VIO 残差函数的构建

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基于滑动窗口的VIO Bundle Adjustment.

$$\min \underbrace{\rho(\|r_p - J_p \chi\|^2_{\Sigma_p})}_{\text{prior}} + \sum_{i\in B} \underbrace{\rho(\|r_B(z_{b_i,b_{i+1}}, \chi)\|^2_{\Sigma_{b_i,b_{i+1}}})}_{\text{IMU error}} + \sum_{(i,j)\in F} \underbrace{\rho(\|r_F(z^{c_i}_{f_j}, \chi\|^2_{\Sigma^{c_i}_{f_j}})}_{\text{image error}}.$$

为了节约计算量采用滑动窗口形式的BA，在i时刻，滑动窗口内需优化的状态量定义为：

逆深度.

$$\chi = [x_n, x_{n+1}, \dots, x_{n+N}, \lambda_m, \lambda_{m+1}, \lambda_{m+M}]$$ → M: 滑动窗口内所有可被观测到的路标点数量

$$x_i = [p^{w}_{wb_i}, q^{w}_{wb_i}, v^{w}_{b_i}, b^{b_i}_a, b^{b_i}_g]^T, \quad i\in[n, n+N]$$ 无角度变量.

平移 旋转 速度

在i时刻，IMU机体在世界坐标系中的位置、速度、姿态，以及IMU机体坐标系中的加速度和角速度的bias估计.

→ 5个3维的变量 ⇒ 15维.

- n, m分别是机身状态量，路标在窗口中的起始位置.

注: q为四元数，但无论用四元数还是欧拉角

视觉重投影误差.

一个特征点在归一化相机坐标系下的估计值与观测值的差.

$$r_c = \begin{bmatrix} \frac{x}{z} - u \\ \frac{y}{z} - v \end{bmatrix}$$

其中，特征点的状态值为特征点的三维空间坐标 $(x, y, z)^T$，观测值为 $(u, v)^T$ 特征点相机归一化平面的坐标.

逆深度参数化

特征点在归一化相机坐标系与相机坐标系下的转换关系为：

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\lambda}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}.$$

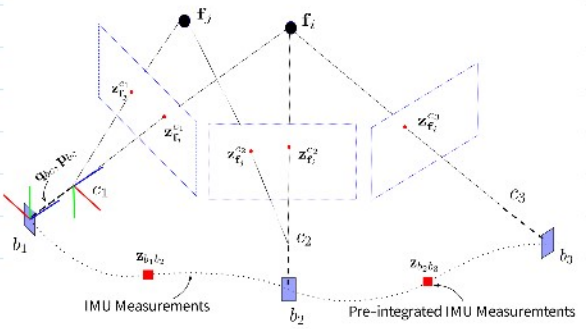
其中 $\lambda = \frac{1}{z}$ 称为逆深度. why: ① x, y, z 比较大. (比如2阶时候...) 大数值对优化的影响较大.

② 减少优化的参数. (0.1 → 1 ?).

VIO中基于逆深度的重投影误差:

特征点逆深度在第i帧中初始化得到，在第j帧中又被观测到，则该点在第j帧中的坐标为：

$$\begin{bmatrix} x_{c_j} \\ y_{c_j} \\ z_{c_j} \\ 1 \end{bmatrix} = T_{bc}^{-1} T_{wb_j}^{-1} T_{wb_i} T_{bc} \begin{bmatrix} \frac{1}{\lambda} u_{c_i} \\ \frac{1}{\lambda} v_{c_i} \\ \frac{1}{\lambda} \\ 1 \end{bmatrix}.$$



视觉重投影误差为：

$$r_c = \begin{bmatrix} \frac{x_{c_j}}{z_{c_j}} - u_{c_j} \\ \frac{y_{c_j}}{z_{c_j}} - v_{c_j} \end{bmatrix}$$

IMU测量值积分:

IMU的真实值为 ω, a. 测量值为 $\tilde{\omega}, \tilde{a}$. 则有：

IMU 测量值积分

IMU的真实值为 ω, a. 测量值为 $\tilde{\omega}, \tilde{a}$. 则有:

$$\tilde{\omega}^b = \omega^b + b^g + n^g \qquad g: \text{gyro} \quad a: \text{acc.}$$

$$\tilde{a}^b = q_{bw}(a^w + g^w) + b^a + n^a$$

PVQ对时间的导数可写成:

$$\dot{p}_{wb_t} = v_t^w$$

$$\dot{v}_t^w = a_t^w$$

$$\dot{q}_{wb_t} = q_{wb_t} \otimes \begin{bmatrix} 0 \\ \frac{1}{2}\omega^{b_t} \end{bmatrix}$$

