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Beyond risk parity – A machine learning-based hierarchical risk parity approach on cryptocurrencies[☆]

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ABSTRACT

It has long been known that estimating large empirical covariance matrices can lead to very unstable solutions, with estimation errors more than offsetting the benefits of diversification. In this study, we employ the Hierarchical Risk Parity approach, which applies state-of-the-art mathematics including graph theory and unsupervised machine learning to a large portfolio of cryptocurrencies. An out-of-sample comparison with traditional risk-minimization methods reveals that Hierarchical Risk Parity outperforms in terms of tail risk-adjusted return, thereby working as a potential risk management tool that can help cryptocurrency investors to better manage portfolio risk. The results are robust to different rebalancing intervals, covariance estimation windows and methodologies.

1. Introduction

Financial markets are often labeled complex systems. While there is no universally accepted definition of "complexity", there is general agreement that complex systems are composed of elements that interact with each other and whose behavior is intrinsically difficult to model.

While modelling complex systems may seem like a daunting task, they often have a structure and are usually organized in a hierarchical manner by being composed of subsystems that in turn have their own subsystems. This property is exploited by so-called hierarchical models. Unfortunately, in the context of the portfolio construction process, investors face one major challenge – the lack of hierarchical structure in the correlation matrix.[1] This problem is exacerbated for large covariance matrices.

To alleviate this problem, Hierarchical Risk Parity (HRP) takes another approach. First proposed by De Prado (2016), HRP uses graph theory and machine learning algorithms to infer the hierarchical relationships between the assets which are then directly utilized for portfolio diversification. By replacing the covariance structure with a hierarchical structure of clusters, HRP fully utilizes the information contained in the covariance matrix and recovers the stability of the weights. Despite its appealing features, the empirical literature on the out-of-sample performance of HRP is extremely scarce.[2] Raffinot (2017) evaluates the performance across three datasets consisting of S&P 500 sectors, multi-assets, and individual stocks. More recently, Lohre et al. (2020) employ the strategy

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[1] To clarify this point, consider an investor who wishes to build a diversified stock portfolio. While some stocks are closely related in terms of industry or region, other stocks are almost completely unrelated to each other. However, in a typical portfolio construction task, all stocks compete with each other for allocation and are potential substitutes, as the correlation matrix disregards any fundamental structure.

[2] As of February 6, 2020, a simple SSRN search (https://www.ssrn.com) for "hierarchical risk parity" generates only seven results.

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in a multi-asset multi-factor allocation context, showing that it achieves better tail risk results. Furthermore, Jain and Jain (2019) apply the strategy to individual stocks that comprise the NIFTY 50 index. Finally, Raffinot (2017) evaluates the performance of different HRP variants (HCCA and HERC).

Other studies examine the performance of traditional portfolios when cryptocurrencies are included or excluded. For instance, Matkovskyy et al. (2019) investigate the role of cryptocurrencies in enhancing portfolio return of poorly performing stocks by applying a probabilistic utility approach. They show that the addition of cryptocurrencies helps to improve performance. Platanakis and Urquhart (2019b) study the relationship between traditional stock and bond portfolios and Bitcoin. They report considerable improvement in terms of Sharpe, Sortino, and Omega ratio when Bitcoin is added. Brauneis and Mestel (2019) analyze cryptocurrency portfolios in a mean-variance framework, showing that Markowitz optimized portfolios show higher Sharpe ratios than single cryptocurrencies, but that naively diversified (1/N) portfolios outperform Markowitz strategies. However, studies by Platanakis and Urquhart (2019a); Burggraf (2019) indicate that this might be due to estimation error in the expected return estimates rather than the superiority of the 1/N strategy itself: Burggraf (2019) applies a set of risk-based optimization strategies, which require information on the covariance matrix only and thereby reduce estimation risk in the optimization process, to a portfolio of cryptocurrencies. The results show that tail-risk (Value at Risk, expected shortfall, maximum drawdown) can be significantly reduced. In line with the aforementioned study, Platanakis and Urquhart (2019a) show that a Black Litterman model with variance based constraints (VBCs) yields superior out-of-sample risk-adjusted returns, supporting the hypothesis that sophisticated portfolio techniques that control for estimation error are preferred over simpler techniques when managing cryptocurrency portfolios.

Another strand of literature explores technical trading rules in the cryptocurrency market. Corbet et al. (2019) compare various technical trading rules in the form of the moving average-oscillator and trading range break-out strategies. Their results provide support for moving average strategies. Employing multivariate econometric tests, the results by Grobys et al. (2020) confirm this, indicating that simple moving average rules produce excess returns. They conclude that the cryptocurrency market is inefficient. In the same vein, Ahmed et al. (2020) study simple moving average trading strategies for a portfolio of the ten most-traded cryptocurrencies. Contrary to previous studies, they find that technical trading rules do not generate positive returns in excess of a buy-and-hold strategy.

