Advanced Signal Processing Techniques for Direction of Arrival Estimation: A Survey

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Abstract

This survey paper delves into advanced signal processing techniques pivotal for enhancing Direction of Arrival (DOA) estimation. It explores subspace methods, compressed sensing, sparse Bayesian learning, and beamforming as integral components that improve accuracy, efficiency, and robustness in complex signal environments. Techniques like TransMUSIC demonstrate superior DOA estimation even with quantized data, while Thresholding-based Subspace Clustering (TSC) effectively clusters noisy data. Innovations such as Subspace Configurable Networks (SCNs) offer high accuracy with fewer dimensions, beneficial for resource-constrained environments. The Space Form PCA (SFPCA) provides robust dimensionality reduction, outperforming traditional methods. In microwave imaging, increasing antenna numbers enhances performance, even virtually. The Improved Subspace Method reduces identification variance and ensures stability, suggesting future research in input design and nonlinear models. Low-rank matrix optimization shows significant convergence speedups for large-scale problems. Additionally, differentially private subspace estimation offers superior accuracy and efficiency. The survey underscores the critical role of these advancements in DOA estimation, highlighting the need for ongoing research to further optimize signal processing capabilities in modern communication and radar systems. Future directions include optimizing adaptive dictionaries, addressing high-dimensional clustering challenges, and exploring novel decomposition techniques for diverse applications.

1 Introduction

1.1 Significance of Signal Processing in Modern Communication

Signal processing serves as a fundamental component of contemporary communication systems, enabling efficient data transmission, reception, and interpretation across diverse platforms. Recent advancements in integrating radar sensing with wireless communication systems illustrate significant performance improvements achievable within modern environments [1]. The advent of 5G technologies has amplified the demand for sophisticated signal processing techniques adept at managing high data rates and complex channel conditions.

In high-dimensional data scenarios, such as sparse classification, traditional methods often falter in feature identification due to data complexity [2]. Techniques like Robust Principal Component Analysis (RPCA) and its dynamic variants have emerged as key solutions, offering robust data representations essential for multimedia applications [3]. Additionally, low-rank matrix optimization techniques have become indispensable, providing computational efficiency and effectiveness in modern data processing [4].

Optimal sampling within reproducing kernel Hilbert spaces (RKHS) is critical for extracting meaningful information from limited samples, emphasizing the necessity of efficient sampling strategies in signal processing [5]. Innovations in audio processing, such as cough detection, exemplify the

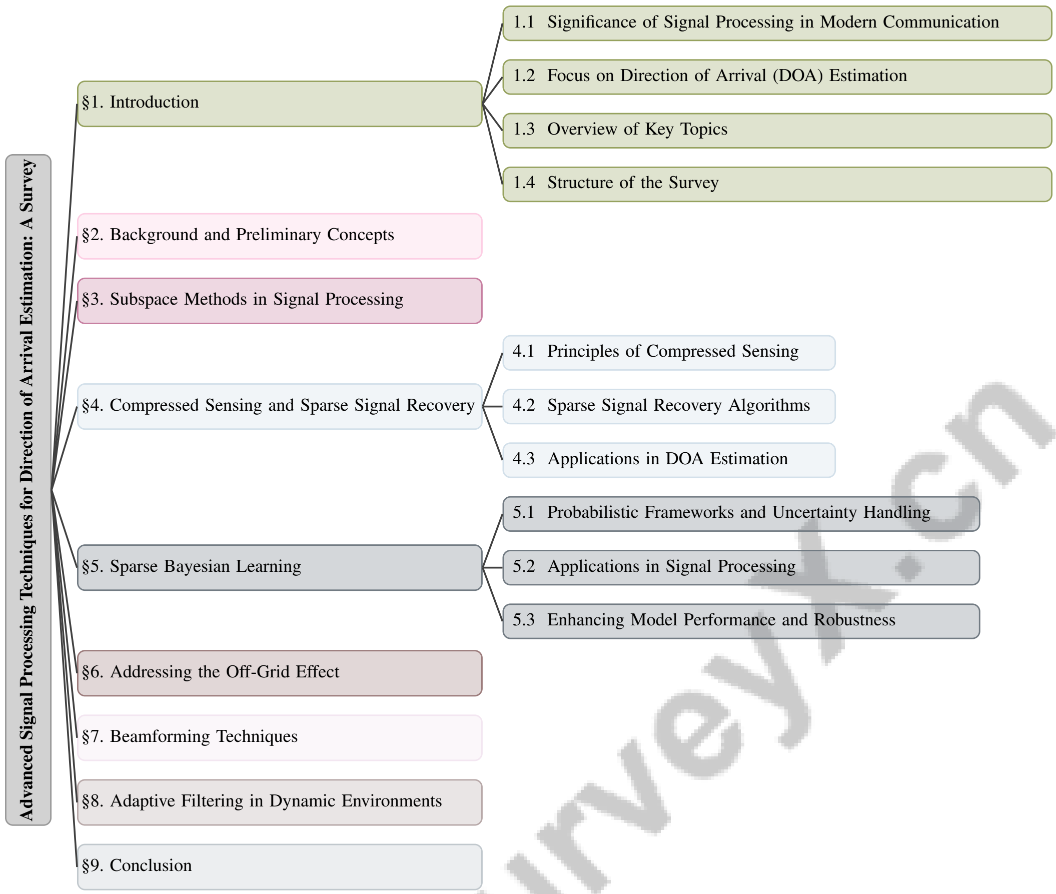

Figure 1: chapter structure

capacity of signal processing techniques to enhance monitoring and analysis capabilities, surpassing limitations of previous methodologies [6].

Robust subspace discovery and low-rank coding have proven pivotal in recovering and denoising image sequences, thus enhancing multimedia applications [7]. The optimization of neural network architectures, particularly for computer vision tasks, underscores the importance of developing efficient models that generalize across various applications [8].

In edge device contexts, ensuring robustness in deep learning models is vital for managing dynamic changes in sensed data, maintaining reliable performance despite sensor drift and variations in data distribution [9]. Addressing blind source separation for spatiotemporal signals is also crucial, highlighting the need for approaches that effectively manage non-stationary data [10]. Continuous development of signal processing methodologies remains essential for overcoming the inherent challenges of modern communication systems, ensuring reliable and efficient information exchange across diverse applications.

1.2 Focus on Direction of Arrival (DOA) Estimation

Direction of Arrival (DOA) estimation is a critical aspect of signal processing, fundamental to radar, sonar, wireless communications, and audio systems. Accurate determination of signal source directions significantly enhances system performance, particularly in multi-source and noisy environments. Traditional DOA estimation techniques, such as the Prediction Error Method (PEM) and subspace methods, often face challenges due to high computational demands and the necessity for large data samples [11].

In massive MIMO systems, DOA estimation is essential for mitigating pilot contamination, which adversely affects channel estimation accuracy [12]. The use of low-resolution analog-to-digital converters (ADCs) further complicates DOA estimation, as quantization distortion presents significant challenges [13]. To address these issues, methods such as projecting tensor signals onto Kronecker-structured subspaces have been proposed, improving detection accuracy in complex signal environments [14].

Recent advancements highlight the importance of robust subspace discovery for recovering multi-subspace structures from noisy and corrupted data, which is vital for accurate DOA estimation [7]. Techniques like RPCA have demonstrated effectiveness in managing outliers, further emphasizing the significance of DOA estimation in dynamic data analysis [3]. Additionally, frequency estimation remains closely linked to DOA estimation, underscoring the necessity for precise frequency and direction determination in complex signal scenarios [15].

