Pose Optimization for Force and Stiffness Control of a redundant Robot Arm

Jieming Chen, Shizeng Zhang

Institute of Control Systems
Electrical and Computer Engineering
University of Kaiserslautern

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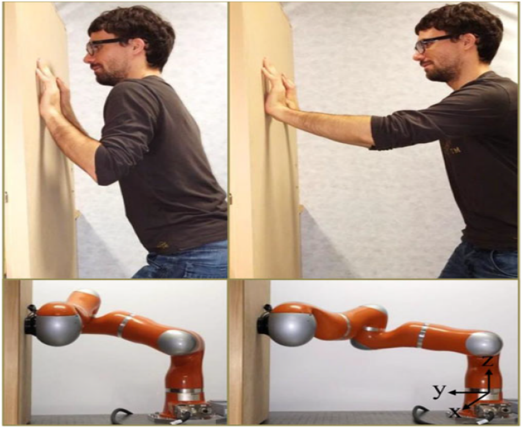
Inhalt

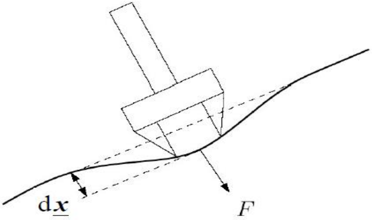
Introduction

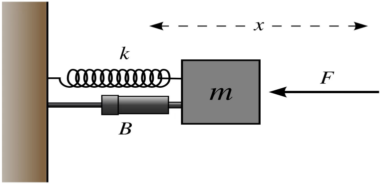


Search Aim

For different arm configurations, the ability to maintain the stiffness are different.

Impedence Control





$$\underline{M_d}\,\underline{\ddot{\delta x}} + \underline{K_d}\,\dot{\delta x} + \underline{K_p}\,\underline{\delta x} = \underline{F_a}$$

$$assume\, \underline{M_d}, \underline{K_d} = 0$$

$$\downarrow$$

$$\underline{F_a} = \underline{K_p}\,\underline{\delta x}$$

$$\downarrow$$

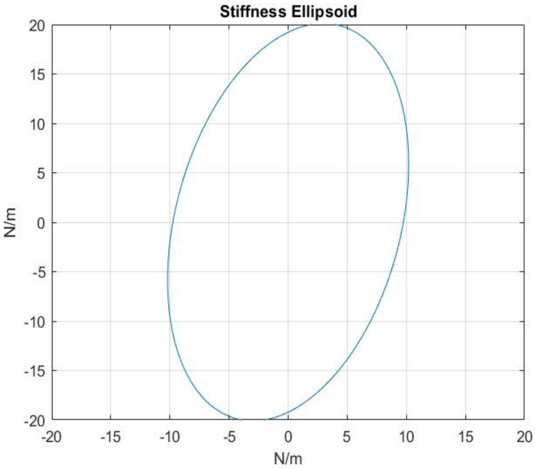
$$\underline{\tau} = \underline{J}^T\,\underline{F_a}$$

$$\downarrow$$

$$\underline{\tau} = \underline{J}^T\,\underline{K_p}\,\underline{\delta x}$$

$\underline{\tau}$ reflects the stiffness.

Stiffness Ellipsoid



$\underline{F_a} = \underline{K_p}\underline{\delta x}$

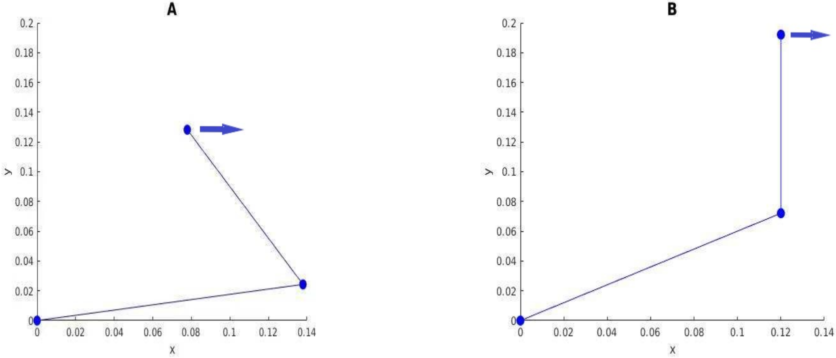
$\underline{\delta x}$: a unit cycle

$\underline{K_p} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2 \end{bmatrix} kN/m$

No consideration about torque limits and joint configurations!