Lastly, related studies by Mensi et al. (2019), Katsiampa et al. (2019) and Bouri et al. (2019c) find evidence for significant spillover effects in volatility across cryptocurrencies and precious metals. Their results have important implications for investors and portfolio managers as they can readily inform their decision-making.

This study contributes to the growing literature on the use of machine learning techniques in the context of asset allocation by employing Hierarchical Risk Parity to a portfolio of cryptocurrencies. While most previous studies focus on stock or multi-asset portfolios, there is a lack of empirical literature on the out-of-sample performance of HRP applied to other asset classes. This study attempts to fill this gap.

There are three main reasons that prompt us to use cryptocurrencies for the portfolio optimization task. First, recent research demonstrates that cryptocurrencies act as a hedge and powerful portfolio diversifier (Liu, 2019; Bouri et al., 2019a; 2017; Akhtaruzzaman et al., 2019). Therefore, research on the state-of-the-art of portfolio diversification methods is of interest for both retail and institutional investors. Second, most cryptocurrencies provide low transaction costs and ample liquidity (Kim, 2017; 2019), which makes them particular suitable for active portfolio optimization strategies that require frequent rebalancing. Third, the cryptocurrency investment universe became extremely large.[3] With an ever-evolving investment landscape, more systematic, algorithm-driven investment solutions are needed. Lastly, cryptocurrencies are known for their elevated volatility and tail risk (Bouri et al., 2019b; Tan et al., 2018; Baur et al., 2018). HRP provides a tail risk management tool to reduce risk.

The remainder is structured as follows. Section 2 introduces the methodology. Section 3 gives an overview of the data and portfolio construction process. Section 4 reports the out-of-sample performance and Section 5 concludes.

2. Methodology

The concept of HRP builds upon graph theory and machine learning techniques and can be separated into three main steps – Tree clustering, quasi-diagonalization and recursive bisection. In the following, we describe each step in more detail.[4]

The first step involves breaking down the assets in the portfolio into different clusters using the Hierarchical Tree Clustering algorithm. For two assets i and j, the correlation matrix is transformed to a correlation-distance matrix D of the following form

$$D(i,j) = \sqrt{0.5 \times (1 - \rho(i,j))}. \tag{1}$$

In the next step, we calculate the Euclidean distance between all the columns in a pair-wise manner. This leaves us with the augmented distance matrix $\overline{D}$

$$\overline{D}(i,j) = \sqrt{\sum_{k=1}^{N} (D(k,i) - D(k,j))^2}. \tag{2}$$

From Eq. (2), we construct clusters using a recursive approach. If we define the set of clusters as U, then the first cluster (i^*, j^*) is

[3] According to coinmarketcap, the total number of cryptocurrencies is 5266 (as of March 28, 2020).

[4] For a more detailed explanation of the algorithm, the reader is referred to Vyas (2019).

calculated as

$$U[1] = \underset{i,j}{\operatorname{argmin}} \overline{D}(i,j). \tag{3}$$

The distance matrix $\overline{D}$ is then updated by calculating the distances of the other assets from cluster $U[1]$ using single linkage clustering. Hence, for any asset i outside the cluster, the distance to the newly formed cluster is calculated as follows

$$\overline{D}(i, U[1]) = min\left(\overline{D}(i,i^*), \overline{D}(i,j^*)\right). \tag{4}$$

In this way, the algorithm recursively combines the assets in the portfolio into clusters and updates the distance matrix until we are left with only one single cluster.

In the second step, quasi-diagonalization of the covariance matrix is used which rearranges the data to show the inherent clusters clearly. The rows and columns of the covariance matrix are reorganized such that similar assets are placed together and dissimilar investments are placed apart. In this way, large covariances are placed along the diagonal of the covariance matrix, while smaller covariances are placed around this diagonal – hence the name *quasi-diagonal*.

The final recursive bisection step involves assigning actual portfolio weights to the assets in the portfolio. Here, the algorithm exploits the portfolio's property that the inverse-variance allocation is optimal for a diagonal covariance matrix. Therefore, given the portfolio weights $w_i = 1, \forall i = 1, \dots, N$, and starting from the final cluster at the end of the tree-clustering step, we split each cluster into two sub-clusters V_1 and V_2 by traversing down the cluster-tree. For each sub-cluster, the variance is calculated such that

$$V_{1,2} = w^{'} V w \tag{5}$$

where

$$w = \frac{diag(V)^{-1}}{trace\left(diag(V)^{-1}\right)}. \tag{6}$$

In this way, HRP takes advantage of the second step which quasi-diagonalised the covariance matrix and the fact that for a diagonal covariance matrix, the inverse-variance allocations are the most optimal. A weighting factor for α_1 and α_2 is calculated, respectively

