Deep Learning Inversion and Nonlinear Inversion in Geophysical Imaging: A Survey

Abstract

Deep learning inversion has significantly advanced geophysical imaging by addressing complex nonlinear inversion problems, particularly in resistivity forward modeling. By integrating deep neural networks (DNNs) and physics-informed neural networks (PINNs), researchers have developed robust models that accurately interpret subsurface resistivity distributions, enhancing imaging resolution and exploration accuracy. These methodologies overcome traditional challenges such as the curse of dimensionality and nonlinearity, offering improved solutions for high-dimensional partial differential equations (PDEs) and seismic imaging. The incorporation of physical principles into neural networks ensures predictions adhere to established laws, improving model reliability and interpretability. Furthermore, advancements in neural network architectures and optimization strategies have increased computational efficiency, crucial for handling large geophysical datasets. Despite these advancements, challenges remain, including data scarcity, model generalization, and computational demands. Future research will focus on refining neural architectures, enhancing data quality, and integrating traditional geophysical methods with deep learning to further improve model performance. As the field evolves, these innovations promise to transform geophysical imaging, providing more precise and reliable solutions for subsurface exploration and resource management.

1 Introduction

1.1 Overview of Deep Learning Inversion in Geophysical Imaging

Deep learning inversion has emerged as a transformative approach in geophysical imaging, utilizing advanced neural network architectures to tackle the complexities of estimating subsurface structures from surface data. Traditional inversion methods often encounter challenges such as nonlinearity and nonuniqueness, relying on computationally intensive iterative algorithms that may introduce errors. In contrast, deep learning techniques, including fully convolutional networks and end-to-end seismic inversion networks, leverage extensive synthetic datasets for rapid and accurate estimations of subsurface properties, such as resistivity and velocity models. These methodologies enhance seismic data utilization by establishing spatial correspondences and exhibit superior performance across various evaluation metrics, thus facilitating real-time applications in geophysical exploration and monitoring [1, 2]. By employing deep neural networks (DNNs), researchers have developed models that effectively map subsurface formations and estimate physical properties, addressing the challenges of nonlinear inversion problems and alleviating the curse of dimensionality, a common hurdle in numerical approximations of high-dimensional fully nonlinear partial differential equations (PDEs).

In seismic imaging, techniques like Seismic Full Waveform Inversion (FWI) have proven instrumental in generating high-resolution subsurface models by minimizing the misfit between simulated and observed seismograms [3]. Additionally, Rational Function Neural Networks (RafNN) have been

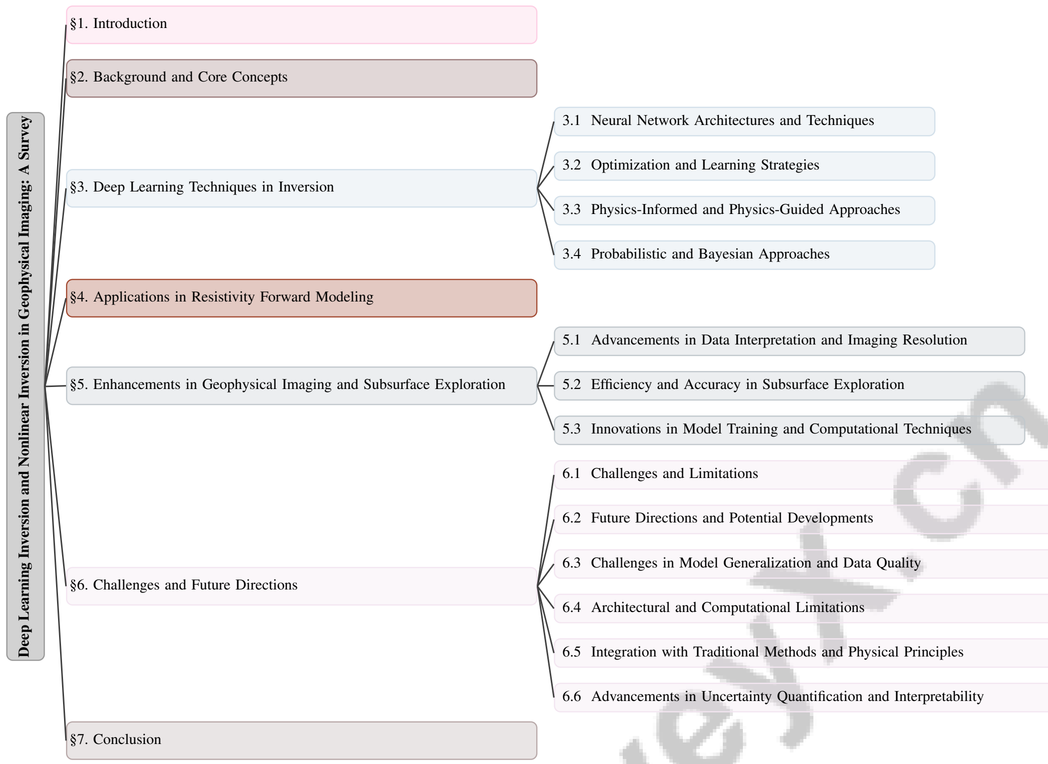

Figure 1: chapter structure

introduced to directly model seismic wave velocity from observational data, effectively addressing the complexities of underground rock characterization [4].

Furthermore, Electrical Resistivity Tomography (ERT) has significantly benefited from deep learning, enabling the construction of detailed 3D resistivity models that uncover critical geological features [5]. The application of deep learning in interpreting noisy logging-while-drilling (LWD) resistivity measurements has enhanced inversion robustness and accuracy [6]. Additionally, deep learning techniques have addressed nonlinear electromagnetic (EM) inverse scattering challenges, providing insights into the internal structures of objects [7].

The integration of physics-informed neural networks (PINNs) has further advanced the field by incorporating governing PDEs as constraints within the loss function, thereby improving the accuracy of physical system approximations. This approach has shown promise in estimating geotechnical parameters from proxy variables, effectively minimizing discrepancies between predicted and observed phenomena, such as landslide occurrences [8].

Moreover, deep learning has played a pivotal role in subsurface geological exploration, with methodologies like transfer learning enhancing the extraction of petrophysical properties from drill core images [9]. The integration of deep learning with time series foundation models has further improved subsurface storage characterization, a critical task given rising energy demands [10].

The increasing demand for large-scale matrix-vector multiplication, essential for artificial intelligence (AI) and DNNs, highlights the growing computational challenges in this domain [11]. Recent advancements in approximating piecewise smooth functions using deep ReLU neural networks underscore the complexity involved and the advantages of depth in achieving optimal approximation rates [12]. Nonlinear image reconstruction methods have also led to significant improvements in imaging systems, traditionally framed as iterative optimization problems [13].

Deep learning inversion signifies a substantial advancement in geophysical imaging, utilizing DNNs to reconstruct velocity models directly from seismic data. This approach addresses the challenges of traditional iterative algorithms, which often struggle with nonlinear mapping and nonuniqueness. Exemplified by end-to-end seismic inversion networks (SeisInvNets), this innovative methodology enhances seismic trace accuracy by incorporating contextual information and spatial relationships.

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Additionally, deep learning methods for electromagnetic inversion provide instantaneous results without gradient calculations, streamlining subsurface property estimation in real-time scenarios. These advancements offer powerful tools for navigating the complexities of nonlinear inversion in geophysical imaging [1, 2]. This survey will explore the diverse techniques and applications of deep learning in geophysical imaging, establishing a foundation for a comprehensive examination of this rapidly evolving field.