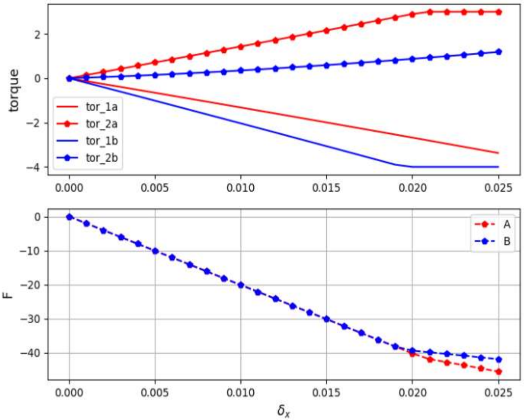
Relationship between Arm Configurations and Boundries(1)

Two Different Arm Configurations



Search Aim: Find the relationship between the e-e displacement and the torque of each joint.

Relationship between Arm Configurations and Boundries(2)



$$divide \quad e-e \quad displacement$$

$$\downarrow$$

$$\underline{F_{des}} = \underline{K_p}\underline{\delta x}$$

$$\downarrow$$

$$\underline{\dot{q}} = \underline{J^{-1}\dot{\delta x}}$$

$$\downarrow$$

$$\underline{\tau_{des}} = \underline{J^T}\underline{F_{des}}$$

$$\downarrow$$

$$|\ \underline{\tau_{des}}\ | \leq |\ \underline{\tau_{limt}}\ |$$

$$\downarrow$$

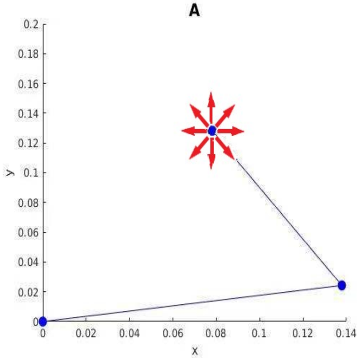
$$|\ \underline{\tau_{real}}\ |$$

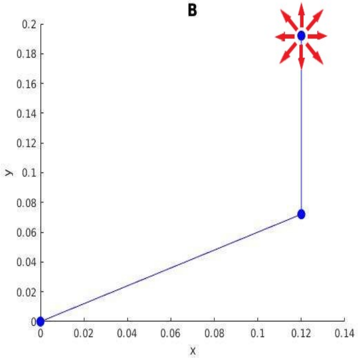
$$\downarrow$$

$$\underline{F_{real}} = \underline{(J^T)^{-1}}\underline{\tau_{real}}$$

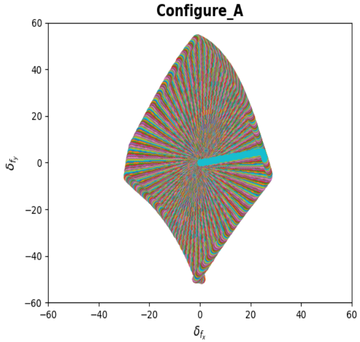
For different configurations, the ability to maintain stiffness are different although they are under the same torque limit.

