# Ultrasound Shear Wave Imaging and Viscoelastic Modeling: A Survey

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## Abstract

Ultrasound Shear Wave Imaging (USWI) has revolutionized medical diagnostics by enabling non-invasive evaluation of tissue biomechanical properties, which is critical for diagnosing and managing various diseases. This survey paper explores the integration of advanced viscoelastic modeling, including fractional derivative models, with ultrasound shear wave imaging to assess shear wave dispersion and characterize tissue elasticity and nonlinear viscoelastic behavior. The paper highlights USWI's significance across diverse medical fields, such as musculoskeletal, neurological, obstetrics, and oncology diagnostics, where it enhances diagnostic accuracy by correlating shear wave speed with tissue stiffness. Viscoelastic modeling, particularly with fractional calculus, offers a comprehensive framework for understanding complex tissue biomechanics, addressing limitations of traditional models by incorporating time-dependent strain responses and nonlinear behaviors. Recent technological advancements, such as quantum light in imaging and deep learning frameworks, have further enhanced the precision and scope of biomechanical assessments. The survey underscores the critical role of these innovations in improving patient care and outcomes by providing non-invasive, precise evaluations of tissue mechanical properties. However, challenges remain, including variability in measurement outcomes and the need for standardized protocols to ensure broader clinical adoption. Future research should focus on refining these technologies and exploring their potential applications in various medical contexts, ultimately contributing to the advancement of diagnostic and therapeutic strategies.


## 1 Introduction

### 1.1 Significance of Ultrasound Shear Wave Imaging in Medical Diagnostics

Ultrasound shear wave imaging (USWI) marks a significant advancement in medical diagnostics by providing a non-invasive method for evaluating tissue biomechanical properties, crucial for diagnosing and managing various diseases. This technique, particularly through shear wave elastography (SWE), generates shear waves in soft tissues and employs ultrasonic tracking to monitor their propagation, thereby enhancing diagnostic accuracy through the correlation between shear wave speed and tissue stiffness. Such insights into the viscoelastic properties of tissues, including those in the liver and breast, facilitate more precise clinical evaluations and interventions [1, 2, 3].

In musculoskeletal applications, USWI characterizes muscle tissues as elastic, incompressible, and transversely isotropic, utilizing advanced ultrasonic rotational 3D shear wave elasticity imaging [4]. This capability is invaluable for musculoskeletal radiologists, providing a detailed overview of ultrasound elastography's state and applications [5]. Additionally, USWI enables non-invasive nerve stiffness measurement during limb movements, essential for assessing neuromuscular conditions [6].

In neurological diagnostics, USWI is vital for characterizing the viscoelastic properties of brain tissue under varying loading conditions [7]. The integration of genetic mutations with biomechanical

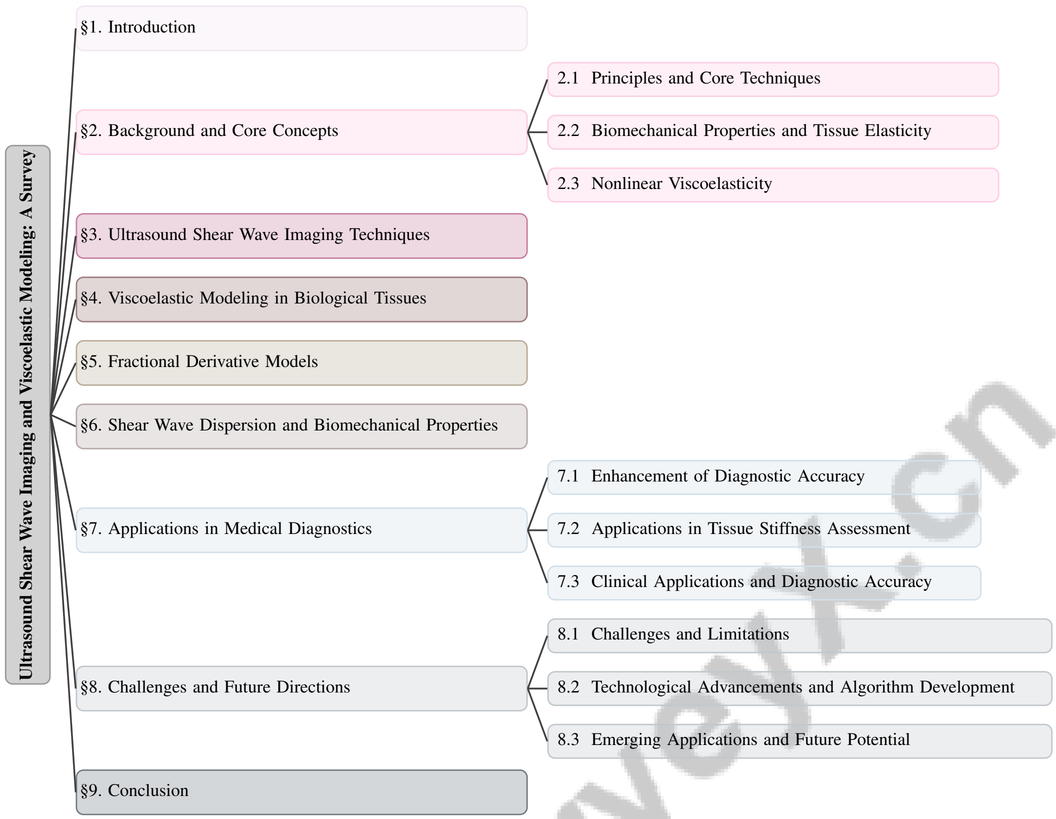


Figure 1: chapter structure



properties using USWI also advances the understanding of vascular diseases, particularly in load-bearing soft tissues [8].

In obstetrics, ultrasound shear-wave elastography offers a non-invasive method to measure placental stiffness, providing quantitative assessments crucial for maternal and fetal health [9]. Similarly, ocular pulse elastography (OPE) employs high-frequency ultrasound to image corneal deformation in response to the ocular pulse, facilitating accurate assessments of corneal biomechanics [10].

In breast ultrasonography, SWE plays a pivotal role in differentiating benign from malignant lesions, enhancing diagnostic performance and aiding in early breast cancer detection [11]. The emergence of optical elastography and advanced imaging techniques in USWI further enhances diagnostic capabilities, addressing challenges such as photo-damage and improving the viability of biological samples during imaging [12].

Moreover, USWI's accessibility is improved through methods that allow elasticity imaging using standard B-mode ultrasound, broadening the technology's availability [13]. As ultrasound elastography gains traction as a diagnostic tool, its application in cancer detection and thyroid disease diagnostics is increasingly recognized [14].

USWI represents a pivotal advancement in contemporary medical diagnostics, significantly enhancing tissue property characterization through innovative methods such as shear wave elastography and cavitation rheology. These techniques provide detailed insights into the mechanical characteristics of various tissues, including lung and pancreatic tissues, facilitating improved disease diagnosis and management across a wide range of clinical applications with minimal sample manipulation [15, 3].

### 1.2 Importance of Viscoelastic Modeling in Tissue Biomechanics

Viscoelastic modeling is essential for elucidating the biomechanics of biological tissues, transitioning from purely elastic descriptions to models that incorporate viscoelastic effects, thus offering a comprehensive analysis of biological materials [16]. This approach is particularly valuable in



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evaluating the mechanical properties of tendons, where traditional methods may inadequately address pathological conditions [17]. In musculoskeletal applications, viscoelastic modeling enhances the assessment of skeletal muscle biomechanics by addressing the limitations of conventional imaging techniques [4].

