PS4_psy

February 4, 2019

1 PS4

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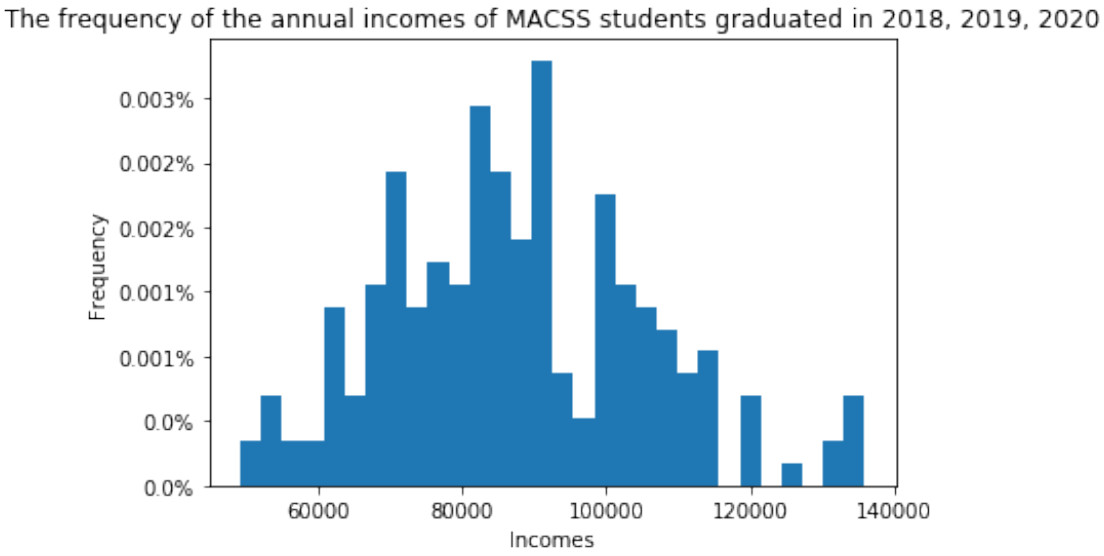
```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import scipy.stats as sts
        from scipy.integrate import quad
        import scipy.optimize as opt
        import sys
        import warnings
        if not sys.warnoptions:
            warnings.simplefilter("ignore")
        from matplotlib.ticker import FuncFormatter
        import matplotlib
```

1.1.1 1

1.1.2 (a)

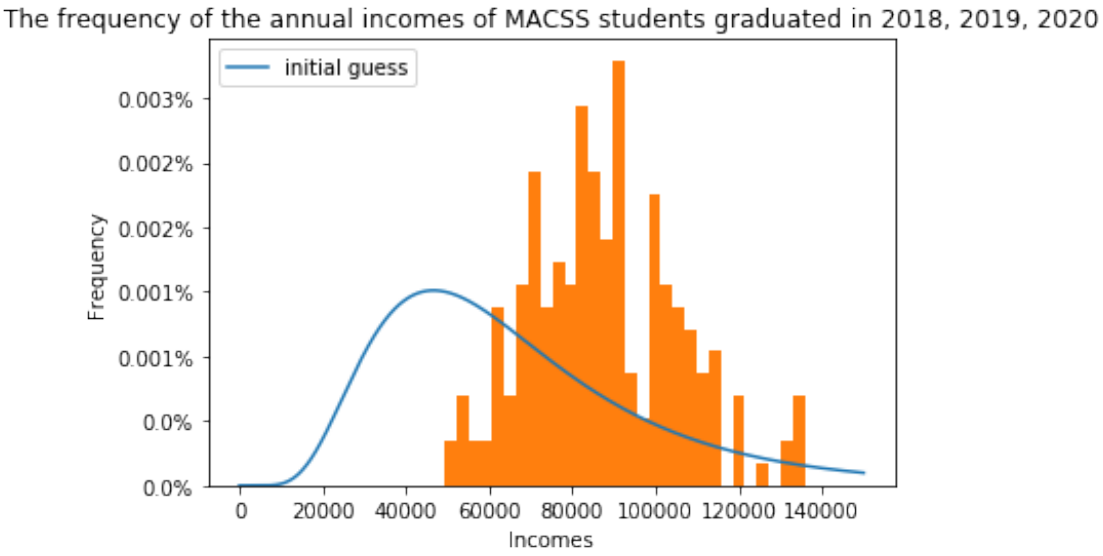
```
In [2]: def to_percent(y, position):
            s = str(round(100 * y, 3))

            # The percent symbol needs escaping in latex
            if matplotlib.rcParams['text.usetex'] is True:
                return s + r'$\%$'
            else:
                return s + '%'
```

```
In [3]: incomes = np.loadtxt('data/incomes.txt')
        fig,ax = plt.subplots(figsize=(6,4))
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('The frequency of the annual incomes of MACSS students graduated in 2018,
        2019, 2020')
        ax.hist(x=incomes, bins=30, normed=True)
        formatter = FuncFormatter(to_percent)

        # Set the formatter
        plt.gca().yaxis.set_major_formatter(formatter)

        plt.show()
```