1.2 Significance of Neural Networks and Deep Learning

Neural networks and deep learning have revolutionized modern geophysical imaging and inversion, offering robust frameworks for addressing complex nonlinear problems. These methodologies effectively tackle challenges inherent in traditional inversion techniques, including the necessity for labeled training data and the integration of physical constraints. The development of models like FWIGAN exemplifies this transformation by utilizing an unsupervised learning framework that merges physics-informed modeling with adversarial learning to estimate velocity models without reliance on labeled data [3].

Data-driven approaches, such as Rational Function Neural Networks (RafNN), illustrate the capability of neural networks to derive velocity models that align with actual data distributions while adhering to rock physics principles [4]. This alignment underscores the importance of neural networks in enhancing the accuracy and reliability of geophysical imaging.

Despite their successes, neural networks encounter challenges related to system latency and power consumption, particularly within traditional digital computing architectures, which can hinder the performance of machine learning models [11]. Addressing these challenges is essential for optimizing computational efficiency and scalability in deep learning applications within geophysical contexts.

The identifiability of neural network architectures, weights, and biases remains a critical area for investigation, as it affects the ability to specify these parameters to realize a given function [14]. Advances in understanding these aspects are crucial for enhancing the predictability and interpretability of neural network models.

Furthermore, integrating physical insights into neural network frameworks, as demonstrated by methods like Physics Guided and Injected Learning (PGIL), enhances classification accuracy and explainability by embedding physical principles into the learning process [15]. This integration is vital for ensuring that neural network models not only perform effectively but also yield insights consistent with established physical theories.

The ability to reconstruct minimum phase preserving operators using recent breakthroughs in stable polynomials further exemplifies the innovative applications of neural networks in geophysical imaging [16]. As the field evolves, the role of neural networks and deep learning in automating complex processes and enhancing model fidelity is expected to expand, driving further innovations in geophysical research and exploration.

1.3 Importance of Resistivity Forward Modeling

Resistivity forward modeling is essential in geophysical imaging, crucial for elucidating subsurface resistivity distributions and interpreting complex geological structures. This technique is particularly vital in applications such as fluid extraction and pressure management, where accurate subsurface models are critical for operational success [17]. The inherently nonlinear and ill-posed nature of electrical resistivity surveys presents significant challenges, necessitating advanced methodologies to enhance the precision and reliability of imaging processes [18].

Traditional forward modeling approaches often face limitations due to high computational demands and inefficiencies, particularly when managing large datasets and three-dimensional scenarios. These constraints underscore the need for innovative solutions that integrate seamlessly with modern computational techniques. The advent of deep learning has revolutionized resistivity forward modeling by providing robust frameworks capable of efficiently processing complex datasets and improving model accuracy [19].

Deep learning techniques, such as the characteristics-informed neural network (CINN), have been developed to hard-encode physical principles into neural network architectures, significantly enhanc-

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ing training efficiency and solution accuracy [20]. This approach not only reduces computational costs but also ensures adherence to underlying physics, thereby improving the fidelity of inversion results. Moreover, the generation of synthetic databases using refined isogeometric analysis (rIGA) exemplifies the synergy between traditional modeling and deep learning, advancing accurate borehole resistivity measurements and geophysical inversion [21].

The integration of physics-informed machine learning frameworks has shown promise in overcoming challenges such as data scarcity and the complexities of seismic data processing, prevalent issues in geophysical tasks [22]. Furthermore, the choice of electrode configuration in Electrical Resistivity Imaging (ERI) is critical for determining result accuracy, emphasizing the importance of optimizing these configurations for reliable geological models [23].

Innovations like SAMERA, which utilizes both passive and active wave sources, have enhanced imaging strategies and resolution capabilities, offering more flexible solutions compared to traditional methods [24]. These advancements in resistivity forward modeling, driven by deep learning, are expected to continue propelling progress in geophysical imaging, providing more accurate and efficient solutions to complex subsurface exploration challenges.

1.4 Structure of the Survey

This survey is meticulously structured to provide a comprehensive exploration of deep learning inversion and nonlinear inversion in the context of geophysical imaging. It begins with an introduction that highlights the significance of integrating neural networks and deep learning techniques to address complex nonlinear inversion problems. The introduction also emphasizes the importance of resistivity forward modeling in enhancing subsurface exploration.

Following the introduction, the survey delves into background and core concepts, offering an overview of fundamental principles underlying deep learning inversion, nonlinear inversion, resistivity forward modeling, geophysical imaging, subsurface exploration, and neural networks. This section addresses challenges and complexities inherent in these concepts and discusses how deep learning offers viable solutions.

The third section focuses on deep learning techniques in inversion, discussing various neural network architectures and techniques employed in inversion processes. It highlights optimization and learning strategies that enhance model performance and explores physics-informed and physics-guided approaches that incorporate physical principles into deep learning models. Probabilistic and Bayesian methods are also introduced to manage uncertainty and improve model reliability.

In the fourth section, the survey examines applications in resistivity forward modeling, illustrating how deep learning inversion techniques enhance the accuracy and efficiency of interpreting subsurface resistivity distributions. This section includes case studies and practical implementations to demonstrate the real-world applicability of these techniques.

The fifth section highlights advancements in geophysical imaging and subsurface exploration, showcasing improvements in data interpretation, imaging resolution, and exploration accuracy due to deep learning inversion techniques. It also discusses potential future developments in this area.

The survey concludes with a discussion of challenges and future directions in implementing deep learning inversion techniques in geophysical applications. This section identifies current limitations and suggests areas for future research, including model generalization, data quality, architectural and computational limitations, and the integration of traditional methods with physical principles. Additionally, advancements in uncertainty quantification and model interpretability are examined.

Each section of this survey is designed to build upon the previous one, providing a cohesive narrative that guides the reader through the complexities and innovations of deep learning inversion in geophysical imaging.The following sections are organized as shown in Figure 1.

2 Background and Core Concepts

2.1 Fundamental Concepts of Deep Learning and Neural Networks

Deep learning and neural networks are pivotal in geophysical inversion, providing sophisticated frameworks to navigate the complexities of subsurface exploration. Central to these advancements is the architecture of deep neural networks (DNNs), which effectively approximate solutions for high-dimensional data, a crucial aspect in geophysical applications due to inherent data complexity and dimensionality challenges [4]. The Rational Function Neural Networks (RafNN) exemplify data-driven modeling, constructing analytical expressions for rock physics models and learning from field datasets, embodying the core principles of deep learning in geophysical inversion [4].

The integration of Physics-Informed Neural Networks (PINNs) marks a significant development, as they embed established physical laws into neural network training, allowing accurate predictions even with sparse data by leveraging physical constraints for effective generalization [3]. This approach is particularly beneficial where traditional data-driven methods falter due to limited data.

The optimization and design of neural networks are evolving to meet geophysical application demands. Techniques like deep convolutional neural networks (CNNs) model complex data relationships relevant to geophysical inversion and other fields, such as radio propagation [11]. Moreover, synthesizing neural networks to optimally model known input-output relationships remains a core challenge, necessitating careful consideration of network architecture and training strategies [14].

Neural networks facilitate parameter estimation by mapping spatial data to parameter estimates for covariance functions in geometrically anisotropic Gaussian random fields, demonstrating their versatility across geophysical tasks from data-driven modeling to parameter estimation [12]. Incorporating adjoint operators within neural network frameworks enhances model performance in both finite and infinite-dimensional spaces, providing a theoretical foundation for understanding neural network learning and optimization [16].

Efforts to explore minimal and optimal neural network architectures highlight ongoing refinements for geophysical applications. By assuming the underlying function of the data is a neural network model, researchers focus on identifying minimal networks that achieve optimal performance [14].