Integrating viscoelastic effects into shear wave elastography, particularly through reverberant shear wave fields, allows for more accurate estimations of shear wave speed and tissue viscoelastic properties, refining biomechanical assessments [2]. In ophthalmology, incorporating viscoelastic models advances clinical imaging technologies, especially in the absence of a gold standard for measuring corneal biomechanics [18]. These models are also crucial for understanding placental stiffness, integral to assessing maternal and fetal health in conditions such as preeclampsia [9].

Recent advancements in computational methodologies, such as deep learning frameworks, further propel the field by enabling the analysis of raw time-domain RF signals for sound speed estimation, thereby enhancing the understanding of tissue properties [14]. This technological progression highlights the importance of viscoelastic modeling in characterizing the frequency-dependent behavior of tissues, significantly influencing shear wave propagation measurements and improving the precision of biomechanical evaluations.

Implementing viscoelastic modeling, particularly through fractional calculus, across a range of scales from micro to intermediate levels is essential for enhancing diagnostic and therapeutic approaches in medicine. This technique not only provides a more accurate characterization of viscoelastic properties of various cell types, including macrophages, but also facilitates a deeper understanding of cellular behaviors and responses to pharmacological interventions. By capturing the complex mechanical responses of cells and extracellular matrices, fractional viscoelastic models can significantly improve the diagnosis and treatment of conditions related to cancer and immune system dysfunctions, broadening their applicability across multiple medical disciplines [15, 19, 20, 21].

### 1.3 Structure of the Survey

This survey is meticulously organized to provide a comprehensive understanding of ultrasound shear wave imaging and viscoelastic modeling in medical diagnostics. The paper begins with an **Introduction** that highlights the significance of ultrasound shear wave imaging and the role of viscoelastic modeling in elucidating tissue biomechanics. Following this, the **Background and Core Concepts** section delves into foundational principles and techniques, establishing a solid basis for subsequent discussions.

The survey progresses to a detailed examination of **Ultrasound Shear Wave Imaging Techniques**, encompassing recent technological advancements and innovative methodologies. This is succeeded by a focus on **Viscoelastic Modeling in Biological Tissues**, which explores the intricate behaviors of biological tissues, specifically how the viscoelastic properties of the extracellular matrix influence cellular processes such as migration, differentiation, and organoid formation. The integration of advanced computational simulations and emerging technologies aims to enhance the understanding of these complex tissue mechanics, ultimately facilitating more accurate predictions of tissue behavior in both health and disease contexts [7, 21].

In **Fractional Derivative Models**, the survey discusses the mathematical foundations and compares these models with traditional ones, highlighting innovations and advantages. The section on **Shear Wave Dispersion and Biomechanical Properties** addresses the fundamentals of shear wave dispersion, associated techniques, and their applications in tissue pathology.

The **Applications in Medical Diagnostics** section reviews the enhancement of diagnostic accuracy and the role of these techniques in assessing tissue stiffness, focusing on clinical applications. Finally, the survey concludes with **Challenges and Future Directions**, identifying current challenges, exploring technological advancements, and discussing emerging applications and future potential in the field.

Each section is crafted to build upon the previous one, ensuring a coherent narrative that guides the reader through the complexities of the subject matter while reflecting on the impact and future directions of ultrasound shear wave imaging and viscoelastic modeling in medical diagnostics.The following sections are organized as shown in Figure 1.

## 2 Background and Core Concepts

### 2.1 Principles and Core Techniques

Ultrasound shear wave imaging (USWI) and viscoelastic modeling are pivotal for non-invasive evaluation of tissue biomechanics, providing insights into tissue stiffness and elasticity. Shear wave elastography (SWE) enhances diagnostic accuracy by quantifying tissue stiffness through shear wave propagation speed, proving beneficial in musculoskeletal evaluations beyond traditional ultrasound [4, 17]. Despite technological advancements, clinical adoption of SWE remains limited [5].

Innovations like Synthetic Shear-Wave Elastography (sSWE) leverage deep learning to generate elastography images from B-mode ultrasound, enhancing accessibility [13]. Shear Wave Absolute Vibro-Elastography (S-WAVE) and Optical Pulse Elastography (OPE) further refine elasticity measurements with multi-frequency excitation and high-frequency ultrasound for corneal biomechanics, respectively [8, 10].

Viscoelastic modeling benefits from advanced mathematical frameworks, such as fractional calculus, improving biomechanical characterizations [22]. Techniques like R-SWE, employing reverberant shear wave fields, refine biomechanical assessments by enhancing shear wave speed and property estimations [2]. Complementary methods like Magnetic Resonance Elastography (MRE) offer non-invasive viscoelastic parameter measurements [23].

Atomic force microscopy (AFM) provides precise viscoelastic property measurements, emphasizing the need for accurate assessment methods [16]. Nonlinear viscoelastic behavior exploration, particularly in liver tissue modeling, underscores the importance of comprehensive constitutive equations [24].

The integration of advanced techniques in USWI and viscoelastic modeling significantly enhances tissue biomechanics assessment accuracy. Innovations in shear wave speed estimation, including reverberant fields, improve diagnostic capabilities in liver, breast, and prostate imaging [1, 2, 25, 26]. These advancements are crucial for both diagnostic and therapeutic applications, providing a comprehensive framework for tissue mechanical property evaluation.

### 2.2 Biomechanical Properties and Tissue Elasticity

Evaluating biomechanical properties and tissue elasticity is essential for understanding mechanical behavior, impacting diagnostic and therapeutic applications. The extracellular matrix (ECM), comprising collagen fibers, proteoglycans, and water, plays a crucial role in maintaining tissue functionality and preventing degenerative diseases [27]. Shear wave elastography (SWE) is fundamental for assessing tissue elasticity by correlating shear wave speed with stiffness, enabling differentiation between healthy and pathological tissues.

Challenges in SWE include measurement variability across ultrasound systems and complex tissue structures lacking depth-dependent elasticity data, particularly in non-linear tissues like the placenta [9]. Advanced methodologies, such as reverberant shear wave elastography (R-SWE), enhance elasticity assessment by providing comprehensive datasets critical for biomechanical evaluations [2].

In clinical settings, in vivo elasticity measurements, such as Young's modulus, are essential, with ultrafast imaging offering precise measurements by correlating shear wave speed with shear modulus [28]. However, reliable shear wave speed measurements are challenging due to musculoskeletal tissue heterogeneity [5]. Deep learning architectures for B-mode ultrasound image analysis represent promising advancements for predicting tissue properties [13].

Non-destructive assessment methods, such as image registration-based techniques, are necessary for exploring mechanical properties in developing 3D tissues and organs [29]. Characterizing viscoelastic properties of soft biological samples, which exhibit both solid- and liquid-like behavior, requires sophisticated mechanical analysis techniques [16].

Understanding biomechanical properties, particularly nerve stiffness changes during limb movements, is crucial for advancing our comprehension of tissue elasticity [6]. These methodologies underscore the critical role of biomechanical property assessments and tissue elasticity evaluations in enhancing medical diagnostics and therapeutic strategies.