从第i时刻的PVQ对IMU的测量值进行积分得到第j时刻的PVQ:

$$p_{wb_j} = p_{wb_i} + v_i^w \Delta t + \iint_{t\in[i,j]} (q_{wb_t} a^{b_t} - g^w)\, \delta t^2$$
$$v_j^w = v_i^w + \int (q_{wb_t} a^{b_t} - g^w)\, \delta t$$
$$q_{wb_j} = \int q_{wb_t} \otimes \begin{bmatrix} 0 \\ \frac{1}{2}\omega^{b_t} \end{bmatrix}$$

问题: 每次 q_{wb_i} 更新时, 都需要重新积分, 计算量大.

⇒ **IMU 预积分** Pre-Integration.

$$\Rightarrow q_{wb_t} = q_{wb_i} \otimes q_{b_i b_t}$$

那么PVQ积分公式中的积分项能成为相对于第i时刻的量吗, 即不是相对于世界坐标系的量吗.

$$p_{wb_j} = p_{wb_i} + v_i^w \Delta t - \frac{1}{2} g^w \Delta t^2 + q_{wb_i} \iint \underbrace{q_{b_i b_t} \cdot a^{b_t}}_{\alpha_{b_i b_j}}\, \delta t^2 \qquad \text{新积分量.}$$

$$v_j^w = v_i^w - g^w \Delta t + q_{wb_i} \int \underbrace{q_{b_i b_t} a^{b_t}}_{\beta_{b_i b_j}}\, \delta t.$$

$$q_{wb_j} = q_{wb_i} \underbrace{\int q_{b_i b_t} \otimes \begin{bmatrix} 0 \\ \frac{1}{2}\omega^{b_t} \end{bmatrix}}_{q_{b_i b_j}}\, \delta t.$$

⇒ 预积分量

定义: 一段时间内IMU将使用预积分量作为测量值, 对两时刻间的状态量进行约束.

$$\begin{bmatrix} r_p \\ r_q \\ r_v \\ r_{ba} \\ r_{bg} \end{bmatrix}_{10\times 1} = \begin{bmatrix} p_{wb_j} - p_{wb_i} - v_i^w \Delta t + \frac{1}{2} g^w \Delta t^2 - q_{wb_i} \cdot \alpha_{b_i b_j} = q_{b_i w}(p_{wb_j} - p_{wb_i} - v_i^w \Delta t + \frac{1}{2} g^w \Delta t^2) - \alpha_{b_i b_j} \\ 2[q_{b_j b_i} \otimes (q_{b_i w} \otimes q_{wb_j})]_{xyz} \\ q_{b_i w}(v_j^w - v_i^w + g^w \Delta t) - \beta_{b_i b_j} \\ b_j^a - b_i^a \\ b_j^g - b_i^g \end{bmatrix}$$

$[\cdot]_{xyz}$ 是将四元数的虚部 (x, y, z) 组成的三维向量

也叫李群 $(\ln(R_i^T R_j))^\vee$

为什么要把预积分量提出来? ⇒ 预积分量才是我们要正确测量出来的结果.

预积分的离散递推 (mid-point 方法. 即两个相邻时刻 k 以 k+1 的过程是已用两个时刻的测量值 a, ω 的平均值来计算的)

$$\omega = \frac{1}{2}(\omega^{b_k} - b_k^g) + (\omega^{b_{k+1}} - b_k^g))$$
$$q_{b_k, k+1} = q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \Delta t \end{bmatrix}$$

$$a = \frac{1}{2}(q_{b_i b_k}(a^{b_k} - b_k^a) + q_{b_i b_{k+1}}(a^{b_{k+1}} - b_k^a))$$
$$\alpha_{b_i b_{k+1}} = \alpha_{b_i b_k} + \beta_{b_i b_k} \Delta t + \frac{1}{2} a \Delta t^2$$
$$\beta_{b_i b_{k+1}} = \beta_{b_i b_k} + a \Delta t$$

$$\alpha_{b_i b_{k+1}} = \alpha_{b_i b_k} + \beta_{b_i b_k}\delta t + \frac{1}{2} a \delta t^2$$
$$\beta_{b_i b_{k+1}} = \beta_{b_i b_k} + a\delta t$$

$$b^a_{k+1} = b^a_k + n_{b^a_k}\delta t$$
$$b^g_{k+1} = b^g_k + n_{b^g_k}\delta t$$

