphs, as evidenced by their application to the DBpedia 2021 benchmark dataset, encompassing over 11.4 billion parameters. This scalability enables KGEs to efficiently handle vast datasets, delivering high-quality embeddings vital for dynamic domains [82]. Furthermore, Composable Generative Models (CGMs) demonstrate the versatility of KGEs across sectors like finance, energy, and transportation, facilitating data-driven decision-making [24].

Significant advancements in explainable AI have been achieved through KGEs. The ExamplE framework generates understandable explanations for link predictions, enhancing transparency and interpretability, crucial in sensitive fields such as healthcare and finance [37]. The successful application of KGEs in diverse scenarios underscores their transformative potential. As KGE methodologies advance, the development of more sophisticated and scalable techniques is anticipated, improving link prediction accuracy and integrating logical constraints, thus driving progress in natural language processing, recommendation systems, and data integration. Addressing the complexities of knowledge graph structures will foster greater innovation and operational efficiency in both academic and applied settings [38, 95, 39, 40, 91].

6.2 Benchmark Evaluations and Comparative Studies

Benchmark	Size	Domain	Task Format	Metric
KG-Geometry[93]	483,142	Knowledge Graphs	Link Prediction	Conicity, Alignment to Mean
LSB[104]	30,000	Human Faces	Image Generation	MSE
MEM[27]	240,000	Handwriting Recognition	Image Generation	Manifold Entropy, Manifold Mutual Information
OD-Benchmark[105]	60,000	Image Classification	Anomaly Detection	BPD, Log-Likelihood
KGE-GNN[103]	17,506	Smart City	Node Classification	F1 Score, Average Precision
KGEM-Bench[89]	1,079,400	Link Prediction	Link Prediction	MRR@10, Hits@10
BioKG[94]	2,067,998	Biomedical	Link Prediction	HITS@10, MRR

Table 5: This table provides a comprehensive overview of various benchmarks employed in evaluating the performance of Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs). It details the size, domain, task format, and evaluation metrics of each benchmark, facilitating a comparative analysis of model effectiveness across diverse datasets.

Benchmark evaluations and comparative studies are crucial for assessing the performance and robustness of Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs). Table 5 presents a detailed summary of the benchmarks utilized in the assessment of CNFs and KGEs, highlighting key attributes such as dataset size, domain, task format, and performance metrics. For CNFs, metrics such as negative log-likelihood (NLL) and Fréchet Inception Distance (FID) quantitatively evaluate the model's ability to generate high-fidelity samples [33]. CNFs like OT-CFM outperform baseline methods such as I-CFM in tasks involving single-cell dynamics and image generation, demonstrating superior sample accuracy [106]. Visual quality assessments and empirical evaluations have highlighted improvements in density estimation and convergence speed [4].

For KGEs, benchmark evaluations typically use datasets like WN18, FB15K, and YAGO, employing metrics such as Mean Reciprocal Rank (MRR), Hits at N (H@N), and Mean Rank (MR) to assess performance in link prediction tasks. Models like ManifoldE have shown significant improvements over state-of-the-art baselines in capturing relational patterns [25]. Similarly, TransG has demonstrated superior performance in link prediction and triple classification tasks, effectively representing complex relationships [35].

Evaluations of Neural Flow Embedding (NFE) models through metrics like MR, MRR, and H@N have illustrated their capacity to effectively capture knowledge graph structures [57]. Neural network-based approaches have been assessed using log-likelihood metrics across multiple trials, ensuring statistical significance and robustness [30].

Benchmark evaluations and comparative studies are vital for advancing generative modeling and knowledge representation techniques, enabling rigorous assessments of model performance and reproducibility across diverse datasets. Large-scale evaluations of knowledge graph embedding models have yielded insights into the influence of model architecture, training methods, and loss functions on performance, while best practices for configuration have been highlighted. The exploration of generative models based on normalizing flows and Kronecker decomposition emphasizes the importance of systematic experimentation in enhancing model efficiency and robustness [88, 89, 38, 42, 48]. These evaluations provide critical insights into the strengths and weaknesses of CNFs and KGEs, guiding ongoing refinement and optimization, ensuring effectiveness across various applications.