## 2.3 Nonlinear Viscoelasticity

Nonlinear viscoelasticity provides a sophisticated modeling approach for biological tissues' mechanical behavior, capturing complex responses under varying loading conditions. Quasi-linear viscoelastic (QLV) models, known for thermodynamic consistency, describe time-dependent behavior of soft solids, ensuring energy dissipation during deformation adheres to thermodynamic principles [30].

Integrating nonlinear viscoelasticity into tissue modeling involves advanced methodologies. Optimization-based techniques and neural networks enhance displacement extraction and shear modulus estimation from MRE images, offering a noise-resistant and efficient approach [23]. Identifying localized elastic regions is critical for understanding nonlinear viscoelasticity in supercooled liquids, emphasizing localized mechanical properties in complex material behaviors [31].

In liver tissue, existing rheological models inadequately characterize nonlinear viscoelastic behavior under large deformations and varying frequencies, necessitating comprehensive models that accurately capture dynamic responses [24]. This challenge extends to materials exhibiting both elastic and viscous characteristics, where accurate modeling under nonlinear conditions remains significant [32].

The equation of motion for one-dimensional nonlinear viscoelasticity, particularly of the strain-rate type, is addressed under the assumption of a -convex stored-energy function, facilitating solid phase transformation modeling critical for understanding materials undergoing significant structural changes [33]. A large deformation poroelastic model for biological membranes (BMs) accounts for finite thickness and hydration, offering a more accurate representation compared to previous models treating them as homogeneous elastic materials [34].

Recent advancements in nonlinear viscoelasticity modeling are crucial for deepening our understanding of tissue mechanics, particularly in soft tissues like the liver, which often experience various mechanical stresses in medical scenarios such as surgeries and trauma. These enhanced models facilitate accurate characterization of tissue responses under diverse loading conditions, guiding the development of precise diagnostic tools and therapeutic strategies for liver injuries and chronic diseases [35, 36, 37, 38, 24]. Ongoing refinement of mathematical models and experimental techniques continues to enhance our comprehension of the intricate mechanical behaviors inherent in biological tissues, contributing to improved healthcare outcomes.

In recent years, the field of ultrasound shear wave imaging has witnessed significant advancements that have transformed both diagnostic and therapeutic applications. As illustrated in Figure 2, this figure depicts the hierarchical structure of these advancements, emphasizing the key improvements in imaging technologies and methodologies. Notably, it highlights enhanced imaging capabilities, innovations in elastography, and the development of advanced imaging techniques. Such progress not only enhances the precision of imaging but also broadens the scope of clinical applications, thereby underscoring the importance of continued research and development in this area.

# 3 Ultrasound Shear Wave Imaging Techniques

## 3.1 Advancements in Imaging Technologies

| Method Name | Technological Innovations | Application Areas | Integration and Adaptability |
|---|---|---|---|
| SBS[39] | Quantum Light Technology | Live-cell Imaging | Squeezed Light |
| S-WAVE[8] | Bandpass Sampling | Prostate Tissue Elasticity | Multi-frequency Excitation |
| SWE[6] | Ultrasound Shear Wave | Nerve Stiffness Measurement | Non-rigid Image Registration |
| R-SWE[2] | Reverberant Shear Wave | IN Vivo Assessments | Noise Reduction Filtering |
| sSWE[13] | Deep Learning | Tissue Elasticity Evaluation | Deep Learning Model |
| NIRT[29] | Synthetic Aperture Method | Vascular Elastography | Non-rigid Image Registration |
| TWENN[23] | Complex-valued Neural | Biomechanical Properties Estimation | Optimization-based Phase |
| PMCBM[34] | Atomic Force Microscopy | Cancer Cell Invasion | Finite Element Simulations |

Table 1: Overview of recent advancements in imaging technologies, highlighting various methods, their technological innovations, application areas, and integration capabilities. The table includes methods such as quantum light technology, bandpass sampling, and deep learning, which have significantly enhanced imaging precision and adaptability across various biomedical fields.

Recent advancements in ultrasound shear wave imaging (USWI) have considerably improved the precision and scope of biomechanical assessments. Quantum light technology has enhanced the signal-to-noise ratio (SNR) over traditional coherent light sources, revolutionizing imaging capabilities [39].

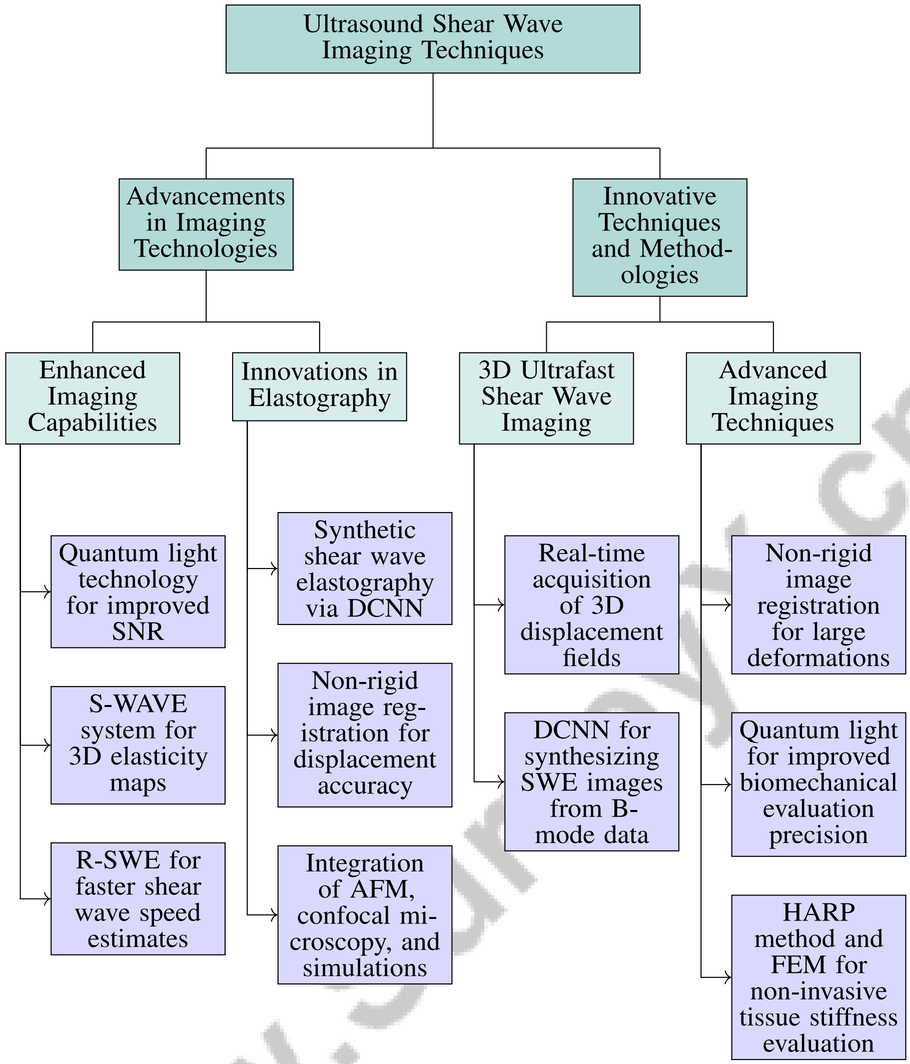


Figure 2: This figure illustrates the hierarchical structure of advancements and innovative techniques in ultrasound shear wave imaging, highlighting key improvements in imaging technologies and methodologies, including enhanced imaging capabilities, innovations in elastography, and advanced imaging techniques.