As research in deep learning and neural networks progresses, these foundational concepts are poised to drive further innovations in geophysical inversion, offering robust solutions to the intricate challenges of subsurface exploration and imaging. The transformative role of deep learning in geophysical investigations is underscored by enhanced accuracy and efficiency in subsurface property estimations, exemplified by methods such as electromagnetic inversion using CNNs, direct mineral content prediction from drill core images, and innovative seismic inversion techniques leveraging both data-driven and physics-informed approaches [25, 1, 9, 2].

2.2 Challenges in Nonlinear Inversion

Nonlinear inversion in geophysical imaging poses significant challenges due to the complex nature of subsurface environments and the computational intensity required for accurate solutions. Modeling seismic wave velocity is particularly challenging, as existing methods often rely on intricate theoretical formulations or overly simplistic empirical models, failing to capture the full complexity of the subsurface [4]. The presence of noise and the nonlinear characteristics of geophysical data further complicate signal recovery, as traditional algorithms struggle to manage these complexities effectively [26].

Non-uniqueness is a prevalent issue in nonlinear inversion, where multiple neural network architectures can represent the same function, complicating the interpretability and reliability of inversion results [14]. This non-uniqueness is exacerbated by the inverse problem of reconstructing operators from insufficient data, a challenge inadequately addressed by current methodologies [16].

Traditional deep learning methods also encounter limitations in accounting for uncertainty, a critical factor in real-world applications, especially in mission-critical domains [27]. The absence of probabilistic frameworks within these models can lead to overconfident predictions, undermining their applicability in uncertain environments.

The high cost of manual annotation and the lack of physical explanations for deep learning predictions further impede the effectiveness of existing methods [15]. This challenge is intensified by the non-smooth nature of popular regularizers and the vast amounts of data requiring processing, making optimization problems particularly challenging to solve [13].

Deep learning offers potential solutions to these challenges by utilizing advanced architectures that accommodate the nonlinearity and non-uniqueness of inverse problems, offering more robust interpretations of geophysical data. However, these models face challenges, including the need for extensive, high-quality training datasets and inherent opacity in their decision-making processes. Addressing these issues is essential for advancing geophysical imaging and ensuring the successful implementation of nonlinear inversion techniques. As research progresses, tackling challenges associated with seismic interpretation and subsurface exploration will be vital for enhancing the applicability and interpretability of deep learning models, particularly in fields such as environmental monitoring, carbon sequestration, and resource extraction. Leveraging machine learning algorithms, including CNNs, can significantly improve seismic data analysis, leading to more accurate assessments of subsurface structures and properties across various geophysical contexts [28, 1, 18, 9, 2].

3 Deep Learning Techniques in Inversion

Category	Feature	Method
Neural Network Architectures and Techniques	Enhanced Model Structures Adaptation and Robustness Parallel and Distributed Processing Efficient Computation and Scalability	DSNN[29], LRFMP[30], DLAO[31] DLIM[6], DL-MCP[9] PRN[32], jInv[33] F-GMM[34], NN-BS[35]
Optimization and Learning Strategies	Probabilistic Techniques Function Approximation Parameter Optimization	RafNN[4] ReLU-NN[12] C-TISTA[26]
Physics-Informed and Physics-Guided Approaches	Physics-Constrained Modeling	PINN[36], SPInProp[37], PGIL[15]

Table 1: This table provides a comprehensive overview of the various deep learning techniques applied in geophysical inversion. It categorizes methods into neural network architectures and techniques, optimization and learning strategies, and physics-informed approaches, highlighting key features and representative methods. The table serves as a reference for understanding the diverse methodologies employed to enhance accuracy, efficiency, and interpretability in geophysical applications.

The integration of deep learning into geophysical inversion has revolutionized the field, offering innovative solutions to complex subsurface challenges. This section examines key methodologies and advancements that have significantly improved accuracy, efficiency, and interpretability in inversion processes. We begin by analyzing neural network architectures and techniques that have been instrumental in advancing deep learning capabilities in geophysical applications. Table 1 presents a detailed classification of deep learning techniques utilized in geophysical inversion, emphasizing the advancements and methodologies critical for improving inversion processes. Additionally, Table 3 offers a comprehensive comparison of various deep learning techniques used in geophysical inversion, emphasizing their contributions to enhancing accuracy, efficiency, and interpretability in the field.

As illustrated in **??**, the hierarchical classification of deep learning techniques in geophysical inversion encompasses various categories, including neural network architectures, optimization strategies, physics-informed methods, and probabilistic approaches. Each of these categories emphasizes specific advancements and methodologies that are crucial for enhancing the accuracy, efficiency, and interpretability of inversion processes. This comprehensive framework not only highlights the diversity of approaches within the field but also underscores the importance of selecting appropriate techniques tailored to specific geophysical challenges.

3.1 Neural Network Architectures and Techniques

The advancement of neural network architectures has been pivotal in solving complex geophysical inversion problems. Convolutional Neural Networks (CNNs) have been particularly effective in extracting intricate patterns from geophysical data, as demonstrated in lithological classification and mineral content prediction [9]. These architectures adeptly handle the nonlinearities in geophysical data, enhancing inversion robustness and accuracy [6].

Innovative architectures like the Piecewise-Linear Residual Network (PRN) offer computational efficiency with reduced complexity while maintaining performance [32]. The Factorized Neural Network (F-NN) optimizes inversion by training on normalized data, streamlining calculations [34]. Incorporating differentiable linear algebra operators into frameworks such as MXNet allows Bayesian methods to enhance model interpretability and reliability [31].

The Linearized Regularized Forward Model Pursuit (LRFMP) utilizes an overcomplete set of basis functions for a more accurate Earth representation, addressing inverse problem challenges [30].

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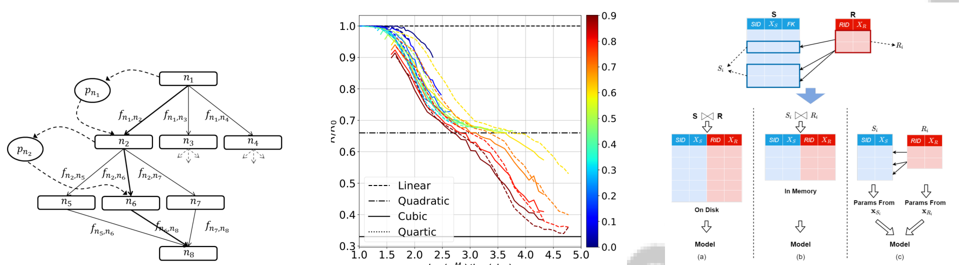
Neural networks' spectral bias, which favors learning lower frequencies, guides the design of architectures that efficiently capture essential geophysical data features [38]. The backward induction scheme with gradient estimation and Hessian approximation offers computational efficiency in solving complex inversion problems [35].

Prototype-based neural network layers improve interpretability and robustness by integrating prototype-based classification [39]. The jInv framework exemplifies the use of parallel computing to reduce computational burdens in solving multiple PDEs [33].

Recent advancements, including end-to-end seismic inversion networks (SeisInvNets) and CNNs, highlight their transformative potential in geophysical inversion, addressing challenges of weak spatial correspondence and seismic data nonlinearity. These methods outperform traditional algorithms, leading to reliable velocity model and subsurface structure estimates [18, 40, 1, 2, 41]. Ongoing research is expected to further enhance neural networks' capabilities in complex geophysical inversion problems.



