Covariance Consistency and Realism in Orbital Mechanics: A Survey

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Abstract

This survey paper explores the interconnected concepts of covariance consistency, covariance realism, uncertainty realism, orbit uncertainty propagation, state estimation, and error analysis in astrodynamics and aerospace engineering. These concepts are crucial for accurately modeling and predicting the uncertainties and dynamics of spacecraft and celestial bodies' trajectories. The paper emphasizes the significance of integrating inertial navigation systems and state estimation algorithms, particularly in the context of nonlinear dynamics and high-dimensional systems. Recent innovations in covariance modeling, such as adaptive fusion matrices and the Least-Squares Kernel Kalman Filter, enhance the robustness and reliability of state estimation. The survey also highlights the role of human-based and physics-based data fusion in tracking Resident Space Objects (RSOs) and the challenges posed by space debris. Advanced methodologies like the Time-Varying Directional State Transition Tensor (TDSTT) and machine learning approaches offer promising directions for improving uncertainty propagation and state estimation accuracy. The paper concludes by underscoring the importance of continued research in developing adaptive algorithms and innovative methodologies to address the complexities and uncertainties inherent in orbital mechanics, ultimately supporting effective decision-making and mission planning.

1 Introduction

1.1 Interconnected Concepts in Astrodynamics

Astrodynamics encompasses a complex interplay of concepts essential for accurately modeling and predicting the trajectories of spacecraft and celestial bodies. Key elements include state estimation and uncertainty modeling, which are crucial for managing dynamics and ensuring mission accuracy. The integration of inertial navigation systems and Inertial Measurement Units (IMUs) with state estimation in three-dimensional space is foundational in orbital mechanics, providing vital data for trajectory analysis [1]. Furthermore, the relevance of state estimation algorithms and kinematic systems modeled on Lie groups illustrates the interconnectedness of these concepts in orbital mechanics [2].

Challenges in estimating rigid body motion significantly affect various mechanical systems, including spacecraft, emphasizing the necessity of accurate pose estimation for position, velocity, and orientation in astrodynamics. The relationship between attitude control and state estimation is particularly critical for small-scale launch vehicles, where precise control is vital for mission success [3]. Additionally, robust estimation in nonlinear state space models underscores the importance of covariance consistency and robustness in state estimation [4].

The fusion of human-based data sources, such as Two-Line Elements (TLEs), with physics-based data sources is essential for tracking Resident Space Objects (RSOs), highlighting the interconnectedness of data sources in orbital mechanics [5]. The complexities of orbit determination and catalog correlation, especially with non-cooperative satellites, further illustrate the challenges posed by these

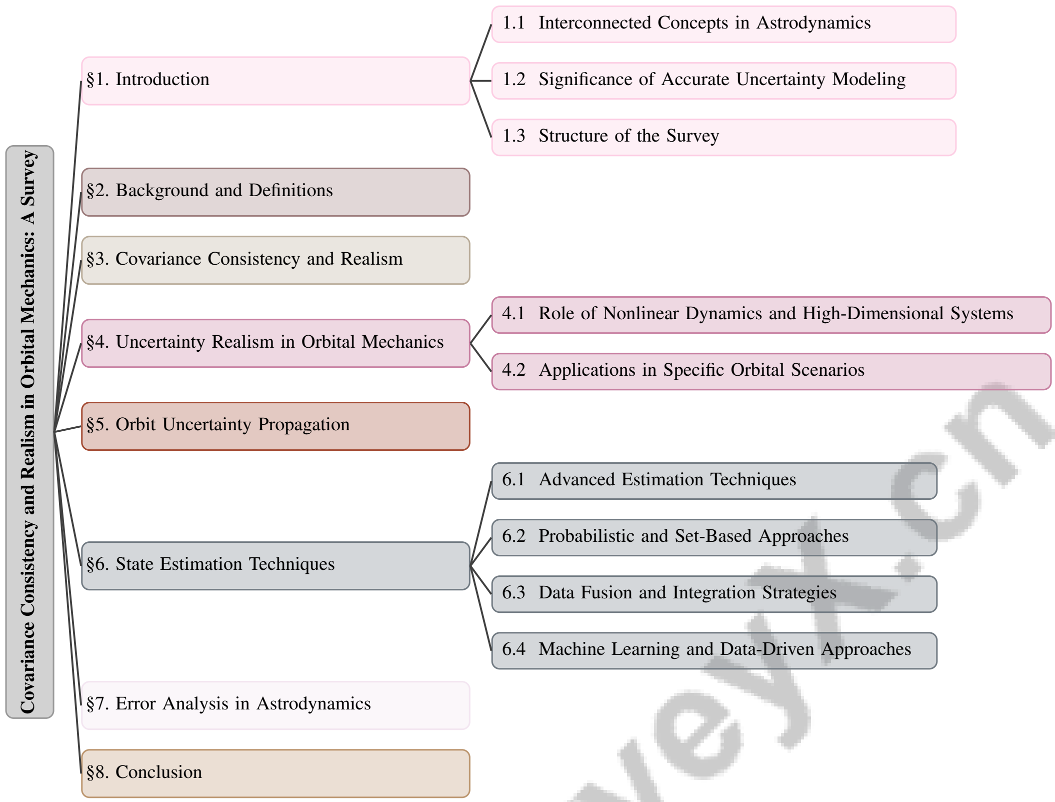

Figure 1: chapter structure

interconnected concepts [6]. The Incremental Covariance Estimation (ICE) method enhances state estimation under degraded observations, thereby improving orbit determination reliability [7].

Automated rendezvous and docking processes exemplify the critical interplay between pose estimation, orbital mechanics, and uncertainty modeling, particularly for free-falling target spacecraft [8]. Furthermore, the optimization of low-thrust trajectories demonstrates the interconnected nature of trajectory optimization, orbital mechanics, and uncertainty modeling [9]. Innovative concepts like the space elevator reflect advancements in astrodynamics, showcasing the synergy between advanced launch methods and orbital mechanics [10].