The S-WAVE system efficiently captures shear wave data, creating 3D elasticity maps of prostate tissue in quasi-real time, which facilitates rapid assessments [8]. Shear-wave elastography has been applied to measure nerve stiffness during limb movements, demonstrating its practical utility in musculoskeletal diagnostics [6]. Moreover, R-SWE eliminates the need for directional filters, offering faster and more reliable shear wave speed estimates [2].

As illustrated in Figure 3, these innovations encompass improvements in ultrasound shear wave imaging precision, synthetic shear-wave elastography utilizing deep learning, and advanced methods such as R-SWE and atomic force microscopy (AFM) integration, all of which contribute to enhanced biomechanical assessments. Table 1 provides a comprehensive overview of the recent advancements in imaging technologies, detailing the methods, technological innovations, application areas, and integration capabilities that have improved the precision and scope of biomechanical assessments. Innovations in synthetic shear wave elastography, particularly through deep fully-convolutional neural networks (DCNN), synthesize SWE images from B-mode data, thereby increasing accessibility and reducing dependency on specialized ultrasound equipment [13]. Non-rigid image registration methods further enhance adaptability and accuracy in measuring displacements under significant compression, overcoming traditional limitations [29]. Combining optimization-based phase unwrapping with a

Traveling Wave Expansion-based Neural Network (TWENN) has improved displacement extraction and modulus estimation accuracy [23].

Additionally, integrating atomic force microscopy (AFM), confocal microscopy, and finite element simulations provides nanoscale insights into biomechanics [34]. These advancements in ultrasonography, particularly shear wave elastography, enhance the accuracy and efficiency of biomechanical assessments in the musculoskeletal system. They enable real-time dynamic evaluations of tissue elasticity, facilitate earlier diagnosis of degenerative conditions, and improve treatment monitoring. The ongoing integration of viscoelasticity considerations and standardization across manufacturers is crucial for maximizing these technologies' potential in clinical practice [40, 41].

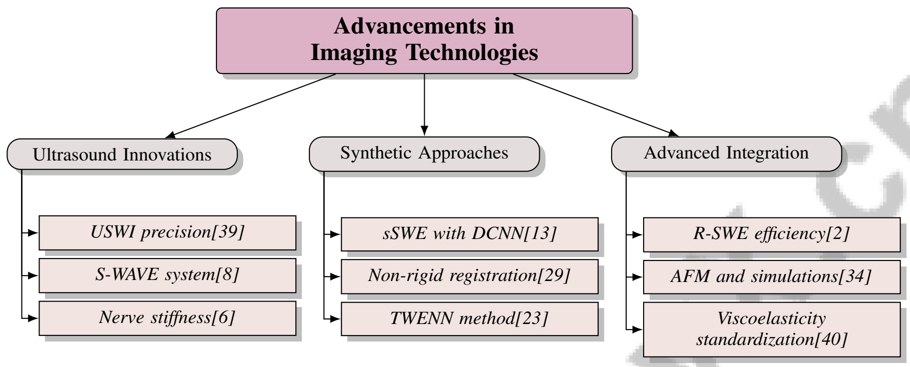


Figure 3: This figure illustrates the recent advancements in imaging technologies, focusing on ultrasound innovations, synthetic approaches, and advanced integration techniques. The innovations include improvements in ultrasound shear wave imaging precision, synthetic shear-wave elastography using deep learning, and advanced methods like R-SWE and AFM integration for enhanced biomechanical assessments.

### 3.2 Innovative Techniques and Methodologies

Innovative techniques in ultrasound shear wave imaging (USWI) have significantly advanced the field, particularly through three-dimensional (3D) ultrafast shear wave imaging, enabling real-time acquisition of 3D displacement fields essential for tissue elasticity estimation [42]. Integrating deep learning frameworks, such as deep fully-convolutional neural networks (DCNN), facilitates the synthesis of shear wave elastography images from B-mode ultrasound data, broadening USWI's applicability and improving elasticity imaging efficiency and accuracy [13].

Non-rigid image registration techniques have addressed challenges associated with large tissue deformations, allowing precise displacement measurements even under significant compression [29]. Systems like the S-WAVE, capable of generating elasticity maps in quasi-real time, exemplify the rapid and accurate assessment of tissue mechanical properties [8]. Incorporating quantum light into imaging technologies has notably improved the signal-to-noise ratio, enhancing the precision of biomechanical evaluations [39].

These methodologies in Ultrasound Elastography and Tagged Magnetic Resonance Imaging (MRI) improve tissue elasticity assessment accuracy and efficiency. Techniques like the harmonic phase (HARP) method and finite element modeling (FEM) expand the scope of medical diagnostics, enhancing non-invasive evaluations of tissue stiffness in various pathologies, including liver fibrosis and tumors, ultimately improving patient care and clinical outcomes [12, 26].

As illustrated in Figure 4, ultrasound shear wave imaging techniques have emerged as innovative methodologies for assessing tissue mechanical properties. The "Medium Amplitude Strain History Protocols and Their Corresponding Stress Responses" technique employs medium amplitude strain protocols to elucidate the relationship between strain history and stress response. The second technique presents a graph plotting stress against strain over varying time intervals, providing insights into the temporal dynamics of tissue response under stress. Lastly, "Muscle Tissue Properties: A Comprehensive Analysis" offers a detailed diagram focusing on the mechanical properties of muscle tissue, particularly the lungs, using diverse measurement methods like micro-indentation. Collectively, these examples highlight the potential of ultrasound shear wave imaging to enhance



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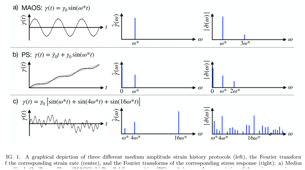
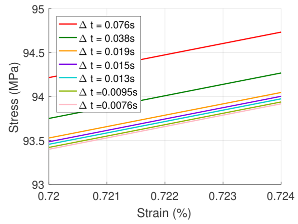
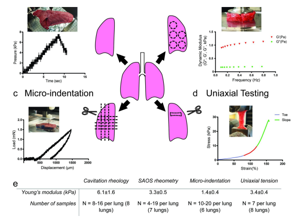


(a) Medium Amplitude Strain History Protocols and Their Corresponding Stress Responses[43]




(b) The image shows a graph plotting stress against strain for different time intervals.[44]




(c) Muscle Tissue Properties: A Comprehensive Analysis[15]

Figure 4: Examples of Innovative Techniques and Methodologies

our understanding of tissue biomechanics, paving the way for improved diagnostic and therapeutic strategies [43, 44, 15].

## 4 Viscoelastic Modeling in Biological Tissues

### 4.1 Modeling Complex Tissue Behaviors

Understanding the complex mechanical behaviors of biological tissues necessitates advanced modeling techniques, particularly for capturing their elastic and viscous characteristics. Traditional models often fall short in representing the intricate responses of tissues under diverse loading conditions. Viscoelastic models, which include time-dependent strain responses, provide a more comprehensive framework for biomechanical analysis [16]. The use of fractional calculus, specifically Caputo-type fractional derivatives, enhances biomechanical characterizations by addressing the history-dependent behavior of materials [22]. This approach is particularly pertinent for modeling tissues that exhibit time- and rate-dependent behaviors, diverging from simple linear elasticity.