Innovations such as the Frequency-Domain Zero-Padding (FDZP) interpolation method enhance imaging accuracy by simulating additional virtual antennas, thereby improving DOA estimation without increasing the number of physical antennas [16]. This is particularly relevant for applications requiring high-resolution imaging and direction finding. Moreover, decomposing mixed time-series signals into spatial and temporal modes while preserving intrinsic mode functions (IMFs) is essential for effective DOA estimation in spatiotemporal signal processing [10].

The ongoing evolution of DOA estimation techniques is crucial for addressing the complex challenges posed by contemporary signal processing applications. Innovations such as the TransMUSIC algorithm, which leverages Transformer models for subspace estimation, and the FRIDA algorithm, which estimates DOA for arbitrary array layouts without grid searches, exemplify this progress. These methods enhance performance by improving resolution at low signal-to-noise ratios and reducing computational burdens through techniques like compressive sensing, ensuring robust and efficient DOA estimation across various contexts [13, 17, 18].

1.3 Overview of Key Topics

This survey investigates several advanced signal processing techniques critical for enhancing Direction of Arrival (DOA) estimation. It begins by examining subspace methods, which form the basis for dimensionality reduction and efficient signal processing. Innovations such as Binary Subspace Chirps (BSSCs) enhance traditional Binary Chirps by introducing a novel structural framework [19].

The discussion then shifts to compressed sensing, a powerful technique for signal acquisition and reconstruction that exploits signal sparsity to improve computational efficiency. This method employs various algorithms, including Orthogonal Matching Pursuit (OMP) and Iterative Hard Thresholding (IHT), achieving optimal recovery guarantees under the Restricted Isometry Property (RIP). Recent advancements have refined the conditions necessary for successful signal recovery, enhancing performance in noisy and corrupted environments. The Accelerated Structured Alternating Projections (ASAP) algorithm exemplifies significant efficiency and robustness in recovering spectrally sparse signals, while new probabilistic frameworks for hard thresholding methods improve sparse signal reconstruction accuracy [20, 21, 22, 23]. Sparse Bayesian learning is also explored, offering a probabilistic framework that excels in uncertainty management and robust estimation, particularly in scenarios demanding high model performance.

Addressing the off-grid effect is another focal point, with techniques developed to mitigate inaccuracies in parameter estimation due to discretization, crucial for maintaining high precision in signal processing tasks. Beamforming techniques are reviewed, emphasizing strategies that enhance DOA estimation by directing signal transmission or reception towards specific spatial sectors [24].

Adaptive filtering is highlighted for its significance in dynamic environments, where filter parameters must be continuously adjusted for optimal performance. The survey also investigates adaptations of classical algorithms such as MUSIC and ESPRIT, modified to handle near-field localization by decoupling DOA and range estimation [25].

This survey thoroughly examines advanced signal processing techniques in DOA estimation, focusing on innovative methods like Transformer-based approaches and compressive sensing (CS) algorithms. By analyzing the current landscape and emerging trends, including the development of the TransMUSIC algorithm, which utilizes low-resolution ADCs and addresses quantization challenges, the survey elucidates the potential for enhanced performance across diverse applications. It also highlights new algorithms that significantly reduce computational complexity while maintaining accuracy, paving the way for robust and efficient DOA estimation in real-world scenarios [13, 17].

1.4 Structure of the Survey

The survey is meticulously structured to provide a comprehensive exploration of advanced signal processing techniques, particularly focusing on Direction of Arrival (DOA) estimation. The introductory section establishes the significance of signal processing in modern communication systems, emphasizing the role of DOA estimation. It offers an overview of key topics covered, including subspace methods, compressed sensing, sparse Bayesian learning, and other pertinent techniques.

Following the introduction, the survey delves into the background and preliminary concepts essential for understanding subsequent discussions. This section lays the groundwork by defining fundamental signal processing concepts and mathematical foundations, ensuring a solid grasp of the technical material presented in later sections.

The core of the survey is divided into thematic sections, each dedicated to a specific advanced technique. The section on subspace methods examines their role in dimensionality reduction and efficient signal processing, focusing on DOA estimation applications. Innovations in subspace techniques are highlighted, showcasing recent advancements and their implications.

Subsequent sections address compressed sensing and sparse signal recovery, exploring principles and applications in DOA estimation. Sparse Bayesian learning is discussed in detail, emphasizing its probabilistic framework and advantages in handling uncertainty.

The survey also tackles the off-grid effect, presenting techniques to mitigate its impact on parameter estimation accuracy. This review delves into advanced beamforming techniques, emphasizing adaptive strategies that significantly enhance DOA estimation through innovative applications. It includes computationally efficient algorithms designed for small sample scenarios, such as a fast adaptive beamforming method utilizing kernel techniques, minimizing computational complexity while maintaining performance. Additionally, it highlights hybrid beamforming approaches that adapt to complex acoustic environments, improving speech intelligibility and quality in augmented reality applications. Furthermore, it discusses the novel compressive sensing (CS) beamformer root-MUSIC algorithm, effectively reducing computational burden while preserving resolution, and introduces the FRIDA algorithm, capable of accurately estimating DOA for wideband sources in arbitrary array layouts without grid searches [18, 17, 26, 27].

Finally, the survey concludes with a section on adaptive filtering in dynamic environments, discussing the importance of adaptive filters in maintaining optimal performance amidst changing conditions. Each section is meticulously referenced, drawing on a broad spectrum of recent research to provide a well-rounded perspective on the current state and future directions of signal processing techniques for DOA estimation.The following sections are organized as shown in Figure 1.

2 Background and Preliminary Concepts

2.1 Fundamental Concepts in Signal Processing

Signal processing involves diverse techniques for data analysis, transformation, and interpretation across applications. Subspace methods, utilizing linear algebra, are crucial for estimating eigenspaces from noisy data, particularly in DOA estimation with low-resolution ADCs that introduce quantization distortion [13]. Dimensionality reduction techniques, such as Principal Component Analysis (PCA), address the complexities of high-dimensional data, though PCA's limitations in non-Euclidean spaces necessitate methods that respect geometric properties [28]. Low-rank matrix optimization is vital for handling computational challenges in eigenvalue decomposition, ensuring efficient data representation [4].

Robust subspace segmentation clusters data vectors from multiple unknown subspaces, maintaining accuracy despite noise and outliers [29]. Subspace-based dimensionality reduction is essential for identifying low-dimensional structures in high-dimensional datasets, crucial for private data analysis [30]. In sparse data scenarios, methods constraining the ℓ_0-norm while minimizing empirical loss are vital for achieving desired sparsity [31]. Techniques for noisy subspace clustering focus on outlier identification within unknown low-dimensional subspaces [32].

Dynamic Mode Decomposition (DMD) decomposes high-dimensional time-series data into spatial modes with linear dynamics, offering insights into complex systems [33]. Guided signal reconstruction uses orthogonal projections onto sampling subspaces for accurate signal reconstruction amidst noise [34]. Estimating maximum Doppler spread in OFDM systems is crucial for mitigating inter-carrier interference and enhancing performance under varying conditions [35]. These concepts form a comprehensive framework for understanding advanced signal processing techniques essential for modern communication and radar systems.

2.2 Mathematical Foundations

Advanced signal processing techniques are grounded in linear algebra, optimization, and statistical inference, providing frameworks for complex estimation problems. Resolving high-dimensional inverse problems is a key challenge, with dimensionality reduction being pivotal. Traditional methods, like finite element and finite difference approaches, often result in computationally expensive systems [36]. Gaussian filters are used to infer states of dynamical systems, managing high-dimensional complexities [37].

