Physics-Informed Neural Networks for Computational Fluid Dynamics: A Survey

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Abstract

Physics-Informed Neural Networks (PINNs) are revolutionizing computational fluid dynamics (CFD) by integrating neural network models with governing physical laws, offering enhanced accuracy and efficiency in solving complex fluid dynamics problems. This survey paper examines the foundational concepts, methodologies, and applications of PINNs in CFD, highlighting their ability to address the limitations of traditional numerical methods. By embedding physical laws into neural network architectures, PINNs improve predictive accuracy, especially in scenarios with sparse data, and efficiently solve partial differential equations (PDEs) governing fluid dynamics. The paper explores the architecture of PINNs, their integration with traditional methods, and innovative variants that enhance simulation capabilities. Applications of PINNs in CFD, such as modeling unsteady flows, multi-fluid interactions, and thermo-fluid dynamics, underscore their versatility and transformative impact. Despite challenges related to training, convergence, and computational costs, advancements in optimization strategies and hybrid approaches promise to overcome these limitations. Future research directions include enhancing PINN methodologies, expanding applications, and integrating advanced computational techniques to further improve their scalability and accuracy. Overall, PINNs represent a pivotal tool in advancing CFD research, providing robust solutions to complex scientific and engineering challenges.

1 Introduction

1.1 Concept and Significance of PINNs

Physics-Informed Neural Networks (PINNs) offer a revolutionary method for solving partial differential equations (PDEs) in complex fluid dynamics, particularly within computational fluid dynamics (CFD) [1]. By directly incorporating physical laws into the neural network architecture, PINNs significantly improve predictive accuracy, especially in scenarios with limited data [2]. This enhancement is vital in CFD, where traditional methods often face challenges in efficiency and accuracy when addressing intricate fluid dynamics problems [3].

The application of PINNs in CFD is propelled by their capability to efficiently tackle linear PDEs with boundary layers, thus overcoming the constraints of conventional numerical methods [4]. Unlike established approaches, PINNs minimize the system's variational energy, which not only boosts computational efficiency but also simplifies the enforcement of boundary conditions [5]. This feature renders PINNs particularly adept at accurately modeling boundary phenomena.

Moreover, the versatility of PINNs is showcased across diverse applications, including real-time control, where the fusion of physics-informed learning with control strategies enhances simulation capabilities [6]. The combination of traditional numerical techniques, such as finite difference methods, with PINNs further illustrates their ability to improve simulation accuracy and efficiency [7].

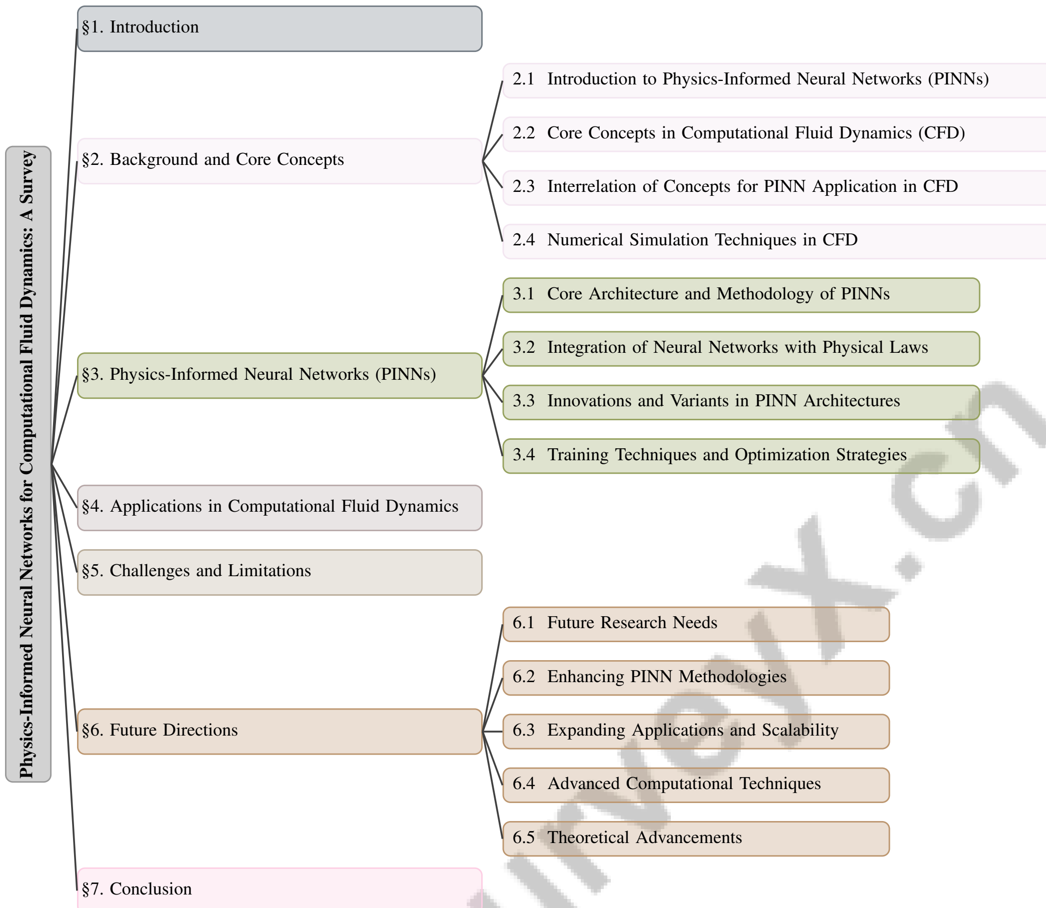


Figure 1: chapter structure

1.2 Structure of the Survey

This survey is meticulously organized to provide a thorough understanding of the role of Physics-Informed Neural Networks (PINNs) in computational fluid dynamics (CFD). It begins with an introductory section that clarifies the foundational concepts and significance of PINNs, establishing the context for their application in CFD. Following this, the survey presents the background and core principles of CFD, elucidating their relationship with PINNs, thereby laying the theoretical groundwork for understanding the integration of neural networks with physical laws.

Next, the survey examines the architecture and methodology of PINNs, emphasizing their innovative incorporation of physical laws to enhance simulation accuracy. An in-depth analysis of PINN applications in CFD follows, particularly focusing on their effectiveness in addressing classical PDEs and complex flow and heat transfer scenarios. This section discusses how PINNs function as parametric surrogate models, integrating existing CFD solutions with governing differential equations to improve computational efficiency, as evidenced in turbulent flow predictions for both internal and external contexts. The challenges faced in employing PINNs for solving the Navier-Stokes equations are also addressed, highlighting their role as complementary tools to traditional CFD solvers and the necessity for ongoing refinement to effectively tackle real-world fluid dynamics issues [8, 9, 10]. Furthermore, the survey critically examines the current challenges and limitations related to PINNs, including training, convergence, computational costs, and accuracy concerns.

In the penultimate section, future research directions and potential advancements in the application and scalability of PINNs are discussed. The paper concludes by summarizing key insights and reiterating the transformative impact of PINNs on advancing CFD research. This structured approach ensures a coherent narrative that guides the reader through the complexities of PINNs and their critical role in contemporary computational fluid dynamics research.The following sections are organized as shown in Figure 1.