The concept of localized elastic regions (LER) further refines our understanding by identifying distinct elastic properties within heterogeneous tissue structures [31]. This is crucial for accurately modeling shear wave propagation, which traditional linear models often inadequately address [2]. Incorporating such concepts into viscoelastic frameworks offers a nuanced perspective on tissue mechanics. Additionally, computational advancements, including optimization-based phase unwrapping and neural networks like the Time-Weighted Energy Neural Network (TWENN), have improved the accuracy of biomechanical property estimations, minimizing errors compared to existing methods [2]. These innovations are essential for capturing the non-linear and frequency-dependent behavior of tissues, vital for precise shear wave propagation analysis.

### 4.2 Integration with Emerging Technologies

Integrating viscoelastic modeling with emerging technologies significantly advances biomechanical analysis, enhancing precision and applicability in clinical and engineering contexts. Recent developments emphasize merging traditional viscoelastic models with innovative technologies to refine biomechanical assessments. Nonlinear anisotropic viscoelasticity has been applied in engineering, utilizing historical insights from viscoelasticity theory for practical applications [45]. Extended Finsler geometry provides a novel framework for understanding the intricate relationships between tissue microstructure and mechanical properties, offering a comprehensive approach to nonlinear soft tissue elasticity and remodeling [46]. Generative modeling frameworks (GMF) enhance flexibility in learning from data without predefined models and quantify uncertainty in predictions, thus improving the reliability of biomechanical property estimations [47].

In ultrasound shear wave imaging, integrating real-time data processing into the ultrasound pipeline is a promising research avenue, particularly for clinical assessments of conditions like liver fibrosis [42]. This approach effectively combines physical laws with data to accurately recover material properties, even with limited information, broadening the applicability of viscoelastic models in clinical settings [48]. Incorporating fitted viscoelastic coefficients derived from experimental data into the nonlinear

Kelvin-Voigt model exemplifies the potential for enhancing the accuracy of pulse wave simulations, crucial for understanding cardiovascular dynamics [49]. The categorization of corneal biomechanics measurement methods into high-magnitude and low-magnitude techniques underscores the need for standardized approaches to improve clinical imaging technologies [18].

Future research should focus on standardizing imaging protocols and exploring factors such as joint position on shear wave elastography (SWE) measurements to facilitate broader clinical applications and enhance diagnostic accuracy [5]. The ongoing integration of viscoelastic modeling with emerging technologies is poised to advance both diagnostic and therapeutic strategies, ultimately improving the understanding and treatment of complex biomechanical conditions.

## 5 Fractional Derivative Models

The exploration of fractional derivative models has significantly advanced the understanding of viscoelastic behavior, especially in biological tissues. These models are pivotal in capturing the complex mechanical responses of materials with both solid-like and liquid-like properties. By delving into the mathematical framework of fractional derivatives, these models extend beyond traditional approaches, offering a comprehensive perspective on viscoelastic phenomena. The following subsection will explore the mathematical foundation of fractional derivative models, emphasizing their relevance in material science and biomechanics.

### 5.1 Mathematical Foundation of Fractional Derivative Models

Fractional derivative models provide a sophisticated framework for capturing the viscoelastic behavior of biological tissues, which often display both solid-like and liquid-like characteristics. Unlike traditional integer-order calculus, fractional derivatives generalize differentiation and integration to non-integer orders, effectively modeling memory effects in materials [32]. This approach accurately describes the time-dependent responses of viscoelastic materials, where classical linear models may be insufficient [20].

A crucial component of these models is the Caputo-type fractional derivative, which facilitates the resolution of nonlinear fractional differential equations [50]. This method enhances classical linear viscoelastic frameworks by providing a nuanced representation of material behavior under dynamic conditions. The integration of fractional calculus into viscoelastic modeling is further exemplified by its connection with fractal geometry, underscoring the need for a spatial approach to modeling complex material behaviors [51].

Applications of fractional derivative models extend to the characterization of polymers and biological tissues, with models capturing viscoelastic behavior utilizing parameters from dynamic mechanical analysis to predict stress-strain relationships at high strain rates [52]. Additionally, a three-dimensional viscoelastic model at finite strain incorporates strain-dependent relaxation times, demonstrating the adaptability of fractional calculus across various material types [53].

In biological tissues, mode coupling theory (MCT) is generalized to account for oscillatory shear strain in complex fluids, providing insights into nonlinear viscoelastic behavior under diverse loading conditions [54]. This advancement is crucial for accurately modeling the mechanical responses of tissues subjected to complex, nonlinear deformations.

The mathematical principles of fractional derivative models enhance the accuracy and relevance of biomechanical assessments in research and clinical applications. These models effectively capture the complex, memory-dependent behaviors of biological tissues and cells, such as the viscoelastic properties of macrophages, reflecting their functional states and responses to pharmacological interventions [19, 20, 32]. By incorporating advanced mathematical concepts and computational techniques, these models provide a robust framework for understanding the intricate mechanical behaviors of biological tissues, informing the development of precise diagnostic and therapeutic strategies.

### 5.2 Comparison with Traditional Models

Fractional derivative models offer a substantial advancement over traditional viscoelastic models due to their ability to capture complex material behaviors with fewer parameters and greater accuracy.

Traditional models, such as the Maxwell and Burgers models, often require multiple parameters to describe viscoelastic properties, which can be cumbersome [55]. In contrast, fractional models utilize derivatives of non-integer order, providing a comprehensive framework for modeling time-dependent behavior in viscoelastic materials, including biological tissues and polymers [20].

A significant advantage of fractional models is their capacity to address anomalous relaxation and diffusion processes more effectively than traditional models. The use of a stretched exponential kernel in fractional derivatives enhances the modeling of these complex processes [56]. This capability is critical for materials exhibiting power-law behavior, where fractional models achieve accurate characterizations with fewer parameters [20].

Despite their advantages, fractional models face challenges, particularly regarding the arbitrary choice of kernels, which can limit practical applications [57]. Furthermore, the equivalence between new fractional models, such as those based on the Prabhakar function, and classical linear viscoelastic models remains an area of ongoing research [58]. These challenges highlight the need for further theoretical developments to fully exploit the potential of fractional derivative models.

The mathematical formulation of fractional models offers a robust framework for viscoelastic characterization. Deriving new spatial fractional models from established wave equations allows direct comparisons with classic models, demonstrating the enhanced predictive capabilities of fractional approaches [51]. This is further supported by the efficiency of fractional models in predicting high strain rate properties of polymers, reducing reliance on extensive experimental testing [52].

Moreover, fractional models have been validated against experimental data and multi-integral viscoelastic models, confirming their efficacy in capturing nonlinear viscoelastic behavior [53]. The existence of weak solutions to the governing equations of nonlinear viscoelasticity, particularly when the viscous stress tensor is defined by nonquadratic polynomial densities, underscores the mathematical rigor and applicability of fractional models in complex scenarios [59].

### 5.3 Innovations and Advantages

Fractional derivative models have emerged as a significant advancement in viscoelastic modeling, offering enhanced capabilities to capture complex behaviors in biological tissues. A primary innovation of these models is their ability to represent a broader spectrum of anomalous behaviors in relaxation and diffusion processes, surpassing the limitations of traditional viscoelastic models. This is particularly evident in the use of the Prabhakar fractional derivative, which provides a novel approach for modeling time-dependent and nonlinear mechanical responses of biological tissues [58].