1.1.3 (b)

```
In [4]: def log_norm(x, mu=11, sigma=0.5):
            f_x = 1 / (x * sigma * np.sqrt(2 * np.pi)) * np.exp(-(np.log(x) - mu) ** 2 / (2 *
        sigma ** 2))
            return f_x
        def log_lik(x, mu, sigma):
            pdf_val = log_norm(x, mu, sigma)
            log_lik = np.log(pdf_val)
            log_lik_val = log_lik.sum()
            return log_lik_val

        x_set = np.linspace(0,150000, 150001)
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('The frequency of the annual incomes of MACSS students graduated in 2018,
        2019, 2020')
        ax.plot(x_set, log_norm(x_set), label = 'initial guess')
        plt.legend(loc='upper left')
        ax.hist(x=incomes, bins=30, normed = True)
        formatter = FuncFormatter(to_percent)

        # Set the formatter
        plt.gca().yaxis.set_major_formatter(formatter)

        plt.show()
        log_lik_val = log_lik(incomes, 11, 0.5)
        print('The value of the log likelihood value for this parameterization of the
        distribution and given this data is', log_lik_val)
```



The value of the log likelihood value for this parameterization of the distribution and given this data is -2385.856997808558

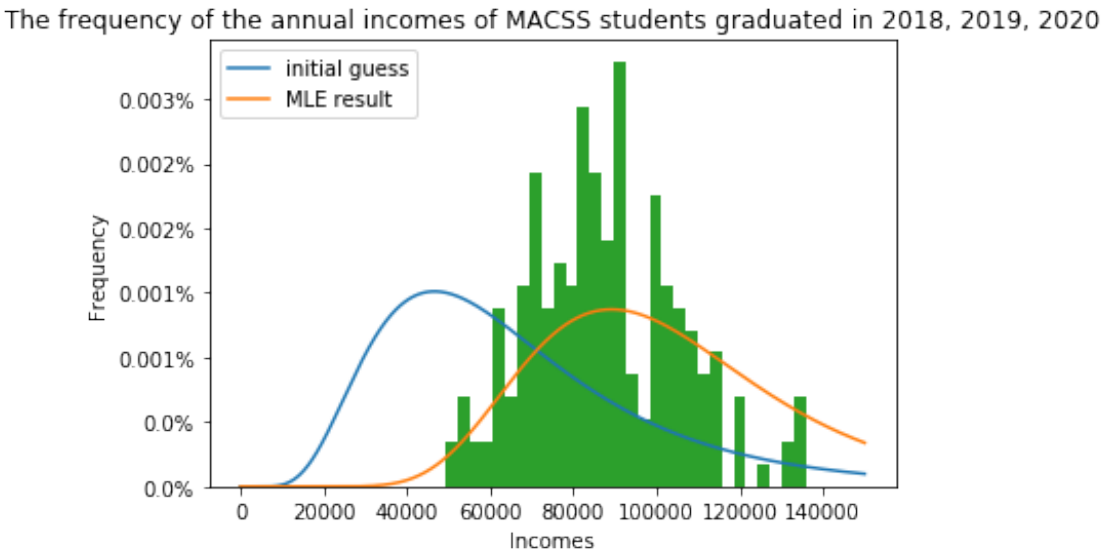
1.1.4 (c)

```
In [5]: def neg_log(params, x):
            mu, sigma = params
            log_lik_val = log_lik(x, mu, sigma)
            neg_log_lik_val = -log_lik_val
            return neg_log_lik_val
```

```
In [6]: init_mu = 11
        init_sig = 0.5
        init_guess = np.array([init_mu, init_sig])
        bnds = ((None, None), (1e-10, None))
        opt_result = opt.minimize(neg_log, init_guess, incomes, bounds=bnds)

        if opt_result.success:
            est_mu, est_sig = opt_result.x
            est_log_lik_val = - opt_result.fun
            var_cov_mat = opt_result.hess_inv.todense()
            print('The value of estimated mu is', est_mu, ' and the value of estimated sigma
        is', est_sig)
            print('The value of the likelihood function is', est_log_lik_val)
            print('The variance-covariance matrix is\n', var_cov_mat)
```