(a) A complex network of interconnected nodes and edges[29]

(b) The image shows the convergence of a neural network model to a target function.[42]

(c) Deep Learning Model for Image Retrieval[34]

Figure 2: Examples of Neural Network Architectures and Techniques

As depicted in Figure 2, neural network architectures and techniques play a crucial role in deep learning inversion, with examples showcasing diverse applications and capabilities in handling complex tasks. These images illustrate the abstract nature of neural networks, model convergence to target functions, and modular architectures for efficient feature extraction, underscoring their versatility and effectiveness in solving inversion problems [29, 42, 34].

3.2 Optimization and Learning Strategies

Method Name	Optimization Techniques	Handling Uncertainty	Model Adaptability
RafNN[4]	Error Back Propagation	Random Noise Added	Varying Signal Characteristics
ReLU-NN[12]	Optimal Approximation Rates	-	Model Complex Relationships
C-TISTA[26]	Deep Unfolding Techniques	Probabilistic Elements	Adapt The Recovery

Table 2: Comparison of optimization techniques, uncertainty handling, and model adaptability across different neural network methods for geophysical inversion. The table highlights the specific strategies employed by RafNN, ReLU-NN, and C-TISTA to enhance performance in handling non-linear data transitions and noisy environments.

Optimization and learning strategies are essential in enhancing deep learning models' performance for geophysical inversion. The Rational Function Neural Networks (RafNN) framework employs forward and error backpropagation, significantly improving model performance [4]. Incorporating probabilistic elements into neural networks enriches model capabilities by handling uncertainty, critical in geophysical inversion where data may be sparse or noisy [27]. Table 2 provides a comprehensive comparison of various neural network methods, focusing on their optimization strategies, uncertainty management, and adaptability to model complex geophysical data.

Deep networks' ability to approximate piecewise smooth functions is particularly beneficial in geophysical contexts with non-linear data transitions [12]. The C-TISTA method uses trainable parameters to adapt recovery processes to signal characteristics, improving accuracy and convergence speed [26].

Advanced optimization and learning strategies significantly enhance deep learning models' performance in inversion tasks, improving accuracy, efficiency, and interpretability across geophysical applications. Techniques like fully convolutional networks and end-to-end seismic inversion networks enable rapid subsurface property estimation, overcoming traditional deterministic methods' limitations. These approaches address nonlinearity and nonuniqueness challenges, demonstrating robustness against measurement noise and facilitating real-time interpretations [1, 41, 6, 2]. As research advances, these strategies will be central to overcoming complex subsurface challenges and ensuring reliable geophysical imaging and exploration.

3.3 Physics-Informed and Physics-Guided Approaches

Physics-Informed Neural Networks (PINNs) have significantly improved inversion accuracy and reliability in geophysical applications by incorporating physical principles. PINNs use governing equations as constraints, ensuring predictions adhere to physical laws, enhancing convergence and accuracy with limited data. Their multi-network architecture offers superior accuracy and convergence compared to single-network approaches [36].

The Fast Low-Rank Neural Representation (FastLRNR) enhances PINNs' efficiency through dimensionality reduction, effectively handling high-dimensional geophysical problems [37]. Physics-guided modules like the Physics Guided Network (PGN) and Physics Injected Network (PIN) exemplify the synergy between data-driven and physics-based approaches, enhancing interpretability and robustness [15].

Theoretical insights into fully-connected layers and learning vector quantization (LVQ) methods offer valuable perspectives on designing neural network architectures incorporating physical principles [39]. Physics-informed and physics-guided approaches represent a transformative advancement in geophysical inversion, providing accurate, interpretable, and reliable solutions.

As subsurface exploration research progresses, innovative approaches such as advanced image processing, machine learning algorithms, and real-time geophysical sensor networks are anticipated to enhance geophysical models' accuracy and address subsurface characterization challenges. These advancements are crucial for applications in oil and gas exploration, environmental monitoring, and carbon sequestration, where precise subsurface delineation is essential for effective decision-making and risk management [28, 10, 24].

3.4 Probabilistic and Bayesian Approaches

Probabilistic and Bayesian approaches in deep learning inversion have become essential for addressing uncertainty and enhancing geophysical models' reliability. Bayesian neural networks offer a principled method for uncertainty estimation by treating model weights as probability distributions, improving interpretability and reliability [43]. Ensemble methods enhance robustness by aggregating predictions from multiple models, providing a comprehensive representation of uncertainty [43].

Test-time augmentation and calibration techniques contribute to uncertainty estimation by adjusting model confidence levels, ensuring predicted probabilities align with observed frequencies [43]. Probabilistic deep learning also includes deep probabilistic models, incorporating probabilistic layers and structures for flexible data distribution modeling [27].

Integrating probabilistic and Bayesian methods into deep learning inversion enhances model reliability by accounting for model and data uncertainties. These approaches refine prediction accuracy and provide insights into nonlinear data-generating processes. By utilizing Bayesian neural networks and variational autoencoders, researchers navigate inversion challenges, ensuring solutions align with geological structures while balancing accuracy and computational feasibility [44, 43, 27]. As the field advances, these approaches will improve geophysical models' accuracy and robustness, leading to more reliable subsurface exploration and imaging.

Feature	Neural Network Architectures and Techniques	Optimization and Learning Strategies	Physics-Informed and Physics-Guided Approaches
Accuracy	High Pattern Recognition	Improved Model Performance	Adheres TO Physical Laws
Efficiency	Reduced Complexity	Fast Convergence Speed	Dimensionality Reduction
Interpretability	Prototype-based Layers	Handles Uncertainty	Physics-based Modules

Table 3: This table provides a comparative analysis of deep learning methodologies applied to geophysical inversion, focusing on neural network architectures, optimization strategies, and physics-informed approaches. It highlights the key features of accuracy, efficiency, and interpretability, showcasing how each method contributes to advancements in inversion processes.

4 Applications in Resistivity Forward Modeling

4.1 Case Studies and Practical Implementations

Deep learning has significantly advanced resistivity forward modeling, as demonstrated by case studies showcasing its efficacy in geophysical imaging. In electromagnetic (EM) inversion, deep learning frameworks have effectively modeled resistivity using synthetic datasets from full 3-D simulations in onshore controlled source electromagnetic scenarios, improving modeling accuracy [1]. Comparative analyses, such as the evaluation of the Deep-RED method against traditional DnCNN on seismic data, have highlighted deep learning's superior performance in denoising and enhancing seismic images, facilitating better subsurface interpretation [45]. Additionally, training with the MNIST dataset alongside EM scattering simulations from digit-like objects has demonstrated deep learning's versatility in managing diverse data types for refined inversion results [7].

In borehole resistivity measurements, deep learning has optimized measurement accuracy and minimized errors, as evidenced by research utilizing datasets from semi-analytic simulators across various logging positions [46]. Studies on synthetic resistivity models under varied noise levels further illustrate deep learning's robustness compared to traditional gradient-based inversion methods [6]. Practical applications also include direct mineral content prediction from drill core images, exemplifying deep learning's value in geological analysis and subsurface composition insights [9].

The integration of Physics-Informed Neural Networks (PINNs) in geophysical tomography has been pivotal for interpreting subsurface resistivity distributions, emphasizing the importance of incorporating physical principles into deep learning frameworks [47]. The jInv framework's evaluation against baseline methods across various datasets further illustrates deep learning's effectiveness in applications like DC resistivity and Full Waveform Inversion (FWI) [33].

These case studies collectively underscore the transformative impact of deep learning in resistivity forward modeling, particularly through methods merging data-driven and physics-driven approaches. Neural networks enhance inversion accuracy by integrating pseudo-physical information regarding electromagnetic field propagation, addressing challenges in nonlinear and ill-posed inverse problems in electrical resistivity surveys. Innovations like the ERSInvNet architecture, employing tier feature maps and depth weighting functions, exemplify deep learning's potential to enhance inversion accuracy and efficiency in complex resistivity modeling scenarios [25, 18]. By leveraging advanced neural network architectures alongside physical principles, these approaches have markedly improved geophysical imaging accuracy, efficiency, and reliability, paving the way for further innovations in subsurface exploration.