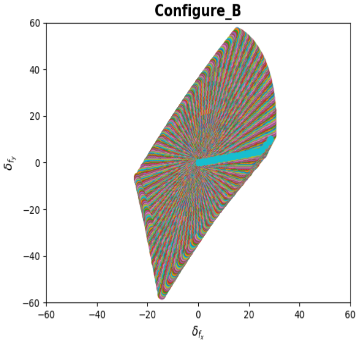
Apply in all Directions





locus of $\| F \|$

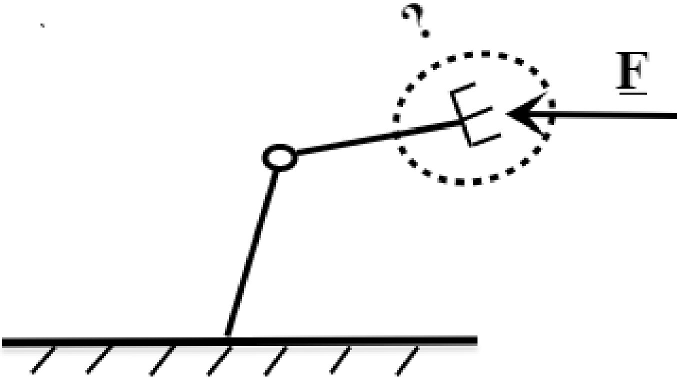




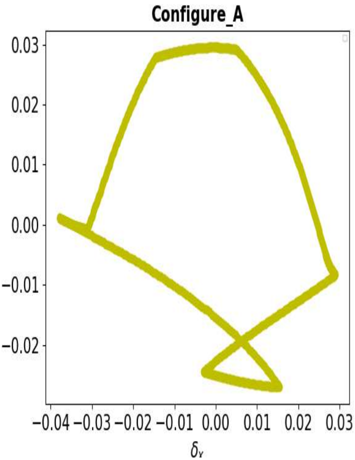
Comparison with Stiffness Matrix

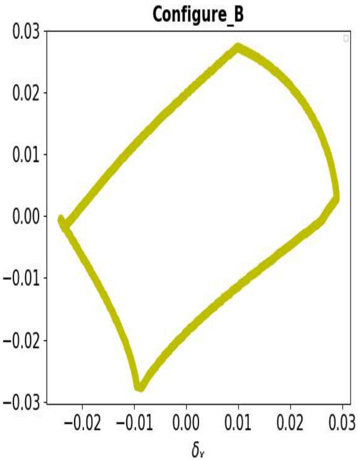
Stiffness Ellipsoid cannot always be realized because of the torque limits.

Application of $\| F \|$



SFR(Stiffness Feasibility Region)(1)





$$\underline{x_{real}} = \underline{K_p^{-1}}\,\underline{F_{real}} \tag{1}$$

SFR(Stiffness Feasibility Regions)(2)

SFR Characteristics

- Jacobian matrix is updated with the growing displacement norm
- most accurate representation of feasible region
- expensive computation and not suitable for real-time applications.

SFP

stiffness feasibility polytope

Definition of SFP :

$$\delta\underline{x} \mid \|\underline{\hat{\tau}}\|_\infty \leq 1$$

Deduction :

$$\underline{W}_\tau = diag[\frac{1}{\tau_{lim_1}} \frac{1}{\tau_{lim_2}} \cdots \frac{1}{\tau_{lim_n}}]$$

$$\downarrow$$

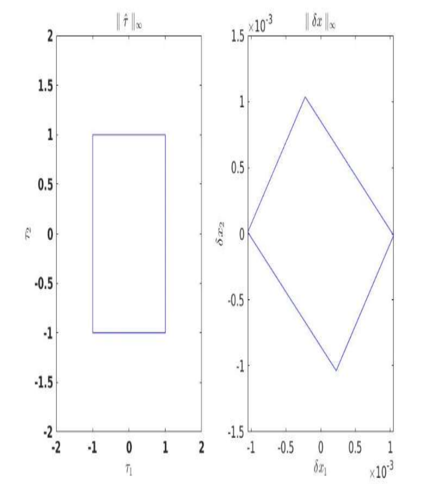
$$\underline{\hat{\tau}} = \underline{W}_\tau \underline{\tau}$$

$$\downarrow$$

$$\|\underline{\hat{\tau}}\|_\infty \leq 1$$

$$\downarrow$$

$$\left\| \underline{W}_\tau \underline{J}(q)^T \underline{K} \delta\underline{x} \right\|_\infty \leq 1$$