Low-rank matrix recovery, especially in noisy, incomplete data, is fundamental. Insights from algebraic geometry, notably Grassmannians, support low-rank matrix completion, facilitating matrix reconstruction from partial observations [38]. In sparse signal processing, the nonconvexity of the ℓ_0-norm complicates optimization, making existing methods inefficient for large-scale problems [31]. The statistical-computational gap in sparse PCA further complicates efficient problem-solving [39]. Adaptive Randomized Sketching Optimization (ARSO) presents promising solutions with adaptive sketching matrices for convex optimization in high-dimensional spaces [40].

Probabilistic modeling, particularly Bayesian inference, is crucial for uncertainty quantification in signal processing. B-FS leverages Bayesian frameworks for robust predictions with limited data, emphasizing probabilistic approaches in managing uncertainty [41]. Random matrix theory aids in analyzing signal and noise subspaces in blind identification tasks, underpinning the discussed techniques [42].

Robust tensor models address the recovery of low-rank tensors from high-dimensional noisy data, tackling unknown noise distributions [43]. Deterministic performance analysis of subspace methods focuses on recovering complex numbers and weights from noisy measurements, offering insights into parameter estimation for sinusoids [44]. In subspace clustering, the graph connectivity problem presents challenges, as methods may over-segment data points, failing to form connected components within clusters [45]. Addressing this requires sophisticated models to ensure effective clustering and dimensionality reduction. The survey categorizes methods into static RPCA, dynamic RPCA, and matrix completion, emphasizing theoretical foundations and empirical performance [3]. The theory focuses on asymptotic characterization of reconstruction errors and clustering algorithm performance in high-dimensional spaces [46].

The mathematical principles supporting advanced signal processing techniques, including RPCA, low-rank matrix recovery, and compressed sensing algorithms, provide frameworks for efficient, resilient solutions. These methodologies enable recovery of spectrally sparse signals, data matrix decomposition amidst outliers, and handling of correlated noise, enhancing performance across applications from communication systems to complex data analysis. The Accelerated Structured Alternating Projections algorithm exemplifies significant efficiency and robustness in recovering corrupted signals, while dynamic RPCA tracks time-varying data amidst sparse disturbances [47, 23, 3, 8, 21].

In recent years, the exploration of subspace methods in signal processing has garnered significant attention due to their versatility and efficacy in handling complex data. These methods are particularly relevant in the context of dimensionality reduction, dynamic environments, high-dimensional data clustering, and advancements in estimation techniques. To elucidate this topic further, Figure 2 illustrates the hierarchical categorization of subspace methods, detailing various techniques, their applications, the challenges faced, and the innovations that have emerged within this field. This figure serves not only as a visual representation of the complexities involved but also enhances our understanding of the interrelationships among these methods, thereby enriching the narrative of this review.

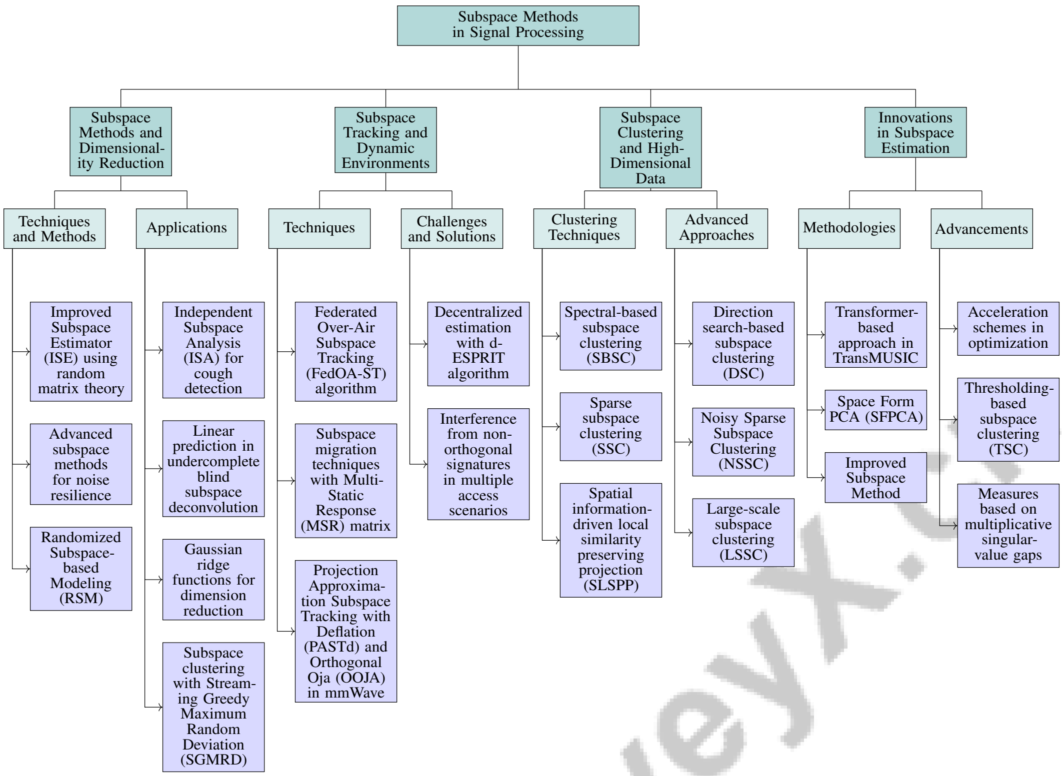

Figure 2: This figure illustrates the hierarchical categorization of subspace methods in signal processing, detailing techniques, applications, challenges, and innovations across dimensionality reduction, dynamic environments, high-dimensional data clustering, and estimation advancements.

3 Subspace Methods in Signal Processing

3.1 Subspace Methods and Dimensionality Reduction

Method Name	Dimensionality Reduction	Noise Resilience	Application Areas
EBFE[15]	Subspace Identification Approach	Additive White Noise	Signal Processing Scenarios
ARSO[40]	Random Subspaces	-	High-dimensional Optimization
SCNs[9]	Low-dimensional Subspace	-	Audio Signal Processing
SFPCA[28]	Complex Data Structures	-	Microbiome Studies
LINEBO[48]	One-dimensional Subspaces	Maintain Safety Constraints	Beam Intensity Optimization
ISM[11]	System Matrix Estimation	Consistent Results	System Identification

Table 1: Overview of various subspace methods, highlighting their approaches to dimensionality reduction, noise resilience, and specific application areas. The table summarizes six methods, detailing their unique techniques and the contexts in which they are applied, providing insights into their effectiveness in handling high-dimensional data.

Subspace methods are integral to signal processing, enabling the decomposition of high-dimensional data into lower-dimensional subspaces for improved computational efficiency and data interpretability. Techniques like the Improved Subspace Estimator (ISE) employ random matrix theory to enhance the estimation of signal and noise subspaces from limited observations [15]. Addressing noise sensitivity, particularly in nonlinear systems, advanced subspace methods have been developed to improve noise resilience and transient phenomenon reconstruction [10, 49]. Randomized Subspace-based Modeling (RSM) exemplifies efficiency by generating random subspaces through variable selection [40]. Table 1 provides a comprehensive summary of different subspace methods, elucidating their dimensionality reduction techniques, noise resilience capabilities, and application domains, thus illustrating their role in enhancing computational efficiency and data interpretation in various fields.

Applications of subspace methods include independent subspace analysis (ISA) for automatic cough detection in audio processing, reducing manual efforts [9]. Linear prediction in undercomplete blind subspace deconvolution efficiently estimates hidden components [28]. Gaussian ridge functions optimize dimension reduction, offering superior accuracy [48]. Subspace clustering further underscores dimensionality reduction's significance, with methods like Streaming Greedy Maximum Random Deviation (SGMRD) enhancing clustering efficiency in data streams [11]. These integrations highlight subspace methods' importance in handling high-dimensional data, reducing computational complexity for robust solutions.