4.2 Integration of Physical Principles

The integration of physical principles into deep learning frameworks has significantly enhanced resistivity modeling, providing a foundation for more accurate and reliable geophysical imaging. This approach ensures models are data-driven while aligned with scientific principles by utilizing the inherent constraints and laws governing physical systems. Physics-Informed Neural Networks (PINNs) exemplify this integration by incorporating differential equations governing physical phenomena into their architecture, enhancing model fidelity to underlying physics during training and improving performance in solving complex systems [48, 49, 50, 36].

In resistivity modeling, PINNs have improved model accuracy and convergence rates by incorporating governing equations into the training process, allowing effective generalization across diverse datasets while maintaining fidelity to underlying physics [51]. This capability is especially advantageous

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in geophysical applications, where subsurface structures' complexity and geological variability challenge traditional modeling methods.

Further advancements include hybrid models merging data-driven insights with physics-based constraints. Models like the Physics Injected Network (PIN) and Physics Guided Network (PGN) utilize physical principles to guide learning, enhancing inversion results' interpretability and robustness [15]. Integrating physical insights augments resistivity models' accuracy and provides a framework for understanding underlying geophysical phenomena mechanisms.

Additionally, differentiable physics engines within neural networks enable efficient and accurate simulation of physical processes, supporting the generation of synthetic datasets adhering to real-world physical constraints, valuable for training and validating deep learning models [37]. The synergy between traditional physics-based modeling and deep learning techniques represents a significant advancement in resistivity forward modeling, offering pathways to more precise and reliable geophysical imaging solutions.

As research progresses, integrating physical principles is expected to play an increasingly central role in developing advanced resistivity models. By merging deep learning frameworks with fundamental physics principles, particularly in geophysical inversion and mineral content prediction, these innovative approaches significantly enhance geophysical models' accuracy and efficiency. This advancement bolsters predictive capabilities for subsurface exploration and optimizes resource management strategies, as illustrated by unsupervised learning schemes addressing ill-posed inverse problems and machine learning techniques analyzing drill core images for mineral content classification [9, 19].

5 Enhancements in Geophysical Imaging and Subsurface Exploration

The application of advanced methodologies in geophysical imaging and subsurface exploration has revolutionized traditional practices, significantly improving data interpretation and imaging resolution. Deep learning techniques have been instrumental in overcoming challenges posed by complex geological structures and noisy data environments. The following subsections explore advancements in data interpretation and imaging resolution, emphasizing deep learning's role in refining these processes to enhance the accuracy and reliability of geophysical imaging.

5.1 Advancements in Data Interpretation and Imaging Resolution

Deep learning has significantly advanced data interpretation and imaging resolution in geophysical applications. The FWIGAN framework outperforms traditional Full Waveform Inversion (FWI) methods by effectively recovering velocity models under noisy and suboptimal conditions [15]. This development underscores deep learning's capability to enhance geophysical imaging's accuracy and reliability.

Physics-Informed Neural Networks (PINNs) have been pivotal in embedding physics into the modeling process, ensuring predictions adhere to physical laws and enhancing model accuracy even with sparse or noisy data [15]. This integration improves inversion reliability and interpretability, allowing better generalization with limited labeled data.

Rational Function Neural Networks (RafNN) further advance data interpretation by reconstructing traditional rock physics models from data, aligning with established theories [12]. This demonstrates deep learning's potential to enhance understanding of complex geological structures and achieve optimal approximation rates efficiently.

In image reconstruction, deep learning methods like C-TISTA improve image quality and stability in nonlinear inverse problems by efficiently recovering complex-valued signals [26]. The PPP-FISTA method also enhances image quality, particularly in handling nonlinear inverse problems [13].

Innovative approaches for identifying minimum phase preserving operators have significant implications for seismic imaging, confirming the effectiveness of reconstructing operators with minimal test functions [16]. These advancements highlight deep learning's transformative impact on geophysical imaging, characterized by enhanced accuracy, reduced computational complexity, and improved model reliability.

5.2 Efficiency and Accuracy in Subsurface Exploration

Deep learning has markedly improved subsurface exploration's efficiency and accuracy. Techniques like L-DeepONet model complex dynamical features of partial differential equations (PDEs) with greater accuracy and generalizability, reducing computational resource requirements [52]. This is crucial in subsurface exploration, where geological complexity demands substantial processing power.

Deep learning models' robustness against noise has advanced subsurface exploration. Techniques such as DSIM-USSL demonstrate resilience to random noise, aiding precise reservoir boundary delineation [53]. The PINN framework also enhances noise robustness and convergence, improving exploration efficiency and accuracy [36].

Deep learning's ability to quantify uncertainties in subsurface structures enhances geophysical models' reliability, vital in safety-critical applications [54]. Uncertainty quantification is crucial for reliable geophysical models [55].

Enriched Physics-Informed Neural Networks (EPINNs) improve computational speed and accuracy in solving Poisson-Nernst-Planck (PNP) systems relevant to subsurface exploration [56]. These networks leverage physical insights to optimize performance, ensuring predictions align with scientific principles.

Data augmentation techniques create accurate data-driven models without extra computational costs, enhancing training datasets and model accuracy [57]. This is beneficial where data scarcity challenges model development.

Advancements in hardware, like reconfigurable linear RF analog systems, offer advantages such as low power consumption, fast processing, and compact form factor suitable for near-sensor applications [11]. These improvements complement deep learning techniques, enhancing subsurface exploration efficiency and accuracy.

Overall, integrating deep learning has substantially improved subsurface exploration's efficiency and accuracy. By integrating advanced image processing techniques, machine learning algorithms, and innovative frameworks for noise resilience and uncertainty quantification, recent advancements in geophysical imaging significantly enhance subsurface resource exploration's accuracy and reliability. These developments enable effective seismic interpretation, allowing researchers to identify critical geological structures for environmental monitoring, carbon sequestration, and oil and gas exploration, while addressing challenges like local minima and computation costs associated with traditional methods [28, 3, 58].

5.3 Innovations in Model Training and Computational Techniques

Recent advancements in model training and computational techniques have significantly enhanced deep learning models' capabilities in geophysical imaging. Innovations in neural network architecture, such as dropout and batch normalization, improve generalization and stability, mitigating overfitting and ensuring robust training [59]. The NN-aPC method integrates neural networks with adaptive polynomial chaos, improving uncertainty quantification and prediction accuracy for stochastic problems [60].

The NN2Poly framework provides a polynomial representation of deep networks, facilitating model coefficient interpretability and insights into underlying data structures [61]. Future research should enhance NN2Poly's scalability and explore its applicability across neural architectures to augment interpretability.

Critical initialization of wide and deep networks enhances trainability, allowing effective architecture design irrespective of initial parameter settings [62]. This ensures optimal network performance, streamlining training and reducing computational overhead.

Exploring the Frequency Principle (F-Principle) through Fourier analysis offers insights into neural networks' learning dynamics, although high-dimensional Fourier transforms' complexity poses challenges [63]. Understanding the F-Principle is crucial for networks capturing essential geophysical data features.