6.3 Scalability and Real-World Applications

The scalability and real-world applications of Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs) are pivotal for managing large-scale data and complex tasks across diverse domains. CNFs demonstrate significant scalability by transforming simple distributions into complex ones through continuous-time dynamics, as seen in applications generating high-quality images with reduced training time [29]. This capability is essential in media generation and computational imaging, where high-dimensional data scenarios are prevalent.

KGEs' scalability is crucial for managing extensive knowledge graphs, evidenced by their application to the DBpedia 2021 benchmark dataset [82]. The proposed framework addresses scalability challenges and continual learning, offering valuable tools for practitioners in fields such as healthcare and manufacturing, where large datasets are common [40]. Future research aims to enhance KGE performance and applicability across larger datasets [7].

Integrating CNFs and KGEs with advanced techniques like Differential Dynamic Equations (DDEs) enhances their scalability, facilitating effective modeling in high-dimensional spaces [11]. This integration is particularly relevant for applications requiring precise modeling of complex data structures in physics, robotics, and biology [6]. Furthermore, proposed models that generate graphs with higher triangle counts and align with input graph statistics demonstrate their real-world applicability [96].

Innovative frameworks such as CatFlow and RVAE exemplify the potential of scalable generative models in graph generation tasks and energy production, respectively. These models showcase superior performance in handling relational dynamics and uncertainty, emphasizing the importance of scalability in real-world applications. Ongoing research continues to focus on the scalability of applications, particularly in image generation [26].

Experiments with models like LGGM have shown improved performance in generating unseen graphs, highlighting the scalability and real-world applications of CNFs and KGEs across various domains [36]. The scalability and applicability of these models are critical for their continued success and innovation. As research progresses, developing more efficient and scalable methodologies promises to expand their impact and utility, driving advancements in generative modeling and knowledge representation.

7 Future Directions and Open Challenges

7.1 Challenges and Future Directions in Knowledge Graph Embeddings

The progression of Knowledge Graph Embeddings (KGEs) involves overcoming several challenges to enhance their robustness, scalability, and applicability. A significant issue is the integration of complex relational characteristics within KGE models, such as improving TransG's performance in data-scarce environments [35]. Incorporating probabilistic embeddings into representation learning can also boost model adaptability, particularly in self-supervised learning [18]. Future research should focus on optimizing computational frameworks for KGEs, enhancing sampling speeds, and integrating task-specific symbolic knowledge to improve performance. Exploring novel energy functions and the generalization behavior of models like Reg-DGM may result in significant advancements in generative model performance [4]. Additionally, enhancing models like MF in higher dimensions and exploring new applications represent promising research avenues [4].

Real-world applications of KGEs present unique challenges. Future studies will explore integrating ExamplE with Graph Neural Networks (GNNs) and seek to improve estimation guarantees for heuristic methods [37]. Interdisciplinary collaboration and standardized protocols in machine learning applications, particularly in healthcare, could significantly amplify the impact of KGEs [40]. Exploring retrieval-augmented generation within the LGGM framework could enhance domain-specific performance, addressing ongoing challenges in KGEs [36]. Investigating frameworks for multi-labeled graphs and thresholds for various distribution families also presents exciting research opportunities [19].

Addressing these challenges and exploring these future directions is essential for advancing KGEs. As research progresses, refining and optimizing KGEs promises to unlock new potentials and applications across various domains, driving innovation and efficiency in knowledge-driven tasks. Future investigations into integrating global latent factors into generative models align with the ongoing exploration of challenges and future directions in KGEs [42]. Additionally, examining the incorporation of stochastic elements into LWGAN could enhance its generative capabilities [43].

7.2 Interdisciplinary Approaches and Standardization

Interdisciplinary approaches and standardization protocols are crucial for advancing Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs). By integrating insights from mathematics, computer science, and specialized domains, researchers can develop advanced models for complex generative modeling and knowledge representation issues. Recent advancements in knowledge graph embedding emphasize hierarchical knowledge representation to improve link prediction accuracy, while innovative methods reinterpret existing models as generative circuits, enhancing efficiency in maximum-likelihood estimation and sampling [38, 44, 75, 35].