$$\alpha_1 = 1 - \frac{V_1}{V_1 + V_2}, \quad \alpha_2 = 1 - \alpha_1. \tag{7}$$

Based on the weighting factors from Eq. (7), the weights for the assets in the portfolio are updated for each subcluster. In this way, only assets within each cluster compete for allocation for the final portfolio construction, rather than with assets from the whole investment universe. Finally, the weights w_1 and w_2 for both subclusters are allocated as follows

$$w_1 = \alpha_1 * w_1, \quad w_2 = \alpha_2 * w_2. \tag{8}$$

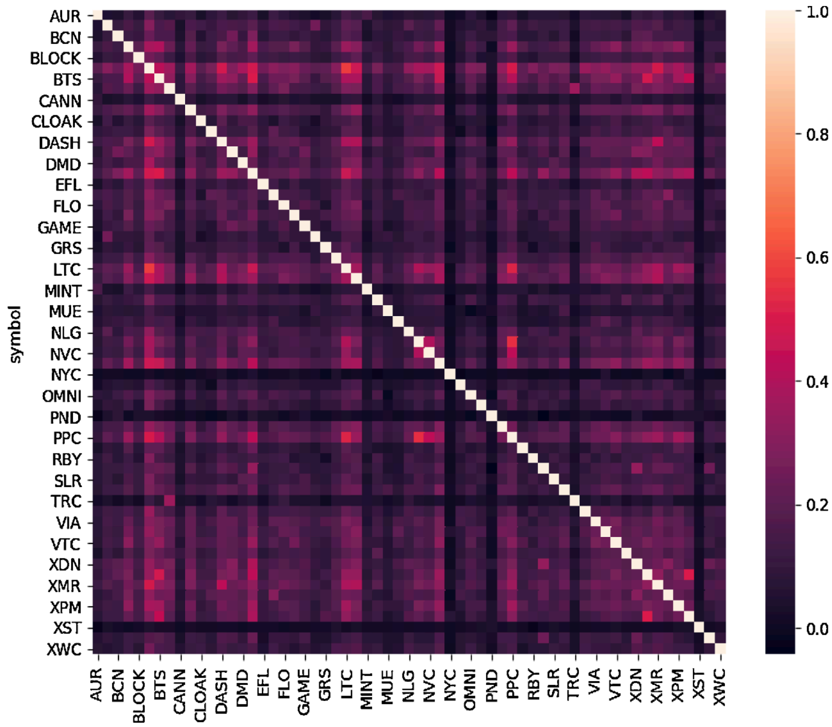
3. Data and descriptive statistics

We use daily cryptocurrency prices for the period January 1, 2015 to November 1, 2019 (1,766 observations). The prices are collected from coinmarketcap.[5] Series with five or more missing data were excluded.[6] Missing values were filled by propagating the last valid observation forward. Thus, our final data sample includes 61 cryptocurrencies. Average daily mean return across all cryptocurrencies is 1.25% and volatility is 27.05%. Table A.1 in the Appendix reports descriptive statistics.

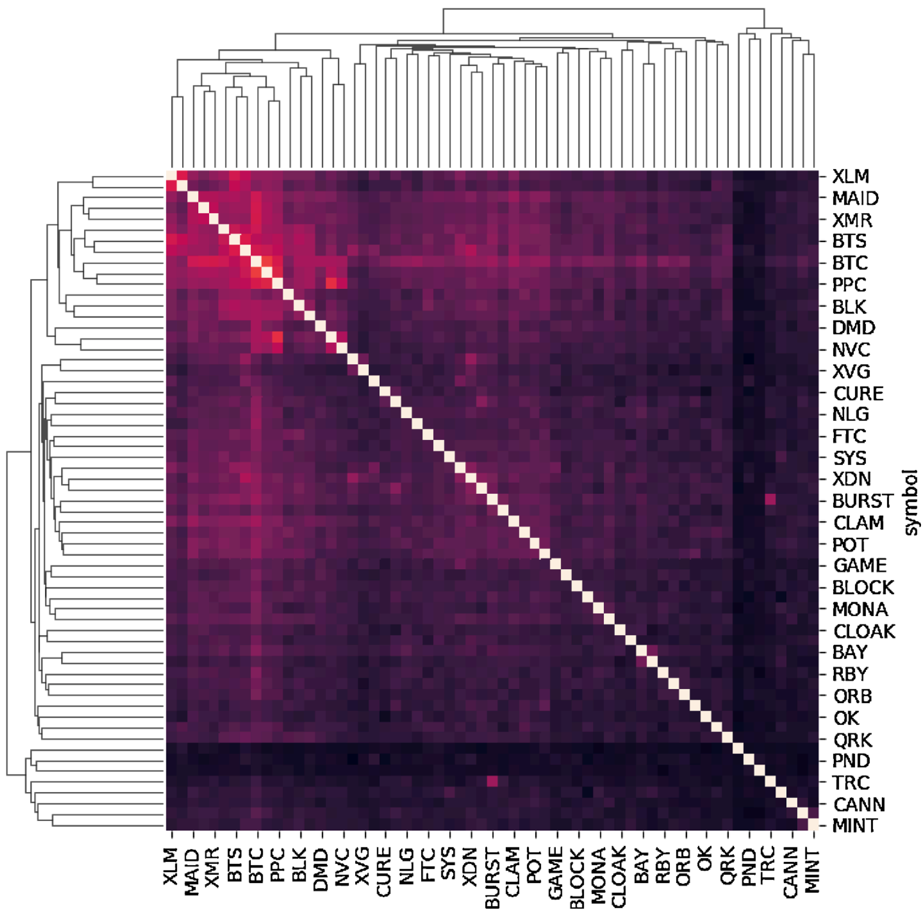
Fig. 1 a shows the original correlation matrix. Fig. 1b depicts the correlation matrix after it has been reorganized in clusters using the hierarchical tree clustering and quasi-diagonalization. The reorganized covariance matrix then serves as the input for the actual asset allocation task. As can be seen, this reordering results in a covariance matrix that groups similar cryptocurrencies together and those dissimilar further away, which helps making more meaningful asset allocation decisions and building more diversified portfolios.