Covariance Propagation:

$y = Ax$ $\quad x \in N(0, \Sigma_x)$ 则有: $\Sigma_y = A\Sigma_x A^T$

$$\Sigma_y = E(Ax(Ax)^T) = E(Axx^TA^T) = A\Sigma_x A^T$$

状态误差的传递: ① 一阶泰勒展开

② 基于误差随时间变化的传递方程.

① 基于一阶泰勒展开 (应用于EKF的推导主要方法) 此处仅用公式

令状态量 $x = \bar{x} + \delta x$. 其中, 真值为 $\bar{x}$, 误差为 δx. 输入量u的噪声为 n.

非线性系统 $x_k = f(x_{k-1}, u_{k-1})$ 的状态误差的传递关系如下:

$$\delta x_k = \boxed{F}\,\delta x_{k-1} + \boxed{G}\,n_{k-1}$$

其中. F是状态量 x_k 对 x_{k-1} 的雅克比矩阵.

G是状态量 x_k 对噪声量 u_{k-1} 的 Jacobi

② 基于误差随时间变化的 △ (论文中常用).

如果我们能够推导误差随时间变化的导数关系:

$$\dot{\delta x} = A\cdot\delta x + Bu.$$

则误差状态的传递方程为:

$$\delta x_k = \delta x_{k-1} + \dot{\delta x}_k\cdot\Delta t.$$
$$= \boxed{(I + A\Delta t)}\,\delta x_{k-1} + \boxed{B\Delta t}\,n_{k-1}.$$

VIO系统中, 已经知道状态的导数和状态之间的转移矩阵.

如: $\dot{v} = Ra^b + g$

$$\Rightarrow \dot{v} + \delta\dot{v} = R(I + [\delta\theta]_\times)(a^b + \delta a^b) + g + \delta g$$
$$\Rightarrow \delta\dot{v} = R\delta a^b + R[\delta\theta]_\times(a^b + \delta a^b) + \delta g$$

基于EKF的预积分误差传递形式的推导

首先回顾预积分的误差递推公式，将测量噪声也考虑进模型：

$$\boldsymbol{\omega} = \frac{1}{2}\left((\boldsymbol{\omega}^{b_k} + \mathbf{n}^g_k - \mathbf{b}^g_k) + (\boldsymbol{\omega}^{b_{k-1}} + \mathbf{n}^g_{k+1} - \mathbf{b}^g_k)\right)$$
$$\mathbf{q}_{b_i b_{k+1}} = \mathbf{q}_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\boldsymbol{\omega}\delta t \end{bmatrix}$$
$$\mathbf{a} = \frac{1}{2}\left(\mathbf{q}_{b_i b_k}(\mathbf{a}^{b_k} + \mathbf{n}^a_k - \mathbf{b}^a_k) + \mathbf{q}_{b_i b_{k-1}}(\mathbf{a}^{b_{k+1}} + \mathbf{n}^a_{k+1} - \mathbf{b}^a_k)\right)$$
$$\boldsymbol{\alpha}_{b_i b_{k+1}} = \boldsymbol{\alpha}_{b_i b_k} + \boldsymbol{\beta}_{b_i b_k}\delta t + \frac{1}{2}\mathbf{a}\delta t^2$$
$$\boldsymbol{\beta}_{b_i b_{k+1}} = \boldsymbol{\beta}_{b_i b_k} + \mathbf{a}\delta t$$
$$\mathbf{b}^a_{k+1} = \mathbf{b}^a_k + \mathbf{n}_{\mathbf{b}^a_k}\delta t$$
$$\mathbf{b}^g_{k+1} = \mathbf{b}^g_k + \mathbf{n}_{\mathbf{b}^g_k}\delta t$$