3.2 Subspace Tracking and Dynamic Environments

In dynamic environments, efficient subspace tracking is crucial, especially with high-dimensional data containing missing entries or outliers. The Federated Over-Air Subspace Tracking (FedOA-ST) algorithm effectively tracks low-dimensional subspaces, maintaining robust performance despite incomplete data [50]. Subspace migration techniques, such as the Multi-Static Response (MSR) matrix, are vital for imaging applications, reconstructing thin inhomogeneities from far-field data [51]. In millimeter-wave (mmWave) channel estimation, adaptations of subspace tracking algorithms like Projection Approximation Subspace Tracking with Deflation (PASTd) and Orthogonal Oja (OOJA) dynamically track subspaces, crucial for maintaining high performance in rapidly changing environments [52].

Decentralized estimation of the direction of arrival (DOA) using the d-ESPRIT algorithm, based on the decentralized Power Method (d-PM), offers scalable solutions without centralized processing, beneficial in distributed systems [53]. However, interference from non-orthogonal signatures in multiple access scenarios complicates subspace tracking, necessitating innovative techniques to enhance robustness and accuracy [19].

3.3 Subspace Clustering and High-Dimensional Data

Subspace clustering is essential for managing high-dimensional data, effectively addressing the curse of dimensionality and noise challenges. Spectral-based subspace clustering (SBSC) methods utilize spectral graph theory to enhance clustering accuracy, with sparse subspace clustering (SSC) promoting sparsity in data representation [29]. Techniques like spatial information-driven local similarity preserving projection (SLSPP) improve classification performance by maintaining local similarities [54, 55]. Multi-view subspace clustering constructs a shared affinity matrix balancing low-rank and sparsity constraints, utilizing consensus information across various data representations.

Direction search-based subspace clustering (DSC) robustly identifies subspaces within a union of unknown linear subspaces, while Noisy Sparse Subspace Clustering (NSSC) ensures cluster connectivity despite noise [32]. Large-scale subspace clustering (LSSC) addresses traditional methods' limitations in handling complex high-dimensional datasets, employing Bayesian approaches like the high-dimensional k-Gaussian mixture model with sparse cluster means [46]. Robust subspace clustering techniques, including those induced by correntropy, focus on accurately clustering noisy data points into their underlying subspaces. Recent advancements, such as sparse subspace clustering and multi-view low-rank approaches, emphasize constructing coefficient and affinity matrices, significantly influencing clustering performance. Dimensionality reduction methods demonstrate maintaining clustering accuracy while simplifying data representation, enhancing computational efficiency across diverse applications [56, 54, 57].

3.4 Innovations in Subspace Estimation

Recent advancements in subspace estimation have introduced methodologies significantly enhancing computational efficiency and accuracy in signal processing. The Transformer-based approach in TransMUSIC exemplifies innovation, offering a faster alternative to traditional eigenvalue decomposition methods [13]. Space Form PCA (SFPCA) leverages geometric properties of Riemannian manifolds for dimensionality reduction, contrasting with heuristic methods and providing a structured framework for subspace estimation [28]. The Improved Subspace Method enhances accuracy and efficiency through a closed-form solution for system matrix estimation and an input design algorithm minimizing identification variance [11].

Acceleration schemes in optimization, such as same-space extrapolation and subspace identification, significantly improve convergence speeds, representing substantial advancements over traditional

Method Name	Computational Efficiency	Dimensionality Reduction	Robustness and Accuracy
TM[13]	Reduces Computational Complexity	Subspace Estimation	Superior Performance
SFPCA[28]	Faster Convergence	Optimal Affine Subspace	Accurate Data Representation
ISM[11]	Closed-form Solution	Projecting System Matrices	Minimize Identification Variance
APG[31]	Reduce Computational Time	Subspace Identification	Improving Solution Quality
TSC[32]	Reducing Computational Complexity	Low-dimensional Subspaces	Tolerate Substantial Noise
ST-MDSE[35]	Lower Computational Complexity	Delay Subspace	Improved Estimation Accuracy
DPSE[30]	Reduced Sample Complexity	Low-dimensional Structures	Improved Accuracy

Table 2: Comparative analysis of recent methodologies in subspace estimation, highlighting advancements in computational efficiency, dimensionality reduction, and robustness and accuracy. The table presents a detailed comparison of seven methods, including TransMUSIC, Space Form PCA, and Improved Subspace Method, showcasing their unique contributions to the field.

methods [31]. In noise-challenged environments, thresholding-based subspace clustering (TSC) reduces computational complexity while maintaining performance guarantees [32]. Combining subspace tracking with estimation processes allows for automatic updates of the delay subspace, enhancing accuracy and efficiency in dynamic environments [35]. Measures based on multiplicative singular-value gaps offer novel perspectives on ensuring privacy in subspace estimation, independent of dimensionality [30].

These advancements illustrate the dynamic evolution of subspace estimation techniques, continually enhancing efficiency and accuracy in signal processing. By incorporating robust algorithms such as Robust Principal Component Analysis (RPCA) and innovative techniques like Accelerated Structured Alternating Projections (ASAP), these methodologies address the complexities of modern signal processing applications. RPCA decomposes data matrices into low-rank and sparse components to manage outliers, while ASAP leverages the low-rank structure of Hankel matrices to ensure high computational efficiency and robustness across various scenarios [23, 3]. Table 2 provides a comprehensive comparison of recent innovations in subspace estimation methods, emphasizing their computational efficiency, dimensionality reduction capabilities, and robustness and accuracy improvements.

4 Compressed Sensing and Sparse Signal Recovery

4.1 Principles of Compressed Sensing

Compressed sensing transforms signal processing by leveraging signal sparsity to enable acquisition and reconstruction from fewer measurements. This method posits that high-dimensional signals can be sparsely represented in a suitable basis, facilitating efficient data processing and storage. The core challenge is reconstructing these sparse signals from compressed measurements, often framed as a non-convex optimization problem involving the ℓ_0-norm minimization [9].

The Restricted Isometry Property (RIP) is crucial in maintaining the geometric structure of sparse signals during measurement, ensuring accurate reconstruction [4]. Traditional RIP methods face difficulties in underdetermined models, leading to advanced algorithms like orthogonal matching pursuit (OMP) and thresholding-based subspace clustering (TSC), which enhance recovery by exploiting inner product relationships, providing robust clustering in noisy environments [32].

Innovations such as Sparse Principal Component Analysis (SFPCA) identify optimal affine subspaces for manifold-valued points, minimizing projection costs [28]. Differentially Private Subspace Estimation (DPSE) uses low-dimensional structures for accurate estimations with fewer samples, showcasing compressed sensing's efficiency in data-scarce scenarios [30].

Applications span various fields, such as microwave imaging systems, where Frequency-Domain Zero-Padding (FDZP) interpolation enhances imaging accuracy without additional antennas [16]. Data-driven approaches like Spatiotemporal Intrinsic Mode Decomposition (STIMD) leverage spatial correlations for non-stationary signal representation, demonstrating compressed sensing's versatility [10].

The effectiveness of these techniques is further supported by subspace-configurable networks (SCNs), which optimize model weights based on input transformations, enhancing reconstruction efficiency [9]. These advancements highlight compressed sensing's robustness, enabling significant measurement

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reductions while maintaining high-fidelity reconstruction, thus forming a robust foundation for modern signal processing.