[5] https://coinmarketcap.com/.

[6] This is done because datasets containing missing values are incompatible with the algorithms. We are aware that this might result in sample selection bias, especially survivorship bias, which in turn lead to upward (downward) biased results in terms of portfolio return (risk). However, we strongly believe that this strategy of dealing with missing data does not erode the legitimacy of our statistical analysis for two reasons. First, the aim of this study is to compare the performance of a novel machine-learning based algorithm for portfolio optimization purposes with other, more traditional algorithms, rather than providing any investment advice based on absolute return or risk metrics. Because all the algorithms have to deal with the same set of data as a first step in the optimization process, the relative performance is not altered and the comparability of the results remains intact. Second, even after restricting the analysis to a subset of the fully observed variables, the sample remains reasonably large with 61 cryptocurrencies. The consensus in the finance literature is that portfolios consisting of 20–30 assets already provide good diversification benefits (Evans and Archer, 1968; Statman, 1987). Hence, the used sample is sufficiently large to ensure that all the algorithms can freely allocate across a broad range of investment opportunities.



(a) Unclustered correlation matrix



(b) Clustered correlation matrix

(caption on next page)

Fig. 1. Correlation matrix before and after quasi-diagonalization.

4. Empirical results

We use three popular traditional risk-based asset allocation strategies to compare and benchmark the HRP strategy, namely Inverse Volatility (IV), Minimum Variance (MV), and Maximum Diversification (MD) portfolios.[7]

Because we use a rolling window of 250 days to estimate the covariance matrix, the first day of backtesting is initialized on September 8, 2015. The window is then rolled forward until we reach the end of the sample period on November 1, 2019. Table 1 Panel A summarizes the out-of-sample performance of the HRP portfolio relative to the other strategies using a 250 days covariance estimation window. The annualized return and volatility of the HRP strategy is 2.891 and 0.882, respectively. Although MD provides significantly more return (4.534), and IV reduces volatility slightly better (2.652), HRP balances out both risk and return most effectively, therefore providing the best risk-return trade-off of all strategies in terms of Sharpe ratio (0.171 compared to 0.158, 0.085 and 0.082 of the IV, MV and MD portfolio, respectively). In addition, HRP yields the best tail-risk-return trade-off measured by the Calmar ratio (average annual rate of return / maximum drawdown). The Calmar ratio for HRP is 6.518 versus 6.042, 3.033 and 5.133 of the benchmark risk-based allocations. Finally, it is the second best performing strategy in terms of Sortino ratio (average annual rate of return / standard deviation of negative returns) with 0.0072.

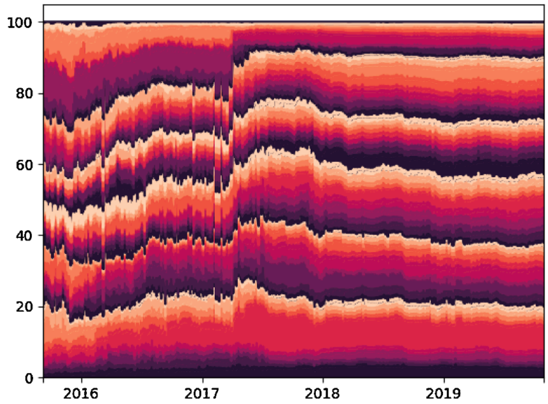
Table 1 Panel B and C report the performance for covariance estimation windows of length 500 and 750 days, respectively, therefore showing how the strategies' mean, standard deviation, and downside risk measures change when the rolling window shifts. While volatility of HRP tends to increase with longer estimation windows, therefore slightly reducing Sharpe ratio (from 0.171 to 0.167 and 0.158, respectively), this applies to all methods, thereby leaving its relative performance untouched and making it the superior strategy. The same applies for the Calmar and Sortino ratio – HRP remains the superior risk-minimization strategy measured

Table 1
Portfolio return and risk performance.