2 Background and Core Concepts

2.1 Introduction to Physics-Informed Neural Networks (PINNs)

Physics-Informed Neural Networks (PINNs) revolutionize deep learning by embedding governing physics equations directly into the training process, significantly enhancing computational fluid dynamics (CFD) applications, where traditional numerical methods often falter with complex multi-scale partial differential equations (PDEs) [11, 12]. This integration enhances predictive accuracy, particularly in data-sparse environments [2], by incorporating PDE residuals and boundary conditions into the loss function, guiding the network towards solutions that adhere to physical laws [3]. This methodology reduces the dependency on labeled data and boosts computational efficiency in fluid dynamics modeling [1].

Advancements like the HLConcPINN method introduce flexibility in model architecture by accommodating various hidden layers and activation functions to better manage complex dynamics [13]. Additionally, integrating control inputs into PINNs facilitates accurate long-range simulations of dynamical systems governed by Ordinary Differential Equations (ODEs), expanding their CFD applicability [6]. However, challenges in predicting sharp solution transitions persist, necessitating new frameworks to enhance generalization and accuracy [14]. The evolving role of PINNs in CFD highlights their potential as robust frameworks for simulating complex fluid dynamics while adhering to physical laws. Approaches like the knowledge-based encoder-decoder (KED) further refine predictions in challenging scenarios, such as two-phase flow in porous media, by utilizing both observed data and physical equations [15].

2.2 Core Concepts in Computational Fluid Dynamics (CFD)

Computational Fluid Dynamics (CFD) involves the numerical simulation of fluid flow and heat transfer, primarily governed by the nonlinear Navier-Stokes equations, essential for accurately modeling fluid behavior across diverse applications, including vortex shedding [16, 17]. Effective simulation of these equations is crucial for capturing laminar and turbulent flow behaviors, vital for understanding fluid dynamics under varying conditions [3].

Traditional CFD methodologies, such as Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and Reynolds-Averaged Navier-Stokes (RANS) models, are widely used but often demand substantial computational resources, especially in high-dimensional or turbulent regimes [16]. These methods face challenges in accurately predicting flow fields in complex geometries and singularities, such as vortex-acoustic lock-in phenomena in combustors [18]. Integrating deep learning techniques, particularly PINNs, offers a promising alternative to enhance predictive accuracy and computational efficiency in CFD by embedding physical laws directly into the neural network framework, improving representation capabilities in capturing multiscale and turbulent behaviors [19, 11]. This approach is especially beneficial in scenarios with incomplete data or intricate geometries, where traditional methods encounter difficulties [4].

Despite their potential, PINNs encounter challenges in efficiently solving high-dimensional PDEs, such as parabolic and hyperbolic equations, due to the curse of dimensionality [13]. The training process can be computationally intensive, often requiring numerous collocation points for accurate solutions [17]. Nonetheless, hybrid approaches integrating finite difference methods with PINNs show promise in overcoming these computational bottlenecks [7].

2.3 Interrelation of Concepts for PINN Application in CFD

The integration of Physics-Informed Neural Networks (PINNs) with Computational Fluid Dynamics (CFD) signifies a major advancement in fluid dynamics simulation, effectively bridging data-driven models with traditional physics-based methodologies. By embedding fluid flow governing equations, such as the Navier-Stokes equations, directly into the neural network framework, PINNs enhance simulation fidelity while maintaining computational efficiency [20]. This integration is particularly advantageous for addressing multiscale and nonlinear fluid dynamics problems, where traditional numerical methods often face limitations.

Advanced PINN models, such as the dual-network approach exemplified by PINN-2, have demonstrated improved capabilities in capturing energy distribution at small scales, crucial for accurately modeling turbulent flows [21]. The NH-PINN method exemplifies this synergy by employing a three-step homogenization process to enhance PINN performance in solving multiscale problems, thereby expanding their applicability in CFD [12]. Incorporating observational data with governing PDEs within the PINN framework enhances flow reconstruction accuracy, enabling effective generalization across diverse fluid dynamics scenarios [22]. Energy-based PINN approaches further illustrate significant computational improvements by alleviating traditional burdens associated with residual-based methods [23], particularly in high-dimensional and complex geometries where traditional methods struggle.

Moreover, the adaptability of PINNs is evident in their application to complex geometries and turbulent flows through the integration of Reynolds-Averaged Navier-Stokes (RANS) equations and dynamic motion equations [24]. This flexibility allows PINNs to incorporate physical constraints into the learning process, optimizing performance in scenarios with limited data availability [25]. Despite these advancements, challenges remain, particularly in achieving convergence when training PINNs with gradient descent methods for PDEs with irregular solutions [26].

Efforts to mitigate the spectral bias of neural networks have focused on enhancing PINN performance for time-dependent multiscale problems [27]. Coupling traditional numerical methods with machine learning techniques further enhances modeling accuracy, providing a robust framework for addressing complex fluid dynamics challenges [28]. These interconnections between the core concepts of PINNs and CFD not only improve simulation accuracy but also pave the way for more efficient and versatile computational frameworks in fluid dynamics research.

2.4 Numerical Simulation Techniques in CFD

Numerical simulation techniques in Computational Fluid Dynamics (CFD) have traditionally relied on methods such as Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and Reynolds-Averaged Navier-Stokes (RANS) models to solve the Navier-Stokes equations governing fluid flow and heat transfer [29]. While effective, these methods often demand substantial computational resources and face challenges in accurately modeling complex geometries and turbulent flows. The integration of Physics-Informed Neural Networks (PINNs) with these traditional methods offers a promising avenue for enhancing simulation accuracy and efficiency.

The F-PINN framework exemplifies this integration by combining physics-based regularization with Fourier features to model fluid dynamics around complex geometries, such as cylinders [29]. This approach captures intricate flow features while maintaining computational efficiency, addressing some limitations of conventional numerical techniques. By embedding governing equations directly into the neural network architecture, F-PINNs enhance simulation fidelity and reduce reliance on extensive computational resources.

Additionally, the development of optimization strategies, such as AutoPINN, facilitates systematic searches for optimal hyperparameters in PINN models, decoupling them and utilizing loss values as a search objective [30]. This optimization process is crucial for ensuring high accuracy and efficiency in solving complex fluid dynamics problems, thereby enhancing PINNs' applicability in CFD.

The Advanced Data Processing Algorithm (ADPA) further contributes to the integration of PINNs with traditional CFD methods by analyzing data in real-time and adjusting processing methods for minimal latency and maximum accuracy [31]. This capability is particularly beneficial in dynamic fluid environments, where rapid adjustments are necessary to maintain simulation accuracy.