Fractional derivative models present several advantages over traditional viscoelastic models. They can capture the complex viscoelastic behavior of tissues with fewer parameters, enhancing efficiency and versatility for modeling purposes [20]. This efficiency is crucial in clinical settings, where rapid and accurate assessments are necessary. Moreover, the capacity of fractional models to describe a broader spectrum of anomalous behaviors in relaxation and diffusion processes further enhances their applicability in diverse biomedical contexts.

Recent innovations in developing multiphase cartilage models have significantly improved biomechanical characterizations, capturing intricate interactions between cartilage structure and surrounding tissues, thus offering a more comprehensive understanding of the biomechanical properties of cartilage [27]. This advancement is critical for developing targeted therapeutic strategies and improving patient outcomes.

Fractional derivative models also provide a more robust mathematical framework for viscoelastic characterization. The advancement of new methodologies, such as the Atangana-Baleanu fractional derivative, has enhanced the precision of biomechanical assessments by incorporating complex physical phenomena into the modeling process [22]. These innovations not only improve the accuracy of viscoelastic models but also expand their potential applications in both research and clinical settings.

## 6 Shear Wave Dispersion and Biomechanical Properties

### 6.1 Fundamentals of Shear Wave Dispersion

Shear wave dispersion is pivotal in biomechanical tissue analysis, primarily through the examination of shear wave propagation to elucidate viscoelastic properties. Advances in shear wave elasticity imaging (SWEI) reveal that variations in shear wave speeds are indicative of tissue viscosity, with Fourier analysis and lookup tables enhancing frequency-dependent phase velocity derivation [1, 25]. This phenomenon, observed when shear waves traverse viscoelastic media, provides critical insights into tissue mechanics.

Shear wave elastography (SWE) is the leading technique for measuring shear wave dispersion, correlating wave speed with tissue stiffness to distinguish between healthy and pathological states [4]. The reverberant shear wave elastography (R-SWE) technique further refines this by utilizing a reverberant field, enhancing shear wave speed and viscoelastic property estimation accuracy [2]. Analyzing frequency-dependent tissue behavior through dispersion is vital for accurate biomechanical assessments, offering insights into microstructural and compositional attributes critical for clinical applications, such as liver fibrosis and breast cancer diagnostics.

Advanced computational techniques, including deep learning and optimization methodologies, have improved shear wave dispersion measurement precision. These innovations enhance wave speed estimation and reduce data noise, aiding in understanding biomechanical properties like elasticity and viscosity. This is particularly relevant for conditions such as diffuse liver disease, where shear wave speed and dispersion slope correlate with fibrosis and necroinflammation. Furthermore, 3D shear wave elasticity imaging (SWEI) facilitates comprehensive assessments of muscle tissue viscoelastic properties, essential for clinical applications [1, 60, 28, 25, 3].

### 6.2 Techniques and Methodologies

Characterizing shear wave dispersion in biological tissues involves advanced techniques crucial for accurately assessing their viscoelastic properties. Shear wave elastography (SWE) remains a primary technique, leveraging shear wave propagation speed to evaluate tissue stiffness and elasticity, effectively distinguishing between healthy and pathological tissues [4].

Reverberant shear wave elastography (R-SWE) enhances shear wave speed and viscoelastic property estimation accuracy by employing a reverberant field, improving biomechanical assessment reliability and reducing directional filtering effects [2].

Integrating deep learning frameworks and optimization methodologies has refined shear wave dispersion analysis, enabling precise wave speed data extraction and enhancing measurement robustness by reducing noise [14]. For example, deep fully-convolutional neural networks (DCNN) synthesize shear wave elastography images from conventional B-mode ultrasound data, expanding shear wave analysis accessibility [13].

Non-rigid image registration techniques address challenges posed by large tissue deformations, allowing accurate displacement measurements under significant compression [29]. These methodologies are crucial for improving shear wave speed estimation reliability, integral to biomechanical assessments.

Exploring quantum light in imaging technologies has shown promise in enhancing the signal-to-noise ratio, marking a significant advancement in biomechanical evaluation precision [39]. Furthermore, atomic force microscopy (AFM) and confocal microscopy facilitate nanoscale mechanical property characterization, providing detailed insights into biological structures' biomechanics [34].

Integrating these advanced techniques significantly enhances tissue elasticity evaluation accuracy and efficiency, broadening ultrasound elastography applications in medical diagnostics. This innovative approach allows non-invasive tissue stiffness assessments across various organs, including the liver, breast, thyroid, and pancreas, improving pathology detection, such as liver fibrosis and solid tumors, ultimately enhancing patient care and health outcomes [3, 26].

### 6.3 Applications in Tissue Pathology

Shear wave dispersion analysis is essential in understanding tissue pathology by elucidating mechanical properties indicative of pathological changes. This technique is particularly valuable in assessing liver fibrosis, where quantifying shear wave speed and dispersion aids in differentiating fibrosis stages, facilitating early diagnosis and treatment planning [25]. Non-invasive liver stiffness evaluation through shear wave elastography (SWE) offers significant advantages over traditional biopsy methods, reducing patient discomfort and risk [2].

In breast cancer diagnostics, shear wave dispersion analysis enhances differentiation between benign and malignant lesions. Correlating shear wave speed variations with tissue stiffness provides critical information for accurate breast tissue pathology characterization, improving diagnostic accuracy and informing treatment decisions [11]. This capability is further bolstered by advanced imaging techniques and deep learning frameworks, which enhance shear wave measurement resolution and reliability [14].

Shear wave dispersion analysis also applies to neurological disorders, assessing brain tissue viscoelastic properties, particularly in traumatic brain injury and neurodegenerative diseases, where stiffness changes reflect underlying pathological processes [7]. Non-invasive monitoring of these changes offers valuable insights into disease progression and therapeutic response.

In musculoskeletal pathology, shear wave dispersion analysis evaluates tendon and muscle stiffness, indicative of conditions like tendinopathy and muscular dystrophy. Quantifying shear wave speed in these tissues provides a non-invasive means of assessing pathological changes, aiding in diagnosis and management [4].

Moreover, shear wave dispersion analysis is instrumental in vascular diseases, assessing arterial wall mechanical properties. Variations in shear wave speed indicate arterial stiffness changes associated with conditions like atherosclerosis and hypertension [8]. This application underscores shear wave dispersion analysis's potential for early cardiovascular disease detection and management.

Shear wave dispersion analysis serves as a sophisticated non-invasive technique for evaluating tissue mechanical properties, enhancing diagnostic accuracy and treatment strategies across diverse pathological conditions. By leveraging shear waves' frequency-dependent behavior, this method provides insights into tissue elasticity and viscosity, critical for understanding various diseases, including liver fibrosis and other soft tissue abnormalities. Quantifying shear wave speed and dispersion slope enables clinicians to obtain valuable information regarding tissue stiffness and necroinflammation, facilitating informed decision-making in patient management [60, 25, 26].