The assessment of asteroid threats necessitates quantitative evaluations of impact distributions, underscoring the importance of orbital mechanics and uncertainty modeling in risk management [11]. Additionally, the issue of space debris poses significant risks to future space operations, particularly from uncontrolled objects in low Earth orbit (LEO) and geostationary orbit (GEO), necessitating robust uncertainty modeling strategies for debris management [12]. Collectively, these interconnected concepts form a comprehensive framework essential for accurate modeling and prediction in orbital mechanics, advancing astrodynamics toward improved uncertainty management and decision-making in space exploration and mission planning.

1.2 Significance of Accurate Uncertainty Modeling

Accurate uncertainty modeling is crucial in orbital mechanics, significantly influencing the precision of trajectory predictions for celestial bodies and spacecraft. The integration of state estimation into the optimization of observer placement, particularly for cislunar Space Domain Awareness (SDA), highlights the necessity for precise uncertainty modeling in satellite constellation design [13]. The increasing number of satellites and the accumulation of orbital debris further emphasize the need for advancements in detection and state estimation, underscoring the importance of precise uncertainty modeling for maintaining space situational awareness [14].

Traditional Inertial Navigation Systems (INS) face limitations due to sensor noise-induced drift, which underscores the critical need for accurate uncertainty modeling in spacecraft dynamics [15].

This necessity is echoed in autonomous navigation in cislunar space, where range measurements have shown superior state estimation capabilities compared to range-rate measurements, thus enhancing navigation accuracy [16].

Accurate motion estimation is vital for autonomous grasping of tumbling satellites in robotic systems, particularly in automated rendezvous and docking scenarios [8]. Similarly, precise modeling of control systems for thrust-vector-controlled small-scale launch vehicles is essential due to their inherent instability and trajectory correction challenges [3]. These aspects highlight the significance of accurate uncertainty modeling for successful mission execution.

Tracking Resident Space Objects (RSOs) poses notable challenges, particularly when integrating uncertain data sources, which is essential for improving trajectory predictions [5]. The development of fast and reliable orbit determination algorithms is critical for real-time applications in space situational awareness, further emphasizing the need for precise uncertainty modeling [6].

In educational contexts, precise simulation is vital for teaching orbital mechanics, as accurate modeling enhances the understanding of celestial trajectories [17]. The robust performance of proposed estimation schemes in accurately estimating rigid body motion addresses challenges posed by nonlinear dynamics and sensor noise, underscoring the importance of precise uncertainty modeling in state estimation processes [18].

The push for advancements beyond current rocket technology, including the concept of a space elevator, highlights the importance of accurate uncertainty modeling in predicting the trajectories of celestial bodies and spacecraft [10]. Active debris removal (ADR) missions further emphasize the necessity of precise uncertainty modeling in mission planning to mitigate risks associated with space debris [12]. Moreover, the complexities of gravitational modeling due to irregular asteroid shapes underscore the importance of accurate uncertainty modeling in trajectory prediction [19].

Recent studies collectively highlight the critical need for sophisticated uncertainty modeling techniques in orbital mechanics, particularly in addressing significant challenges posed by atmospheric density variations and other stochastic factors. Employing advanced methodologies such as probabilistic machine learning models, separated representations for uncertainty propagation, and innovative filtering techniques like the covariance sigma point filter enhances the accuracy and reliability of orbital predictions. This improved precision is vital for effective space situational awareness and safe navigation of spacecraft, ultimately ensuring mission success and mitigating risks associated with space debris [20, 21, 22, 23, 24].

1.3 Structure of the Survey

This survey is structured to provide a comprehensive examination of interconnected concepts within orbital mechanics, focusing on covariance consistency, covariance realism, uncertainty realism, orbit uncertainty propagation, state estimation, and error analysis. The paper begins with an introduction to essential concepts in astrodynamics and aerospace engineering, establishing foundational principles necessary for understanding the intricate dynamics of space systems, including space elevator design, the historical context of Kepler's equations, and implications of robotic manipulation in space exploration. By elucidating these core ideas, the paper highlights their critical significance in advancing technologies for orbital mechanics, satellite servicing, and enhancing space situational awareness [25, 26, 27, 10, 28].

The second section delves into background and definitions, offering detailed explanations of each concept and their applications. It also explores the broader context of uncertainty in orbital mechanics and introduces unified mathematical frameworks that integrate these concepts.

The third section focuses on covariance consistency and realism, discussing principles that ensure realistic and consistent state variable estimations over time. It reviews recent innovations in covariance modeling and addresses current challenges and future research directions.

In the fourth section, the survey examines uncertainty realism in orbital mechanics, analyzing the role of nonlinear dynamics and high-dimensional systems. This section discusses specific applications of uncertainty realism in various orbital scenarios and missions.

The fifth section addresses orbit uncertainty propagation, discussing techniques and mathematical frameworks used to propagate uncertainties over time. It reviews both stochastic and analytical

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methods, as well as numerical techniques and frameworks, emphasizing their importance for reliable decision-making and mission planning.

The sixth section reviews state estimation techniques, comparing various methods and their effectiveness in handling uncertainties and providing accurate trajectory predictions. It explores advanced estimation techniques, probabilistic and set-based approaches, data fusion and integration strategies, and the application of machine learning and data-driven methods.

The seventh section analyzes error analysis in astrodynamics, examining sources of error, the impact of computational complexity, and techniques for mitigating errors to enhance prediction reliability.

The conclusion synthesizes the primary findings of the survey, emphasizing the critical roles of covariance consistency, realistic modeling, and precise uncertainty propagation in orbital mechanics. These factors significantly enhance the accuracy and reliability of orbital predictions, particularly in the context of nonlinear uncertainty propagation and challenges posed by atmospheric density variations [21, 29, 23, 20]. It also suggests potential areas for future research and development in this field.The following sections are organized as shown in Figure 1.

2 Background and Definitions

2.1 Broader Context of Uncertainty in Orbital Mechanics

Uncertainty in orbital mechanics presents significant challenges in predicting and managing spacecraft trajectories. A major contributor to these uncertainties is the drift and inaccuracies in state estimation, which are often inadequately addressed by existing methods due to the intricate nature of orbital dynamics [15]. The integration of uncertain data sources, such as Two-Line Elements (TLEs) and radar observations, further complicates the tracking and prediction of Resident Space Objects (RSOs) [5].

