MAE 259B Group 2 Final Presentation

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Review of our midterm presentation

Started from simplest 2D DER

- Performance optimization (pre-computing reused terms)
- Initial curvature
- Circular structure
- Inflation pressure (models as force-per-unit-length)
- Surface contact (naïve predictor-corrector)
- Damping (relative to center-of-mass)
- Adaptive time stepping (limit momentum change per step)
- Dynamic friction (partially Euler-forward)

Review of our midterm presentation

Bouncing ring structure on surface w/o and w/ friction

no-friction.mp4

friction-0.2.mp4

Not all PDF viewers can play videos. Videos files can be alternatively found at

`https://github.com/kmxz/mae259b/tree/master/final-presentation/video`.

Review of our midterm presentation

Damping

Naïve damping $\underline{F}_{d,i} \propto -\underline{u}_i$ ruins bulk motion (as our bulk velocity is high)

Use velocity relative to center of mass: $\underline{F}_{d,i} \propto -(\underline{u}_i - \underline{u}_{cm})$

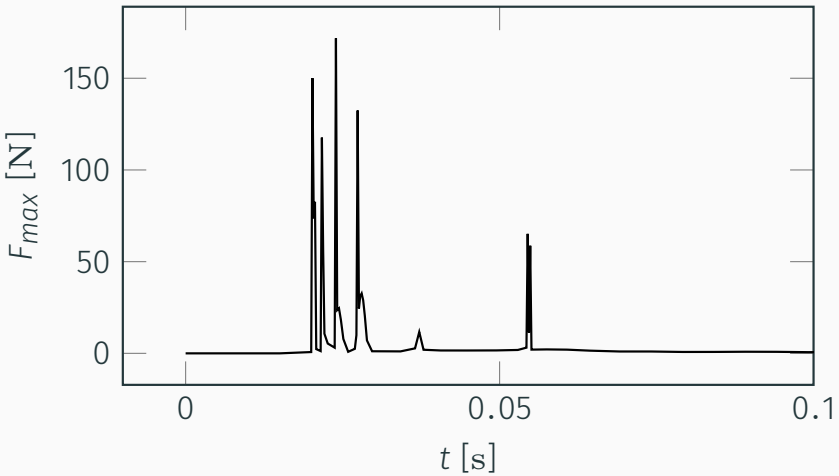
No damping	Naïve damping	CM-Relative damping
no-damp.mp4	bad-damp.mp4	good-damp.mp4

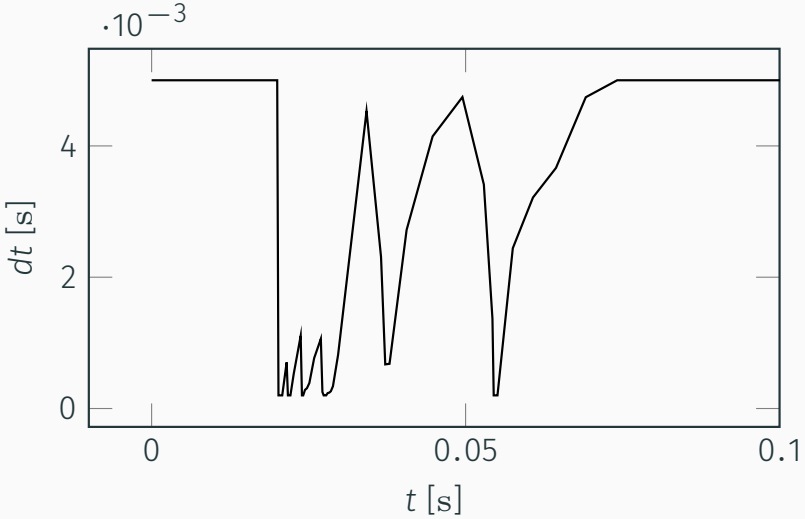
Review of our midterm presentation

Adaptive time stepping

After trying Δx, Δv and contact as criteria for determining time step size, we resulted in using momentum change as the optimal criteria.

Step size should be inverse proportional to the maximum nodal reaction force from the ground.
We limit $(F_{max})(dt)$ to a threshold ($\approx 0.005\,\mathrm{N}\cdot\mathrm{s}$) for momentum change per step.





Review of our midterm presentation

Adaptive time stepping:

ats.mp4

After midterm presentation

We spent quite some time trying to write a 3-D discrete shell simulation in C++.

