PS2_1

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1 Problem Set 2

1.1 Siyuan Peng

```
In [1]: import sympy as sy
        import numpy as np
        import pandas as pd
        import warnings
        warnings.filterwarnings("ignore")
        from matplotlib import pyplot as plt
```

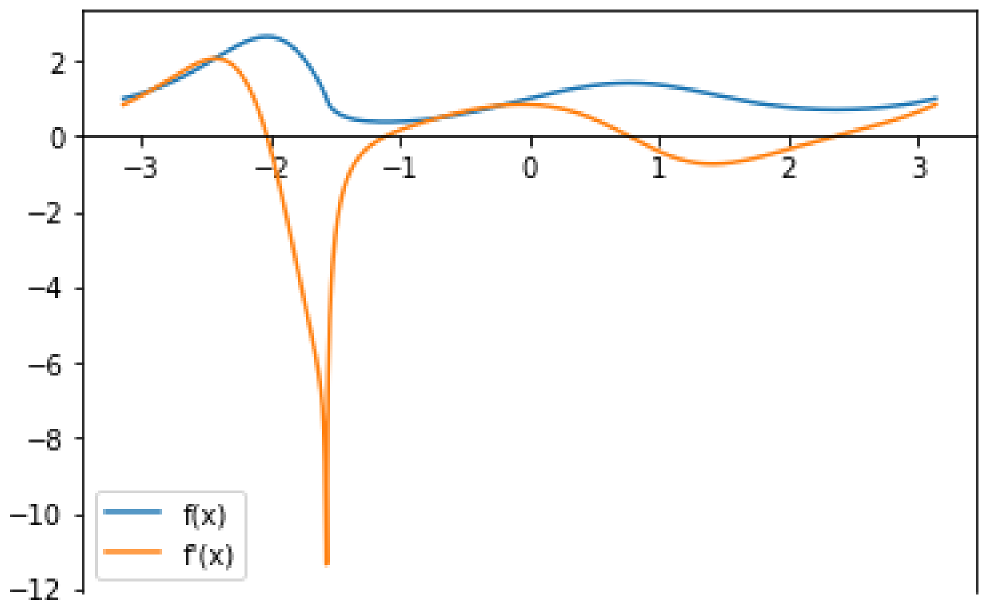
1.1.1 Problem 1

```
In [2]: x = sy.symbols('x')
        f_x = (sy.sin(x)+1) ** (sy.sin(sy.cos(x)))
        f_prime = sy.diff(f_x, x)
        f_prime
```

```
Out[2]: (-log(sin(x) + 1)*sin(x)*cos(cos(x)) + sin(cos(x))*cos(x)/(sin(x) + 1))*(sin(x) + 1)**s
```

```
In [3]: # plot f(x) and its derivative
        x_space = np.linspace(-np.pi,np.pi,1000)
        ax = plt.gca()
        ax.spines['bottom'].set_position("zero")

        f = sy.lambdify(x, f_x)
        fx_prime = sy.lambdify(x, f_prime)

        ax.plot(x_space,f(x_space),label='f(x)')
        ax.plot(x_space,fx_prime(x_space),label='f\'(x)')
        ax.legend()
```

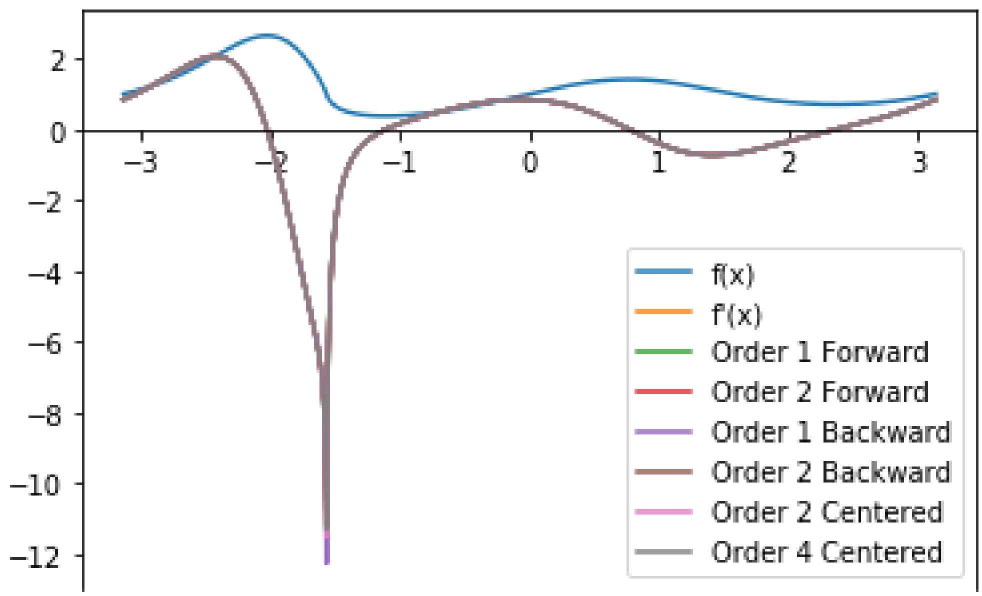
```
Out[3]: <matplotlib.legend.Legend at 0x1fe7e731828>
```