3 Physics-Informed Neural Networks (PINNs)

The integration of advanced methodologies and neural network architectures has become crucial in computational fluid dynamics. This section explores the core architecture and methodology of Physics-Informed Neural Networks (PINNs), which leverage deep learning while adhering to governing physical laws. As illustrated in **??**, the hierarchical structure of PINNs showcases their

Category	Feature	Method
Core Architecture and Methodology of PINNs	Integration Techniques Differentiation and Search Strategies	QPINN[32], BICIM[33] AP[30], HFD-PINN[7]
Integration of Neural Networks with Physical Laws	Physics-Guided Approaches	PINN[2], PINN[5], PINC[6], OL-PINN[14], KED[15]
Innovations and Variants in PINN Architectures	Bio-Inspired Models Task-Enhanced Learning Constraint-Driven Techniques Generative Frameworks	BIMT[34] ATL-PINNs[35] PINN-Proj[36] GA-PINN[37]
Training Techniques and Optimization Strategies	Resource and Efficiency Focus Precision and Accuracy Enhancement Loss Function Optimization Input and Feature Enhancement	PPINN[38], IS-PINN[39] DT-PINNs[40], FE-PINN[41] QPGD[42], E-PINNs[43], BC-PINN[44], EPINN[45] DaPINN[46]

Table 1: This table provides a comprehensive overview of the categorization and specific methodologies employed in the development and optimization of Physics-Informed Neural Networks (PINNs). It details the core architecture and methodology, integration with physical laws, innovations and variants, as well as training techniques and optimization strategies. Each category is further broken down into features and methods, highlighting the diverse approaches and advancements in the field.

core architecture and methodology, highlighting the integration with physical laws, as well as the various innovations and variants that have emerged. Table 1 presents a detailed categorization and summary of various methodologies and innovations in Physics-Informed Neural Networks (PINNs), demonstrating the integration of neural networks with physical laws and the advancements in training and optimization strategies. Additionally, Table 4 offers further insights by presenting a detailed categorization and summary of these methodologies and innovations, underscoring the integration of neural networks with physical laws and the advancements in training and optimization strategies. The figure also details training techniques and optimization strategies, emphasizing the versatility and effectiveness of PINNs in addressing complex fluid dynamics challenges. By analyzing the design and operational principles of PINNs alongside this visual representation, we can appreciate their transformative potential in the field, setting the stage for a deeper exploration of their architecture and methodology.

3.1 Core Architecture and Methodology of PINNs

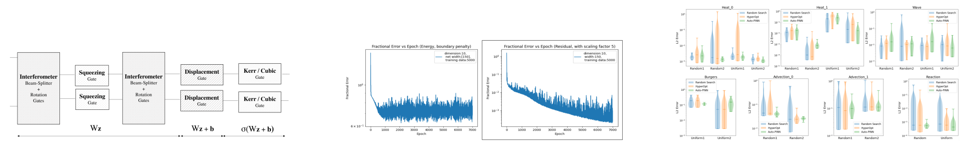
Method Name	Core Architecture	Methodological Advancements	Application Scenarios
OL-PINN[14]	Hybrid Framework	Operator Learning	Sharp Solutions
HFD-PINN[7]	Fully Connected Network	Hybrid Finite Difference	Complex Geometries
KED[15]	U-Net Architecture	Knowledge-Based Encoder-Decoder	Two-phase Flow
QPINN[32]	Quantum Neural Networks	Hybrid Variational Solver	Differential Equations
BICIM[33]	Neural Network Architecture	Boundary And Initial Condition Inclusion	Solving Partial Differential Equations
AP[30]	Physical Laws Integration	Automated Hyperparameter Optimization	Partial Differential Equations

Table 2: Summary of recent advancements in Physics-Informed Neural Networks (PINNs) methodologies, highlighting core architectures, methodological innovations, and application scenarios. This table provides a comparative overview of six different methods, showcasing their unique contributions to solving complex differential equations in various contexts.

Physics-Informed Neural Networks (PINNs) incorporate governing physical laws directly into their framework by embedding PDE residuals and boundary conditions into the loss function, guiding the network to approximate solutions that respect these constraints [3]. This approach is effective in addressing complex fluid dynamics problems where traditional methods may struggle with efficiency and accuracy. Recent advancements, such as the OL-PINN method, enhance the standard PINN framework by combining DeepONet's predictive capabilities, improving accuracy in solving PDEs with sharp transitions [14]. Hybrid methods, employing finite differences in regular domains and automatic differentiation in complex regions, further boost prediction accuracy [7].

In data handling, the KED framework integrates observed data with discretized governing equations to predict pressure and saturation fields in two-phase flow problems [15]. This exemplifies the versatility of PINNs in various CFD contexts. The evolution of PINNs reveals their transformative potential in CFD, generating parametric surrogate models that significantly enhance accuracy and efficiency in solving complex turbulent flows. For instance, the can-PINN framework shows marked improvements in accuracy, especially in challenging scenarios like flow mixing and cavity-driven flows, paving the way for real-world applications [? 9, 29, 10].

Table 2 presents a comprehensive comparison of recent methodologies in Physics-Informed Neural Networks (PINNs), detailing their core architectures, methodological advancements, and specific application scenarios.



(a) Interferometer Squeezing and Displacement Gate System[32]

(b) Fractional Error vs Epoch (Energy, boundary penalty)[33]

(c) Comparison of L2 Error for Different Optimization Methods on Various Benchmark Functions[30]

Figure 2: Examples of Core Architecture and Methodology of PINNs

As illustrated in Figure 2, PINNs solve differential equations by embedding physical laws into the learning process. The "Interferometer Squeezing and Displacement Gate System" highlights quantum operations, while "Fractional Error vs Epoch" assesses penalty methods. The comparison of L2 error across optimization methods underscores PINNs' versatility in minimizing errors [32, 33, 30].

3.2 Integration of Neural Networks with Physical Laws

Method Name	Integration Approach	Application Domains	Stabilization Techniques
PINN[2]	Loss Function	Concrete Strength Prediction	Heuristic Optimization Methods
PINN[5]	Variational Energy Minimization	Fracture Mechanics Problems	Transfer Learning Integration
PINC[6]	Physical Laws	Dynamic Systems	Pre-trained Operators
OL-PINN[14]	Pre-trained Operator	Fluid Dynamics	Pre-trained Operators
KED[15]	Discretized Equations	Porous Media	Boundary Conditions

Table 3: Summary of various methods integrating neural networks with physical laws, detailing their integration approaches, application domains, and stabilization techniques. This table highlights the diversity in methodologies, demonstrating the adaptability of Physics-Informed Neural Networks (PINNs) across different scientific and engineering problems.

Integrating physical laws into the PINN framework enhances simulation accuracy and efficiency by embedding governing equations into the neural network's loss function, which penalizes deviations from established principles. This integration allows PINNs to leverage both data-driven insights and physical laws, ensuring solutions adhere to fundamental physics [2]. This is particularly advantageous in fluid dynamics, where traditional methods often struggle with dynamic phenomena like vortex shedding [17]. Table 3 presents a comprehensive overview of different methods that integrate neural networks with physical laws, emphasizing their approaches, application domains, and stabilization techniques.

Fully-connected neural networks in PINNs facilitate fluid flow modeling while ensuring compliance with governing equations, enhancing predictive accuracy [3]. By minimizing variational energy, PINNs respect the physics of the problem, crucial for accurate predictions in scenarios like crack propagation [5]. Additionally, PINNs integrate known physical laws with control inputs and variable initial conditions, enabling indefinite time horizon simulations [6].