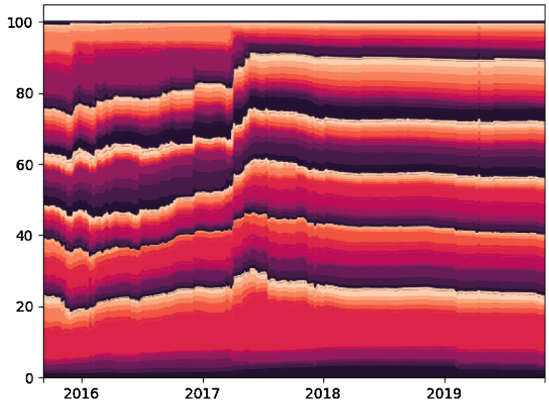
	Sample covariance matrix				Shrinkage covariance matrix			
	HRP	IV	MV	MD	HRP	IV	MV	MD
Pandel A: Window = 250								
Annualized return	2.8913	2.6522	2.3518	4.5343	3.4178	2.6522	2.3515	3.4646
Annualized volatility	0.8819	0.8779	1.4456	2.8673	0.9805	0.8779	1.0602	1.5449
Value-at-Risk (5%)	0.0098	0.0005	0.0043	0.0130	0.0135	0.0005	0.0010	0.0046
Conditional Value-at-Risk (5%)	0.0019	0.0011	0.0005	0.0019	0.0049	0.0011	0.0004	0.0019
Drawdown	0.3171	0.3540	0.7069	0.8459	0.3817	0.3540	0.5822	0.7181
Maximum drawdown	0.4435	0.4389	0.7755	0.8834	0.5152	0.4389	0.6920	0.7917
Sharpe ratio	0.1716	0.1581	0.0852	0.0828	0.1825	0.1581	0.1161	0.1174
Calmar ratio	6.5185	6.0423	3.0326	5.1325	6.6337	6.0423	3.3983	4.3760
Sortino ratio	0.0072	0.0066	0.0068	0.0121	0.0081	0.0066	0.0063	0.0092
Panel B: Window = 500								
Annualized return	2.9839	2.7675	2.2262	4.7684	3.4379	2.7675	2.2100	3.6598
Annualized volatility	0.9310	0.9295	0.9744	2.6048	1.0319	0.9295	0.8484	1.5313
Value-at-Risk (5%)	0.0175	0.0182	0.0160	0.0229	0.0039	0.0182	0.0269	0.0215
Conditional Value-at-Risk (5%)	0.0132	0.0116	0.0081	0.0131	0.0130	0.0116	0.0099	0.0125
Drawdown	0.3171	0.3539	0.4944	0.8169	0.3817	0.3539	0.2857	0.6883
Maximum drawdown	0.4435	0.4389	0.6134	0.8611	0.5152	0.4389	0.4251	0.7714
Sharpe ratio	0.1677	0.1558	0.1195	0.0958	0.1744	0.1558	0.1363	0.1251
Calmar ratio	6.7275	6.3050	3.6293	5.5371	6.6729	6.3050	5.1982	4.7444
Sortino ratio	0.0077	0.0071	0.0066	0.0130	0.0084	0.0071	0.0060	0.0098
Panel C: Window = 750								
Annualized return	2.9737	2.8767	2.2161	5.5427	3.4330	2.8767	2.7677	3.9270
Annualized volatility	0.9832	1.0010	0.9142	2.7918	1.0737	1.0010	1.2177	1.6674
Value-at-Risk (5%)	0.0650	0.0596	0.0804	0.0793	0.0546	0.0596	0.0684	0.0706
Conditional Value-at-Risk (5%)	0.0077	0.0060	0.0048	0.0095	0.0062	0.0060	0.0057	0.0074
Drawdown	0.3171	0.3539	0.3213	0.8069	0.3817	0.3539	0.5395	0.6784
Maximum drawdown	0.4059	0.4389	0.4792	0.8525	0.4719	0.4389	0.6651	0.7633
Sharpe ratio	0.1583	0.1504	0.1268	0.1039	0.1674	0.1504	0.1190	0.1233
Calmar ratio	7.3246	6.5537	4.6244	6.5014	7.2750	6.5537	4.1616	5.1448
Sortino ratio	0.0079	0.0076	0.0067	0.0153	0.0087	0.0076	0.0076	0.0106

Note: The table presents portfolio performance and risk statistics using rolling-window sample and shrinkage covariance matrix estimates of 250, 500, and 750 days. The shrinkage covariance matrix is calculated following Ledoit and Wolf (2004), where the shrinkage target S is a diagonal matrix of only variances with zeros elsewhere and the shrinkage constant is $\delta = 0.3$. Return and volatility is annualized using an annualization factor of 365 trading days. All series are in logarithmic first differences.