Interdisciplinary collaboration is particularly beneficial for CNFs, where mathematical frameworks like differential geometry and manifold learning enhance model expressiveness and scalability, broadening their applicability across fields such as physics, biology, and finance [12]. Integrating probabilistic modeling techniques with CNFs improves their ability to capture uncertainty and variability in high-dimensional data [10]. In KGEs, interdisciplinary methods facilitate the integration of domain-specific knowledge into embedding frameworks, crucial for capturing intricate relational patterns and logical constraints in fields like healthcare and social sciences [40]. Exploring novel energy functions and the generalization behavior of models like Reg-DGM highlights interdisciplinary collaboration's potential to drive innovation in KGE methodologies [4].

Standardization efforts ensure consistency and comparability of CNF and KGE models across applications and datasets. Establishing standardized protocols for model evaluation and benchmarking creates a framework for systematically assessing the performance and robustness of generative models and knowledge embeddings. This framework aids in identifying optimal hyperparameter configurations and promotes reproducibility in model comparisons, guiding the refinement and optimization processes crucial for advancing generative modeling techniques [47, 81, 38, 89]. Standardization is critical for facilitating the widespread adoption of CNFs and KGEs in real-world applications, ensuring their reliability and effectiveness across diverse domains.

The integration of interdisciplinary approaches and standardization protocols is pivotal for advancing CNFs and KGEs. By fostering collaboration and establishing standardized evaluation frameworks, researchers can significantly enhance generative modeling and knowledge representation techniques. This approach supports the development of innovative methods, such as Linear combinations of Latent variables (LOL) for effective manipulation of latent spaces, and integrates robust frameworks like variational autoencoders and diffusion processes for likelihood estimation. Ultimately, these advancements lead to sophisticated and reliable machine learning solutions that address critical challenges, such as controlling output generation and ensuring compliance with logical constraints in applications like knowledge graph embeddings and generative image modeling [38, 65, 66, 47].

7.3 Theoretical Advancements and Novel Architectures

Exploring theoretical advancements and novel architectures in Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs) is essential for enhancing their capabilities and expanding their applicability. Developing hybrid models that integrate diverse methodologies could lead to more robust and versatile generative frameworks [107]. By combining various approaches, researchers can leverage the unique strengths of each, resulting in models that are both computationally efficient and adept at capturing complex data structures.

In the CNF domain, refining the VGrow framework by investigating additional divergences and applying it to a broader range of generative modeling tasks could significantly improve performance [108]. Enhancing model architectures by incorporating more complex probability distributions and optimizing the training process could improve performance across applications [58]. Exploring alternative conditioning mechanisms for CNFs and optimizing latent dimension selection also represent potential avenues for advancing model architecture. Theoretical advancements in the Sinkhorn divergence, particularly regarding its sample complexity and positivity, present opportunities for significant progress in generative modeling [69]. Understanding these theoretical aspects could lead to more efficient and accurate generative models, enhancing their applicability in real-world scenarios. Extending frameworks like OT-Flow to other architectures and applications, along with improving regularization techniques, could further bolster performance [109].

In KGEs, integrating domain-specific knowledge into embedding models remains a critical area of research. Developing interpretable models capable of effectively incorporating this knowledge is essential for improving the transparency and applicability of KGEs [107]. Integrating information-theoretic principles, such as the information bottleneck, into probabilistic embeddings offers promising avenues for enhancing model robustness and adaptability [18].

The ongoing investigation into theoretical advancements and innovative architectures in Knowledge Graphs (KGs) and Knowledge Graph Embeddings (KGEs) is essential for fostering innovation and enhancing the performance of generative models and knowledge embeddings. This exploration is particularly important given the complexities of KG structures, which can introduce biases and influence KGE effectiveness. Recent studies highlight the need for a deeper understanding of the relationship between KGE models and KG structure, as well as the potential for KGEs to be reinterpreted as energy-based models that facilitate maximum-likelihood estimation and efficient sampling. Incorporating advanced relation modeling techniques and multimodal information can further enrich KGE applications, ultimately driving more effective knowledge-augmented solutions [91, 38, 39, 40]. By focusing on hybrid models, enhancing computational efficiency, and incorporating domain-specific knowledge, researchers can unlock new potentials and applications across various fields.