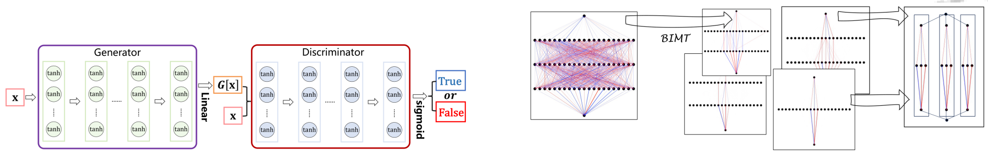
PINNs' mesh-free nature allows on-demand solution computation and simultaneous solving of forward and inverse problems [1]. Methods like OL-PINN use pre-trained operators as regularization terms to stabilize training [14]. The integration of physical laws maintains accuracy in the presence of discontinuities, as demonstrated in knowledge-based convolutional networks [15].

This integration significantly enhances simulation accuracy, enabling effective approximation of PDE solutions and broadening applicability to complex challenges. It stabilizes training by incorporating boundary and initial conditions algebraically, facilitating consistent predictions beyond the training domain [47, 33].

3.3 Innovations and Variants in PINN Architectures

Recent advancements in PINN architecture have introduced innovations enhancing capabilities in solving complex fluid dynamics problems. Auxiliary-task learning modes within the PINN framework optimize the integration of physics-based constraints with data-driven learning, improved by the gradient cosine similarity algorithm, which balances multiple tasks [35].

The implementation of a projection technique as a hard constraint ensures network predictions adhere to conservation laws, providing a robust alternative to soft constraints [36]. These innovations improve simulation fidelity by ensuring compliance with fundamental physical principles, expanding the applicability of PINNs to a broader range of problems. Sophisticated methodologies like the can-PINN framework and parametric PINNs elevate performance, enhancing utility in CFD research for efficient and accurate predictions [48, 9, 29, 49].



(a) A Generative Adversarial Network (GAN) architecture[37]

(b) Deep Learning Network Architecture with BIMT[34]

Figure 3: Examples of Innovations and Variants in PINN Architectures

As shown in Figure 3, PINNs offer innovative solutions for differential equations by embedding physical laws into neural network architectures. Advancements include a GAN architecture enhancing data generation and validation, and a deep learning network with a BIMT module optimizing multi-task learning [37, 34].

3.4 Training Techniques and Optimization Strategies

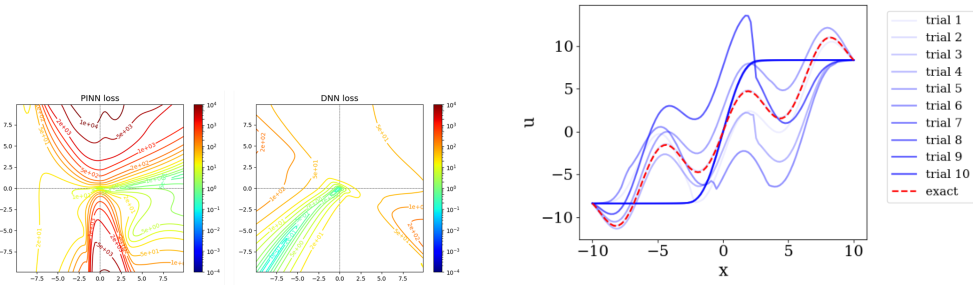
Training PINNs involves sophisticated techniques and optimization strategies enhancing performance in solving complex fluid dynamics problems. The EPINN method adaptively balances loss contributions, improving learning capability and accuracy [45]. Precision is enhanced by employing double precision in DT-PINNs, accelerating training compared to traditional single precision [40]. Strategically selecting collocation points based on loss function distribution focuses resources on regions with higher error [39].

Curriculum PINN initiates training with simpler PDE constraints, progressively increasing complexity for better optimization and convergence [50]. The FE-PINN method combines a novel loss function with preprocessing steps to enhance convergence speed [41]. Optimization strategies like QPGD ensure convergence by balancing loss components [42]. DaPINN enhances modeling of complex behaviors by adding features to the input layer and modifying the loss function [46].

Architectural strategies, as assessed by Saratchandran et al., optimize training and testing phases by evaluating performance metrics like L2 error and convergence rates [43]. The PPINN framework uses a coarse solver for initial predictions, allowing fine PINNs to operate on smaller datasets and converge rapidly [38]. Integrating soft loss-based and exact distance function-based approaches enhances precision in handling boundary constraints [44].

Combining traditional optimization methods, such as Adam and Newton's method, for parameter estimation across different datasets yields optimal results based on the lowest loss and relative error metrics [51]. These diverse training techniques and optimization strategies advance PINNs, enabling them to tackle increasingly complex fluid dynamics challenges with greater accuracy and efficiency.

As depicted in Figure 4, PINNs integrate governing physical laws into training, offering a promising alternative to traditional models. The first part compares loss functions of PINNs and DNNs, highlighting complexities from incorporating physical laws. The second part presents a plot of function u against variable x, illustrating PINNs' capability to approximate solutions across trials, emphasizing optimized training techniques for accurate results [52, 53].



(a) Comparison of Loss Functions for PINN and DNN[52]

(b) The image shows a plot of the function u against the variable x, with different trials represented by different colors and a red dashed line representing the exact solution.[53]

Figure 4: Examples of Training Techniques and Optimization Strategies

Feature	Core Architecture and Methodology of PINNs	Integration of Neural Networks with Physical Laws	Innovations and Variants in PINN Architectures
Core Architecture	Pde Residual Embedding	Fully-connected Networks	Auxiliary-task Learning
Integration with Physical Laws	Direct Embedding	Loss Function Penalty	Projection Technique
Training and Optimization	Hybrid Methods	Mesh-free Computation	Gradient Cosine Similarity

Table 4: This table provides a comparative analysis of various methodologies in Physics-Informed Neural Networks (PINNs), focusing on their core architectures, integration with physical laws, and recent innovations. It highlights the different approaches to embedding physical laws into neural networks and explores advancements in training and optimization strategies. The table serves as a comprehensive resource for understanding the diverse methods and their applications in computational fluid dynamics.

4 Applications in Computational Fluid Dynamics

4.1 Applications and Case Studies

Physics-Informed Neural Networks (PINNs) have emerged as a transformative tool in computational fluid dynamics (CFD), providing innovative solutions to complex flow problems. A notable application includes the simulation of unsteady flows around moving bodies, where frameworks such as MB-PINN and MB-IBM-PINN have effectively reconstructed velocity fields and pressure data in dynamic environments [54]. This capability underscores the proficiency of PINNs in managing intricate boundary interactions and fluctuating conditions.

In multi-fluid flow scenarios, the integration of PINNs with finite element methods has shown promise, particularly in benchmark problems like the rising bubble, enhancing simulation accuracy and efficiency beyond traditional methods [28]. This highlights PINNs' ability to improve predictive accuracy in complex fluid interactions and interfaces.

The Enhanced Physics-Informed Neural Networks (EPINN) framework has demonstrated superior performance in hyperelasticity problems compared to traditional PINNs, achieving this without extensive data generation [45]. This adaptability illustrates the potential of PINNs in addressing complex material behaviors.