The synchronization of motion between spacecraft and manipulators, compounded by environmental disturbances, exemplifies the complexities inherent in orbital mechanics [30]. These factors necessitate robust decision-making frameworks capable of real-time operation, highlighting the broader implications of uncertainty in space operations.

Uncertainties in model parameters and external disturbances critically affect the control accuracy of small-scale launch vehicles, adding complexity to the uncertainty landscape in orbital mechanics [3]. The limitations of current space launch methods, particularly concerning space elevators, underscore inefficiencies that contribute to uncertainty in interplanetary travel, with broader implications for future exploration [10].

The Kessler syndrome illustrates the potential for exponential increases in space debris due to collisions, emphasizing the risks posed by uncertainty to sustainable space operations [12]. Additionally, the challenges presented by irregularly shaped asteroids complicate gravitational modeling and trajectory prediction, further demonstrating the pervasive nature of uncertainty in this field [19].

These challenges in space missions underscore the urgent need for sophisticated methodologies and innovative strategies to enhance prediction accuracy and reliability in orbital mechanics. Addressing complex uncertainties, particularly those arising from atmospheric density variations, is crucial for space situational awareness. Recent advancements, such as probabilistic machine learning models like HASDM-ML, CHAMP-ML, and MSIS-UQ for atmospheric density estimation, have been explored to improve orbit uncertainty quantification. Techniques like the consider covariance sigma point (CCSP) filter and separated representations for orbit uncertainty propagation can mitigate computational burdens and enhance prediction accuracy, providing a robust framework for navigating the multifaceted uncertainties in this domain [21, 22].

2.2 Unified Mathematical Frameworks

Unified mathematical frameworks are essential for integrating covariance, uncertainty, and error analysis concepts in orbital mechanics, thereby enhancing modeling and prediction precision. The Ensemble Kalman Filter (EnKF) exemplifies the application of Bayesian frameworks in uncertainty quantification, effectively coupling ensemble members through empirical covariance for robust estimation in ill-posed scenarios [31]. Complementing this is the Time-Varying Directional State

Transition Tensor (TDSTT) method, which integrates eigenvalues and eigenvectors of the covariance matrix, providing a unified approach to uncertainty propagation [32].

The kernel smoothing-based mixture Kalman filter (MKF-KS) introduces a novel state estimation approach that leverages an expectation-maximization algorithm based on disturbance smoothing to enhance parameter estimation [33]. This method, along with the Incremental Covariance Estimation (ICE) technique utilizing a Gaussian Mixture Model (GMM) for adaptive measurement uncertainty estimation, illustrates the integration of statistical techniques into state estimation frameworks [7].

The Projected Cumulative Distribution (PCD) method systematically approaches deterministic sampling by projecting continuous density functions onto univariate distributions and matching their cumulatives, contributing to the unification of uncertainty modeling techniques [34]. Additionally, the log-linear error state model based on matrix Lie group properties allows for the analysis of Inertial Navigation System (INS) initial alignment without first-order approximations, further demonstrating the integration of advanced mathematical constructs into state estimation [35].

To address the limitations of existing methods like the extended Kalman filter (EKF), a proposed methodology for designing a globally optimal filter for nonlinear systems enhances reliability in state estimation through global guarantees [36]. Employing symplectic geometric tools to formulate the optimal orbit stabilization problem underscores the role of geometric methods in enhancing stability and accuracy in orbital mechanics models [37].

The Regularized 4D-Var optimization approach synchronizes a master system (truth) and a slave system (model) to minimize a cost function under observational constraints, showcasing the integration of optimization techniques in uncertainty modeling [38]. Moreover, the extended state distributed Kalman filter estimates both the original state and nonlinear dynamics, providing a comprehensive framework for addressing complex dynamical systems in orbital mechanics [39].

These frameworks significantly advance the field of orbital mechanics by offering sophisticated methodologies for integrating covariance, uncertainty, and error analysis, particularly concerning atmospheric density modeling and orbit uncertainty propagation. By employing advanced machine learning techniques and robust statistical methods, such as Monte Carlo simulations and separated representations, these frameworks enable more precise assessments of orbital state predictions, ultimately improving accuracy and reliability in forecasting spacecraft and space debris trajectories. Models like HASDM-ML, CHAMP-ML, and MSIS-UQ facilitate a nuanced understanding of atmospheric density uncertainties, while Kalman filtering techniques allow for efficient orbit uncertainty propagation, collectively contributing to safer and more effective space missions [40, 21, 23, 22].

3 Covariance Consistency and Realism

3.1 Innovations in Covariance Modeling

Recent advancements in covariance modeling have significantly enhanced state estimation in orbital mechanics. The introduction of nine new dynamical equations has improved understanding of particle motion within the gravitational fields of rotating asteroids, offering a refined approach to covariance modeling in complex gravitational settings [19]. Moreover, adaptive fusion matrices and filtering gains in distributed Kalman filters facilitate real-time evaluation of estimation accuracy and ensure covariance boundedness under mild assumptions, thus increasing model robustness in dynamic and uncertain environments [39]. The least-squares kernel Kalman filter (LSK-KF) addresses scalability and computational demands, providing an efficient solution for large-scale models [41].

As illustrated in Figure 2, these innovations highlight key advancements in covariance modeling, including the new dynamical equations for asteroid gravitational fields, adaptive Kalman filters designed for real-time evaluation, and scalable solutions that enhance computational efficiency in large-scale models. These developments integrate advanced probabilistic machine learning models and uncertainty propagation methods, improving atmospheric density estimations and managing complex uncertainties, thereby enhancing the realism and reliability of trajectory forecasts crucial for space situational awareness and collision avoidance in low Earth orbit [21, 22, 29, 42, 5].