4.2 Sparse Signal Recovery Algorithms

Sparse signal recovery is pivotal in compressed sensing, focusing on reconstructing signals from limited measurements by exploiting their inherent sparsity. Various algorithms enhance recovery efficiency and accuracy. The Coherence Pursuit (CoP) algorithm uses robust PCA to identify inliers by assessing dataset coherence, employing a Gram matrix for effective comparisons [58]. This method excels in distinguishing inliers from outliers, crucial for accurate reconstruction.

Structured Dictionary Learning (StructDL) combines single-task and multi-task learning to improve classification through structured sparsity, proving effective in sparse recovery [59]. This approach adeptly manages complex data, ensuring robust recovery amid noise.

The Accelerated Projected Gradient (APG) algorithm merges projected gradient steps with extrapolation and subspace optimization, efficiently tackling sparsity-constrained problems [31]. This enhances computational efficiency and convergence, making it suitable for large-scale tasks.

Dimensionality-reduced subspace clustering, benchmarked against TSC, SSC, and SSC-OMP, is vital for managing high-dimensional data by promoting sparsity in representation [56]. These methods enhance clustering accuracy and efficiency by identifying informative subspaces.

In high-dimensional state estimation, Gaussian filters operate in two phases: offline subspace identification and online filtering, effectively addressing high-dimensional challenges [37]. Randomness in efficient sampling, exemplified by the Fast Johnson-Lindenstrauss Transform, preserves differential privacy while enabling efficient recovery [60]. Bo et al.'s approach evaluates subspace significance based on prediction error reduction, advancing sparse recovery algorithms [49].

These innovations collectively enhance sparse recovery algorithms' capabilities, expanding their applicability in modern signal processing. Advanced algorithms like Accelerated Structured Alternating Projections (ASAP) and dimensionality reduction methods such as Robust Principal Component Analysis (RPCA) leverage the Restricted Isometry Property (RIP) to offer multifaceted solutions to sparse recovery challenges. ASAP addresses spectrally sparse signals corrupted by noise, while RPCA ensures accurate reconstruction amid outliers [20, 23, 3, 61, 21].

4.3 Applications in DOA Estimation

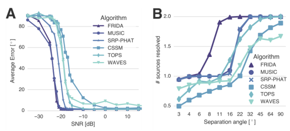
Compressed sensing has significantly advanced Direction of Arrival (DOA) estimation in sparse signal and limited data scenarios. This approach reduces hardware complexity while ensuring robust performance, as seen in MIMO systems where waveform optimization enhances DOA precision [13]. The integration of compressed sensing with cyclic rank minimization has improved DOA performance through covariance matrix reconstruction, illustrating novel applications in signal processing [16].

In decentralized systems, algorithms like the decentralized Power Method (d-PM) and d-ESPRIT provide scalable DOA solutions without centralized processing, enhancing performance in distributed networks [35]. Transmit beamspace energy focusing techniques further improve accuracy by concentrating energy, increasing the signal-to-noise ratio (SNR) [48].

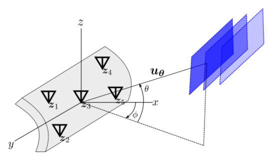
Dimensionality-reduced subspace clustering methods maintain performance while reducing data complexity, essential for accurate DOA estimation in high-dimensional environments [31]. Techniques like the Empirical Bayes approach have been adapted for multiple frequency and DOA estimation, underscoring their relevance in compressed sensing applications [13]. Deterministic performance analysis offers insights into damped and undamped sinusoid scenarios, enhancing DOA understanding under varied conditions.

Compressed sensing's robustness is further exemplified in blind identification tasks, where it enhances performance under low SNR conditions, critical for accurate DOA estimation. Optimal sampling strategies significantly reduce reconstruction errors, improving DOA precision [16].

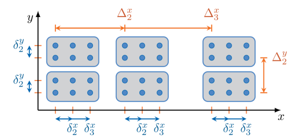
Compressed sensing provides a powerful framework for addressing high-dimensional data and limited sample challenges in DOA estimation. By leveraging advancements such as improved performance guarantees for sparse recovery through methods like Orthogonal Matching Pursuit (OMP) and developing novel algorithms like the CS beamformer root-MUSIC, computational efficiency and robustness are enhanced. These methodologies reduce the computational burden of traditional techniques while maintaining high resolution in estimating signal directions, even with limited samples [20, 23, 17, 21, 62]. By integrating advanced mathematical frameworks and adaptive algorithms, these techniques substantially improve estimation accuracy and efficiency, ensuring robust performance across diverse signal processing applications.



(a) Comparison of Source Separation Algorithms in Audio Signal Processing[18]



(b) Three-Dimensional Coordinate System and Surface Representation[63]



(c) Grid of Rectangles with Delta Notations[64]

Figure 3: Examples of Applications in DOA Estimation

As illustrated in Figure 3, compressed sensing and sparse signal recovery have emerged as powerful techniques in signal processing, particularly in applications like DOA estimation. The first figure compares various source separation algorithms in audio signal processing, highlighting performance differences in terms of average error across varying signal-to-noise ratios, including algorithms such as FRIDA and MUSIC. The second figure depicts a three-dimensional coordinate system with surface and cylindrical representations, essential for visualizing spatial relationships in DOA estimation scenarios. Lastly, the grid of rectangles with delta notations illustrates the precision and structure involved in spatial sampling, critical for accurate signal recovery and interpretation. Together, these visual examples underscore the significance of compressed sensing and sparse signal recovery in enhancing the accuracy and efficiency of DOA estimation within complex signal environments [18, 63, 64].

5 Sparse Bayesian Learning

Sparse Bayesian Learning (SBL) offers a sophisticated probabilistic approach to managing uncertainty and enhancing model performance across diverse applications. This section explores SBL's foundational aspects, emphasizing its probabilistic frameworks that effectively address uncertainty in sparse models. By examining these frameworks, their significance in improving model robustness and accuracy, particularly in dealing with incomplete and noisy data, becomes evident. The subsequent subsection delves into the methodologies employed within these frameworks, highlighting their efficacy in uncertainty management and implications for practical model performance.

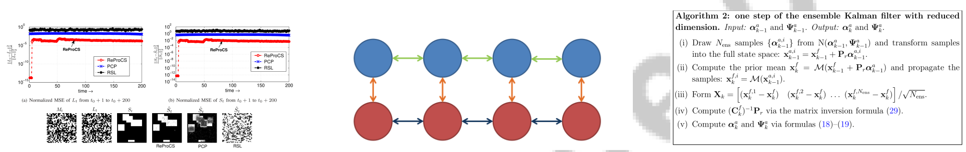
5.1 Probabilistic Frameworks and Uncertainty Handling

Probabilistic frameworks are crucial in SBL for managing uncertainty, offering robust methodologies for inferring sparse models from incomplete and noisy data. These frameworks utilize Bayesian inference to provide reliable estimates under challenging conditions, such as non-differentiable loss functions or adversarial environments. The ALPCAH method, for instance, leverages all available data, including noisy samples, to estimate noise variances, enhancing PCA robustness against noise [65]. In high-dimensional settings, Bayesian frameworks retain more posterior distribution information than traditional PCA methods, crucial for maintaining accuracy and robustness in uncertain environments [66]. The B-FS method exemplifies Bayesian approaches' strength in fully quantifying epistemic uncertainty, enhancing predictive accuracy with limited data and effectively recovering active subspaces [41].

The decentralized power method demonstrates the effectiveness of local averaging in converging to global averages, enhancing distributed systems' robustness against uncertainty [53]. Similarly, the SI-SPARROW method improves estimation accuracy in uncertain environments by exploiting shift-invariance properties to enhance robustness against source correlation [64]. Correntropy-induced methods significantly enhance clustering accuracy in non-Gaussian noise and outlier scenarios, showcasing the robustness of probabilistic frameworks in challenging conditions [67]. This robustness is further highlighted by the second-order difference subspace approach, which captures both velocity and acceleration in subspace dynamics, providing a comprehensive framework for managing dynamic uncertainties [68].