8 Conclusion

This survey has delved into the complex realms of Continuous Normalizing Flows (CNFs) and Knowledge Graph Embeddings (KGEs), underscoring their pivotal contributions to generative modeling and knowledge representation. CNFs facilitate the transformation of basic probability distributions into intricate ones via continuous-time dynamics, employing Neural Ordinary Differential Equations to effectively capture complex data dependencies. This functionality is crucial for applications demanding precise density estimation and sampling within high-dimensional spaces. Simultaneously, KGEs have been vital in embedding entities and relationships from knowledge graphs into continuous vector spaces, enabling tasks such as link prediction and entity classification. The fusion of geometric and probabilistic methodologies has bolstered the robustness and scalability of these models, underscoring the importance of probabilistic deep learning frameworks in uncertain environments.

The survey identifies key challenges and future directions, including the optimization of computational frameworks, improvement of model scalability, and the integration of domain-specific knowledge into embedding models. As research advances, the exploration of theoretical developments and innovative architectures is anticipated to reveal new potentials and applications across diverse fields. The synergy between CNFs, KGEs, and other generative models offers a promising path for enhancing the expressiveness and flexibility of generative frameworks, driving innovation in machine learning and data science. The continuous refinement and optimization of CNFs and KGEs are essential for the progression of generative modeling, paving the way for more advanced and dependable solutions across various applications.

References

[1] Lars Ruthotto and Eldad Haber. An introduction to deep generative modeling. *GAMM-Mitteilungen*, 44(2):e202100008, 2021.

[2] Tom White. Sampling generative networks, 2016.

[3] Aude Genevay, Gabriel Peyré, and Marco Cuturi. Learning generative models with sinkhorn divergences, 2017.

[4] Noam Rozen, Aditya Grover, Maximilian Nickel, and Yaron Lipman. Moser flow: Divergence-based generative modeling on manifolds, 2021.

[5] Linfeng Li, Peng Wang, Yao Wang, Jinpeng Jiang, Buzhou Tang, Jun Yan, Shenghui Wang, and Yuting Liu. A method to learn embedding of a probabilistic medical knowledge graph: Algorithm development, 2020.

[6] Christian Weilbach, William Harvey, and Frank Wood. Graphically structured diffusion models, 2023.

[7] Zhipeng Tan, Baifan Zhou, Zhuoxun Zheng, Ognjen Savkovic, Ziqi Huang, Irlan-Grangel Gonzalez, Ahmet Soylu, and Evgeny Kharlamov. Literal-aware knowledge graph embedding for welding quality monitoring: A bosch case, 2023.

[8] Mingze Gong, Lei Chen, and Jia Li. Progen: Revisiting probabilistic spatial-temporal time series forecasting from a continuous generative perspective using stochastic differential equations, 2024.

[9] Kashif Rasul, Abdul-Saboor Sheikh, Ingmar Schuster, Urs Bergmann, and Roland Vollgraf. Multivariate probabilistic time series forecasting via conditioned normalizing flows. *arXiv preprint arXiv:2002.06103*, 2020.

[10] George Papamakarios, Eric Nalisnick, Danilo Jimenez Rezende, Shakir Mohamed, and Balaji Lakshminarayanan. Normalizing flows for probabilistic modeling and inference, 2021.

[11] Gianluigi Silvestri, Emily Fertig, Dave Moore, and Luca Ambrogioni. Embedded-model flows: Combining the inductive biases of model-free deep learning and explicit probabilistic modeling, 2022.

[12] Aaron Lou, Derek Lim, Isay Katsman, Leo Huang, Qingxuan Jiang, Ser-Nam Lim, and Christopher De Sa. Neural manifold ordinary differential equations, 2020.

[13] Yong Zhong, Hongtao Liu, Xiaodong Liu, Fan Bao, Weiran Shen, and Chongxuan Li. Deep generative modeling on limited data with regularization by nontransferable pre-trained models, 2023.

[14] Sitan Chen, Jerry Li, and Yuanzhi Li. Learning (very) simple generative models is hard, 2022.

[15] Zachary M. Ziegler and Alexander M. Rush. Latent normalizing flows for discrete sequences, 2019.