But we gave up, considering the limited time and difficulty in debugging.

We continued improving the 2-D DER model.

Improving visualization tool

Let viewport track the ring:

vp-track.mp4

Baraff-Witkin mass modification

For surface contact, we used Baraff-Witkin mass modification method to replace our naïve predictor-corrector method.

Why?

- Our previous implementation need to extract sub-matrices of different sizes in each time step. With mass modification method, implementation can be neater.
- Our previous implementation only handles horizontal surface. With mass modification method, sloped surface can be included.

Baraff-Witkin mass modification

- Detect DOFs to be constrained, just as before.
- Compute "imposed velocity change" $\underline{z}$ for those DOFs to stay above contact surface.
- In time marching, use v_i instead of x_i:

$$\frac{\delta v_i}{\delta t} = \frac{x_i(t_{k+1}) - x_i(t_k)}{(dt)^2} - \frac{v_i(t_k)}{dt}$$

- Therefore, EOM becomes: $\Delta \underline{v} - (dt)\underline{\underline{w}}\,\underline{F} - \underline{z} = 0$
- Inversed mass matrix $\underline{\underline{w}}$ is composed by 2×2 component matrices $\underline{\underline{w}}_i$, which is $\frac{1}{m_i}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for unconstrained nodes, $\frac{1}{m_i}(\underline{\underline{I}} - \underline{n}\,\underline{n}^T)$ for nodes constrained in $\underline{n}$ direction.

Baraff-Witkin mass modification

Newton-Raphson iteration

Since we are solving for $\Delta\underline{v}$, in each iteration:

$$\Delta\underline{v} \leftarrow \Delta\underline{v} - \underline{\underline{J}} \backslash \underline{f}$$

where $\underline{f} = \Delta\underline{v} - (dt)\underline{\underline{w}}\,\underline{F} - z$ and $\underbrace{\underline{\underline{J}}}_{\frac{\partial \underline{F}}{\partial \Delta \underline{v}}} = \underline{\underline{I}} - (dt)^2 \underbrace{\underline{\underline{J}}^{old}}_{\frac{\partial \underline{F}}{\partial \underline{x}}}$

Baraff-Witkin mass modification

Yeah, sloped surface.

sloped.mp4

Baraff-Witkin mass modification

Yeah, sloped surface.

sloped-track.mp4

Baraff-Witkin mass modification

The procedure of handling **non-zero friction**, as covered in class, is:

1. Constrain both directions (x, y) of a node (to emulate static friction)
2. Decompose reaction force $\underline{f}_i$ into normal force $\underline{f}_i^{normal}$ and $\underline{f}_i^{friction}$ (by $\underline{f}^{normal} = \underline{f}_i \cdot \underline{n}_{wall}$ and $\underline{f}^{friction} = \underline{f}_i - \underline{f}_i^{normal}$)
3. Test if $f_i^{friction} > \mu_{static} f_i^{normal}$. If so, release x-constraint and apply dynamic friction $\mu_{dynamic} f_i^{normal}$ instead.

However, in our scenario, normal force is always relatively small. $f_i^{friction} > \mu_{static} f_i^{normal}$ happens all the time. Static friction never get applied. Dynamic pressure is simply added to $F^{external}$.

Variable inflation pressure

Recall that we discretized the inflation pressure as below:

However, instead of constant f, we may expect the pressure to get higher when the volume inside the structure is reduced.

For ideal gas:

$$pV = nRT$$

Assume the temperature is constant. Then $p \propto \frac{1}{V}$.

Variable inflation pressure

For computing the internal volume, we take the area of the 2D ring. Since we already discretized the structure into a polygon, we can use shoelace formula to calculate area:

$$A = \frac{1}{2}\left[\sum_{i=1}^{n-1} x_i y_{i+1} + x_n y_1 - \sum_{i=1}^{n-1} x_{i+1} y_i - x_1 y_n\right]$$

Then, for each time frame, $p(t_k) = \frac{A(t_0)}{A(t_k)} p(t_0)$

We simple ignore this effects' contribution to Jacobian (otherwise the Jacobian will no longer be banded), it turns out we can still get convergence.

Variable inflation pressure

Variable inflation pressure does make results more realistic!

pressure-inv.mp4

pressure-var.mp4