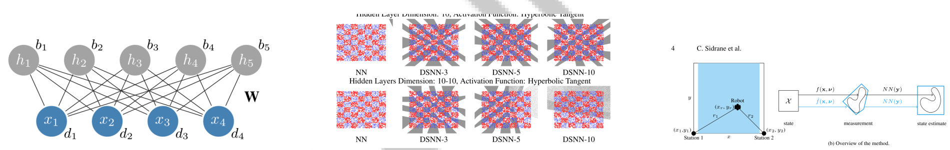
Bilevel Physics-Informed Neural Networks (BPN) provide a stable, efficient approach by decoupling optimization processes, outperforming traditional methods in stability and computational efficiency [64]. This is advantageous in geophysical imaging, requiring sophisticated modeling.

The RBF-PINN method integrates radial basis functions with physics-informed neural networks, improving convergence rates and generalization in high-dimensional geophysical problems [50]. This enhances inversion process accuracy and efficiency, facilitating reliable subsurface exploration.

Developing shallow network architectures simplifies design, improving optimization efficiency without sacrificing performance [32]. This simplification reduces computational demands, ensuring deep learning models' scalability in geophysical applications.

A unified understanding of adjoint operators enhances their applicability across fields, providing a foundation for interdisciplinary research and applications [65]. The SPInProp framework reduces computational complexity, accelerates training, and maintains accuracy in solving complex scientific problems, underscoring its potential in geophysical contexts [37]. Future work could extend identifiability results to complex nonlinearities and explore practical implications for neural network training and architecture design [14].

Recent advancements in model training and computational techniques, particularly in image processing and machine learning, significantly enhance geophysical imaging capabilities. These innovations offer effective solutions for complex subsurface exploration challenges, such as identifying geological structures critical for environmental monitoring, carbon sequestration, and oil and gas exploration. Integrating deep learning methods with traditional wave equation techniques, like least-squares reverse time migration (LSRTM), shows promise in producing high-resolution images of subsurface structures efficiently and accurately, addressing computational costs and imaging artifacts [28, 58]. As research progresses, these techniques are expected to further enhance deep learning models' capabilities and reliability in geophysical contexts.



(a) A neural network architecture with multiple layers and connections[66]

(b) Hidden Layer Dimension: 10, Activation Function: Hyperbolic Tangent[29]

(c) An Overview of a Method for Estimating the State of a Robot in a Dynamic Environment[54]

Figure 3: Examples of Innovations in Model Training and Computational Techniques

As shown in Figure 3, the example highlights significant advancements in geophysical imaging and subsurface exploration, focusing on innovations in model training and computational techniques. The first subfigure illustrates a complex neural network architecture featuring multiple layers and interconnections, as described by de Oliveira et al. (2022), showcasing the intricate connectivity that enhances computational efficiency and accuracy in processing geophysical data. The second subfigure delves into the architecture of neural networks (NN) and deep sequential neural networks (DSNN) with specific attention to hidden layer dimensions and activation functions, particularly the hyperbolic tangent function, as explored by Denoyer et al. (2014). This comparison underscores the role of architectural choices in optimizing model performance for subsurface exploration tasks. Finally, the third subfigure offers an overview of a method for estimating the state of a robot in a dynamic environment, as detailed by Sidrane et al. (2023), demonstrating the application of neural network models in real-time decision-making and navigation in complex terrains. Collectively, these examples underscore the transformative impact of advanced computational techniques and model training methodologies in enhancing the precision and efficacy of geophysical and subsurface exploration endeavors [66, 29, 54].

6 Challenges and Future Directions

6.1 Challenges and Limitations

The application of deep learning in geophysical inversion faces several critical challenges that limit its effectiveness and broader adoption. A significant issue is the dependence on extensive, high-quality training datasets necessary for model generalization, especially at parameter space boundaries. The scarcity of labeled data in geophysical settings hampers supervised learning efficacy in complex scenarios [15]. This limitation is exacerbated by neural networks' tendency to specialize in specific acquisition geometries, reducing their generalizability across diverse conditions.

Another major concern is the interpretability of deep learning models, which often function as black boxes, obscuring decision-making processes and undermining trust in critical applications. Although Physics-Informed Neural Networks (PINNs) incorporate physical laws, they lack the interpretative clarity of traditional numerical solvers, complicating deployment in high-stakes environments [12]. Furthermore, PINNs may struggle where physical attributes do not offer sufficient discrimination, requiring further refinement.

Scalability is another constraint, as deep learning methods incur high computational costs, particularly during the training of large models or integration of multiple models for complex scenarios. Frameworks like FWIGAN highlight these limitations, especially regarding memory usage. Additionally, overfitting risks, particularly without prior knowledge about model structure, challenge model robustness and accuracy [26].

The non-linear and often non-identifiable nature of neural network models complicates model selection [16]. Assumptions like low rank may not hold across various architectures or domains, limiting applicability. Furthermore, adaptive solutions may lack mathematical exactness, affecting accuracy [13].

Techniques like PLNet in geophysical inversion heavily depend on input data quality, underscoring challenges like weak spatial correspondence and uncertain seismic data-model parameter relationships. These issues necessitate advanced deep learning techniques capable of leveraging available data while navigating seismic inversion complexities, as recent studies on end-to-end seismic inversion networks and physical information incorporation into deep learning frameworks demonstrate [25, 18, 40, 2, 6]. Fixed discrete phase states of phase shifters may also limit classification region orientation flexibility, impacting accuracy.

Effectively modeling uncertainty remains a critical challenge, as current frameworks often fail to account for data and model structure uncertainty, hindering reliable deep learning systems. PINNs' limitations in solving differential equations stem from reliance on Fourier-based feature mappings, which exhibit spectral bias and propagation failures, complicating geophysical inversion applications. Recent studies suggest alternative feature mapping techniques, such as conditionally positive definite Radial Basis Functions, could enhance PINNs' expressivity and generalization capabilities, improving performance in this challenging domain [3, 50, 38, 67, 16].

Advancing deep learning in geophysical inversion requires addressing challenges associated with weak spatial correspondence, uncertain reflection-reception relationships, and seismic data's time-varying nature. Recent studies propose innovative solutions like end-to-end seismic inversion networks (SeisInvNets) and fully convolutional neural networks, leveraging comprehensive seismic data to enhance model reconstruction and improve subsurface property estimation accuracy, overcoming nonlinearity and nonuniqueness issues plaguing traditional methods [1, 2]. Future research should prioritize integrating higher-order statistical descriptors into models and developing numerical simulations of elastic wave propagation through complex microstructures to bolster robustness and accuracy of inference processes. Continued efforts are necessary to enhance model robustness, generalization, and efficiency to ensure deep learning methods effectively address subsurface exploration complexities and yield reliable geophysical insights.

6.2 Future Directions and Potential Developments

Future deep learning research for geophysical imaging aims to explore promising avenues to enhance model performance and applicability. A critical focus is refining neural network architectures to accommodate larger parameter spaces and applicability to three-dimensional models and spatiotemporal data. Enhancing input tensor design and optimizing neural network architecture can inform potential developments in geophysical imaging applications. Exploring alternative deep neural network (DNN) architectures, particularly those applicable to a broader class of partial differential equations (PDEs), remains a priority for enhancing performance in geophysical imaging. Future work could extend frameworks to include more complex applications and investigate adjoint operators' implications in emerging fields like machine learning and data science [65]. Additionally, future research could focus on extending results to deep neural networks and exploring methods for selecting optimal layer numbers in deep architectures [68].