[16] Charilaos Mylonas, Imad Abdallah, and Eleni Chatzi. Relational vae: A continuous latent variable model for graph structured data, 2021.

[17] Gerrit J. J. van den Burg and Christopher K. I. Williams. On memorization in probabilistic deep generative models, 2021.

[18] Denis Janiak, Jakub Binkowski, Piotr Bielak, and Tomasz Kajdanowicz. Unveiling the potential of probabilistic embeddings in self-supervised learning, 2023.

[19] Pablo Robles-Granda, Katherine Tsai, and Oluwasanmi Koyejo. Goodness-of-fit of attributed probabilistic graph generative models, 2023.

[20] Marin Biloš and Stephan Günnemann. Scalable normalizing flows for permutation invariant densities, 2021.

[21] Luca Falorsi. Continuous normalizing flows on manifolds, 2021.

[22] Seyedeh Fatemeh Razavi, Mohammad Mahdi Mehmanchi, Reshad Hosseini, and Mostafa Tavassolipour. Out-of-distribution detection using normalizing flows on the data manifold, 2025.

[23] Samet Oymak, Zalan Fabian, Mingchen Li, and Mahdi Soltanolkotabi. Generalization guarantees for neural networks via harnessing the low-rank structure of the jacobian. *arXiv preprint arXiv:1906.05392*, 2019.

[24] Johan Leduc and Nicolas Grislain. Composable generative models, 2021.

[25] Han Xiao, Minlie Huang, and Xiaoyan Zhu. From one point to a manifold: Knowledge graph embedding for precise link prediction, 2017.

[26] Heli Ben-Hamu, Samuel Cohen, Joey Bose, Brandon Amos, Aditya Grover, Maximilian Nickel, Ricky T. Q. Chen, and Yaron Lipman. Matching normalizing flows and probability paths on manifolds, 2022.

[27] Daniel Galperin and Ullrich Köthe. Analyzing generative models by manifold entropic metrics, 2024.

[28] Tuc Nguyen-Van, Dung D. Le, and The-Anh Ta. Improving heterogeneous graph learning with weighted mixed-curvature product manifold, 2023.

[29] Vikram Voleti, Chris Finlay, Adam Oberman, and Christopher Pal. Multi-resolution continuous normalizing flows, 2021.

[30] Chin-Wei Huang, David Krueger, Alexandre Lacoste, and Aaron Courville. Neural autoregressive flows. In *International conference on machine learning*, pages 2078–2087. PMLR, 2018.

[31] Chen Xu, Xiuyuan Cheng, and Yao Xie. Normalizing flow neural networks by jko scheme, 2024.

[32] Maximilian Schmidt and Marko Simic. Normalizing flows for novelty detection in industrial time series data, 2019.

[33] Michael S Albergo and Eric Vanden-Eijnden. Building normalizing flows with stochastic interpolants. *arXiv preprint arXiv:2209.15571*, 2022.

[34] Carl Remlinger, Joseph Mikael, and Romuald Elie. Conditional loss and deep euler scheme for time series generation, 2021.

[35] Han Xiao, Minlie Huang, Yu Hao, and Xiaoyan Zhu. Transg : A generative mixture model for knowledge graph embedding, 2017.

[36] Yu Wang, Ryan A. Rossi, Namyong Park, Huiyuan Chen, Nesreen K. Ahmed, Puja Trivedi, Franck Dernoncourt, Danai Koutra, and Tyler Derr. Large generative graph models, 2024.

[37] Adrianna Janik and Luca Costabello. Explaining link predictions in knowledge graph embedding models with influential examples, 2022.

[38] Lorenzo Loconte, Nicola Di Mauro, Robert Peharz, and Antonio Vergari. How to turn your knowledge graph embeddings into generative models, 2024.

[39] Guanglin Niu. Knowledge graph embeddings: A comprehensive survey on capturing relation properties, 2024.

[40] Jeffrey Sardina, John D. Kelleher, and Declan O'Sullivan. A survey on knowledge graph structure and knowledge graph embeddings, 2024.

[41] Matthew M. Graham and Amos J. Storkey. Asymptotically exact inference in differentiable generative models, 2017.

[42] Shaohua Li, Jun Zhu, and Chunyan Miao. A generative word embedding model and its low rank positive semidefinite solution, 2015.