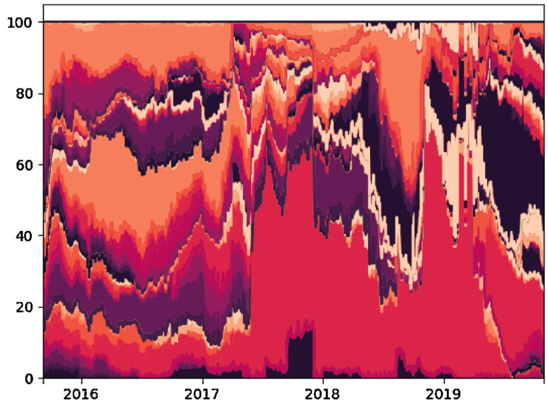
[7] We use risk-based strategies over market capitalization weighted indices as the benchmark for two reasons. First, it is common industry practice and in line with previous studies (see, for example, Kremer et al., 2018; Lohre et al., 2020; Burggraf, 2019). Second, since they focus on diversification, risk-based strategies tend to be tougher to beat, thereby providing a fair benchmark.



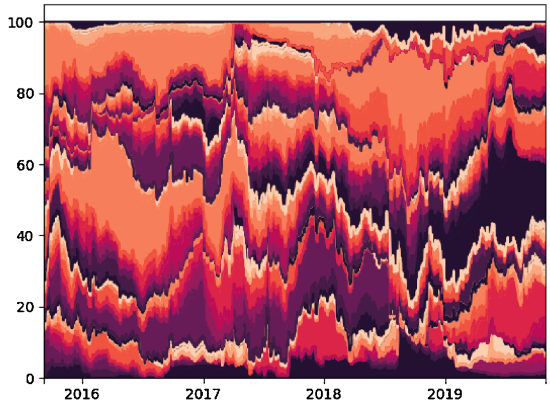
(a) Hierarchical Risk Parity



(b) Inverse volatility



(c) Minimum variance



(d) Maximum diversification

Fig. 2. Portfolio weight decomposition of asset allocation strategies. The figure plots the weight decomposition over time for the four risk-based asset allocation strategies. The values represent the investment period from September 8, 2015 to November 1, 2019, considering a rolling window covariance estimation approach of one trading year (250 days). The y-axis shows the weight allocation on a scale from 0% to 100%.

by the Calmar ratio and the second best by Sortino ratio across all estimation windows. As an additional test, we use shrinkage covariance matrix estimation as suggested by Ledoit and Wolf (2004, 2000). Shrinkage estimators pull extreme covariance values towards certain target values, which reduces noise and guarantees minimum mean squared error, among others. The results are presented on the right handside of Table 1, providing consistent evidence that the results are independent of the selected covariance estimation method. Lastly, to alleviate the potential concern that frequent position rebalancing and the associated transaction costs might erode the strategy's profitability, we repeat our analysis for lower rebalancing frequencies of one week, one month, and one quarter, respectively. Transaction costs should become negligible with lower frequencies. The results remain robust – HRP outperforms the other strategies in terms of risk-adjusted return across all rebalancing periods. Table A.2 in the Appendix summarizes the results.

In Fig. 2, we plot the weight allocation of the different strategies, providing further intuition on how the strategies are allocation their weights. There are several points that are worth noting. First, from Fig. 2a and b, it is obvious that HRP and IV are closely related in terms of how they allocate assets. However, while IV has more stable weight distributions, HRP seems to make adjustments to its weight allocation more frequently. This has the advantage of more dynamic weight allocations, but also results in higher turnover and therefore transaction costs. Fig. 2c shows the allocation of the MV portfolio, having by far the most concentrated weight allocation among all competing risk-based strategies. While MV neglects allocating to certain cryptocurrencies all together, it significantly overweights others. Lastly, MD in Fig. 2d shows well-diversified weight allocations, but the concentration of certain cryptocurrencies varies significantly over time, resulting excessive turnover.

5. Conclusion

In this study, we employed a novel asset allocation method – Hierarchical Risk Parity. Applied to a portfolio of cryptocurrencies, our results show that HRP better navigates volatility and tail risk compared to traditional risk-based strategies. In addition, HRP has the most desirable diversification properties – while IV portfolios tend to be too static, MV results in too concentrated portfolios. Our results survive many robustness tests, including different estimation windows, covariance estimation methodologies, and rebalancing periods.