Additionally, incorporating physical coefficients within PINNs significantly enhances predictions for linear PDEs with boundary layers, as evidenced by the HLConcPINN method [4]. This integration is particularly beneficial in scenarios where boundary phenomena are critical.

In the context of thermoacoustic interactions within combustors, PINNs have effectively modeled combustion instability dynamics, providing accurate predictions that inform combustor design improvements [18]. This application highlights the potential of PINNs in enhancing the understanding and control of complex fluid-thermal interactions.

The NH-PINN method has been successfully employed in solving various homogenization problems, showing improved performance over classical PINN approaches and alignment with reference solutions from finite element methods [12]. This exemplifies the efficacy of PINNs in addressing multiscale problems that challenge traditional methods.

Recent experiments on various PDE problems, including nonlinear diffusion-reaction equations, Burgers equations, and the Navier-Stokes equations at high Reynolds numbers, further showcase the versatility of PINNs in tackling diverse CFD challenges [14]. The KED approach also effectively combines physical knowledge with observed data to enhance accuracy in predicting two-phase flow dynamics in porous media compared to non-physically aware models [15].

These applications and case studies collectively illustrate the transformative impact of PINNs in advancing CFD research, offering robust solutions across a broad spectrum of scientific and engineering challenges. The ongoing development of innovative PINN methodologies holds significant potential for improving the accuracy and efficiency of fluid dynamics simulations, particularly in challenging scenarios like solving the Navier-Stokes equations without extensive data reliance. Recent studies have highlighted their effectiveness in accurately predicting flow fields near wall regions with minimal sampling points, while also addressing current limitations such as substantial training times and challenges in capturing complex phenomena like vortex shedding [49, 10].

4.2 PINNs in Solving Classical PDEs

PINNs have emerged as a powerful tool for solving classical partial differential equations (PDEs) in CFD, offering enhanced accuracy and efficiency compared to traditional numerical methods. Their application in solving biharmonic equations in elasticity exemplifies their superior accuracy and computational efficiency, positioning them as a viable alternative to conventional approaches [55].

The integration of PINNs with hybrid methods, such as the Hybrid Finite Difference-PINN (HFD-PINN), has shown promise in solving classical PDEs like the Poisson equation and Burgers equation, achieving improved performance over standard PINN methods [7]. This hybrid approach highlights the versatility of PINNs in capturing intricate flow behaviors in complex fluid dynamics scenarios.

Despite their potential, challenges remain in maintaining local and global mass balance accuracy. While PINNs can achieve low mean squared error (MSE) values, they often exhibit higher mass balance errors compared to finite volume methods [56]. Addressing these challenges is crucial for enhancing the applicability of PINNs in CFD.

Innovations such as the SPINN framework have demonstrated significant improvements in accuracy for solving classical PDEs, including the heat equation and the Korteweg-de Vries (KdV) equation, utilizing fewer training points and simpler architectures [57]. This advancement illustrates the potential of PINNs to efficiently solve complex PDEs with reduced computational resources.

The Distributed Physics-Informed Neural Networks (DPINN) approach has also been validated with classical equations like Burgers' equation and the Navier-Stokes equations, achieving lower prediction errors with less training data [16]. This validation emphasizes the efficiency of PINNs in leveraging distributed learning to enhance predictive accuracy.

Furthermore, the application of PINNs in predicting crack paths in brittle materials illustrates their capability in solving classical PDEs related to fracture mechanics, offering superior accuracy and computational efficiency [5]. This demonstrates the versatility of PINNs across various domains within CFD.

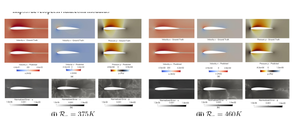
The ongoing advancement of innovative PINN methodologies, particularly their integration with traditional CFD techniques, holds significant potential for improving fluid dynamics simulations' accuracy and efficiency. While currently viewed as complementary to conventional CFD solvers, their capacity to address the Navier-Stokes equations without reliance on existing data remains a focal point of research. Recent studies highlight both the promise and challenges of applying PINNs in fluid simulations; for instance, a benchmark test involving the 2D Taylor-Green vortex demonstrated acceptable accuracy after extensive training, yet the method struggled with the 2D cylinder flow problem, failing to capture essential physical phenomena like vortex shedding. The development of parametric PINNs for predicting turbulent flows offers a computationally efficient alternative to traditional RANS formulations, enabling near real-time predictions while leveraging existing CFD data and governing equations. These advancements underscore the transformative potential of PINNs in addressing classical PDEs in CFD, although further refinement is necessary to enhance their practical applicability in real-world scenarios [9, 10].

4.3 Applications in Turbulent and Laminar Flow

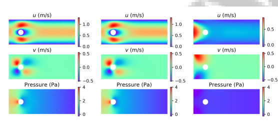
PINNs have demonstrated significant promise in modeling both turbulent and laminar flow regimes, providing an innovative approach to overcoming the limitations faced by traditional CFD methods. In laminar flows, PINNs have effectively simulated boundary layer flows, as evidenced in experiments involving laminar boundary layer flow over a flat plate with Reynolds numbers ranging from 500 to 100,000 [49]. These experiments underscore the capability of PINNs to accurately capture laminar flow dynamics, which is crucial for applications such as aerodynamic design and optimization.

In turbulent flow regimes, the integration of PINNs with complex physical laws, such as those described by the Reynolds-Averaged Navier-Stokes (RANS) equations, facilitates the development of parametric surrogate models that efficiently predict flow characteristics like velocity and pressure. Advanced training techniques, including novel loss function balancing and domain-specific sampling strategies, enhance the accuracy and convergence of PINNs in capturing intricate turbulent flow dynamics, making them powerful tools for real-time fluid dynamics applications [58, 59, 48, 9]. This capability allows for accurate modeling of intricate flow patterns and interactions characteristic of turbulence. The integration of PINNs with traditional turbulence models has been explored to improve predictive accuracy and computational efficiency, leveraging the strengths of both approaches.

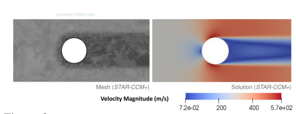
The versatility of PINNs in handling laminar and turbulent flows highlights their potential as a transformative tool in CFD. By embedding the governing equations of fluid dynamics directly into the neural network architecture, PINNs provide a mesh-free alternative to traditional methods, reducing computational costs while maintaining high accuracy. Their adaptability makes them particularly suitable for diverse applications, including aerospace engineering and environmental modeling, where they can efficiently predict complex turbulent flow patterns and boundary layer behaviors, thus enabling accurate and computationally efficient simulations crucial for design optimization and understanding fluid dynamics in real-world scenarios [58, 49, 9, 10].