[43] Yixuan Qiu, Qingyi Gao, and Xiao Wang. Adaptive learning of the latent space of wasserstein generative adversarial networks, 2024.

[44] Ying Nian Wu, Ruiqi Gao, Tian Han, and Song-Chun Zhu. A tale of three probabilistic families: Discriminative, descriptive and generative models, 2018.

[45] George Papamakarios, Eric Nalisnick, Danilo Jimenez Rezende, Shakir Mohamed, and Balaji Lakshminarayanan. Normalizing flows for probabilistic modeling and inference. *Journal of Machine Learning Research*, 22(57):1–64, 2021.

[46] Rick Farouni. A contemporary overview of probabilistic latent variable models, 2017.

[47] Ali Zand and Milad Nasr. Avoiding generative model writer's block with embedding nudging, 2024.

[48] Ivan Kobyzev, Simon JD Prince, and Marcus A Brubaker. Normalizing flows: An introduction and review of current methods. *IEEE transactions on pattern analysis and machine intelligence*, 43(11):3964–3979, 2020.

[49] Sumit Pai and Luca Costabello. Learning embeddings from knowledge graphs with numeric edge attributes, 2021.

[50] Zhiqing Sun, Zhi-Hong Deng, Jian-Yun Nie, and Jian Tang. Rotate: Knowledge graph embedding by relational rotation in complex space, 2019.

[51] Siyu Yao, Ruijie Wang, Shen Sun, Derui Bu, and Jun Liu. Joint embedding learning of educational knowledge graphs, 2019.

[52] Daniel T. Chang. Probabilistic deep learning with probabilistic neural networks and deep probabilistic models, 2021.

[53] Cosimo Gregucci, Mojtaba Nayyeri, Daniel Hernández, and Steffen Staab. Link prediction with attention applied on multiple knowledge graph embedding models, 2023.

[54] Kai Wang, Yu Liu, Dan Lin, and Quan Z. Sheng. Hyperbolic geometry is not necessary: Lightweight euclidean-based models for low-dimensional knowledge graph embeddings, 2021.

[55] P Manisha and Sujit Gujar. Generative adversarial networks (gans): What it can generate and what it cannot?, 2019.

[56] Anson Bastos, Kuldeep Singh, Abhishek Nadgeri, Saeedeh Shekarpour, Isaiah Onando Mulang, and Johannes Hoffart. Hopfe: Knowledge graph representation learning using inverse hopf fibrations, 2021.

[57] Changyi Xiao, Xiangnan He, and Yixin Cao. Knowledge graph embedding by normalizing flows, 2024.

[58] Yaron Lipman, Ricky TQ Chen, Heli Ben-Hamu, Maximilian Nickel, and Matt Le. Flow matching for generative modeling. *arXiv preprint arXiv:2210.02747*, 2022.

[59] Yaron Lipman, Ricky T. Q. Chen, Heli Ben-Hamu, Maximilian Nickel, and Matt Le. Flow matching for generative modeling, 2023.

[60] Yuan Gao, Jian Huang, Yuling Jiao, and Shurong Zheng. Convergence of continuous normalizing flows for learning probability distributions, 2024.

[61] Randall Balestriero, Sebastien Paris, and Richard G. Baraniuk. Analytical probability distributions and em-learning for deep generative networks, 2020.

[62] Floor Eijkelboom, Grigory Bartosh, Christian Andersson Naesseth, Max Welling, and Jan-Willem van de Meent. Variational flow matching for graph generation. *Advances in Neural Information Processing Systems*, 37:11735–11764, 2024.

[63] Anh Do, Duy Dinh, Tan Nguyen, Khuong Nguyen, Stanley Osher, and Nhat Ho. Improving generative flow networks with path regularization, 2022.

[64] High-dimensional density estimation with tensorizing flow.

[65] Chin-Wei Huang, Jae Hyun Lim, and Aaron C Courville. A variational perspective on diffusion-based generative models and score matching. *Advances in Neural Information Processing Systems*, 34:22863–22876, 2021.

[66] Erik Bodin, Alexandru Stere, Dragos D. Margineantu, Carl Henrik Ek, and Henry Moss. Linear combinations of latents in generative models: subspaces and beyond, 2025.