SFE

stiffness feasibility ellipsoid

Definition of SFE :

$$\delta \underline{x} \mid ||\hat{\underline{\tau}}||_2 \leq 1$$

Deduction :

$$\underline{W}_\tau = diag[\frac{1}{\tau_{lim_1}} \frac{1}{\tau_{lim_2}} \cdots \frac{1}{\tau_{lim_n}}]$$

$$\downarrow$$

$$\hat{\underline{\tau}} = \underline{W}_\tau \underline{\tau}$$

$$\downarrow$$

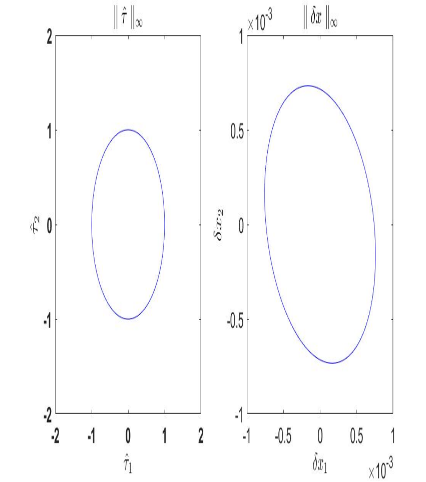
$$||\hat{\underline{\tau}}||_2 \leq 1$$

$$\downarrow$$

$$\hat{\underline{\tau}}^T \hat{\underline{\tau}} \leq 1$$

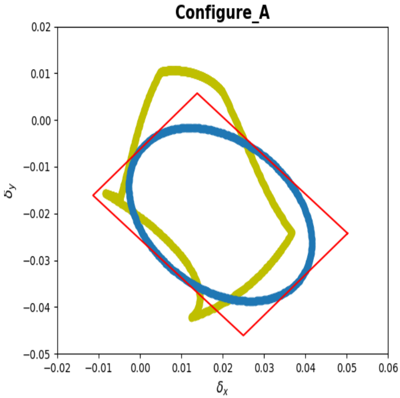
$$\downarrow$$

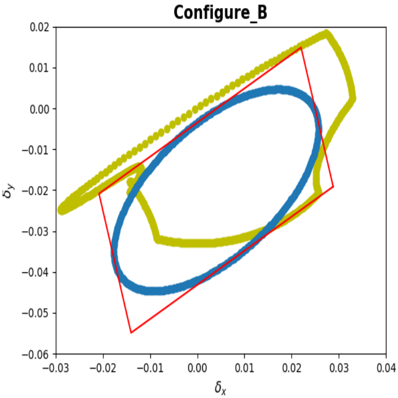
$$\delta \underline{x}^T \quad \underline{K}^T \underline{J}(q) \underline{W}_\tau^T \underline{W}_\tau \underline{J}(q)^T \underline{K} \quad \delta \underline{x} \leq 1$$



SFR,SFP,SFE Comparison

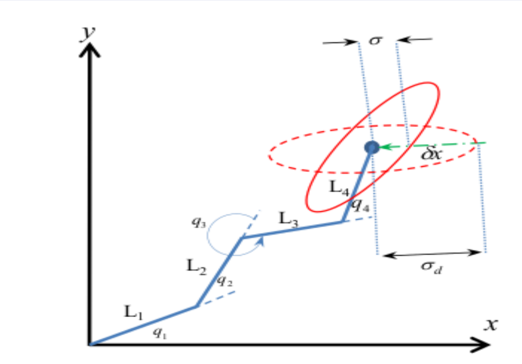
The yellow plots SFR and red plots SFP and blue plot SFE.





Pose Optimization for Force and Stiffness Control(1)

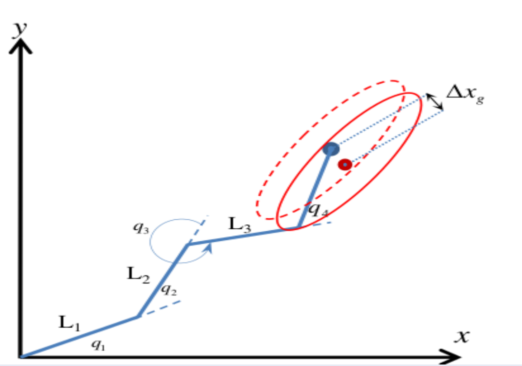
optimization objective(1): SFE Geometry



σ: the length in which the robot arm could maintain stiffness in ceartin direction. The aim is to try to enlarge this σ by changing configuration and rotating this ellipsoid.