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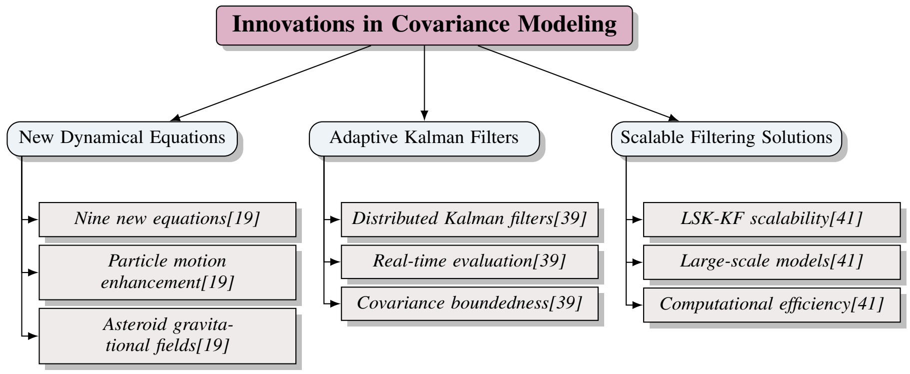


Figure 2: This figure illustrates key innovations in covariance modeling, highlighting new dynamical equations for asteroid gravitational fields, adaptive Kalman filters for real-time evaluation, and scalable solutions to enhance computational efficiency in large-scale models.

3.2 Challenges and Future Directions

Achieving covariance consistency and realism in state estimation presents challenges, particularly in computational complexity and adaptability for real-time applications. High computational demands hinder rapid decision-making in systems with numerous states [41]. Traditional methods struggle with correlations from process noise and sequential measurements, affecting optimal state estimation and prediction accuracy [43]. Kalman methods' limitations in incorporating constraints naturally reduce robustness and physical validity in dynamic modeling [44]. In decentralized scenarios, methods like Covariance Intersection may produce overly conservative estimates, slowing convergence and reducing accuracy due to inflated covariances from fusion methods [45, 46]. Inconsistencies from state-dependent propagation Jacobians further complicate achieving covariance consistency [47].

Future research should focus on integrating advanced mathematical tools to overcome these challenges. Leveraging symplectic geometry could enhance optimal control for periodic orbits [37], while developing methods to inherently account for constraints within Kalman frameworks would improve robustness and validity. Machine learning advancements offer promising paths for enhancing model accuracy without prior system knowledge. Recent methodologies combining learned simulators with symbolic regression indicate potential in rediscovering orbital mechanics, suggesting a promising direction for refining state estimation frameworks [40]. Extending the Ballistic Multibody Estimator (BME) to closed-kinematic chains and enhancing its robustness in real-world applications could yield significant benefits [48].

Addressing these challenges requires innovative methodologies and sophisticated computational tools to achieve covariance consistency and realism, crucial for accurate and reliable state estimation. Advanced ensemble modeling techniques addressing atmospheric density uncertainties, alongside machine learning methods for orbit prediction, show promise in improving state estimation accuracy. Utilizing separated representations for orbit uncertainty propagation can alleviate computational burdens associated with high-dimensional stochastic differential equations. By integrating these advanced techniques, researchers can enhance the robustness and precision of orbital state estimations, improving space situational awareness and operational effectiveness across aerospace applications [17, 20, 21, 22, 40].

4 Uncertainty Realism in Orbital Mechanics

Uncertainty realism in orbital mechanics requires an in-depth understanding of the factors complicating trajectory predictions, particularly the interplay of nonlinear dynamics and high-dimensional systems. These dynamics influence spacecraft and celestial body motion, necessitating sophisticated methodologies to address associated challenges.

4.1 Role of Nonlinear Dynamics and High-Dimensional Systems

Nonlinear dynamics and high-dimensional systems are pivotal in shaping uncertainty realism in orbital mechanics, directly impacting trajectory prediction accuracy. The complexity of nonlinear dynamics can lead to unpredictable behaviors, necessitating advanced methodologies for effective management. Traditional linearization techniques often fail under extreme conditions, such as those encountered by thrust-vector-controlled launch vehicles, inadequately capturing the full dynamics [3].

Robust approaches like Regularized Moving Horizon Estimation (RMHE) enhance state estimation reliability by employing regularization to counteract measurement noise and parametric uncertainty [49]. Incremental Covariance Estimation (ICE) is crucial for realistic uncertainty quantification, effectively managing nonlinear dynamics and high-dimensional systems [7]. Ensuring accurate observability and consistency in state estimation requires rendering the propagation Jacobian independent of the state, crucial for representing nonlinear dynamics precisely [47]. The space elevator concept further illustrates the importance of nonlinear dynamics in achieving realistic uncertainty quantification in trajectory predictions [10].

Advanced sampling methods, including deterministic sampling, generate numerous samples for continuous density functions accurately without gradient-based optimization, addressing high-dimensional system challenges [34]. The log-linear error state model enhances Inertial Navigation Systems (INS) alignment across various scenarios, underscoring the need for advanced methodologies in managing nonlinear dynamics [35]. The stable manifold method's success in point stabilization suggests potential adaptations for periodic orbit stabilization, indicating promising directions for managing nonlinear dynamics [37]. Irregularly shaped asteroids exemplify the impact of nonlinear dynamics and high-dimensional systems on uncertainty realism, necessitating comprehensive approaches for accurate gravitational modeling and trajectory prediction [19].

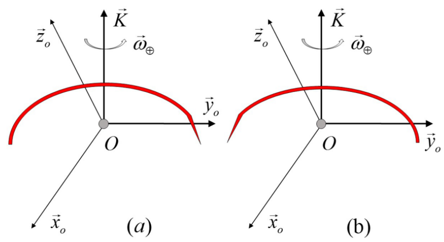
The Extended State Distributed Kalman Filter (ESDKF), which relies on system observability and strong network topology connectivity to ensure bounded estimation covariances, highlights the importance of advanced filtering techniques in addressing nonlinear dynamics and high-dimensional systems [39]. The Ballistic Multibody Estimator (BME) outperforms traditional methods like the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) in scenarios with parameter uncertainty, showcasing its potential as a scalable state estimation solution for free-flying open kinematic chains [48]. These methodologies underscore the critical role of addressing nonlinear dynamics and high-dimensional systems in enhancing uncertainty realism, leading to more reliable trajectory predictions.