The empirical Bayes approach in frequency estimation aligns with the probabilistic frameworks used in SBL, enhancing estimation accuracy by incorporating uncertainty [15]. Moreover, optimizing ISAC waveforms illustrates the application of probabilistic frameworks in improving performance by effectively managing uncertainty in integrated sensing and communication systems [1]. Subspace Configurable Networks (SCNs) are particularly beneficial for edge devices due to their low resource requirements and rapid adaptability to varying input transformations without extensive retraining [9]. This adaptability is crucial for managing uncertainty in dynamic environments. Additionally, the application of probabilistic frameworks in differentially private subspace estimation emphasizes their role in handling uncertainty, especially regarding dataset 'easiness' [30].

Integrating these probabilistic frameworks underscores SBL's effectiveness in addressing uncertainty, enhancing robustness and efficiency while facilitating optimal recovery of sparse solutions in underdetermined linear systems. This is exemplified through advanced algorithms like CoSaMP, Subspace Pursuit, and Iterative Hard Thresholding, extending to robust principal component analysis that accommodates outlier detection and real-time tracking in high-dimensional data across diverse signal processing applications [69, 21, 61]. By integrating robust algorithms and leveraging Bayesian inference strengths, these frameworks provide a comprehensive approach to tackling uncertainty challenges in modern signal processing environments.



(a) ReProCS: A Novel Approach for Time Series Prediction with Sparse Latent Variables[69]

(b) Sequential Flow of Information[70]

(c) Algorithm 2: One step of the ensemble Kalman filter with reduced dimension.[37]

Figure 4: Examples of Probabilistic Frameworks and Uncertainty Handling

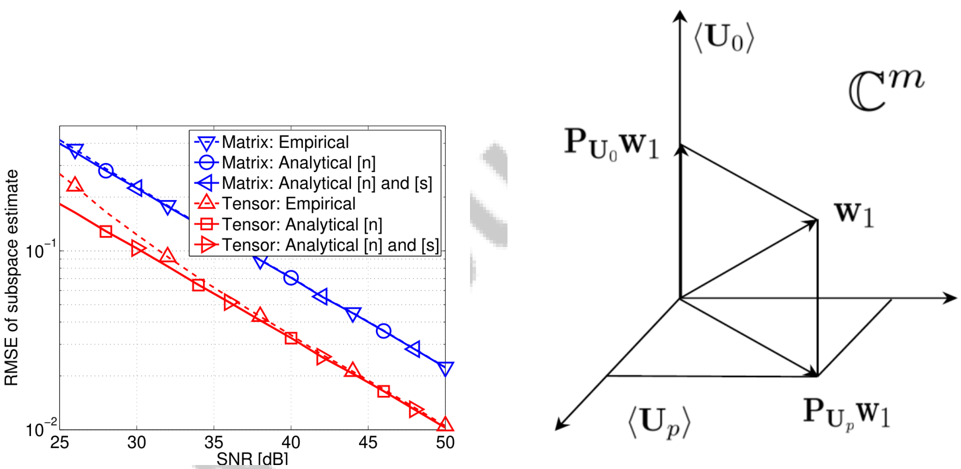
As illustrated in Figure 4, the "Sparse Bayesian Learning; Probabilistic Frameworks and Uncertainty Handling" is depicted through three distinct visual representations that encapsulate various methodologies for managing uncertainty. The first image, "ReProCS: A Novel Approach for Time Series Prediction with Sparse Latent Variables," showcases a comparative analysis of three methods—ReProCS, PCP, and RSL—used for predicting time series data, evaluated through normalized mean squared error across time steps, highlighting the efficacy of ReProCS in managing sparse latent variables for improved prediction accuracy. The second image, "Sequential Flow of Information," presents a flowchart depicting the progression of information through interconnected stages, underscoring the systematic handling of data. The final image, "Algorithm 2: One step of the ensemble Kalman filter with reduced dimension," details a step-by-step procedure of the ensemble Kalman filter, emphasizing the processes of transformation, propagation, and computation involved in dimension reduction. Collectively, these illustrations provide a comprehensive overview of the frameworks and techniques employed in sparse Bayesian learning to address uncertainty in data-driven environments [69, 70, 37].

5.2 Applications in Signal Processing

Sparse Bayesian Learning (SBL) is pivotal in signal processing, leveraging its probabilistic framework to enhance performance and robustness in dynamic and uncertain environments. A notable application of SBL is in Direction of Arrival (DOA) estimation, where techniques like transmit energy focusing improve the signal-to-noise ratio (SNR) per virtual element, increasing DOA estimation accuracy and reducing computational complexity [24]. In federated learning, SBL's scalability is demonstrated through algorithms like DSSAL1, which efficiently manage large datasets while minimizing communication overhead, crucial in distributed networks [71]. The Online Supervised Subspace Tracking (OSDR) method exemplifies SBL's adaptability in handling dynamic data and missing values, utilizing supervised information to improve prediction accuracy in real-time applications [72].

The robustness of SBL in high-dimensional settings is further emphasized by the Random Subspace Ensemble (RaSE) method, which outperforms existing ensemble methods through strong consistency and robustness, even in complex data environments [2]. This robustness is vital for high-dimensional data analysis applications, such as video processing, speech enhancement, and machine learning, where guided signal reconstruction techniques can be effectively applied [34]. SBL's capacity to handle a wide range of nonlinear transformations without restrictive assumptions on input bounds is significant, particularly in applications involving various neural network architectures [73]. This flexibility is crucial for enhancing learning accuracy and robustness across diverse signal processing tasks, as demonstrated in Fast Analog Transmission (FAT) contexts [74].

Moreover, the deterministic performance analysis framework provides a comprehensive benchmark that extends beyond asymptotic results, offering valuable insights into the practical applications of SBL in subspace methods [44]. In massive MIMO systems, SBL addresses uncertainty in channel estimation by mitigating pilot contamination, underscoring its importance in enhancing signal processing tasks in complex communication environments [12]. Despite certain limitations, such as potential suboptimal performance in high noise levels or insufficient channel sparsity [75], the versatility and robustness of SBL continue to position it as a valuable asset in advancing signal processing methodologies. Additionally, structured dictionary learning (StructDL) enhances classification performance through improved label consistency and robustness, particularly with small datasets [59]. Techniques like LINEBO further illustrate the efficacy of SBL by efficiently addressing high-dimensional optimization problems while adhering to safety constraints, providing practical solutions to challenging optimization scenarios [48].



(a) The image shows a graph comparing the root mean square error (RMSE) of subspace estimates for different methods and SNR values.[76]

(b) Geometric Representation of Three-Dimensional Space[77]

Figure 5: Examples of Applications in Signal Processing

As depicted in Figure 5, Sparse Bayesian Learning (SBL) serves as a powerful tool in signal processing, providing a robust framework for managing sparse data representations. This technique is particularly beneficial in scenarios with limited or incomplete data, enabling more accurate and efficient signal reconstruction. The accompanying figures illustrate two distinct applications of SBL in signal processing. The first image presents a graph comparing the root mean square error (RMSE) of subspace estimates across various methods and signal-to-noise ratio (SNR) values, highlighting the effectiveness of different approaches in minimizing error under varying noise conditions. The second image offers a geometric perspective, showcasing a three-dimensional space with labeled axes U0, Up, and U1, featuring vectors PU0W1 and PUPW1, which emphasize their direction and magnitude within a boundary marked as 'Cm.' Together, these examples underscore the versatility and applicability of Sparse Bayesian Learning in enhancing signal processing methodologies, providing clearer insights and more precise outcomes in complex data environments [76, 77].