1.1.2 Problem 2

In [4]:
```
def Forward1(x, h = 0.001):
    return (f(x+h)-f(x))/h
def Forward2(x, h = 0.001):
    return (4*f(x+h)-3*f(x)-f(x+2*h))/(2*h)
def Backward1(x, h = 0.001):
    return (f(x)-f(x-h))/h
def Backward2(x, h = 0.001):
    return (3*f(x)-4*f(x-h)+f(x-2*h))/(2*h)
def Centered2(x, h = 0.001):
    return (f(x+h)-f(x-h))/(2*h)
def Centered4(x, h = 0.001):
    return (f(x-2*h)-8*f(x-h)+8*f(x+h)-f(x+2*h))/(12*h)

ax2 = plt.gca()
ax2.spines['bottom'].set_position("zero")

ax2.plot(x_space,f(x_space),label='f(x)')
ax2.plot(x_space,fx_prime(x_space),label='f\'(x)')
ax2.plot(x_space,Forward1(x_space),label='Order 1 Forward')
ax2.plot(x_space,Forward2(x_space),label='Order 2 Forward')
ax2.plot(x_space,Backward1(x_space),label='Order 1 Backward')
ax2.plot(x_space,Backward2(x_space),label='Order 2 Backward')
ax2.plot(x_space,Centered2(x_space),label='Order 2 Centered')
ax2.plot(x_space,Centered4(x_space),label='Order 4 Centered')
ax2.legend()
```

Out[4]: <matplotlib.legend.Legend at 0x1fe7e852ef0>

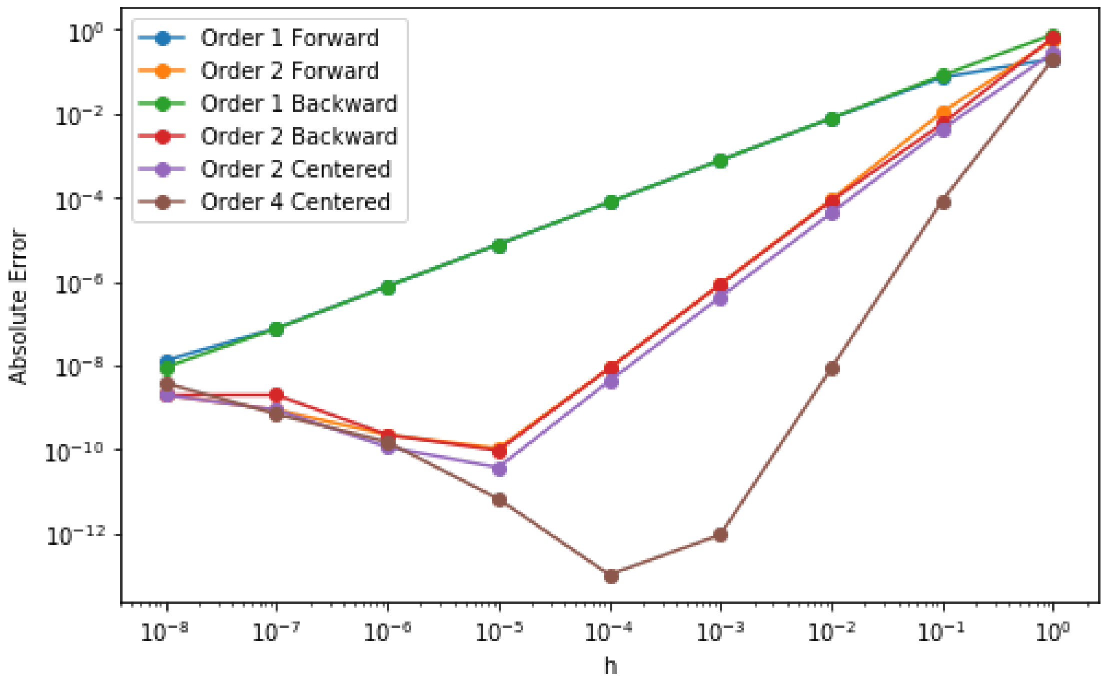


We could see from the above graph that seven results generated by different methods are almost the same. Seven curves overlap each other.