The value of estimated mu is 11.495016102470231 and the value of estimated sigma is 0.3108792150154579
The value of the likelihood function is -2285.8999545081624
The variance-covariance matrix is
[[4.59466263e-04 -6.65453178e-05]
[-6.65453178e-05 1.71221449e-03]]

```
In [7]: fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('The frequency of the annual incomes of MACSS students graduated in 2018,
        2019, 2020')
        ax.plot(x_set, log_norm(x_set), label = 'initial guess')
        plt.legend(loc='upper left')
        ax.plot(x_set, log_norm(x_set, est_mu, est_sig), label = 'MLE result')
        plt.legend(loc='upper left')
        ax.hist(x=incomes, bins=30, normed=True)
        formatter = FuncFormatter(to_percent)

        # Set the formatter
        plt.gca().yaxis.set_major_formatter(formatter)

        plt.show()
```



1.1.5 (d)

```
In [8]: log_lik_h0 = log_lik(incomes, init_mu, init_sig)
        print('Hypothesis value log likelihood is', log_lik_h0)
        log_lik_mle = log_lik(incomes, est_mu, est_sig)
        print('MLE log likelihood is', log_lik_mle)
        LR_val = 2 * (log_lik_mle - log_lik_h0)
        print('Likelihood ratio value is', LR_val)
        pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
        print('Chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
Hypothesis value log likelihood is -2385.856997808558
MLE log likelihood is -2285.8999545081624
Likelihood ratio value is 199.91408660079105
Chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

1.1.6 (e)

```
In [9]: p_high = 1 - quad(lambda x: log_norm(x, mu=est_mu,sigma=est_sig), 0, 100000)[0]
        p_low = quad(lambda x: log_norm(x, mu=est_mu,sigma=est_sig), 0, 75000)[0]
        print('The probability of earning more than $100,000 is', '%.2f%%' % (p_high * 100))
        print('The probability of earning less than $75,000 is', '%.2f%%' % (p_low * 100))
```

```
The probability of earning more than $100,000 is 47.70%
The probability of earning less than $75,000 is 19.28%
```

1.2 2

```
In [10]: # Import the data
         data = []
         with open('data/sick.txt') as sick:
             for line in sick:
                 data.append(line.rsplit()[0].split(','))
         data = np.array(data)
         df = pd.DataFrame(data[1:], columns=['sick', 'age', 'children',
         'temp_winter']).astype('float64')
         df.head()
```

Out[10]:

	sick	age	children	temp_winter
0	1.67	57.47	3.04	54.10
1	0.71	26.77	1.20	36.54
2	1.39	41.85	2.31	32.38
3	1.37	51.27	2.46	52.94
4	1.45	44.22	2.72	45.90

1.2.1 (a)