## 7 Applications in Medical Diagnostics

### 7.1 Enhancement of Diagnostic Accuracy

The integration of ultrasound shear wave imaging (USWI) and advanced viscoelastic modeling significantly improves diagnostic accuracy by enabling non-invasive, patient-specific evaluations of tissue stiffness and elasticity. Techniques such as synthetic shear wave elastography (sSWE), which leverages deep learning for precise tissue elasticity imaging from conventional B-mode ultrasound, and 3D rotational shear wave elasticity imaging (SWEI) aid in characterizing muscle properties through shear wave speed measurements. These methodologies enhance the assessment of mechanical properties across various tissues, facilitating early detection and management of conditions like cancer and musculoskeletal disorders [2, 28, 13, 42]. The insights derived from these techniques are crucial as they often reflect pathological changes, thereby enhancing diagnostic evaluations in clinical practice.

In breast cancer diagnostics, shear wave elastography (SWE) provides significant advantages by offering patient-specific stiffness estimates that enhance surgical intervention accuracy [61]. In prostate cancer, the S-WAVE system exhibits strong correlations between elasticity values and cancer presence, further improving diagnostic precision [8]. Additionally, SWE's application in placental assessments enables non-invasive evaluations of placental health, which is vital for maternal and fetal outcomes [9].

The correlation between elasticity measurements from ultrasound shear wave elastography (US-SWE) and traditional biopsy methods underscores the potential of these techniques to offer complementary diagnostic information [62]. This is particularly beneficial in diagnosing chronic obstructive pulmonary disease (COPD), where the YM 1-3 biomarker based on lung tissue elasticity improves diagnostic accuracy and sensitivity compared to conventional methods [63].

In ophthalmology, Optical Pulse Elastography (OPE) provides a non-invasive means to assess corneal biomechanics, aligning well with established techniques and serving as a valuable clinical diagnostic tool [10]. The advent of reverberant shear wave elastography (R-SWE) simplifies data acquisition and processing, enabling rapid estimates without requiring detailed knowledge of wave propagation direction, thereby enhancing diagnostic efficiency [1].

Recent advancements in ultrasound-based tissue characterization, including model-based feature extraction for lesion classification and synthetic elastography utilizing deep learning, highlight their significant contributions to diagnostic accuracy across various medical conditions. These techniques capitalize on the unique elastic properties of soft tissues, allowing precise differentiation between healthy and pathological states, which is critical for effective patient management and treatment planning. Innovations such as model-based approaches and advanced mechanical characterization techniques provide valuable insights into the biomechanical properties of tissues, representing a transformative shift in diagnostic imaging and assessment [13, 64, 15].

### 7.2 Applications in Tissue Stiffness Assessment

Assessing tissue stiffness is crucial in medical diagnostics, offering insights into the mechanical properties indicative of pathological conditions. Ultrasound shear wave imaging (USWI) and viscoelastic modeling are essential non-invasive techniques that enable precise evaluations of tissue stiffness. USWI employs advanced imaging methods to measure shear wave propagation, allowing for the assessment of parameters such as longitudinal and transverse shear moduli, vital for characterizing tissues, especially muscles, as elastic, incompressible, and transversely isotropic [2, 28, 26]. This high precision in tissue stiffness assessment is invaluable for diagnosing and monitoring various pathological conditions.

Shear wave elastography (SWE) has emerged as a fundamental technique for evaluating tissue stiffness, particularly in liver fibrosis. By measuring shear wave speed, SWE provides quantitative estimates of liver stiffness, essential for staging fibrosis and monitoring disease progression. This non-invasive approach, utilizing advanced imaging techniques such as ultrasound elastography and computed tomography-based elasticity modeling, significantly enhances lesion classification by accurately assessing tissue stiffness, thereby reducing patient discomfort and risk compared to traditional biopsy methods. SWE has demonstrated accuracy improvements of 5

In musculoskeletal applications, SWE evaluates tendon and muscle stiffness, crucial for diagnosing conditions like tendinopathy and muscular dystrophy. Accurate tissue stiffness measurements aid in assessing injury severity and monitoring rehabilitation progress [4]. Furthermore, assessing nerve stiffness during limb movements with SWE provides valuable insights into neuromuscular conditions, enhancing diagnostic capabilities [6].

In breast cancer diagnostics, SWE plays a vital role in differentiating benign from malignant lesions by correlating shear wave speed with tissue stiffness, thereby improving detection accuracy and informing treatment decisions [11]. Similarly, in prostate cancer, the S-WAVE system shows strong correlations between elasticity values and cancer presence, facilitating early detection and intervention [8].

The application of reverberant shear wave elastography (R-SWE) further enhances tissue stiffness assessment by providing rapid, reliable estimates without complex boundary conditions or explicit knowledge of wave propagation direction [1]. This innovation simplifies data acquisition and processing, making it a valuable tool for clinical diagnostics.

The integration of USWI and viscoelastic modeling in tissue stiffness assessment represents significant advancements in medical diagnostics. These advanced imaging techniques, including ultrasound elastography and model-based feature extraction, offer comprehensive, non-invasive evaluations of tissue mechanical properties, crucial for accurate diagnosis and treatment planning. By assessing variations in tissue stiffness associated with various pathologies, these methods enhance clinical decision-making and optimize patient care across diverse medical conditions, such as liver fibrosis, breast cancer, and other soft tissue lesions. Their capacity to provide quantitative, patient-specific stiffness measurements not only improves surgical intervention precision but also supports the development of biomechanical models predicting tissue behavior during treatment [61, 64, 26].

### 7.3 Clinical Applications and Diagnostic Accuracy

The incorporation of ultrasound shear wave imaging (USWI) and advanced viscoelastic modeling techniques into clinical practice has markedly improved diagnostic accuracy by enabling the assessment of tissue biomechanical properties across various medical applications, including the evaluation of organs such as the liver, breast, and prostate, and facilitating the identification of malignant lesions through enhanced tissue characterization [42, 13, 3]. These advanced imaging techniques provide non-invasive, quantitative assessments of tissue mechanical properties, offering critical insights into the structural integrity and pathological status of tissues, essential for accurate diagnosis and treatment planning.

In breast cancer diagnostics, shear wave elastography (SWE) has proven effective in differentiating between benign and malignant lesions. By correlating shear wave speed with tissue stiffness, SWE provides a reliable metric for assessing tumor characteristics, thereby improving diagnostic specificity and sensitivity. This capability is crucial for early detection and intervention, ultimately enhancing patient outcomes [11].

The application of USWI in liver disease, particularly in assessing liver fibrosis, exemplifies its clinical utility. Shear Wave Elastography (SWE) serves as a non-invasive alternative to traditional liver biopsy by accurately measuring liver stiffness, a critical parameter for assessing fibrosis extent and tracking liver disease progression. This imaging technique utilizes acoustic radiation force to generate shear waves, enabling healthcare professionals to obtain reliable elasticity estimates, essential for effective disease management and monitoring. Recent advancements in SWE technology have expanded its application beyond liver assessments to other organs, enhancing its clinical utility in diagnosing various conditions [13, 26, 60, 3, 42]. This approach mitigates the risks associated with invasive procedures while providing reliable diagnostic information.

In musculoskeletal disorders, USWI facilitates the evaluation of tendon and muscle stiffness, aiding in diagnosing and managing conditions such as tendinopathy and muscular dystrophy. The ability to assess tissue mechanical properties in vivo enhances the understanding of injury mechanisms and supports targeted rehabilitation strategies [4]. Additionally, measuring nerve stiffness during limb movements with SWE provides valuable insights into neuromuscular conditions, contributing to more accurate diagnoses and improved therapeutic outcomes [6].