4.2 Applications in Specific Orbital Scenarios

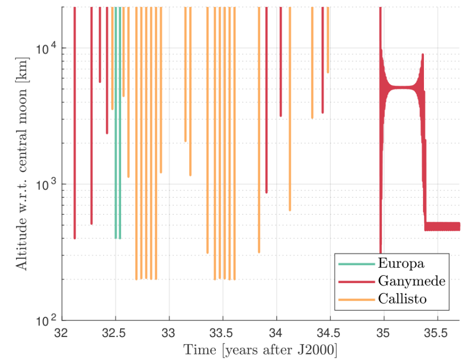
Uncertainty realism is crucial across various orbital scenarios, where precise trajectory prediction is essential for mission success. In low-thrust orbit-raising missions, high-fidelity models with high-order geopotential harmonics assess trajectory uncertainties, with the Cramér-von Mises test validating uncertainty propagation robustness [24]. In cooperative localization, minimizing communication costs while ensuring accuracy is vital, especially in missions involving multiple spacecraft [47]. Decentralized state estimation techniques, particularly in atmospheric dispersion scenarios, illustrate the significance of robust connectivity and managing disconnections, akin to space mission challenges [50].

The RMHE method's robustness in state estimation is demonstrated by its stable estimates even without persistent excitation, crucial in dynamic orbital environments with external disturbances [49]. Advanced Monte Carlo simulation methods, such as the MAC method, are effective in two-dimensional target tracking, outperforming optimal fusion and covariance intersection techniques [51]. These applications emphasize the role of uncertainty realism in reliable decision-making and mission planning across diverse scenarios. By addressing inherent uncertainties in the dynamic space environment, methodologies like separated representations and state transition tensors (STTs) enable accurate orbit uncertainty propagation and sensitivity analysis. This adaptability ensures that missions, including satellite formation flying, can respond to potential risks and variations in satellite states, enhancing mission success and safety [21, 29].

As illustrated in Figure 3, managing uncertainties in orbital mechanics is crucial for accurate celestial movement predictions. "Uncertainty Realism" addresses these uncertainties in specific scenarios. The



(a) Rotation of a Plane about an Axis[52]



(b) The image shows a graph plotting the altitude of the central moon with respect to time for three celestial bodies: Europa, Ganymede, and Callisto, all of which are orbiting Jupiter.[53]

Figure 3: Examples of Applications in Specific Orbital Scenarios

"Rotation of a Plane about an Axis" example visually demonstrates the dynamics of a plane rotating around a fixed axis, emphasizing accurate modeling of rotational movements and their uncertainties. The second example shows the altitudinal changes of Jupiter's moons—Europa, Ganymede, and Callisto—over time, highlighting the complexities in predicting orbital paths and altitudinal variations, where minor uncertainties can significantly affect long-term prediction accuracy. These examples underscore the necessity of incorporating uncertainty realism into orbital mechanics to enhance celestial modeling and forecasting precision [52, 53].

5 Orbit Uncertainty Propagation

5.1 Stochastic and Analytical Methods

Orbit uncertainty propagation is essential in orbital mechanics, relying on advanced stochastic and analytical methods for precise spacecraft trajectory predictions. The Time-Varying Directional State Transition Tensor (TDSTT) method exemplifies the integration of innovative mathematical constructs into stochastic modeling, reducing computational complexity while maintaining accuracy, thus offering advantages over traditional techniques [32]. Analytical methods also play a crucial role, with the analysis of space elevator tiers providing valuable insights into managing uncertainties in orbital mechanics [10].

Hybrid approaches like the Ballistic Multibody Estimator (BME) combine stochastic and analytical techniques, demonstrating superior performance over traditional methods such as the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) in terms of execution time and accuracy, as evidenced by root-mean-square error (RMSE) metrics [48]. These hybrid methodologies enhance computational efficiency and accuracy in orbit uncertainty propagation.

The integration of advanced stochastic and analytical methods significantly advances orbital mechanics by providing refined tools for propagating orbit uncertainties. These methodologies address computational challenges, such as the curse of dimensionality, affecting traditional methods when dealing with high-dimensional, non-Gaussian inputs. Separated representations enable efficient propagation of orbit-state probability density functions, scaling linearly with uncertain parameters. Stochastic processes improve atmospheric density uncertainty characterization, a major error source in orbit predictions, thereby facilitating reliable trajectory predictions for spacecraft and space debris across various missions [21, 23].

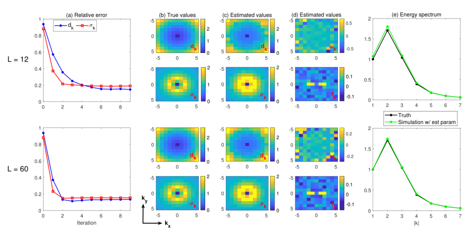
5.2 Numerical Techniques and Frameworks

Advancements in numerical techniques and frameworks are pivotal for effective orbit uncertainty propagation, enhancing accuracy and computational efficiency in orbital mechanics. The multifidelity method proposed by Foss et al. achieves significant computational efficiency improvements, offering speedups of 15-20 times over traditional high-fidelity methods while maintaining accuracy in uncertainty propagation [54]. This approach integrates multiple fidelity levels, balancing computational costs with precision.

Combining State Transition Tensors (STTs) with Gaussian Mixture Models (GMMs) creates a robust framework for propagating the probability density function (PDF) of uncertainties, effectively addressing limitations in existing techniques by improving the representation of uncertainty distributions in nonlinear dynamical systems [29]. GMMs facilitate a more flexible and accurate depiction of the uncertainty landscape, enhancing trajectory predictions.

Analytical and machine learning methods, as discussed by Acciarini et al., demonstrate effectiveness in orbit uncertainty propagation, particularly in low-thrust orbit transfers. The integration of machine learning with traditional analytical approaches yields innovative solutions for managing uncertainties in complex orbital scenarios [9]. The evaluation of numerical techniques through metrics such as the minimum eigenvalue of the Fisher Information Matrix and the trace of the covariance matrix provides quantitative assessments of performance, indicating the potential applicability of such frameworks in orbit uncertainty propagation [55].