(a) Comparison of Predicted and Ground Truth Flow Fields for a Turbulent Flow Simulation[9]



(b) Flow Visualization of a Circular Object in a Fluid Flow Field[60]



(c) STAR-CCM+ Simulation of a Sphere in a Flow Field[58]

Figure 5: Examples of Applications in Turbulent and Laminar Flow

As shown in Figure 5, the study of turbulent and laminar flows is pivotal for understanding and predicting fluid behavior in various engineering and scientific applications. The examples illustrate diverse methodologies employed in this field. The first example compares predicted and actual flow fields in a turbulent flow simulation, highlighting computational predictions' accuracy against established ground truths. The second example visualizes fluid dynamics around a circular object, showcasing the intricate interplay of velocity and pressure fields. The third example employs STAR-CCM+ software to simulate a sphere within a flow field, demonstrating advanced meshing techniques and color-coded velocity representations to analyze flow characteristics. Collectively, these examples underscore the critical role of CFD in advancing our understanding of turbulent and laminar flow phenomena, facilitating efficient and innovative solutions in fluid dynamics research and applications. [9, 60, 58]

4.4 PINNs in Heat Transfer and Thermo-Fluid Problems

PINNs have increasingly been applied to heat transfer and thermo-fluid dynamics problems, providing innovative solutions that enhance simulation accuracy and efficiency. By integrating PINNs with traditional heat transfer models, established physical principles and constraints are directly embedded into the neural network architecture, significantly improving the model's capability to accurately represent complex heat transfer phenomena encountered in coupled moving boundary problems, multi-scale scenarios, and non-linear conservation laws [11, 61, 62, 63, 33].

In heat transfer, PINNs have effectively modeled complex scenarios involving conduction, convection, and radiation. By integrating physical principles into their architecture, they tackle intricate governing equations and boundary conditions that often challenge traditional modeling techniques. Recent studies have demonstrated PINNs' ability to accurately capture phenomena such as moving boundary dynamics, boundary layer effects, and radiative transfer, offering robust solutions even in cases characterized by high variability and sharp transitions in physical processes. This approach enhances predictive accuracy and streamlines modeling of transient multiphysics problems, making it a promising alternative to conventional methods [11, 4, 33, 64, 8]. By embedding the residuals of these equations into the loss function, PINNs ensure that solutions adhere to fundamental heat transfer principles, resulting in more accurate and reliable predictions.

One notable application of PINNs in thermo-fluid dynamics is simulating heat exchangers, where accurate prediction of temperature fields and heat fluxes is critical for optimizing performance. The ability of PINNs to model intricate geometries and complex boundary conditions without extensive mesh generation makes them advantageous for various applications. This approach significantly lowers computational costs while ensuring high accuracy in solving partial differential equations (PDEs). Recent advancements, such as incorporating algebraic boundary and initial conditions, enhance PINNs' stability and performance, leading to reduced error rates compared to traditional methods. Techniques like hard boundary condition enforcement and singularity enrichment further improve solution precision in challenging scenarios, including those involving singularities and moving boundaries. Overall, PINNs provide a powerful alternative to conventional numerical methods, enabling efficient and accurate simulations across diverse scientific and engineering problems [8, 9, 65, 33, 66].

Furthermore, PINNs have been applied to solve coupled thermo-fluid problems, where the interaction between fluid flow and heat transfer is significant. Integrating PINNs with traditional CFD techniques enhances the ability to simulate fluid dynamics and heat transfer simultaneously, offering a comprehensive understanding of the coupled processes. This approach leverages existing CFD solutions and governing differential equations to predict outcomes efficiently, addressing traditional methods' limitations, such as high computational costs in achieving accuracy in turbulent flow simulations. Recent studies highlight PINNs' potential as parametric surrogate models, particularly in complex flow scenarios, while acknowledging ongoing challenges in achieving reliable results without sufficient data [9, 10]. This capability is essential for applications like designing cooling systems and analyzing thermal management in electronic devices.

The application of PINNs in heat transfer and thermo-fluid dynamics marks a substantial advancement in the field, providing a sophisticated framework for tackling complex problems characterized by moving boundaries and multi-physics interactions. By integrating physical laws into the neural network architecture, PINNs enhance predictive accuracy and efficiency, as demonstrated in various studies. They have successfully captured intricate composition profiles in binary alloy solidification and accurately predicted near-wall flow fields, even with limited measurement data. The mixed-variable PINN methodology has shown promise in developing surrogate models for incompressible laminar flow with heat transfer in two-dimensional domains, achieving good agreement with CFD results. This innovative approach addresses challenges associated with low-fidelity experimental data and holds potential for solving transient multiphysics problems in diverse applications [11, 49, 61]. The continuous development of PINN methodologies promises to further expand their applicability, providing robust solutions to a wide range of scientific and engineering challenges in heat transfer and thermo-fluid dynamics.

5 Challenges and Limitations

The application of Physics-Informed Neural Networks (PINNs) in computational fluid dynamics (CFD) is marked by notable challenges and limitations that impact their effectiveness. Key issues include training and convergence difficulties, computational costs, accuracy and generalization challenges, and architectural constraints. These factors significantly influence the practical implementation of PINNs in complex fluid dynamics scenarios.

5.1 Training and Convergence Issues

Training and convergence present significant challenges for PINNs in CFD applications. The generation of suitable meshes for complex geometries often leads to inaccuracies, exacerbated by the multiscale nature of fluid dynamics problems that traditional PINN formulations struggle to capture effectively [7, 12]. Enforcing boundary conditions remains problematic, impacting computational efficiency and accuracy, while complicating the optimization process [5]. Extrapolating PINNs beyond their training range requires intensive hyperparameter tuning, often leading to computational challenges [4]. Dynamic instabilities, such as vortex shedding, add complexity to PINN applications, necessitating advancements to enhance predictive capabilities [17]. The reliance on extensive training data further limits PINN performance in data-scarce environments, highlighting unresolved theoretical challenges [15, 1]. Innovative methods like OL-PINN show promise in improving accuracy and generalization with fewer residual points, indicating potential enhancements for PINN performance [14].

5.2 Computational Costs and Efficiency

The computational demands of PINNs are a critical consideration in their CFD applications. While embedding physical laws into neural networks enhances accuracy, it also increases computational costs. Methods like gPINNs, which incorporate higher-order derivatives, can significantly raise costs compared to traditional PINNs, posing challenges for real-time applications [67]. The integration of physics-based loss terms, although beneficial for accuracy, further burdens computational resources [68, 11]. Advancements such as the FE-PINN approach offer speed advantages by reducing hyperparameter tuning needs and effectively balancing loss functions [41]. Despite these innovations, the complexity of architecture search and conflicting loss functions continue to impact computational efficiency, underscoring the need for improved optimization strategies [42]. Compared to traditional numerical methods, PINNs may struggle with higher-order nonlinear effects, highlighting ongoing challenges in achieving competitive performance [69, 70]. Nevertheless, recent innovations, including the RANS-PINN framework and can-PINN methodology, demonstrate how PINNs can leverage physics-based regularization and advanced differentiation techniques to enhance training efficiency and accuracy [29, 48, 9, 58, 10].