To enhance monitoring capabilities in hydraulic fracturing scenarios, improving electromagnetic methods' sensitivity and incorporating multi-method approaches is crucial, as these strategies can provide more accurate detection of fluid movements and subsurface structure changes. Recent advancements in electromagnetic monitoring, like time-lapse magnetotelluric measurements, demonstrate that subtle resistivity alterations can be detected during fracking, influenced by the current stress field and fracture orientation. Moreover, integrating advanced imaging techniques and machine learning algorithms can further refine seismic interpretation and facilitate real-time subsurface condition analysis, leading to more effective hydraulic fracturing activity monitoring and management [28, 69, 1, 25]. Future research should focus on advanced modeling techniques capable of effectively handling noise and extending current methods to multidimensional inversion problems. Directions for future research include optimizing the Fast Low-Rank Neural Representation (FastLRNR) method, testing its applicability to a wider range of problems, and enhancing robustness against generalization issues. Developing enriched physics-informed neural networks (EPINNs) for prolonged and large-scale dynamic problems also represents a promising direction, potentially applicable to other complex systems to improve efficiency and accuracy. Furthermore, future research will aim to enhance frameworks like FWIGAN's computational efficiency, explore alternative distance measures for optimization, and extend applications to elastic full waveform inversion.

Additionally, future research should focus on refining RafNN to enhance robustness across various geological contexts, suggesting potential advancements in deep learning applications for geophysical imaging. Developing more efficient algorithms for probabilistic inference, exploring hybrid models combining different approaches' strengths, and enhancing probabilistic deep learning frameworks' scalability are critical areas for future exploration. Future research could investigate further enhancements to trainable shrinkage functions and C-TISTA's applicability beyond signal recovery, such as image processing and machine learning [26]. Enhancements to denoising techniques used alongside PPP-FISTA and their applicability to a broader range of inverse problems should also be explored [13].

Future efforts could focus on enhancing phase shifters' reconfigurability, investigating continuous phase control, and integrating more complex neural network architectures. Improving the physics-injected learning strategy to address identified limitations and enhance classification performance will be vital for advancing the field. Future research could explore improvements in approximation rates and developing methods capable of handling more complex function classes [12]. Moreover, future studies should examine minimum phase preserving operator reconstruction methods' implications in practical seismic imaging applications and investigate improvements to these techniques [16].

These future directions and potential developments highlight deep learning's transformative potential in revolutionizing geophysical imaging, offering more precise, efficient, and versatile subsurface exploration and resource management solutions. As geophysical applications research evolves, innovations like deep learning techniques for mineral content prediction from drill core images, electromagnetic inversion, and seismic interpretation are expected to significantly enhance deep learning models' accuracy, efficiency, and reliability. These advancements leverage methods like transfer learning and convolutional neural networks (CNNs) to process complex datasets, enabling real-time analysis and improved subsurface structure understanding. Integrating these technologies is poised to transform mining, oil and gas exploration, and environmental monitoring practices, ultimately leading to more effective decision-making and resource management [28, 1, 9, 25].

6.3 Challenges in Model Generalization and Data Quality

Generalizing deep learning models in geophysical applications is complex due to subsurface environments' variability and intricacy. A major issue is the limited availability of high-quality labeled data, crucial for training robust models capable of generalizing across different geological settings [15].

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This scarcity often leads to overfitting, where models excel on training data but fail to generalize to new, unseen scenarios. Neural networks' specificity to particular acquisition geometries exacerbates this problem, as models trained on specific datasets may not transfer effectively to other contexts [26].

Input data quality significantly influences model performance, with inaccuracies or noise potentially undermining deep learning models' reliability. In geophysical applications, data collection often occurs under challenging conditions, leading to noise and errors that can degrade model accuracy [15]. Techniques like noise reduction and data augmentation are vital for mitigating these issues, though their effectiveness is often limited by geophysical data's inherent variability.

Moreover, neural network models' non-linear and frequently non-identifiable nature presents challenges in ensuring effective generalization [16]. The spectral bias inherent in many neural network architectures, which favors learning lower frequency components, further restricts models' ability to capture geophysical data's full complexity [63]. This bias can result in models inadequately representing high-frequency details essential for accurate subsurface imaging.

Addressing these challenges necessitates a multifaceted approach, involving sophisticated data preprocessing techniques, robust regularization strategies, and hybrid models blending data-driven and physics-informed approaches. Enhancing model interpretability and reliability through physical constraints incorporation, as exemplified by Physics-Informed Neural Networks (PINNs), offers a promising pathway to overcoming generalization challenges [36]. As research progresses, ongoing efforts to improve data quality and model generalization will be essential for advancing deep learning application in geophysical imaging and subsurface exploration.

6.4 Architectural and Computational Limitations

Implementing deep learning models in geophysical applications is often hindered by architectural and computational limitations affecting efficiency and scalability. A primary challenge is the computational intensity required to train and deploy deep learning models, particularly in high-dimensional geophysical data settings. This demand for extensive computational resources can limit these models' accessibility and applicability in resource-constrained environments [27].

Neural networks' architectural complexity, while offering potential accuracy and performance improvements, introduces challenges related to model interpretability and optimization. Deep networks, characterized by extensive layers and parameters, present significant hurdles during training, often necessitating advanced optimization techniques like mini-batch stochastic gradient descent and momentum-based learning, alongside substantial computational resources. These complexities arise from issues like vanishing and exploding gradients, impeding convergence, as well as the need for careful hyperparameter tuning and initialization strategies to enhance training effectiveness. Additionally, deep architectures excel in expressive power and generalization, but recent findings indicate that parallel shallow networks with residual connections can achieve comparable performance, potentially simplifying training processes and improving optimization efficiency [38, 59, 68, 32]. This complexity can also lead to overfitting, where models perform well on training data but fail to generalize to new, unseen data.

Scalability of deep learning models is a significant concern, particularly with large datasets common in geophysical applications. As data size and dimensionality increase, the computational demands for training and inference in machine learning applications, particularly those involving normalized relational data, become increasingly challenging. This underscores the urgent need for developing more efficient algorithms and architectures effectively managing these complexities. Recent advancements, like systematic decomposition of computations in Gaussian Mixture Models and Neural Networks, demonstrate potential to enhance training speed without sacrificing accuracy. While traditional methods may struggle with the curse of dimensionality, neural networks can leverage low-dimensional structures to outperform kernel methods in specific tasks. Innovative strategies, like prioritizing perplexing samples in active learning, can further optimize performance by focusing on features yet to be learned, reducing the overall computational burden [70, 34, 42].

Another limitation is reliance on specialized hardware, such as GPUs, to achieve necessary computational speed and efficiency. While deep learning technologies have significantly improved geophysical data analysis capabilities, high costs and limited accessibility can hinder implementation in research and industry applications. This financial barrier restricts broader deep learning technique adoption in critical geophysical tasks like seismic interpretation, essential for environmental monitoring, carbon sequestration, and oil and gas exploration. Consequently, these limitations may impede efforts to accurately delineate subsurface structures, predict geological events, and optimize resource exploration and management [28, 9].

Furthermore, integrating probabilistic models into deep learning frameworks, while offering benefits in uncertainty quantification and model reliability, often struggles with computational efficiency. Many probabilistic models face challenges scaling to high-dimensional data, common in geophysical applications [27]. This limitation underscores the need for continued research into more scalable and computationally efficient probabilistic modeling techniques.

To effectively advance deep learning application in geophysical imaging, overcoming architectural and computational limitations currently hindering accurate subsurface structure reconstruction from seismic data is essential. These challenges include addressing weak spatial correspondence between seismic data and velocity models, nonuniqueness of inversion solutions, and significant computational demands associated with high-resolution imaging in complex subsurface environments. By developing innovative deep learning frameworks, like SeisInvNets, and leveraging physics-based models, researchers can enhance seismic inversion methods' efficiency and accuracy, ultimately leading to improved interpretations of subsurface geological features critical for applications in environmental monitoring, carbon sequestration, and resource exploration [58, 28, 40, 2, 41]. Future research should focus on developing more efficient network architectures, optimizing training algorithms, and exploring alternative computational frameworks reducing deep learning models' resource demands while maintaining accuracy and reliability.