1.1.3 Problem 3

In [5]:
```
def conver(x0):
    h = np.logspace(-8, 0, 9)
    b = fx_prime(x0)
    plt.figure(figsize=(8,5))
    plt.loglog(h, np.abs(Forward1(x0,h)-b),label='Order 1 Forward',marker='o')
    plt.loglog(h, np.abs(Forward2(x0,h)-b),label='Order 2 Forward',marker='o')
    plt.loglog(h, np.abs(Backward1(x0,h)-b),label='Order 1 Backward',marker='o')
    plt.loglog(h, np.abs(Backward2(x0,h)-b),label='Order 2 Backward',marker='o')
    plt.loglog(h, np.abs(Centered2(x0,h)-b),label='Order 2 Centered',marker='o')
    plt.loglog(h, np.abs(Centered4(x0,h)-b),label='Order 4 Centered',marker='o')
    plt.xlabel('h')
    plt.ylabel('Absolute Error')
    plt.legend(loc='upper left')
    plt.show()
```

In [6]:
```
conver(1)
```



1.1.4 Problem 4

In [13]:
```
# Load data
plane = np.load('plane.npy')
plane = pd.DataFrame(plane,columns = ['time', 'alpha', 'beta'])

# Convert degree to rad
plane['alpha'] = np.deg2rad(plane['alpha'])
plane['beta'] = np.deg2rad(plane['beta'])

# Calculate x(t) and y(t)
a=500
plane['x(t)'] = a * np.tan(plane['beta']) / (np.tan(plane['beta']) -
np.tan(plane['alpha']))
plane['y(t)'] = a * np.tan(plane['beta']) * np.tan(plane['alpha']) /
(np.tan(plane['beta']) - np.tan(plane['alpha']))

# Calculate x'(t) and y'(t) according to the problem
plane['x(t)_prime'] = 0.0
plane['y(t)_prime'] = 0.0

plane['x(t)_prime'][0] = plane['x(t)'][1] - plane['x(t)'][0]
plane['y(t)_prime'][0] = plane['y(t)'][1] - plane['y(t)'][0]

plane['x(t)_prime'][7] = plane['x(t)'][7] - plane['x(t)'][6]
plane['y(t)_prime'][7] = plane['y(t)'][7] - plane['y(t)'][6]

for i in range(1,7):
    plane['x(t)_prime'][i] = (plane['x(t)'][i + 1] - plane['x(t)'][i - 1]) / 2
    plane['y(t)_prime'][i] = (plane['y(t)'][i + 1] - plane['y(t)'][i - 1]) / 2

# Calulate the speed
plane['speed'] = np.sqrt(plane['x(t)_prime'] ** 2 + plane['y(t)_prime'] ** 2)

plane
```

```
Out[13]:    time     alpha      beta         x(t)         y(t)  x(t)_prime  y(t)_prime  \
         0   7.0  0.981748  1.178795  1311.271337  1962.456239   44.665140   12.658266
         1   8.0  0.969181  1.161866  1355.936476  1975.114505   45.323531   12.444889
         2   9.0  0.956440  1.144761  1401.918398  1987.346016   47.280265   12.863104
         3  10.0  0.943525  1.127308  1450.497006  2000.840713   48.360976   13.083197
         4  11.0  0.930959  1.110378  1498.640350  2013.512411   46.650974   12.475760
         5  12.0  0.919614  1.095020  1543.798955  2025.792234   49.700516   13.739086
         6  13.0  0.906524  1.077217  1598.041382  2040.990583   51.898569   14.636669
         7  14.0  0.895005  1.061509  1647.596093  2055.065571   49.554711   14.074988

                speed
         0  46.424201
         1  47.001039
         2  48.998805
         3  50.099442
         4  48.290351
         5  51.564559
         6  53.923034
         7  51.514801
```

1.1.5 Problem 5

```
In [8]: from sympy import Symbol
        def jacob(func, point, h):
            num_func = len(func)
            dim = len(point)
            J = np.zeros((num_func, dim))
            variables = set()
            for sub_func in func:
                variables = variables | sub_func.atoms(Symbol)
            for i, sub_func in enumerate(func):
                for j, val in enumerate(variables):
                    do_sub_func = sy.simplify((sub_func.subs(val, val + h) - sub_func.subs(val,
        val - h)) / (2 * h))
                    sub_func_prime = sy.lambdify(val, do_sub_func)
                    J[i, j] = sub_func_prime(point[j])
            return J
```

```
In [9]: x = sy.symbols('x')
        y = sy.symbols('y')
        func_1 = x ** 2
        func_2 = x ** 3 - y
        jacob([func_1, func_2], [1,1], 0.0001)
```

```
Out[9]: array([[ 0.        ,  2.        ],
               [-1.        ,  3.00000001]])
```