The incorporation of advanced numerical techniques and frameworks marks a significant advancement in astrodynamics. Methodologies such as separated representations, multifidelity approaches, and nonlinear analytical methods offer robust tools for managing the complexities of orbit-state probability density functions, especially in high-dimensional, non-Gaussian uncertainty contexts. By employing strategies like linear scaling with respect to input uncertainties and the use of Gaussian mixture models, these techniques enhance the accuracy and reliability of trajectory predictions across diverse space mission scenarios, thereby improving satellite formation flying and conjunction analysis while mitigating computational costs [21, 54, 29].



(a) Comparison of True and Estimated Values in a Simulated System[56]

Algorithm 1 summarizes the SR approximation process.

$r = 0$;
while $\gamma > \epsilon$ **do**
  $r = r + 1$;
  Initialize $\boldsymbol{c}_k^r$, $k = 1, \ldots, d$, $\boldsymbol{u}_0^r$, and s^r;
  while γ *decreases more than* δ *(See end of Section 3.3)* **do**
    for $k = 1$ **to** d **do**
      Solve for $\boldsymbol{c}_k^l$ as elements of $\boldsymbol{z}$ using least squares problem (14)
      Update s^l and $\boldsymbol{c}_k^l$ using (19);
    end
    Solve for $\boldsymbol{u}_0^l$ as columns of $\boldsymbol{Z}$ using least squares problem (20);
    Update s^l and $\boldsymbol{u}_0^l$ using (25);
    Generate $\hat{\boldsymbol{q}}(\boldsymbol{\xi})$ using (10);
    Calculate γ using (26);
  end

(b) Algorithm 1 summarizes the SR approximation process.[21]

Figure 4: Examples of Numerical Techniques and Frameworks

As shown in Figure 4, understanding and predicting the behavior of complex systems under uncertainty is crucial in orbit uncertainty propagation. The first figure illustrates a comparison between true and estimated values within a simulated system, highlighting the impact of varying noise levels on estimation accuracy. True values are represented by blue squares, while red squares indicate estimated values, providing a visual representation of how noise levels, such as L = 12 and L = 60, affect the system's parameters, d$_k and_k. The energy spectrum, a critical aspect of the system's behavior, is also depicted, allowing for comprehensive anal$ 21].

In the realm of state estimation techniques, a comprehensive understanding of the various methodologies is crucial for advancing research and applications in orbital mechanics and complex dynamical systems. Figure 5 illustrates the hierarchical categorization of these techniques, highlighting advanced estimation methods, probabilistic and set-based approaches, data fusion strategies, and machine learning applications. Each category encompasses specific techniques and methodologies that contribute to enhancing accuracy and efficiency, thereby providing a structured overview that aids in the selection of appropriate strategies for specific applications. This visual representation not only clarifies the relationships between different estimation methods but also underscores the evolution of techniques in response to the growing complexity of dynamical systems.

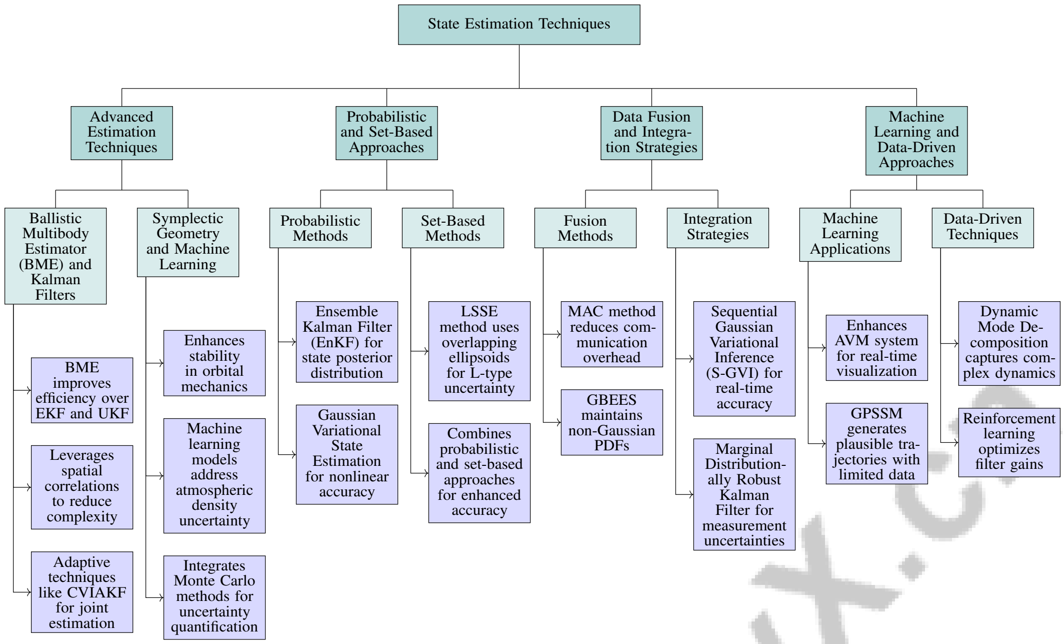


Figure 5: This figure illustrates the hierarchical categorization of state estimation techniques, highlighting advanced estimation methods, probabilistic and set-based approaches, data fusion strategies, and machine learning applications. Each category encompasses specific techniques and methodologies that contribute to enhancing accuracy and efficiency in orbital mechanics and complex dynamical systems.

6 State Estimation Techniques

6.1 Advanced Estimation Techniques

Advanced estimation techniques are pivotal in enhancing accuracy within complex, nonlinear dynamical systems. The Ballistic Multibody Estimator (BME) exemplifies efficiency and performance improvements over traditional methods like the Extended and Unscented Kalman Filters (EKF and UKF) [48], crucial for maintaining precision in dynamic environments. Nouwens et al. propose leveraging strong spatial correlations among state elements to reduce computational complexity, benefiting high-dimensional systems with limited resources [41]. Optimizing Kalman Filter Equations (KFEs) through structural enhancements reduces complexity and improves performance, with adaptive techniques like the Conjugate-Variational Inference Adaptive Kalman Filter (CVIAKF) enabling joint estimation of system states and noise parameters [57, 58].