Thus, HRP provides a meaningful alternative to traditional asset allocation approaches and an important risk management tool for cryptocurrency investors. Because HRP is an extremely novel approach, there is a lack of empirical studies. Future research directions might include testing the out-of-sample performance of other asset classes or combining it with robust portfolio optimization techniques.

Appendix A

Tables A.1 and A.2.

Table A.1
Descriptive statistics.

	Mean	SD	Min	25%	50%	75%	Max
AUR	0.0125	0.1819	−0.6352	−0.0475	−0.0014	0.0448	3.2339
BAY	0.0103	0.1440	−0.5957	−0.0495	0.0000	0.0512	2.0986
BCN	0.0097	0.1643	−0.5976	−0.0446	0.0000	0.0389	3.9423
BLK	0.0045	0.0928	−0.3869	−0.0364	−0.0018	0.0363	1.2330
BLOCK	0.0103	0.1386	−0.5826	−0.0577	−0.0021	0.0573	1.8873
BTC	0.0027	0.0392	−0.2115	−0.0114	0.0021	0.0178	0.2525
BTS	0.0030	0.0756	−0.3241	−0.0298	−0.0018	0.0290	0.6820
BURST	0.0053	0.0978	−0.3806	−0.0428	0.0000	0.0438	1.5189
CANN	0.0229	0.3092	−0.8041	−0.0503	−0.0019	0.0453	6.9771
CLAM	0.0034	0.0866	−0.6230	−0.0338	0.0000	0.0384	1.3024
CLOAK	0.0133	0.1733	−0.6428	−0.0615	−0.0010	0.0629	2.7356
CURE	0.0076	0.1346	−0.5778	−0.0463	−0.0034	0.0410	3.0379
DASH	0.0038	0.0597	−0.2159	−0.0243	−0.0014	0.0266	0.5492
DGB	0.0079	0.1154	−0.3497	−0.0390	−0.0026	0.0372	2.2079
DMD	0.0041	0.0911	−0.6108	−0.0361	0.0000	0.0381	1.2840
DOGE	0.0034	0.0645	−0.3891	−0.0211	0.0000	0.0191	0.6792
EFL	0.0092	0.1363	−0.5745	−0.0488	0.0000	0.0476	1.6781
EMC2	0.0092	0.1269	−0.4787	−0.0423	0.0000	0.0402	1.8608
FLO	0.0095	0.1229	−0.4379	−0.0519	−0.0050	0.0523	1.2922
FTC	0.0057	0.1164	−0.6940	−0.0426	−0.0058	0.0339	1.1719
GAME	0.0087	0.1306	−0.4796	−0.0452	−0.0043	0.0407	1.9913
GLC	0.0110	0.1718	−0.6956	−0.0512	−0.0033	0.0476	3.9037
GRS	0.0131	0.1522	−0.4068	−0.0488	−0.0046	0.0425	1.5154
IOC	0.0073	0.1070	−0.4082	−0.0508	−0.0019	0.0543	1.0100
LTC	0.0035	0.0608	−0.4019	−0.0183	0.0000	0.0193	0.6659
MAID	0.0027	0.0668	−0.3157	−0.0317	0.0006	0.0353	0.4093
MINT	0.0368	0.3078	−0.6723	−0.0807	0.0000	0.0780	3.3333
MONA	0.0059	0.0962	−0.4142	−0.0292	−0.0034	0.0240	1.3448
MUE	0.0206	0.2893	−0.8105	−0.0528	−0.0059	0.0533	8.9083
NAV	0.0118	0.2015	−0.7797	−0.0483	−0.0026	0.0458	6.7875
NLG	0.0039	0.0803	−0.4234	−0.0333	−0.0043	0.0288	0.6835
NMC	0.0038	0.0910	−0.6859	−0.0286	−0.0020	0.0298	1.0250
NVC	0.0039	0.0890	−0.8912	−0.0277	−0.0029	0.0259	0.9010
NXT	0.0026	0.0774	−0.4516	−0.0321	−0.0048	0.0268	0.8006
NYC	0.3260	7.4174	−0.9856	−0.1000	0.0000	0.0952	284.7143
OK	0.0169	0.2245	−0.6636	−0.0557	−0.0026	0.0589	6.6655
OMNI	0.0070	0.1279	−0.5495	−0.0496	−0.0024	0.0482	1.1649
ORB	0.0094	0.1277	−0.6907	−0.0449	0.0000	0.0522	1.1669
PND	0.0813	0.4929	−0.8922	−0.1138	0.0000	0.1176	7.0000
POT	0.0050	0.0930	−0.5773	−0.0395	−0.0028	0.0412	0.9575
PPC	0.0017	0.0670	−0.4868	−0.0278	−0.0011	0.0265	0.4996
QRK	0.0122	0.1771	−0.8290	−0.0434	0.0000	0.0387	3.1941
RBY	0.0099	0.1400	−0.5993	−0.0397	0.0006	0.0438	2.1733
RDD	0.0115	0.1676	−0.7891	−0.0532	0.0000	0.0556	3.3235
SLR	0.0073	0.1231	−0.5948	−0.0474	−0.0019	0.0439	1.5117
SYS	0.0078	0.1230	−0.6674	−0.0430	−0.0017	0.0432	3.0000
TRC	0.0213	0.3977	−0.8991	−0.0578	−0.0014	0.0563	14.1111
UNO	0.0060	0.0906	−0.5976	−0.0360	−0.0007	0.0388	0.8346
VIA	0.0058	0.1040	−0.3459	−0.0462	−0.0019	0.0439	1.1377
VRC	0.0059	0.1072	−0.4838	−0.0462	−0.0033	0.0444	1.0496
VTC	0.0067	0.1105	−0.4496	−0.0409	−0.0036	0.0354	1.4053
XCP	0.0052	0.1188	−0.4938	−0.0487	−0.0051	0.0429	2.3097
XDN	0.0071	0.1190	−0.4014	−0.0457	0.0000	0.0436	1.7501
XLM	0.0044	0.0831	−0.3067	−0.0295	−0.0033	0.0264	1.0608
XMR	0.0050	0.0697	−0.2777	−0.0274	−0.0002	0.0336	0.7943
XMY	0.0087	0.1338	−0.6149	−0.0480	0.0000	0.0426	1.8043
XPM	0.0041	0.0963	−0.4337	−0.0372	−0.0027	0.0345	1.4117
XRP	0.0039	0.0793	−0.4600	−0.0206	−0.0034	0.0170	1.7937
XST	0.0307	0.9407	−0.9683	−0.0559	−0.0047	0.0521	39.0834
XVG	0.0179	0.2161	−0.5000	−0.0556	0.0000	0.0500	5.8000
XWC	0.0116	0.1601	−0.4467	−0.0527	−0.0011	0.0503	2.0571