误差状态量

噪声量

$$\begin{bmatrix} \delta\boldsymbol{\alpha}_{b_{k+1}b'_{k+1}} \\ \delta\boldsymbol{\theta}_{b_{k+1}b'_{k+1}} \\ \delta\mathbf{b}^a_{k+1} \end{bmatrix} = \mathbf{F}\begin{bmatrix} \delta\boldsymbol{\alpha}_{b_k b'_k} \\ \delta\boldsymbol{\theta}_{b_k b'_k} \\ \delta\mathbf{b}^a_k \end{bmatrix} + \mathbf{G}\begin{bmatrix} \mathbf{n}^a_k \\ \mathbf{n}^g_k \\ \mathbf{n}^a_{k+1} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{I} & \mathbf{f}_{12} & \mathbf{I}\delta t & -\frac{1}{4}(\mathbf{q}_{b_i b_k} - \mathbf{q}_{b_i b_{k+1}})\delta t^2 & \mathbf{f}_{15} \\ \mathbf{0} & \mathbf{I} - [\boldsymbol{\omega}]_\times & \mathbf{0} & \mathbf{0} & -\mathbf{I}\delta t \\ \mathbf{0} & \mathbf{f}_{32} & \mathbf{I} & -\frac{1}{2}(\mathbf{q}_{b_i b_k} - \mathbf{q}_{b_i b_{k+1}})\delta t & \mathbf{f}_{35} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix}\delta\boldsymbol{\alpha}_{b_{k+1}b'_{k+1}}\\ \delta\boldsymbol{\theta}_{b_{k+1}b'_{k+1}}\\ \delta\boldsymbol{\beta}_{b_{k+1}b'_{k+1}}\\ \delta\mathbf{b}^a_{k+1}\\ \delta\mathbf{b}^g_{k+1}\end{bmatrix} = \mathbf{F}\begin{bmatrix}\delta\boldsymbol{\alpha}_{b_kb'_k}\\ \delta\boldsymbol{\theta}_{b_kb'_k}\\ \delta\boldsymbol{\beta}_{b_kb'_k}\\ \delta\mathbf{b}^a_k\\ \delta\mathbf{b}^g_k\end{bmatrix} + \mathbf{G}\begin{bmatrix}\mathbf{n}^a_k\\ \mathbf{n}^g_k\\ \mathbf{n}^a_{k+1}\\ \mathbf{n}^g_{k+1}\\ \mathbf{n}_{\mathbf{b}^a_k}\\ \mathbf{n}_{\mathbf{b}^g_k}\end{bmatrix}$$

速度位移

$$\mathbf{F} = \begin{bmatrix}\mathbf{I} & \mathbf{f}_{12} & \mathbf{I}\delta t & -\frac{1}{4}(\mathbf{q}_{b_ib_k}+\mathbf{q}_{b_ib_{k+1}})\delta t^2 & \mathbf{f}_{15}\\ 0 & \mathbf{I}-[\boldsymbol{\omega}]_\times & 0 & 0 & -\mathbf{I}\delta t\\ 0 & \mathbf{f}_{32} & \mathbf{I} & -\frac{1}{2}(\mathbf{q}_{b_ib_k}+\mathbf{q}_{b_ib_{k+1}})\delta t & \mathbf{f}_{35}\\ 0 & 0 & 0 & \mathbf{I} & 0\\ 0 & 0 & 0 & 0 & \mathbf{I}\end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix}\frac{1}{4}\mathbf{q}_{b_ib_k}\delta t^2 & \mathbf{g}_{12} & \frac{1}{4}\mathbf{q}_{b_ib_{k+1}}\delta t^2 & \mathbf{g}_{14} & 0 & 0\\ 0 & \frac{1}{2}\mathbf{I}\delta t & 0 & \frac{1}{2}\mathbf{I}\delta t & 0 & 0\\ \frac{1}{2}\mathbf{q}_{b_ib_k}\delta t & \mathbf{g}_{32} & \frac{1}{2}\mathbf{q}_{b_ib_{k+1}}\delta t & \mathbf{g}_{34} & 0 & 0\\ 0 & 0 & 0 & 0 & \mathbf{I}\delta t & 0\\ 0 & 0 & 0 & 0 & 0 & \mathbf{I}\delta t\end{bmatrix}$$