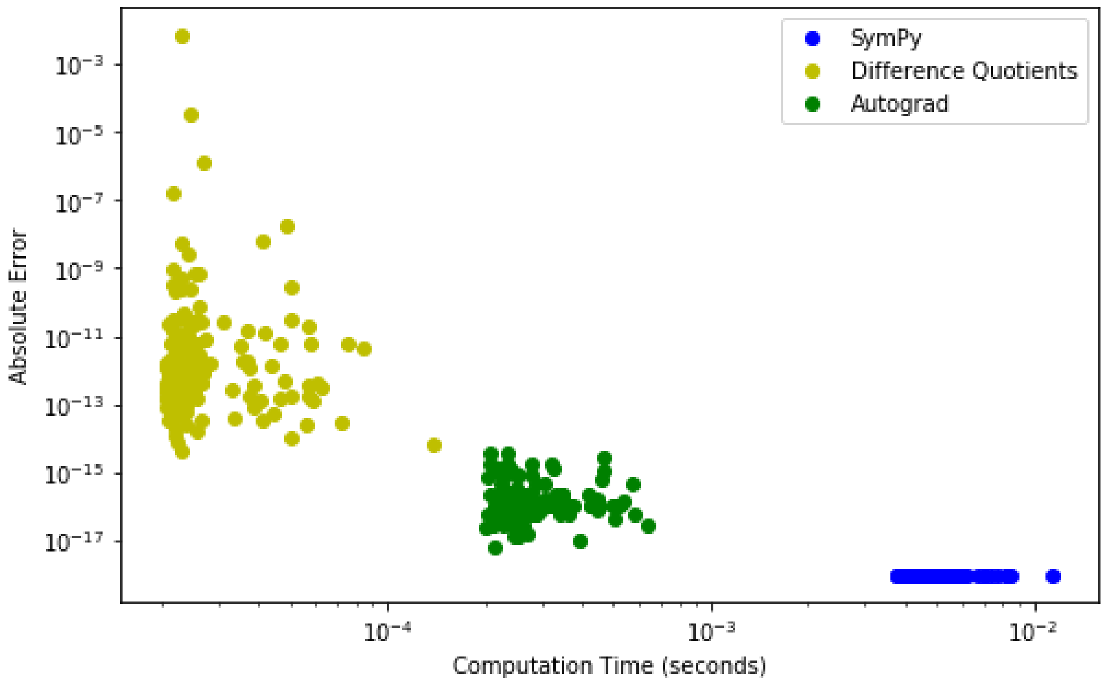
1.1.6 Problem 7

```
In [10]: import time
         from autograd import numpy as anp
         from autograd import grad
```

```
In [11]: def time_error(N):
             timer_1 = np.zeros(N,dtype='float')
             timer_2 = np.zeros(N,dtype='float')
             timer_3 = np.zeros(N,dtype='float')
```

```
        error_1 = 1e-18 * np.ones(N,dtype='float')
        error_2 = np.zeros(N,dtype='float')
        error_3 = np.zeros(N,dtype='float')

        x = sy.symbols('x')
        f_x = (sy.sin(x)+1) ** (sy.sin(sy.cos(x)))

        g = lambda x: (anp.sin(x)+1)**(anp.sin(anp.cos(x)))
        dg = grad(g)

        for i in range(N):
            chosen_x = np.random.uniform(-np.pi, np.pi)
            timer_1_1 = time.clock()
            f_prime = sy.diff(f_x, x)
            fx_prime = sy.lambdify(x, f_prime)
            result_1 = fx_prime(chosen_x)
            timer_1_2 = time.clock()
            timer_1[i] = timer_1_2 - timer_1_1

            timer_2_1 = time.clock()
            result_2 = Centered4(chosen_x)
            timer_2_2 = time.clock()
            timer_2[i] = timer_2_2 - timer_2_1
            error_2[i] = abs(result_2 - result_1)

            timer_3_1 = time.clock()
            result_3 = dg(chosen_x)
            timer_3_2 = time.clock()
            timer_3[i] = timer_3_2 - timer_3_1
            error_3[i] = abs(result_3 - result_1)

        plt.figure(figsize=(8,5))
        plt.loglog(timer_1,error_1,'bo',label="SymPy")
        plt.loglog(timer_2,error_2,'yo',label="Difference Quotients")
        plt.loglog(timer_3,error_3,'go',label="Autograd")
        plt.legend(loc='upper right')
        plt.xlabel("Computation Time (seconds)")
        plt.ylabel("Absolute Error")
        plt.show()
```

```
In [12]: time_error(200)
```


SymPy
Difference Quotients
Autograd
Absolute Error
Computation Time (seconds)
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-4}
10^{-3}
10^{-2}

```
In [ ]:
```