5.3 Accuracy and Generalization Challenges

Ensuring accuracy and generalization remains a significant challenge for PINNs in CFD. Reliable predictions beyond the training domain are limited by spectral bias, complicating training in both advection-dominated and diffusion-dominated regimes [20]. Mass balance constraints lead to discrepancies in local and global mass conservation, particularly at high Reynolds numbers, where deviations from CFD results are common. Methods like semi-analytic PINNs (sf-PINNs) struggle with complex PDE problems, especially those involving stiff differential equations and sharp transitions. Recent research introducing enriched PINN methods with corrector functions has shown promise in improving accuracy for singularly perturbed boundary value problems [8, 33]. Enhanced PINNs (ePINNs) face generalization limitations, particularly with nonlinearities and higher-dimensional states. Separable Physics-Informed Neural Networks (SPINN) offer advancements in solving PDEs with reduced resource requirements but face challenges with equations not conforming to low-rank tensor structures [47, 33, 71, 72]. Addressing these challenges is crucial for improving PINNs' effectiveness in CFD applications, ensuring reliable predictions while maintaining performance comparable to traditional methods [58, 49, 9, 10].

5.4 Architectural and Methodological Constraints

PINNs are constrained by architectural and methodological limitations that impact their broader application in fluid dynamics. Hyperparameter tuning significantly influences methods like Region-Optimized PINNs (RoPINNs), requiring careful adjustments for optimal results [73]. The integration of graph neural networks with physics-informed frameworks enhances modeling capabilities but faces challenges with complex geometries or extreme parameter variations [74]. Distributed Physics-Informed Neural Networks (DPINNs) show promise but are currently limited to low Reynolds number problems, requiring further research for application in unsteady, turbulent flows [16]. Addressing these constraints is essential for enhancing PINNs' robustness and versatility, particularly in predicting complex flow phenomena like vortex shedding. Continued research is crucial for establishing PINNs as transformative tools in CFD, facilitating their application in real-world turbulent flow simulations and other intricate fluid dynamics challenges [17, 9, 10].

6 Future Directions

6.1 Future Research Needs

Advancing Physics-Informed Neural Networks (PINNs) in computational fluid dynamics (CFD) requires targeted research to overcome existing challenges and broaden their applicability. Key areas include enhancing theoretical foundations, exploring novel architectures, and addressing methodological limitations [1]. This involves refining operator learning integration and developing adaptive training techniques to boost predictive performance [14]. Investigating the Knowledge-Based Encoder-Decoder (KED) model alongside architectures like Generative Adversarial Networks (GANs) could improve predictive accuracy [15]. Optimizing loss function dynamics and integrating traditional PDE-solving techniques with machine learning are crucial for improving generalization and prediction accuracy [7].

Developing automatic adaptive algorithms for training PINN and Variational PINN (VPINN) methods is essential for real-world applicability. Future studies should explore transfer learning to optimize PINNs for turbulent flow simulations, particularly in engineering applications like downburst outflow and heat transfer. Transfer learning can enhance training robustness and convergence, facilitating effective modeling across diverse fluid dynamics scenarios while minimizing required training data and time [62, 59]. Further, refining methods for unknown solid body dynamics and applying the framework to complex flow scenarios can significantly enhance PINN utility.

Integrating PINNs with traditional numerical methods and investigating innovative boundary condition impositions, particularly through algebraic inclusion, can improve stability and performance. Employing semi-analytic techniques to tackle singularly perturbed boundary value problems and coupled moving boundary PDEs is promising [33, 11, 8]. Research could further explore frameworks like HLConcPINN across a wider range of PDE problems, enhancing architecture and activation functions. Extending projection methods for diverse boundary conditions and improving computational efficiency are essential for advancing PINN methodologies.

Exploring distributed training strategies, multi-objective optimization, and applying Particle Swarm Optimization-PINN (PSO-PINN) to complex problems, including inverse mapping and high-dimensional PDEs, represents promising future avenues. Enhancing the Distributed Physics-Informed Neural Network (DPINN) approach for inverse problems, particularly in unsteady and three-dimensional Navier-Stokes equations, is critical for accurately modeling fluid flows with intricate boundary conditions. The DPINN framework can efficiently estimate parameters and reconstruct flow fields using limited data, addressing the limitations of traditional CFD methods [16, 51, 10, 75, 8]. Further refinements to oversampling strategies for a broader range of multiscale problems should also be investigated.

Enhancing PINN versatility involves optimizing architectures, novel reinitialization strategies, and applying these methods to complex fluid dynamics challenges, such as the Navier-Stokes equations. Research indicates that PINN generalization capabilities are significantly influenced by algorithmic setups, including boundary and initial conditions that stabilize training and enhance performance. Developing advanced frameworks like coupled-automatic-numerical differentiation can further increase training efficiency and accuracy, enabling effective solutions for intricate fluid dynamics problems that traditional solvers struggle to address [47, 33, 48, 10]. Future research will also extend the PINC framework to handle Differential-Algebraic Equations (DAEs) and Partial Differential Equations (PDEs), as well as apply it to industrial control problems with uncertain parameters.

Innovative research directions, such as algebraic inclusion of boundary and initial conditions and analysis of PINNs' generalization capabilities beyond their training domain, are poised to significantly enhance the performance and reliability of PINNs. These advancements aim for more stable training processes and improved accuracy in solving complex scientific and engineering problems, ultimately leading to robust and efficient solutions across diverse applications [47, 33].

6.2 Enhancing PINN Methodologies

Enhancing the methodologies of Physics-Informed Neural Networks (PINNs) is essential for improving performance and expanding applicability across various scientific and engineering domains. Integrating advanced activation functions and network architectures can significantly enhance PINNs' ability to capture complex fluid dynamics phenomena. The exploration of novel activation functions, such as sinusoidal or periodic functions, may yield improved convergence properties and better representation of periodic behaviors in fluid dynamics [14].

Another enhancement avenue involves developing hybrid approaches that combine PINNs with traditional numerical methods. For instance, integrating finite element methods (FEM) with PINNs leverages FEM's accuracy in handling complex geometries while benefiting from neural networks' data-driven insights [7]. This hybridization can lead to more robust simulations, particularly in scenarios with intricate boundary conditions or multiscale phenomena.

Optimizing training techniques is critical for enhancing PINN methodologies. Adaptive training strategies, such as curriculum learning, can gradually increase the complexity of training tasks, improving the network's generalization across different fluid dynamics scenarios [50]. Additionally, employing advanced optimization algorithms like Quadratic Programming Gradient Descent (QPGD) can effectively balance different loss components, ensuring stable and efficient training [42].

Incorporating multi-fidelity approaches within the PINN framework represents another potential enhancement. Utilizing data from simulations of varying fidelity levels allows PINNs to achieve higher accuracy while reducing computational costs. This approach is particularly beneficial in applications where high-resolution data is scarce or costly to obtain [27].

Exploring distributed training strategies and parallel computing techniques can significantly enhance the scalability and efficiency of PINNs, enabling them to tackle larger and more complex fluid dynamics problems [16]. By addressing these methodological enhancements, PINNs can provide more accurate, efficient, and versatile solutions to complex challenges in computational fluid dynamics and beyond.

6.3 Expanding Applications and Scalability

Expanding the applications and scalability of Physics-Informed Neural Networks (PINNs) in computational fluid dynamics (CFD) is crucial for future research and development. Broadening PINNs' scope allows for the realization of their transformative potential in solving complex scientific and engineering problems. One promising direction involves integrating PINNs with high-performance computing (HPC) resources to manage large-scale simulations, significantly enhancing scalability and enabling the tackling of extensive and intricate CFD problems with improved efficiency [69].