$$\mathbf{f}_{12} = \frac{\partial\boldsymbol{\alpha}_{b_ib_{k+1}}}{\partial\delta\boldsymbol{\theta}_{b_kb'_k}} = -\frac{1}{4}(\mathbf{R}_{b_ib_k}[\mathbf{a}^{b_k}-\mathbf{b}^a_k]_\times\delta t^2 + \mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times(\mathbf{I}-[\boldsymbol{\omega}]_\times\delta t)\delta t^2)$$

$$\mathbf{f}_{32} = \frac{\partial\boldsymbol{\beta}_{b_ib_{k+1}}}{\partial\delta\boldsymbol{\theta}_{b_kb'_k}} = -\frac{1}{2}(\mathbf{R}_{b_ib_k}[\mathbf{a}^{b_k}-\mathbf{b}^a_k]_\times\delta t + \mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times(\mathbf{I}-[\boldsymbol{\omega}]_\times\delta t)\delta t)$$

$$\mathbf{f}_{15} = \frac{\partial\boldsymbol{\alpha}_{b_ib_{k+1}}}{\partial\delta\mathbf{b}^g_k} = -\frac{1}{4}(\mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times\delta t^2)(-\delta t)$$

$$\mathbf{f}_{35} = \frac{\partial\boldsymbol{\beta}_{b_ib_{k+1}}}{\partial\delta\mathbf{b}^g_k} = -\frac{1}{2}(\mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times\delta t)(-\delta t)$$

$$\mathbf{g}_{12} = \frac{\partial\boldsymbol{\alpha}_{b_ib_{k+1}}}{\partial\mathbf{n}^g_k} = \mathbf{g}_{14} = \frac{\partial\boldsymbol{\alpha}_{b_ib_{k+1}}}{\partial\mathbf{n}^g_{k+1}} = -\frac{1}{4}(\mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times\delta t^2)(\frac{1}{2}\delta t)$$

$$\mathbf{g}_{32} = \frac{\partial\boldsymbol{\beta}_{b_ib_{k+1}}}{\partial\mathbf{n}^g_k} = \mathbf{g}_{34} = \frac{\partial\boldsymbol{\beta}_{b_ib_{k+1}}}{\partial\mathbf{n}^g_{k+1}} = -\frac{1}{2}(\mathbf{R}_{b_ib_{k+1}}[(\mathbf{a}^{b_{k+1}}-\mathbf{b}^a_k)]_\times\delta t^2)(\frac{1}{2}\delta t)$$

$\beta_{b_ib_{k+1}} = \beta_{b_ib_k} + a\,\delta t$

$= \beta_{b_ib_k} + \frac{1}{2}(q_{b_ib_k}(a^{b_k} - b^a_k) + q_{b_ib_{k+1}}(a^{b_{k+1}} - b^a_k))\delta t$

$= \beta_{b_ib_k} + \frac{1}{2}(q_{b_ib_k}(a^{b_k} - b^a_k) + q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\omega\delta t\end{bmatrix}(a^{b_{k+1}} - b^a_k))\delta t$

δ_{32}: 速度用旋转矩阵表示时候使用旋转矩阵用 Jacobian

$$\frac{\partial\beta_{b_ib_{k+1}}}{\partial\theta_{b_kb'_k}} = \frac{\partial a\,\delta t}{\partial\delta\theta_{b_kb'_k}}$$

其中 $a\delta t = \frac{1}{2}q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\delta\theta_{b_kb'_k}\end{bmatrix}(a^{b_k} - b^a_k)\delta t$ → 即上一个时刻的状态

$+ \frac{1}{2}q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\delta\theta_{b_kb'_k}\end{bmatrix}\otimes\begin{bmatrix}1\\ \frac{1}{2}\omega\delta t\end{bmatrix}(a^{b_{k+1}} - b^a_k)\delta t$

$= \underbrace{\frac{1}{2}R_{b_kb'_k}\exp([\delta\theta_{b_kb'_k}]_\times)(a^{b_k} - b^a_k)\delta t}_{\text{Part 1}}$ → 四元数 → SO(3) 形式

$+ \underbrace{\frac{1}{2}R_{b_ib_k}\exp([\delta\theta_{b_kb'_k}]_\times)\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t}_{\text{Part 2}}$

怎样简化的呢:

$$\frac{\partial x_b}{\partial\delta\theta} = \lim_{\delta\theta\to 0}\frac{R_{ab}\exp([\delta\theta]_\times)x_b - R_{ab}x_b}{\delta\theta}$$

$$\Rightarrow \frac{\partial x_b}{\partial\theta} = \frac{\partial R_{ab}\exp([\delta\theta]_\times)x_b}{\partial\delta\theta}$$

β 对 参数 $\delta\theta$ 的 Jacobian 推导, 即 F 第三列.