USWI also plays a pivotal role in cardiovascular diagnostics, where it assesses arterial wall stiffness. Variations in arterial stiffness, measured by shear wave speed, can indicate vascular diseases such as atherosclerosis and hypertension, facilitating early detection and intervention [8].

The integration of advanced computational techniques, including deep learning frameworks, into USWI has further enhanced its diagnostic capabilities. These innovations improve shear wave speed measurement accuracy and efficiency, reducing noise and enhancing image quality, thus providing clinicians with more reliable diagnostic information [14].

The clinical applications of USWI and viscoelastic modeling underscore their transformative impact on medical diagnostics. By utilizing advanced ultrasound elastography techniques that provide detailed, non-invasive assessments of tissue mechanical properties, clinicians can achieve enhanced diagnostic accuracy and make more informed decisions in patient care. These techniques leverage tissue stiffness variations associated with various medical conditions, such as liver fibrosis and breast cancer, to deliver both qualitative and quantitative data. The ability to measure tissue stiffness through specialized imaging modes, including strain and shear wave imaging, enables real-time evaluations at the bedside, improving clinical outcomes across a diverse range of pathologies [61, 26].

## 8 Challenges and Future Directions

### 8.1 Challenges and Limitations

Despite the progress in ultrasound shear wave imaging (USWI) and viscoelastic modeling, challenges such as variability in measurement outcomes due to differences in ultrasound systems and the complexity of biological tissues impede the establishment of standardized diagnostic criteria [13]. Traditional models like Maxwell and Burgers require multiple parameters for accurate characterization, often resulting in inefficiencies and a failure to capture nonlinear tissue behaviors, highlighting the need for advanced modeling techniques [55, 20]. While fractional calculus offers a promising approach by representing time-dependent strain responses and memory effects, its complexity and the need for standardization limit its clinical application [57, 32].

Dynamic Distributed Learning's dependence on robust network infrastructure can hinder performance in resource-limited settings [14]. Additionally, assumptions of isotropy and homogeneity in models like Magnetic Resonance Elastography (MRE) challenge modulus estimation accuracy [23]. Nonlinear viscoelastic behaviors under varying conditions are not well-represented by existing models, which underscores the necessity for comprehensive models that accurately depict the dynamic mechanical responses of tissues [24, 32]. The practical implementation of fractional calculus in viscoelastic modeling remains challenging due to its mathematical complexity, requiring further simplification for broader clinical use [32].

### 8.2 Technological Advancements and Algorithm Development

Recent technological advances in USWI and viscoelastic modeling, such as the use of quantum light for improved signal-to-noise ratios, have significantly enhanced biomechanical assessments [39]. The development of 3D shear wave imaging allows real-time acquisition of displacement fields, offering comprehensive insights into tissue biomechanics [42, 8]. Deep learning frameworks, including deep fully-convolutional neural networks, have expanded USWI's applicability by enhancing the efficiency and accuracy of shear wave elastography image synthesis from conventional ultrasound data [13].

Non-rigid image registration techniques improve displacement measurements under significant tissue compression, facilitating more accurate non-invasive vascular assessments [29]. In viscoelastic modeling, hybrid models that integrate synthetic and experimental data enhance training efficiency and model robustness [65]. Optimization-based approaches, such as phase unwrapping combined with neural networks, have advanced displacement extraction and modulus estimation, offering noise-resistant methodologies [23]. Future research could extend these results to higher dimensions and explore new applications in clinical settings [33].

Techniques like atomic force microscopy (AFM), confocal microscopy, and finite element simulations provide nanoscale insights into the biomechanics of biological structures [34]. Future studies should focus on the mechanical properties of biological membranes across developmental stages and pathological conditions, as well as the implications of poroelastic behavior on cellular signaling and drug delivery [34]. Recent model-based feature extraction techniques using Computed Tomography (CT) have demonstrated high efficacy in distinguishing malignant from benign lesions, showing significant improvements over existing classification methods [64, 12]. By addressing current challenges and exploring innovative approaches, the field continues to evolve, ultimately contributing to improved patient care and outcomes.

### 8.3 Emerging Applications and Future Potential

USWI and viscoelastic modeling are poised for transformative advancements in medical diagnostics and therapeutic strategies. Emerging applications and ongoing research are enhancing the capabilities of robotic technologies in diagnostics and interventions. Notable advancements include the use of computed tomography (CT) for machine learning-based lesion classification, achieving high area under the curve scores for colon polyps and lung nodules, which surpass existing methods by 5

Refining experimental protocols for medium amplitude parallel superposition and integrating advanced data analysis techniques can broaden material characterization [43]. The integration of deep learning frameworks into USWI methodologies enhances accessibility and reduces the need for specialized equipment [13].

In ocular diagnostics, refining USWI systems for clinical use and exploring their applicability in various ocular diseases can significantly enhance understanding of corneal biomechanics, especially in the context of refractive surgeries [66, 29]. Future research should elucidate the relationships between viscoelasticity and biological processes, with the aim of developing new biomaterials for innovative therapeutic applications [67, 45]. Optimizing viscoelastic gels and exploring their applications in biological contexts may provide insights into disease mechanisms related to altered tissue mechanics [67].

The exploration of hybrid models that combine synthetic and experimental data holds promise for improving training efficiency and incorporating physical constraints to enhance model robustness [65]. Extending nonlinear viscoelasticity frameworks to complex material models and in vivo applications could explore the impact of different excitation configurations on measurement accuracy [25, 58]. The integration of advanced computational techniques, such as deep learning frameworks and optimization-based methodologies, has further enhanced diagnostic capabilities by improving wave speed estimations and reducing noise [68]. Future research should focus on refining optimization algorithms, exploring additional radial basis function types, and validating approaches across a wider range of materials and loading conditions [69]. The ongoing refinement of mathematical models and experimental techniques continues to deepen our understanding of the intricate mechanical behaviors inherent in biological tissues, contributing to improved healthcare outcomes.

# 9 Conclusion

The convergence of ultrasound shear wave imaging (USWI) with viscoelastic modeling marks a pivotal advancement in medical diagnostics, offering non-invasive means to assess tissue mechanical properties with precision. These methodologies have significantly enriched the understanding of tissue biomechanics, enhancing diagnostic accuracy across a spectrum of medical conditions. In musculoskeletal diagnostics, shear wave elastography (SWE) facilitates comprehensive evaluation of muscle properties, thereby improving patient outcomes. Its application extends to breast ultrasonography, where it plays a crucial role in refining BI-RADS categories and predicting breast cancer prognosis, underscoring its clinical utility.

Innovative imaging technologies, such as optical elastography, are enhancing diagnostic capabilities, particularly in fields like oncology and ophthalmology, where accurate tissue characterization is critical. Despite these advancements, the variability in measurement outcomes due to diverse ultrasound systems and the complexity of biological tissues underscores the need for standardized practices to ensure measurement consistency and reliability.

The integration of advanced computational techniques, such as the Dynamic Distributed Learning method, holds promise for real-time applications by significantly reducing processing time while maintaining accuracy. This development complements the role of USWI in medical diagnostics, facilitating rapid and reliable assessments that support timely clinical decision-making.

The progress in USWI and viscoelastic modeling underscores their transformative impact on medical diagnostics. By providing detailed insights into tissue biomechanics, these techniques refine diagnostic evaluations and contribute to enhanced patient care and outcomes. Future research should focus on refining these technologies and developing standardized protocols to expand their clinical applications and efficacy.

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