Pose Optimization for Force and Stiffness Control(2)

optimization objective(2): Minimize Gravity Effect



The aim is to reduce the translation of the SFE due to gravity in the task space.

Pose Optimization for Force and Stiffness Control(3)

Build optimization problem

- Cost function for SFE Geometry:

$$\sigma^T \frac{\delta \underline{x}}{\|\delta \underline{x}\|}^T \quad \underline{K}^T \underline{J}(q) \underline{W}_\tau^T \underline{W}_\tau \underline{J}(q)^T \underline{K} \quad \frac{\delta \underline{x}}{\|\delta \underline{x}\|} \sigma = 1$$

$$V_1 = \frac{\delta \underline{x}^T}{\|\delta \underline{x}\|} \underline{K}_c \underline{J}(q) \underline{W}_\tau^2 \underline{J}(q)^T \underline{K}_c \frac{\delta \underline{x}}{\|\delta \underline{x}\|}$$

- Cost function for minimizing gravity effect:

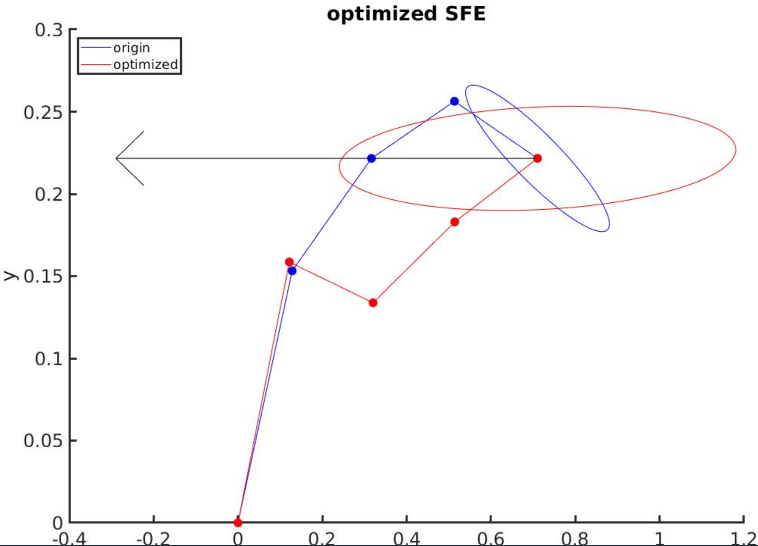
$$\underline{G}_q = (\underline{J}^T)^{-1} \underline{\tau}_g \quad \textit{and} \quad \triangle \underline{x}_g = \underline{K}_c^{-1} \underline{G}_q$$

$$V_2 = \triangle \underline{x}_g^T \triangle \underline{x}_g$$

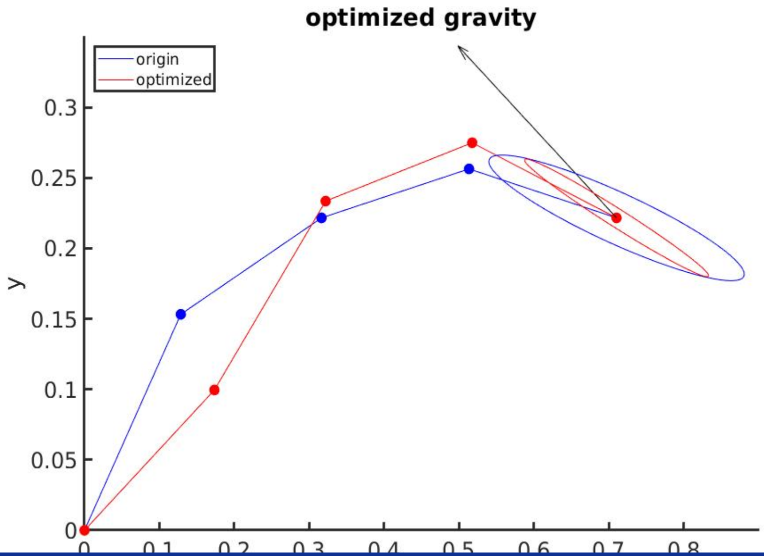
- Optimazation problem:

$$\begin{aligned} \min_{\underline{q}} \quad & V = weightingfactor_1 * V_1 + weightingfactor_2 * V_2, \\ \text{s.t.} \quad & forwardKinematic(\underline{q}) = \underline{X}_0, \\ & \underline{q_{min}} < \underline{q} < \underline{q_{max}}. \end{aligned}$$

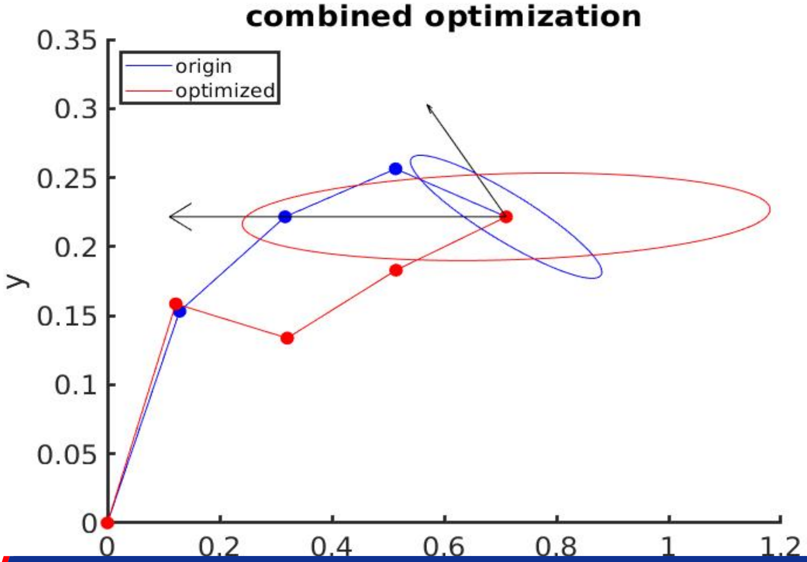
Result of Optimizing SFE geometry



Result of minimizing the gravity effect



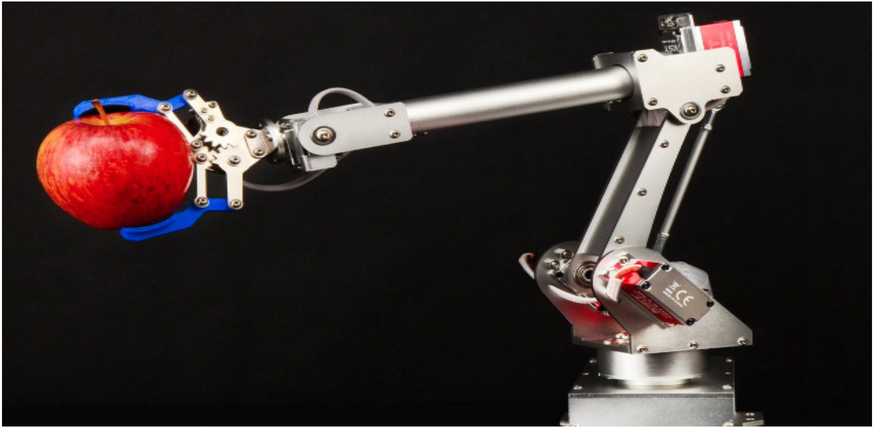
Result of Combined Optimization



Conclusion

- How large is the area that the robot could maintain stiffness , under the practical torque limitations and gravity effect?
- SFR and Explain the role of arm configuration for stiffness.
- SPE, SFP
- Pose Optimization for Stiffness Control and Minimizing the gravity effect for SFE.

Prospect



When the robot arm holding a stuff tracks trajectory, we could involve the maintaining stiffness ability in the cost function.

Thanks for listening!