速度位移方程是 β 的递推形式.

$$\beta_{b_ib_{k+1}} = \beta_{b_ib_k} + a\cdot\delta t$$
$$= \beta_{b_ib_k} + \frac{1}{2}(q_{b_ib_k}(a^{b_k} - b^a_k) + q_{b_ib_{k+1}}(a^{b_{k+1}} - b^a_k))\delta t$$

δ_{31}: 对上一时刻速度位移方程的 Jacobian

$$\delta_{31} = \frac{\partial\beta_{b_ib_{k+1}}}{\partial\delta\beta_{b_kb'_k}} = I_{3\times3}$$

δ_{32}: 对角度旋转方程的 Jacobian:

$$\beta_{b_ib_{k+1}} = \beta_{b_ib_k} + \frac{1}{2}(q_{b_ib_k}(a^{b_k} - b^a_k) + q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\omega\delta t\end{bmatrix}(a^{b_{k+1}} - b^a_k))\delta t$$

$$\boxed{\frac{\partial\beta_{b_ib_{k+1}}}{\partial\delta\theta_{b_kb'_k}}}$$

Part 1:

$$\frac{\partial R_{b_ib_k}\exp([\delta\theta_{b_kb'_k}]_\times)(a^{b_k} - b^a_k)\delta t}{\partial\delta\theta_{b_kb'_k}}$$

$$= \frac{\partial R_{b_ib_k}(I + [\delta\theta_{b_kb'_k}]_\times)(a^{b_k} - b^a_k)\delta t}{\partial\delta\theta_{b_kb'_k}}$$ → 泰勒展开

$$= \frac{\partial(R_{b_ib_k}[\delta\theta_{b_kb'_k}]_\times(a^{b_k} - b^a_k)\delta t)}{\partial\delta\theta_{b_kb'_k}}$$

$$= -\frac{\partial - R_{b_ib_k}[(a^{b_k} - b^a_k)\delta t]_\times\delta\theta_{b_kb'_k}}{\partial\delta\theta_{b_kb'_k}}$$ 这不用展开. $a\times b = -b\times a$.

$$= -R_{b_ib_k}[(a^{b_k} - b^a_k)\delta t]_\times$$

Part 2:

$$\frac{\partial R_{b_ib_k}\exp([\delta\theta_{b_kb'_k}]_\times)\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta\theta_{b_kb'_k}}$$

$$= \frac{\partial R_{b_ib_k}(I + [\delta\theta_{b_kb'_k}]_\times)\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta\theta_{b_kb'_k}}$$

$$= \frac{\partial R_{b_ib_k}[\delta\theta_{b_kb'_k}]_\times\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta\theta_{b_kb'_k}}$$

$$= -\frac{\partial R_{b_ib_k}[\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t]_\times\delta\theta_{b_kb'_k}}{\partial\delta\theta_{b_kb'_k}}$$ ⊗

$$= -R_{b_ib_k}(\exp([\omega\delta t]_\times)(a^{b_{k+1}} - b^a_k)\delta t)_\times\exp([\omega\delta t]_\times)$$

$$= -R_{b_ib_{k+1}}(a^{b_{k+1}} - b^a_k)\delta t]_\times\exp([-\omega\delta t]_\times)$$

$$\approx -R_{b_ib_{k+1}}[(a^{b_{k+1}} - b^a_k)\delta t]_\times(I - [\omega\delta t]_\times)$$

δ_{36}: 速度位移方程对上时刻速度 bias 的 Jacobian

$$U = \frac{1}{2}(\omega^{b_k} - b^g_k) + (\omega^{b_{k+1}} - b^g_k)) = \frac{1}{2}(\omega^{b_k} + \omega^{b_{k+1}}) - b^g_k$$

$$\beta_{b_ib_{k+1}} = \beta_{b_ib_k} + \frac{1}{2}(q_{b_ib_k}(a^{b_k} - b^a_k) + q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\omega\delta t\end{bmatrix}(a^{b_{k+1}} - b^a_k))\delta t$$