6.5 Integration with Traditional Methods and Physical Principles

Integrating deep learning techniques with traditional geophysical methods and physical principles has emerged as a transformative approach significantly enhancing geophysical imaging accuracy and reliability, particularly in complex scenarios like seismic full waveform inversion (FWI) and magnetotelluric (MT) data interpretation. Recent advancements leverage generative adversarial networks and neural networks to effectively address conventional FWI limitations, like local minima challenges and initial model sensitivity, by utilizing unsupervised learning paradigms requiring no labeled training data. Additionally, incorporating pseudo-physical information through neural network-based forward modeling in MT deep learning inversion has demonstrated improved inversion accuracy and reduced overfitting, making these methods more robust and adaptable for real-world applications [25, 3]. This synergy leverages data-driven and physics-based methodologies' strengths, providing a comprehensive framework for interpreting complex subsurface phenomena.

Traditional geophysical methods, like seismic and electromagnetic surveys, offer well-established subsurface exploration techniques but often face resolution and computational efficiency limitations when applied to complex geological settings. By incorporating deep learning techniques, these traditional methods can be augmented to overcome such limitations, offering improved imaging capabilities and more accurate subsurface models.

Physics-Informed Neural Networks (PINNs) exemplify integrating physical principles into deep learning models, embedding differential equations governing geophysical processes directly into neural network architecture. This integration ensures model predictions are consistent with known physical laws, enhancing interpretability and reliability of inversion results even in data-scarce environments. PINNs' implementation in geophysical applications has led to notable model accuracy and convergence rate enhancements, particularly in contexts where traditional data-driven methods struggle due to insufficient data. For instance, in landslide prediction, PINNs have successfully integrated physical constraints with data-driven architectures, enabling geotechnical parameter retrieval from common proxy variables and producing reliable susceptibility assessments along with regional geotechnical property maps. Additionally, PINNs have shown promise in uncertainty quantification for systems with limited observations, like subsurface water bodies, by effectively satisfying physical principles while accommodating high-dimensional solution spaces. This innovative approach positions PINNs as a robust alternative to conventional modeling techniques, particularly in scenarios where data scarcity poses significant challenges [71, 50, 8, 22].

Moreover, developing hybrid models combining data-driven insights with physics-based constraints, like Physics Guided Network (PGN) and Physics Injected Network (PIN), further exemplifies integrating deep learning with traditional methods [15]. These models utilize physical principles to guide the learning process, enhancing inversion results' robustness and interpretability. Integrating physical insights not only improves geophysical models' accuracy but also provides a framework for understanding underlying mechanisms driving subsurface phenomena.

Additionally, applying differentiable physics engines within neural networks facilitates simulating physical processes in a computationally efficient and physically accurate manner. This integration allows generating synthetic datasets adhering to real-world physical constraints, providing a valuable resource for training and validating deep learning models [37]. Combining traditional geophysical methods with advanced deep learning techniques represents a significant advancement in resistivity forward modeling, offering a pathway to more precise and reliable geophysical imaging solutions.

As geophysical modeling research advances, deep learning techniques' convergence with traditional methodologies and established physical principles is anticipated to become increasingly pivotal. Recent studies highlight integrating data-driven approaches with physics-informed frameworks' effectiveness, like in 3-D magnetotelluric deep learning inversion, where incorporating pseudo-physical information significantly enhances inversion accuracy. Additionally, unsupervised learning schemes leveraging physical laws, like those governing electric field propagation, are proving valuable in addressing ill-posed inverse problems' challenges in resistivity surveys. These innovations suggest advanced geophysical models' future will heavily rely on deep learning to improve subsurface property estimation precision and efficiency, particularly in real-time applications [25, 1, 19]. By aligning deep learning frameworks with fundamental physics laws, these approaches promise to enhance geophysical models' predictive capabilities, ultimately leading to more effective subsurface exploration and resource management.

6.6 Advancements in Uncertainty Quantification and Interpretability

Recent advancements in deep learning models' uncertainty quantification and interpretability have significantly enhanced their geophysical context applications. These advancements are crucial for improving models' reliability and safety, especially in high-risk applications where decision-making depends on accurate and interpretable predictions. Current research has made substantial progress in understanding and quantifying uncertainty in deep neural networks (DNNs), leading to improved safety and reliability [43]. Effectively quantifying uncertainty allows for better risk assessment and management, essential in geophysical exploration, where data is often sparse and noisy.

Physics-Informed Neural Networks with Uncertainty Quantification (PINN-UU) represent a significant leap forward in this domain. PINN-UU is designed to handle high-dimensional parameter spaces efficiently and provides uncertainty quantification without extensive data [71]. This capability is particularly beneficial in geophysical applications, where subsurface structures' complexity and geological conditions' variability pose significant challenges to traditional modeling approaches. By embedding physical laws directly into the neural network framework, PINN-UU ensures predictions are consistent with known physical principles, enhancing models' interpretability and reliability.

Moreover, integrating probabilistic methods into deep learning frameworks has further enriched uncertainty quantification capacity. Techniques like Bayesian neural networks treat model weights as probability distributions, allowing prior knowledge incorporation and predictive distributions generation. This probabilistic treatment not only improves model interpretability but also provides a robust framework for quantifying uncertainty in model predictions [43].

Advancements in uncertainty quantification and model interpretability are anticipated to significantly enhance deep learning models' reliability and applicability in geophysical applications, particularly in areas like seismic interpretation, where accurate subsurface analysis is crucial for environmental monitoring, carbon sequestration, and resource exploration. Recent research highlights addressing uncertainties in neural network predictions' importance, as these can lead to biases in critical applications like earthquake source inversion and well navigation during drilling. By employing techniques like Bayesian methods, ensemble approaches, and noise-robust deep learning architectures, researchers aim to improve model outputs' calibration and confidence, ultimately fostering greater trust in deep learning technologies across various geophysical contexts [28, 55, 43, 6]. As research evolves, these developments promise to provide more precise, reliable, and interpretable solutions for subsurface exploration and resource management, ultimately leading to more effective decision-making in geophysical applications.

7 Conclusion

Deep learning inversion has emerged as a pivotal innovation in geophysical imaging, offering solutions to the intricate challenges of nonlinear inversion. Leveraging sophisticated neural network architectures, researchers have developed frameworks that substantially enhance the interpretation of subsurface resistivity profiles, thus advancing the field of geophysical imaging. The integration of Physics-Informed Neural Networks (PINNs) with probabilistic approaches has further enriched this domain by embedding physical laws into models and quantifying uncertainties, thereby improving both interpretability and reliability.

Advancements in optimizing neural network structures and learning methodologies have significantly increased the efficiency and precision of inversion processes. Incorporating physical principles within deep learning frameworks has not only elevated model performance but also deepened the understanding of subsurface dynamics. The collaboration between deep learning and traditional geophysical approaches has resulted in more accurate and dependable imaging techniques.

The future of this research area is promising, with ongoing efforts focused on refining neural network architectures, improving data quality, and enhancing model generalization. Although challenges persist regarding the scalability and practical implementation of deep learning methods in geophysical contexts, the potential for analog deep learning to impact consumer-level applications suggests notable advancements on the horizon. The continued evolution of deep learning inversion techniques is expected to spur further breakthroughs in geophysical imaging and subsurface exploration, offering robust solutions to complex geophysical issues.

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