Integrating symplectic geometry with control theory enhances periodic orbit stability through feedback control, providing robust frameworks for state estimation in orbital mechanics. This integration results in stable estimation schemes resilient to sensor noise without re-tuning [17, 37, 59, 60, 18]. Machine learning and probabilistic models further enhance state estimation accuracy in orbital mechanics, addressing challenges like atmospheric density uncertainty. Approaches such as HASDM-ML and CHAMP-ML improve unmeasured object characterization and non-conservative force effects [42, 22]. Integrating Monte Carlo methods with advanced filtering techniques like the Covariance Sigma Point (CCSP) filter enhances uncertainty quantification, crucial for space situational awareness and collision prevention.

6.2 Probabilistic and Set-Based Approaches

Probabilistic and set-based approaches offer complementary methodologies for managing uncertainties in orbital mechanics. Table 1 presents a comprehensive comparison of different probabilistic and set-based estimation methods employed in orbital mechanics, elucidating their methodological approaches and application contexts. The Ensemble Kalman Filter (EnKF) effectively approximates

Method Name	Methodology Type	Estimation Techniques	Application Context
LSSE[61]	Set-based Methods	Minimax Filter	Complex Flow Scenarios
LM-MAP[14]	Hybrid Approach	Ransac-based Ellipse	Autonomous Docking Operations
GGF[62]	Probabilistic Methods	Invariant Extended Kalman	Orbital Mechanics

Table 1: Table 1 provides a comparative analysis of various estimation methods used in orbital mechanics. It highlights the methodology type, estimation techniques, and specific application contexts for each method, offering insights into their suitability for different scenarios.

the state posterior distribution, outperforming traditional methods in non-stiff systems and complex dynamical contexts [40, 19]. The Gaussian Variational State Estimation method leverages variational inference to enhance nonlinear state estimation accuracy [63, 64]. Set-based approaches, such as the LSSE method, use overlapping ellipsoids to manage L-type uncertainty, improving state estimation in scenarios with limited observability [61].

Combining probabilistic and set-based approaches utilizes sensitivity information to identify relevant states and parameters for estimation, enhancing accuracy [14]. Ge et al. propose approximating distributions at various points using Gaussian fusion, enhancing robustness in nonlinear dynamics [62]. The UKF-M method adapts the Unscented Kalman Filter for manifolds, minimizing worst-case estimation error and ensuring reliable predictions [65, 36]. These approaches provide powerful tools for enhancing state estimation accuracy, addressing complexities and uncertainties in orbital mechanics. By integrating advanced mathematical frameworks and diverse data sources, they bolster trajectory prediction reliability [22, 5].

6.3 Data Fusion and Integration Strategies

Method Name	Computational Efficiency	Real-Time Integration	Robustness to Uncertainty
MAC[51]	Reduce Communication Overhead	-	Unknown Correlations
S-GVI[66]	Efficient Sequential Inference	Sequential Updates	Robustness Against Nonlinearity
MC-MDRKF[67]	Efficient Moment-constrained	-	Distributional Uncertainties
SSVBAKF[68]	Recursive Bayesian Framework	Recursive Bayesian Framework	Flexibility IN Handling

Table 2: Comparison of data fusion and integration methods based on computational efficiency, real-time integration capabilities, and robustness to uncertainty. The table evaluates four methods: MAC, S-GVI, MC-MDRKF, and SSVBAKF, highlighting their unique strengths and areas of application in enhancing state estimation performance.

Data fusion and integration strategies enhance state estimation performance in complex environments. Table 2 presents a comparative analysis of various data fusion and integration strategies, focusing on their computational efficiency, real-time integration potential, and robustness to uncertainty, which are critical for improving state estimation in complex environments. The MAC method improves fusion estimates by reducing communication overhead and enhancing computational efficiency [51]. Grid-Based Estimation with Efficient Sampling (GBEES) maintains high-fidelity non-Gaussian PDFs while reducing computational costs [58]. The Sequential Gaussian Variational Inference (S-GVI) method enhances real-time data integration and state estimate accuracy [66]. Moving Horizon Estimation (MHE) ensures robust stability in data fusion, critical for integrating diverse data sources [69]. The Marginal Distributionally Robust Kalman Filter accounts for measurement uncertainties, enhancing reliability [67]. Evaluating variational nonlinear Kalman filtering methods through metrics like RMSE validates data fusion strategies [68]. Future research should explore adaptive ensemble strategies and new heuristics for data fusion, particularly in scenarios involving multiple UAVs [31, 55]. These strategies effectively manage uncertainties, improving trajectory prediction accuracy and supporting robust solutions for tracking Resident Space Objects [5, 70].

6.4 Machine Learning and Data-Driven Approaches

Machine learning and data-driven methods transform state estimation in orbital mechanics, enhancing trajectory prediction precision and efficiency. Integrating these approaches into existing frameworks improves real-time satellite operations and space situational awareness. Machine learning enhances the Automated Visualization and Monitoring (AVM) system, facilitating real-time satellite behavior visualization and decision-making [27]. The Gaussian Process State-Space Model (GPSSM) generates plausible trajectories with limited data, beneficial in data-scarce or high-dimensional scenarios [71].

Dynamic Mode Decomposition (DMD) captures complex dynamics in nonlinear periodic systems [72]. Reinforcement learning techniques, like the Approximate Optimal Filter (AOF), optimize filter gains without complex algebraic solutions [73]. Machine learning approaches consistently outperform analytical methods in accuracy and speed, especially in long-flight scenarios [9]. Future research should enhance these methods' robustness in high-dimensional systems and explore real-time satellite operations applications [29]. Optimizing square-root Kalman filters could improve computational efficiency in large-scale problems [58]. Developing filtering techniques for stiff stochastic systems remains crucial, emphasizing understanding theoretical performance differences among filtering methods [74]. These advancements support effective decision-making and mission planning in orbital mechanics.