Exploring multi-physics scenarios, where interactions between different physical phenomena are critical, further expands PINN applications. Coupling PINNs with thermo-fluid dynamics models can provide insights into complex heat transfer processes, enhancing predictive capabilities in areas such as energy systems and environmental modeling [15]. Additionally, applying PINNs in biofluid dynamics shows promise for advancing medical diagnostics and treatment planning by accurately simulating blood flow and other physiological processes.

To improve scalability, developing multi-fidelity approaches within the PINN framework is essential. Leveraging data from simulations of varying fidelity levels allows for higher accuracy while reducing computational costs, making PINNs more accessible across a broader range of applications [27]. This approach is particularly advantageous in scenarios where high-resolution data is scarce or expensive to obtain.

Enhancing scalability can also be achieved through distributed training strategies and parallel computing techniques. These methodologies significantly reduce training times and enable PINNs to handle more complex fluid dynamics problems, thereby broadening their applicability in various scientific and engineering domains [16].

6.4 Advanced Computational Techniques

Integrating advanced computational techniques with Physics-Informed Neural Networks (PINNs) represents a promising frontier for enhancing their performance and applicability in computational fluid dynamics (CFD). A significant advancement is incorporating multi-fidelity and domain decomposition strategies, which efficiently handle complex, multiscale problems by leveraging data of varying fidelity levels [27]. This approach improves computational efficiency and enhances simulation accuracy by optimizing data use.

Another promising technique involves using parallel computing and distributed training strategies, significantly reducing the computational time required for training PINNs. Distributing the computational load across multiple processors enables handling larger datasets and more complex simulations, expanding PINN scalability in CFD applications [16]. This capability is particularly beneficial for high-dimensional problems where traditional methods encounter significant computational bottlenecks.

The integration of neural architecture search (NAS) techniques with PINNs, as demonstrated by the NAS-PINN framework, offers a systematic approach to optimizing neural network architectures for specific CFD problems [76]. Automating the search for optimal network configurations enhances PINN performance, ensuring the architecture is well-suited to the problem.

Moreover, applying advanced optimization algorithms, such as Quadratic Programming Gradient Descent (QPGD), provides a robust framework for balancing different loss components within the PINN training process [42]. This strategy is crucial for ensuring convergence and stability, especially in complex fluid dynamics scenarios where multiple physical laws and constraints must be satisfied simultaneously.

Incorporating machine learning techniques, such as reinforcement learning and transfer learning, into the PINN framework holds potential for advancing capabilities. These techniques enhance PINNs' ability to learn complex dynamic behaviors in fluid dynamics, particularly for turbulent flow scenarios, by developing parametric surrogate models that integrate existing CFD solutions and governing differential equations. This integration improves prediction accuracy and efficiency across various scenarios—such as internal and external flows—while significantly boosting PINN generalization, broadening applicability to diverse scientific and engineering challenges, including offshore structure design and heat exchanger optimization. Innovative sampling methods ensure effective training convergence, further solidifying model reliability in practical applications [9, 29, 10].

The integration of advanced computational techniques with PINNs holds significant potential to enhance performance, scalability, and applicability in addressing complex fluid dynamics challenges. By leveraging parametric surrogate models and innovative training approaches, such as the RANS-PINN framework and the coupled-automatic-numerical differentiation method (CAN-PINN), researchers can improve simulation accuracy and efficiency in CFD. These advancements facilitate the resolution of the Navier-Stokes equations without extensive data requirements and enable real-time predictions of turbulent flows, paving the way for more accurate and efficient simulations in CFD research and practical applications [58, 48, 9, 10].

6.5 Theoretical Advancements

Theoretical advancements in Physics-Informed Neural Networks (PINNs) are essential for their development and applicability in computational fluid dynamics (CFD) and other scientific domains. A significant area of exploration involves developing robust frameworks for integrating governing physical laws into neural network architectures, requiring a deep understanding of the mathematical properties of partial differential equations (PDEs) and their interaction with neural network training dynamics [1].

A promising direction for theoretical advancements is exploring novel loss functions that better capture the physics of complex fluid dynamics scenarios. These loss functions should enhance the stability and convergence of PINNs, particularly in high-dimensional and nonlinear problems. Developing such loss functions necessitates a comprehensive understanding of the interplay between data-driven learning and physics-based modeling [14].

Furthermore, advancements in the theoretical understanding of neural network architectures, such as the impact of network depth and width on approximating PDE solutions, are crucial for optimizing PINN performance. Insights into the spectral bias of neural networks, particularly through Neural Tangent Kernel (NTK) analysis, can guide the development of innovative architectures, such as Spectral PINNs, better equipped to accurately capture fluid systems' intricate dynamics across various scales. This understanding reveals that spectral bias often hinders learning high-frequency behaviors, especially in advection-dominated regimes, highlighting the potential of employing periodic activation functions and a decoupling solving paradigm to enhance multiscale phenomena representation [77, 78].

The theoretical exploration of multi-fidelity approaches within the PINN framework also holds potential for enhancing efficiency and accuracy. Leveraging data from simulations of varying fidelity levels can guide developing strategies that optimize available data use, improving PINN generalization capabilities [27].

Additionally, investigating hybrid methods that combine PINNs with traditional numerical techniques, such as finite element methods (FEM), can provide valuable insights into the strengths and limitations of these approaches. Understanding the theoretical underpinnings of such hybrid methods can lead to more effective integration strategies, resulting in improved simulation accuracy and computational efficiency [7].

Theoretical advancements in PINN methodologies, loss functions, network architectures, and hybrid approaches are vital for their development. These advancements, particularly the development of parametric surrogate models and the coupled-automatic-numerical differentiation (CAN-PINN) framework, significantly enhance the applicability and performance of CFD solutions. By integrating existing CFD data with governing differential equations, these methods enable rapid and accurate predictions of turbulent flow variables in internal and external flow scenarios. The CAN-PINN framework, which combines automatic and numerical differentiation strengths, improves training efficiency and accuracy, allowing for robust solutions to complex fluid dynamics problems like flow mixing, lid-driven cavity flow, and channel flow over obstacles. This progress streamlines the simulation process and provides precise and efficient approaches to tackling intricate challenges in CFD and related fields [48, 9].

7 Conclusion

The exploration of Physics-Informed Neural Networks (PINNs) in the realm of computational fluid dynamics (CFD) underscores their transformative potential in resolving intricate fluid dynamics issues by integrating physical laws directly into neural network frameworks. This integration significantly enhances predictive accuracy and computational efficiency, addressing the constraints of conventional methods. PINNs demonstrate remarkable versatility in managing both forward and inverse problems, providing robust solutions across a wide range of scientific and engineering fields. Recent advancements, such as automated hyperparameter tuning and auxiliary-task learning, have further improved the precision and stability of PINNs, streamlining the training process. These developments underscore the crucial contribution of PINNs in advancing CFD research, offering innovative approaches that enhance the precision and efficiency of fluid dynamics simulations. The continuous evolution and application of PINNs are set to further advance the field, solidifying their role as a pivotal element in contemporary computational fluid dynamics research.

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