既是 δb^g_k 的函数 δb^g_k 的函数

$$\Rightarrow \delta_{36} = \frac{\partial\beta_{b_ib_{k+1}}}{\partial\delta b^g_k} = \frac{\partial\frac{1}{2}q_{b_ib_k}\otimes\begin{bmatrix}1\\ \frac{1}{2}\omega\delta t\end{bmatrix}\otimes\begin{bmatrix}1\\ \frac{1}{2}\delta b^g_k\delta t\end{bmatrix}(a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta b^g_k}$$

$$= \frac{1}{2}\cdot\frac{\partial R_{b_ib_k}\exp([\omega\delta t]_\times)\exp([-\delta b^g_k\delta t])(a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta b^g_k}$$

$$= \frac{1}{2}\cdot\frac{\partial R_{b_ib_{k+1}}[I + [-\delta b^g_k\delta t]_\times](a^{b_{k+1}} - b^a_k)\delta t}{\partial\delta b^g_k}$$

$$= \frac{1}{2}\cdot\frac{\partial - R_{b_ib_{k+1}}([(a^{b_{k+1}} - b^a_k)\delta t]_\times](-\delta b^g_k\delta t)}{\partial\delta b^g_k}$$

$$= \frac{1}{2} \cdot \frac{\partial R_{b_i b_{k+1}} [I + [-\delta b_k^a \delta t]_\times] (a^{b_{k+1}} - b_k^a) \delta t}{\partial \delta b_k^a}$$

$$= \frac{1}{2} \frac{\partial -R_{b_i b_{k+1}} ([(a^{b_{k+1}} - b_k^a)\delta t]_\times)(-\delta b_k^a \delta t)}{\partial \delta b_k^a}$$

$$= -\frac{1}{2} R_{b_i b_{k+1}} [(a^{b_{k+1}} - b_k^a)\delta t]_\times (-\delta t).$$

旋转平移积分的 Jacobian.

$$\omega = \frac{1}{2}((\omega_b^k - b_k^g) + (\omega_b^{k+1} - b_k^g))$$

$$q_{b_i b_{k+1}} = q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}$$

$$= q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}(\frac{1}{2}((\omega_b^k + \omega_b^{k+1}) - b_k^g)\delta t \end{bmatrix}$$

求证：前一时刻的旋转误差 $\delta\theta_{b_k b_k'}$，如何影响当前时刻的旋转误差：$\delta\theta_{b_{k+1} b_{k+1}'}$

引入：

假设两个时刻的真值为 $q_{b_i b_{k+1}}$，$q_{b_i b_k}$，两个时刻间的增量真值为 $q_{b_k b_{k+1}}$.

推导过程为考虑旋转误差 $\delta\theta_{b_k b_k'}$ 的影响，暂不考虑测量值角速度 bias 误差影响.

可以假设 $q_{b_k b_{k+1}} \approx \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}$.

另外，三元组四元数乘积有如下性质：

$$q \otimes p \otimes q^* = [q]_L [q]_R^T p = \begin{bmatrix} p_w \\ R p_v \end{bmatrix}$$

其中，R 是 q 对应的旋转矩阵，p_w 为 p 的实部，p_v 为 p 的虚部.

$$q_{b_i b_{k+1}} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_{k+1} b_{k+1}'} \end{bmatrix} = q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_k b_k'} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}.$$

$b_{k+1} \to b_i \to b_k$.

$$\begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_{k+1} b_{k+1}'} \end{bmatrix} = q^*_{b_i b_{k+1}} \otimes q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_i b_k'} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}$$

$$= q_{b_{k+1} b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_k b_k'} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}$$

$$\Updownarrow$$

$$\approx \begin{bmatrix} 1 \\ -\frac{1}{2}\omega \delta t \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta_{b_k b_k'} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \frac{1}{2} R \delta\theta_{b_k b_k'} \end{bmatrix}$$

$$\Rightarrow \delta\theta_{b_{k+1} b_{k+1}'} = R \delta\theta_{b_k b_k'} = \exp([-\omega \delta t]_\times) \delta\theta_{b_k b_k'}$$

$$\approx (I - [\omega \delta t]_\times) \delta\theta_{b_k b_k'}$$