[67] Will Grathwohl, Ricky TQ Chen, Jesse Bettencourt, Ilya Sutskever, and David Duvenaud. Ffjord: Free-form continuous dynamics for scalable reversible generative models. *arXiv preprint arXiv:1810.01367*, 2018.

[68] Shian Du, Yihong Luo, Wei Chen, Jian Xu, and Delu Zeng. To-flow: Efficient continuous normalizing flows with temporal optimization adjoint with moving speed, 2022.

[69] Aude Genevay, Gabriel Peyré, and Marco Cuturi. Learning generative models with sinkhorn divergences. In *International Conference on Artificial Intelligence and Statistics*, pages 1608–1617. PMLR, 2018.

[70] Bastian Boll, Daniel Gonzalez-Alvarado, and Christoph Schnörr. Generative modeling of discrete joint distributions by e-geodesic flow matching on assignment manifolds, 2024.

[71] Gleb Ryzhakov, Svetlana Pavlova, Egor Sevriugov, and Ivan Oseledets. Explicit flow matching: On the theory of flow matching algorithms with applications, 2024.

[72] Alberto Cabezas, Louis Sharrock, and Christopher Nemeth. Markovian flow matching: Accelerating mcmc with continuous normalizing flows, 2024.

[73] Lucas Liebenwein, Ramin Hasani, Alexander Amini, and Daniela Rus. Sparse flows: Pruning continuous-depth models, 2021.

[74] Alexander Vidal, Samy Wu Fung, Luis Tenorio, Stanley Osher, and Levon Nurbekyan. Taming hyperparameter tuning in continuous normalizing flows using the jko scheme, 2022.

[75] Cunxiang Wang, Feiliang Ren, Zhichao Lin, Chenxv Zhao, Tian Xie, and Yue Zhang. Domain representation for knowledge graph embedding, 2019.

[76] Wenqiang Liu, Hongyun Cai, Xu Cheng, Sifa Xie, Yipeng Yu, and Hanyu Zhang. Learning high-order structural and attribute information by knowledge graph attention networks for enhancing knowledge graph embedding, 2019.

[77] Yichen Liu, Jiawei Chen, Defang Chen, Zhehui Zhou, Yan Feng, and Can Wang. Confidence-aware self-semantic distillation on knowledge graph embedding, 2024.

[78] Zelong Li, Jianchao Ji, Zuohui Fu, Yingqiang Ge, Shuyuan Xu, Chong Chen, and Yongfeng Zhang. Efficient non-sampling knowledge graph embedding, 2021.

[79] Qiuyu Liang, Weihua Wang, Feilong Bao, and Guanglai Gao. Fully hyperbolic rotation for knowledge graph embedding, 2024.

[80] Ines Chami, Adva Wolf, Da-Cheng Juan, Frederic Sala, Sujith Ravi, and Christopher Ré. Low-dimensional hyperbolic knowledge graph embeddings. *arXiv preprint arXiv:2005.00545*, 2020.

[81] Oliver Lloyd, Yi Liu, and Tom Gaunt. Assessing the effects of hyperparameters on knowledge graph embedding quality, 2022.

[82] Caglar Demir and Axel-Cyrille Ngonga Ngomo. Hardware-agnostic computation for large-scale knowledge graph embeddings, 2022.

[83] Yanhui Peng and Jing Zhang. Lineare: Simple but powerful knowledge graph embedding for link prediction, 2021.

[84] Takuma Ebisu and Ryutaro Ichise. Toruse: Knowledge graph embedding on a lie group, 2017.

[85] Yihua Zhu and Hidetoshi Shimodaira. 3d rotation and translation for hyperbolic knowledge graph embedding, 2024.

[86] Chengjin Xu, Mojtaba Nayyeri, Sahar Vahdati, and Jens Lehmann. Multiple run ensemble learning with low-dimensional knowledge graph embeddings, 2021.

[87] Caglar Demir and Axel-Cyrille Ngonga Ngomo. A physical embedding model for knowledge graphs, 2020.

[88] Caglar Demir, Julian Lienen, and Axel-Cyrille Ngonga Ngomo. Kronecker decomposition for knowledge graph embeddings, 2022.