5.3 Enhancing Model Performance and Robustness

Enhancing model performance and robustness in sparse Bayesian learning involves integrating advanced methodologies that effectively address challenges posed by noise, high-dimensionality, and sparse data environments. Techniques such as Robust Block-Diagonal Low-Rank (rBDLR) representation excel in providing block-diagonal structures in coefficient matrices, significantly improving the interpretability and accuracy of recovered representations [7]. This capability is essential for maintaining model integrity in complex data scenarios. Robust Principal Component Analysis (RPCA) and its dynamic variants play a pivotal role in enhancing robustness against non-uniform outlier distributions, dynamically adjusting to changing data characteristics to ensure robust performance across various signal processing tasks [3]. The implementation of Low-Rank Representation with Positive Semidefinite constraints (LRR-PSD) further enhances segmentation accuracy by producing valid affinity matrices that are both low-rank and positive semidefinite, contributing to improved clustering outcomes in high-dimensional data environments [29].

In optimization contexts, methods like Adaptive Safe Bayesian Optimization (ASBO) significantly enhance exploration capabilities, particularly in the presence of complex constraints, extending the applicability of Bayesian optimization to high-dimensional problems and improving model robustness and performance [48]. The identification variance minimization method offers improved convergence rates and stability compared to existing methods, presenting a robust alternative in system identification [11]. The Recursive Projected Compressive Sensing (ReProCS) algorithm is instrumental in managing larger support sizes and correlated outliers, making it highly suitable for real-time applications such as video surveillance, where dynamic environments and rapid data changes are prevalent. This algorithm enhances model robustness by effectively managing outliers and ensuring consistent performance in fluctuating conditions [69]. In signal processing applications, the Doppler spread estimation method provides improved noise resilience, lower computational demands, and the ability to track dynamic changes in the channel environment, critical for maintaining robust performance in dynamic signal processing contexts [35].

Recent advancements in sparse Bayesian learning, particularly in robust principal component analysis (RPCA) and its dynamic variants, highlight a significant evolution in techniques designed to improve model performance and resilience against outliers. Developments in robust subspace tracking and accelerated optimization algorithms have led to faster, more efficient solutions for identifying low-rank structures in corrupted data. These innovations enhance the accuracy of principal component estimation and facilitate real-time tracking and recovery of signals in high-dimensional spaces, showcasing a comprehensive approach to addressing challenges in sparse data environments [31, 23, 3, 61]. By integrating robust algorithms, innovative dimensionality reduction techniques, and novel mathematical frameworks, these advancements provide comprehensive solutions to the challenges of modern signal processing applications.

6 Addressing the Off-Grid Effect

6.1 Understanding the Off-Grid Effect

The off-grid effect presents notable challenges in signal processing, especially in parameter estimation where continuous parameters are discretized onto a predefined grid. This mismatch between actual signal parameters and grid points leads to estimation inaccuracies. In millimeter-wave (mmWave) channel estimation, the issue is pronounced, as traditional methods struggle with grid mismatches in angle of arrival (AoA) and angle of departure (AoD), compromising the performance of full-dimensional MIMO systems [78].

A significant challenge is subspace swaps, where noise subspace components better model the data than signal subspace components, particularly under low signal-to-noise ratio (SNR) conditions [77]. This misalignment results in errors as true signal parameters are inadequately captured by the discretized model. Fast fading in high-mobility environments exacerbates the off-grid effect, causing rapid channel condition changes that misalign actual signal parameters with grid points. The Fast Analog Transmission (FAT) method improves robustness against fast fading, crucial for maintaining accurate parameter estimation in dynamic environments [74].

Moreover, differential privacy preservation can be compromised by the off-grid effect, as traditional random matrix distributions satisfying the Johnson-Lindenstrauss property may not efficiently maintain privacy under grid mismatches [60]. This highlights the need for innovative strategies to manage the off-grid effect while ensuring privacy and accuracy in parameter estimation.

The off-grid effect can significantly deviate mean-squared error from the Cramer-Rao bound, particularly when SNR falls below a critical threshold, often due to subspace swap events where data better approximates components from an orthogonal subspace rather than the intended signal subspace. Consequently, estimation techniques like maximum likelihood estimation and robust PCA may suffer, necessitating advanced methodologies to mitigate these effects and enhance estimation accuracy [47, 23, 3, 77, 8]. Developing techniques to effectively mitigate grid mismatches is essential for robust performance across diverse applications.

6.2 Techniques to Mitigate the Off-Grid Effect

Addressing the off-grid effect is crucial for improving parameter estimation accuracy, particularly in high-dimensional signal processing applications. Atomic norm minimization is a promising approach that effectively addresses grid mismatch issues by enabling precise localization of signal paths. This technique is especially beneficial in mmWave beamformed full-dimensional MIMO systems, where accurate AoA and AoD estimation is vital for optimal performance [78].

Coherence Pursuit (CoP) is another effective technique, offering superior speed and robustness compared to traditional methods. CoP excels at recovering the true subspace amidst numerous unstructured outliers and additive noise, making it a robust choice for mitigating the off-grid effect in noisy environments. Its ability to manage large numbers of outliers without sacrificing subspace estimation accuracy is particularly valuable in dynamic signal processing scenarios [58].

These advanced methodologies collectively contribute to mitigating the off-grid effect, ensuring more accurate parameter estimation and enhancing the reliability of signal processing systems in diverse environments. By integrating robust principal component analysis (RPCA) techniques, which decompose data matrices into low-rank and sparse components, alongside advanced parameter estimation methods under compression, practitioners can significantly enhance performance and resilience in applications requiring precise parameter estimation, particularly in noise and outlier-prone environments. This approach facilitates real-time tracking of low-rank subspaces, mitigates adverse effects from subspace swaps, and bolsters estimation reliability, as evidenced by empirical studies in fields such as computer vision, bioinformatics, and signal processing [23, 3, 61, 77].

7 Beamforming Techniques

7.1 Introduction to Beamforming

Beamforming is a critical signal processing technique that focuses transmission or reception patterns to direct energy, thereby enhancing signal quality and minimizing interference. This approach is vital in modern communication systems such as radar, wireless communications, and sonar, where precise directional control is essential [12, 24]. By employing antenna arrays, beamforming creates interference patterns that concentrate signal energy and suppress unwanted interference.

The primary advantage of beamforming is its ability to improve the signal-to-noise ratio (SNR) and spatial resolution, which are crucial for high-precision applications. In wireless systems, beamforming enables efficient spectrum use by targeting energy towards specific receivers, reducing power consumption and interference, especially in environments with multiple users and high data traffic [13].

Recent innovations emphasize adaptive strategies that maintain performance across varying conditions, incorporating algorithms that optimize beam patterns in real-time, accounting for user mobility and channel changes [1]. Additionally, integrating beamforming with advanced techniques like compressed sensing and sparse Bayesian learning further optimizes data acquisition and processing [16].

7.2 Adaptive Beamforming Techniques

Adaptive beamforming techniques dynamically modify antenna array patterns in response to environmental changes, optimizing signal transmission and reception. These techniques are particularly beneficial in dynamic environments where user mobility, multipath propagation, and interference impact communication performance. By enhancing the ability to sustain high signal quality, adaptive beamforming is crucial for reliable communication [24].