Note: The table presents descriptive statistics for the 61 cryptocurrencies for the period January 1, 2015 – November 1, 2019. Mean is mean return, SD is standard deviation. All series are in logarithmic first differences.

Table A.2
Portfolio return and risk performance for different rebalancing intervals.

	HRP	IV	MV	MD
Pandel A: Weekly rebalancing				
Annualized return	2.7612	1.9956	2.1727	4.2117
Annualized volatility	1.0393	0.9314	1.2752	2.5061
Value-at-Risk (5%)	0.0036	0.0238	0.0016	0.0253
Conditional Value-at-Risk (5%)	0.0207	0.0076	0.0039	0.0110
Drawdown	0.5913	0.5756	0.5762	0.7767
Maximum drawdown	0.7256	0.7181	0.8203	0.9151
Sharpe ratio	0.3684	0.2971	0.2362	0.2330
Calmar ratio	3.8053	2.7788	2.6483	4.6023
Sortino ratio	0.0513	0.0377	0.0484	0.0920
Panel B: Monthly rebalancing				
Annualized return	2.7217	1.9669	2.2537	4.0675
Annualized volatility	1.2603	1.1260	1.8823	2.7141
Value-at-Risk (5%)	0.1580	0.0731	0.0797	0.1366
Conditional Value-at-Risk (5%)	0.1248	0.0721	0.0265	0.1033
Drawdown	0.0914	0.0817	0.0152	0.0833
Maximum drawdown	0.7198	0.7102	0.8438	0.8983
Sharpe ratio	0.6233	0.5042	0.3456	0.4326
Calmar ratio	3.7808	2.7695	2.6709	4.5276
Sortino ratio	0.2857	0.2124	0.2734	0.5290
Panel C: Quarterly rebalancing				
Annualized return	2.8173	2.0549	2.0421	3.9611
Annualized volatility	1.6289	1.36203	1.8764	2.9320
Value-at-Risk (5%)	0.3910	0.2649	0.0915	0.3790
Conditional Value-at-Risk (5%)	–	–	–	–
Drawdown	–	–	–	–
Maximum drawdown	0.8932	0.8570	0.8851	0.9283
Sharpe ratio	0.8647	0.75438	0.5441	0.6754
Calmar ratio	3.1540	2.3977	2.3071	4.2667
Sortino ratio	1.0739	0.8775	0.9004	2.0101

Note: The table presents portfolio performance and risk statistics for weekly, monthly, and quarterly rebalancing periods, respectively. Similar to the base case analysis, we use 250 day rolling-window sample covariance matrix estimates for the optimization. Return and volatility is annualized using annualization factors of 52, 12, and 4, respectively. All series are in logarithmic first differences.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.frl.2020.101523 .

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