7 Error Analysis in Astrodynamics

7.1 Sources of Error in State Estimation

State estimation in astrodynamics is inherently complex, with errors significantly impacting trajectory prediction accuracy. The exponential growth of configurations in space debris scenarios complicates solution verification, necessitating robust methodologies for optimal solutions [12]. Additionally, assumptions about network connectivity and observability in distributed Kalman filter frameworks often do not hold, leading to inaccuracies [39]. This highlights the need for adaptable estimation methods.

High-dimensional systems pose computational challenges, with traditional methods struggling to maintain accuracy without prohibitive costs. The Least-Squares Kernel Kalman Filter (LSK-KF) offers promise in achieving accurate estimates with low computational complexity, yet scaling remains challenging [41]. Furthermore, degraded observations contribute to estimation errors, as revealed by metrics like root mean square error (RMSE) and normalized estimation error squared (NEES) [7, 47].

Existing inertial navigation models, typically designed for small initial misalignments, exacerbate errors in scenarios with larger discrepancies, necessitating more robust models [35]. Addressing key error sources, such as uncertainties in atmospheric density and initial state conditions, is crucial for refining state estimation processes and enhancing reliability. Advanced probabilistic machine learning models like HASDM-ML, CHAMP-ML, and MSIS-UQ improve predictions in low Earth orbit, where satellite conjunctions and debris collisions pose significant risks. Innovative approaches, including the consider covariance sigma point (CCSP) filter and separated representations for uncertainty propagation, facilitate better handling of complex stochastic inputs, leading to more informed decision-making in space operations [21, 42, 22].

7.2 Impact of Computational Complexity

Computational complexity critically affects error analysis and state estimation accuracy in astrodynamics, where real-time decision-making is paramount. The intricate nature of orbital mechanics demands sophisticated algorithms, yet these often face substantial computational demands. For instance, the robust motion planning framework by Lindemann et al. is constrained by the computational burden of constraint checking during tree construction, impacting real-time performance [75]. Similarly, Laouar et al. highlight challenges in determining the maximally feasible consensus horizon within model predictive control frameworks, where increased complexity affects error analysis and precision [76].

Balancing computational efficiency with accuracy remains a recurring theme in state estimation techniques, as real-time processing demands often conflict with advanced algorithms' intensity. Innovative approaches are necessary to mitigate computational complexity's impact on state estimation accuracy. Enhancing algorithmic efficiency and leveraging recent computational advancements can improve reliability without sacrificing precision. Machine learning and advanced Monte Carlo methods address uncertainties in atmospheric density and orbital parameters, refining orbit predictions and enhancing space situational awareness. Optimization strategies for Kalman filter extensions can reduce computational demands while maintaining high estimation accuracy, essential for mitigating risks like Kessler Syndrome and ensuring successful space missions [21, 22, 9, 42, 58].

7.3 Error Mitigation Techniques

Error mitigation in astrodynamics is vital for enhancing prediction reliability amid uncertainties and complex dynamical systems. The Consistent Batch State Estimation method by Yoon et al. effectively models time-correlated noise, improving estimator consistency [77]. The Least Squares Data Assimilation Filter (LSDAF) integrates prior knowledge and observational data, enabling statistically optimal estimation [78]. This method refines state estimates by integrating diverse data sources, mitigating error impacts on trajectory predictions.

Silvestre et al. discuss the Exact Set-Valued Estimation method, highlighting potential future research to enhance order reduction methods for Convex Constraint Games (CCGs), improving computational efficiency [79]. The Localized Sequential State Estimation (LSSE) method demonstrates improved state estimates in complex flow scenarios by managing non-Gaussian uncertainties [61]. The 3DVAR parametrized partial differential method offers a robust framework for data assimilation, maintaining stability against noise amplification [80].

A limitation of the Least-Squares Kernel Kalman Filter (LSK-KF) is designing the kernel matrix L to reflect spatial correlations accurately, necessitating careful consideration [41]. Addressing this limitation is essential for ensuring precision in high-dimensional systems.

Integrating advanced techniques in astrodynamics, including machine learning for orbit estimation, analytical approximations for low-thrust trajectory optimization, and sophisticated uncertainty propagation methods, enhances error mitigation strategies. These approaches improve trajectory prediction reliability and accuracy, enabling effective decision-making and mission planning in complex space operations, such as asteroid belt exploration and space debris tracking. By leveraging data-driven insights and innovative modeling techniques, these methods address key challenges like atmospheric density uncertainties and the curse of dimensionality, contributing to safer and more efficient space missions [27, 21, 22, 9, 42].

8 Conclusion

This survey highlights the pivotal role of covariance consistency and realism, alongside precise uncertainty propagation, in refining the accuracy of orbital mechanics, crucial for state estimation and trajectory prediction in space exploration. Innovative mathematical frameworks, such as the Time-Varying Directional State Transition Tensor (TDSTT), have demonstrated significant efficiency improvements, reducing computational demands while maintaining accuracy. Similarly, the Extended State Distributed Kalman Filter (ESDKF) showcases effective distributed state estimation for nonlinear systems, offering bounded estimation covariance and real-time evaluation capabilities.

The integration of machine learning and data-driven approaches presents a transformative opportunity to enhance predictive reliability and automate the discovery of fundamental principles. Future research should focus on refining these models and exploring hybrid methodologies that combine analytical and data-driven techniques. The development of novel dynamical equations has advanced the understanding of particle dynamics near rotating asteroids, providing a foundation for further exploration. Additionally, the use of symplectic geometry and normally hyperbolic invariant manifolds (NHIMs) in stabilizing periodic orbits marks a significant methodological advancement.

The concept of a space elevator, with its potential to improve transit times and launch capabilities, opens new avenues for research in orbital mechanics. Furthermore, the application of quantum annealing in optimizing active debris removal (ADR) missions exemplifies the potential of advanced computational techniques in managing space debris, optimizing propellant usage while adhering to mission parameters. These insights underscore the need for continuous research to enhance the precision and reliability of orbital mechanics, advocating for the development of adaptive algorithms and innovative methodologies to address the complexities inherent in space exploration.

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