```
In [11]: def norm_pdf(x, sigma):
             sigma = np.abs(sigma)
             pdf_vals = (1 / (sigma * np.sqrt(2 * np.pi)) *
                             np.exp( - x ** 2 / (2 * sigma ** 2)))
             return pdf_vals

         def log_lik(y, x1, x2, x3, b0, b1, b2, b3, sigma):
             error = y - b0 - x1 * b1 - x2 * b2 - x3 * b3
             pdf = norm_pdf(error, sigma)
             log_lik = np.log(pdf)
             log_lik_val = log_lik.sum()
             return log_lik_val

         def crit(params, *args):
             b0, b1, b2, b3, sigma = params
             y, x1, x2, x3 = args
             neg_log_lik = -log_lik(y, x1, x2, x3, b0, b1, b2, b3, sigma)
             return neg_log_lik

In [12]: init_b0, init_b1, init_b2, init_b3 = (1.0, 0.0, 0.0, 0.0)
         init_sig = 0.1
         init_guess = np.array([init_b0, init_b1, init_b2, init_b3, init_sig])
         y = df['sick']
         x1, x2, x3 = df['age'], df['children'], df['temp_winter']
         var = (y, x1, x2, x3)
         results = opt.minimize(crit, init_guess, args = var)

In [13]: MLE_b0, MLE_b1, MLE_b2, MLE_b3, MLE_sig = results.x
         var_cov_mat = results.hess_inv
```

```
        est_log_lik_val = -results.fun
        print('The MLE result is b0 = {:.4f}, b1 = {:.4f}, b2 = {:.4f}, b3 = {:.4f}, sigma =
        {:.4f}'.format(MLE_b0, MLE_b1, MLE_b2, MLE_b3, MLE_sig))
        print('The value of the log likelihood function is ', est_log_lik_val)
        print('The estimated variance covariance matrix of the estimates is \n', var_cov_mat)
```

```
The MLE result is b0 = 0.2516, b1 = 0.0129, b2 = 0.4005, b3 = -0.0100, sigma = 0.0030
The value of the log likelihood function is  876.8650462886415
The estimated variance covariance matrix of the estimates is
 [[ 2.54544044e-08 -1.78744714e-09  2.41230516e-08  2.25060709e-10
  -7.41365711e-08]
 [-1.78744714e-09  5.42683251e-10 -1.76464432e-09 -3.86107909e-10
   5.59234008e-09]
 [ 2.41230516e-08 -1.76464432e-09  2.41466056e-08  2.62174332e-10
  -7.16161474e-08]
 [ 2.25060709e-10 -3.86107909e-10  2.62174332e-10  3.55008410e-10
  -1.04143527e-09]
 [-7.41365711e-08  5.59234008e-09 -7.16161474e-08 -1.04143527e-09
   2.17760694e-07]]
```

1.2.2 (b)

```
In [14]: log_lik_h0 = log_lik(y, x1, x2, x3, init_b0, init_b1, init_b2, init_b3, init_sig)
         print('Hypothesis value log likelihood is', log_lik_h0)
         log_lik_mle = log_lik(y, x1, x2, x3, MLE_b0, MLE_b1, MLE_b2, MLE_b3, MLE_sig)
         print('MLE log likelihood is', log_lik_mle)
         LR_val = 2 * (log_lik_mle - log_lik_h0)
         print('Likelihood ratio value is', LR_val)
         pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
         print('Chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
Hypothesis value log likelihood is -2253.700688042125
MLE log likelihood is 876.8650462886415
Likelihood ratio value is 6261.131468661533
Chi squared of H0 with 2 degrees of freedom p-value =  0.0
```