A key benefit of adaptive beamforming is its capacity to improve SNR by focusing energy on desired signals while suppressing interference from other directions. Techniques like the Minimum Variance Distortionless Response (MVDR) beamformer reduce noise power while maintaining a distortionless response towards the target direction [12].

In situations with unknown direction of arrival (DOA), adaptive algorithms estimate DOA in real-time, adjusting beam patterns as needed, which is vital in highly mobile or rapidly changing environments like urban areas with significant multipath effects or vehicular communication systems [1].

Recent advancements incorporate machine learning to enhance the adaptability and precision of beamforming algorithms, enabling predictive beam pattern adjustments for anticipated environmental changes [13]. Furthermore, sparse Bayesian learning frameworks improve handling of uncertainty and noise, ensuring robust performance across various applications [16].

Adaptive beamforming is a significant advancement in signal processing, enhancing signal quality and system adaptability. Advanced algorithms, including computationally efficient methods for small sample support and hybrid beamformers for complex acoustic environments, address challenges like high computational complexity and variable noise fields. By using low-dimensional signal subspace models and innovative methods such as compressive sensing, adaptive beamforming reduces computational demands while maintaining high resolution and performance, making it indispensable for modern communication systems, including millimeter-wave MIMO technologies and augmented reality audio [17, 63, 27, 75, 26].

7.3 Innovative Beamforming Applications

Recent innovations in beamforming have expanded its applications across various signal processing domains, enhancing both performance and adaptability. The integration of machine learning with traditional beamforming methods has led to more intelligent systems capable of predicting environmental changes and adjusting beam patterns proactively, thus improving communication robustness and efficiency [13].

In radar and wireless communications, beamforming is crucial for enhancing spatial resolution and signal quality. Adaptive algorithms that optimize beam patterns in real-time allow precise targeting and reduced interference, particularly in high-density user environments [1]. Sparse Bayesian learning frameworks further ensure robust handling of uncertainty and noise across diverse applications [16].

Beamforming techniques are also key in developing integrated sensing and communication (ISAC) systems, where shared hardware resources achieve both sensing and communication objectives. This integration improves spectrum efficiency and system performance, as shown in studies on waveform optimization and energy focusing strategies [24], enhancing SNR and detection capabilities in applications like autonomous vehicles and smart city infrastructure.

In millimeter-wave (mmWave) communications, where high-frequency bands offer substantial bandwidth but face challenges like signal attenuation and path loss, beamforming extends the range and reliability by directing energy effectively [12]. This is particularly beneficial in urban settings, where physical obstructions and multipath effects can degrade signal quality.

Recent advancements in beamforming techniques have broadened their applicability and effectiveness in modern signal processing, providing enhanced capabilities for various applications. Innovations such as robust adaptive beamformers for augmented reality audio, computationally efficient algorithms for small sample support, and novel broadband beamforming approaches enhance noise suppression, improve signal intelligibility, and reduce computational complexity. These advancements address challenges posed by complex acoustic environments and limited data, paving the way for more efficient and scalable communication solutions [17, 63, 27, 26].

8 Adaptive Filtering in Dynamic Environments

8.1 Concept and Importance of Adaptive Filtering

Adaptive filtering is fundamental to signal processing, enabling automatic filter parameter adjustment in response to input signal or environmental changes. This adaptability is crucial in dynamic environments where signal characteristics fluctuate due to noise, interference, and channel variations. Applications like noise cancellation, echo suppression, and channel equalization utilize adaptive filters to continuously optimize filter coefficients, enhancing signal quality [12]. By improving the signal-to-noise ratio (SNR) and mitigating interference, adaptive filtering enhances communication system reliability and clarity. In non-stationary scenarios, adaptive filters dynamically adjust to changes, ensuring consistent performance [24], especially in wireless systems where they manage multipath fading and Doppler shifts effectively [13].

Recent advancements focus on enhancing convergence speed and computational efficiency, making adaptive filtering suitable for real-time applications. Machine learning integration has further augmented their predictive and adaptive capabilities, resulting in more resilient signal processing systems [1]. Additionally, combining adaptive filtering with advanced techniques like beamforming and compressed sensing has broadened its applicability in modern communication and radar systems [16]. Adaptive filtering offers the flexibility and robustness needed to navigate contemporary communication complexities, underscored by advanced algorithms like Accelerated Structured Alternating Projections and fast adaptive beamforming techniques, ensuring reliable and efficient signal processing across diverse applications, including robust recovery of spectrally sparse signals and effective parameter estimation from compressed data [20, 23, 77, 8, 26].

8.2 Techniques and Algorithms for Adaptive Filtering

Adaptive filtering techniques are pivotal in dynamic signal processing, allowing real-time filter parameter adjustments to optimize performance amid changing signal conditions. The Least Mean Squares (LMS) algorithm is foundational, iteratively updating filter coefficients to minimize mean square error, making it popular for echo cancellation and noise reduction due to its simplicity and robustness [24]. The Recursive Least Squares (RLS) algorithm, known for rapid convergence and tracking swift signal changes, minimizes a weighted sum of squared errors, providing superior performance in non-stationary environments despite its computational complexity, which can be mitigated through efficient implementations [12].

The intersection of machine learning and adaptive filtering has led to advanced algorithms that leverage data-driven insights to enhance filtering performance. Neural network-based adaptive filters learn complex signal patterns and adjust parameters adaptively, proving effective in scenarios with nonlinear and time-varying dynamics [13]. Subspace-based adaptive filtering techniques, decomposing signals into lower-dimensional subspaces, achieve computational efficiency while maintaining high accuracy [1]. Compressed sensing principles in adaptive filtering minimize measurement requirements without sacrificing signal quality, particularly relevant in resource-constrained environments [16].

The evolution of adaptive filtering techniques, including robust PCA, dynamic subspace tracking, and efficient beamforming methods, underscores their essential role in modern signal processing. These advancements enable improved performance in noise reduction, outlier management, and real-time data analysis [23, 3, 77, 8, 26]. By integrating traditional and emerging methodologies, these techniques deliver robust and efficient solutions, ensuring optimal performance in complex and dynamic environments.

9 Conclusion

This survey has provided a comprehensive analysis of recent advances in signal processing techniques that significantly enhance Direction of Arrival (DOA) estimation. Techniques such as subspace methods, compressed sensing, sparse Bayesian learning, and beamforming have shown substantial improvements in accuracy, efficiency, and robustness when dealing with complex signal environments. Innovations like TransMUSIC have demonstrated exceptional performance in DOA estimation using one-bit quantized data, surpassing traditional methods that depend on unquantized data. Thresholding-based Subspace Clustering (TSC) has proven effective in clustering noisy data, maintaining robust performance even under significant noise levels, especially when subspaces are low-dimensional.

The randomized subspace-based approach has improved prediction accuracy and variable selection in high-dimensional datasets, while Subspace Configurable Networks (SCNs) offer high accuracy with fewer configuration dimensions, making them suitable for resource-constrained environments. Space Form PCA (SFPCA) has outperformed conventional PCA methods in speed and accuracy, proving to be a valuable tool for dimensionality reduction in complex data structures.

In microwave imaging, the addition of antennas, whether physical or virtual, significantly enhances imaging performance in noisy settings. The Improved Subspace Method has shown reduced identification variance and increased stability, suggesting potential for future research in input design for partially observable systems and extensions to nonlinear models. Further exploration could enhance algorithm performance in low SNR scenarios and integrate these techniques with other channel estimation methods.

Advancements in low-rank matrix optimization have markedly reduced convergence times for large-scale problems, showcasing progress in optimization strategies. Additionally, the development of a differentially private subspace estimation method has outperformed traditional approaches in both accuracy and efficiency, offering a new framework for high-dimensional contexts.

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