[89] Mehdi Ali, Max Berrendorf, Charles Tapley Hoyt, Laurent Vermue, Mikhail Galkin, Sahand Sharifzadeh, Asja Fischer, Volker Tresp, and Jens Lehmann. Bringing light into the dark: A large-scale evaluation of knowledge graph embedding models under a unified framework. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(12):8825–8845, 2021.

[90] Donghan Yu, Yiming Yang, Ruohong Zhang, and Yuexin Wu. Knowledge embedding based graph convolutional network, 2021.

[91] Manita Pote. Survey on embedding models for knowledge graph and its applications, 2024.

[92] Bilal Abu-Salih, Marwan Al-Tawil, Ibrahim Aljarah, Hossam Faris, and Pornpit Wongthongtham. Relational learning analysis of social politics using knowledge graph embedding, 2020.

[93] Aditya Sharma, Partha Talukdar, et al. Towards understanding the geometry of knowledge graph embeddings. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 122–131, 2018.

[94] Aryo Pradipta Gema, Dominik Grabarczyk, Wolf De Wulf, Piyush Borole, Javier Antonio Alfaro, Pasquale Minervini, Antonio Vergari, and Ajitha Rajan. Knowledge graph embeddings in the biomedical domain: Are they useful? a look at link prediction, rule learning, and downstream polypharmacy tasks, 2023.

[95] Borui Cai, Yong Xiang, Longxiang Gao, Di Wu, He Zhang, Jiong Jin, and Tom Luan. From wide to deep: Dimension lifting network for parameter-efficient knowledge graph embedding, 2024.

[96] Shohei Nakazawa, Yoshiki Sato, Kenji Nakagawa, Sho Tsugawa, and Kohei Watabe. A tunable model for graph generation using lstm and conditional vae, 2021.

[97] Naixing Xu, Qian Li, Xu Wang, Bingchen Liu, and Xin Li. Learn to unlearn: Meta-learning-based knowledge graph embedding unlearning, 2024.

[98] Mengyu Dai and Haibin Hang. Manifold matching via deep metric learning for generative modeling, 2021.

[99] Vikram Voleti. Conditional generative modeling for images, 3d animations, and video, 2023.

[100] Tan M. Nguyen, Animesh Garg, Richard G. Baraniuk, and Anima Anandkumar. Infocnf: An efficient conditional continuous normalizing flow with adaptive solvers, 2019.

[101] Zifan Shi, Sida Peng, Yinghao Xu, Andreas Geiger, Yiyi Liao, and Yujun Shen. Deep generative models on 3d representations: A survey, 2023.

[102] Rui Li, Chaozhuo Li, Yanming Shen, Zeyu Zhang, and Xu Chen. Generalizing knowledge graph embedding with universal orthogonal parameterization, 2024.

[103] Rohith Teja Mittakola and Thomas Hassan. A study on knowledge graph embeddings and graph neural networks for web of things, 2023.

[104] Andrea Asperti and Valerio Tonelli. Comparing the latent space of generative models, 2022.

[105] Eric Nalisnick, Akihiro Matsukawa, Yee Whye Teh, Dilan Gorur, and Balaji Lakshminarayanan. Do deep generative models know what they don't know? *arXiv preprint arXiv:1810.09136*, 2018.

[106] Alexander Tong, Kilian Fatras, Nikolay Malkin, Guillaume Huguet, Yanlei Zhang, Jarrid Rector-Brooks, Guy Wolf, and Yoshua Bengio. Improving and generalizing flow-based generative models with minibatch optimal transport, 2024.

[107] Benyamin Ghojogh, Ali Ghodsi, Fakhri Karray, and Mark Crowley. Factor analysis, probabilistic principal component analysis, variational inference, and variational autoencoder: Tutorial and survey, 2022.

[108] Yuan Gao, Yuling Jiao, Yang Wang, Yao Wang, Can Yang, and Shunkang Zhang. Deep generative learning via variational gradient flow, 2019.

[109] Derek Onken, Samy Wu Fung, Xingjian Li, and Lars Ruthotto. Ot-flow: Fast and accurate continuous normalizing flows via optimal transport, 2021.

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