Introduction to the sizing of multi-rotor drones with Jupyter Notebooks

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Contents

1st part: Tutorial

Case study presentation: multi-rotor drones and eVTOL

Sizing Scenarios: Definition of scenarios corresponding to main design criteria of components

System level models: Equations of sizing scenarios

Component level models: Establishment of estimation models from scaling laws or linear regression

Sizing procedure definition: under-constraint, over-constraint and algebraic loops

Design space exploration & optimization: effect of increase of drone size

2nd part: Discussions and exchanges



Case study

Electric Vertical Takeoff And Landing (eVTOL)



Joby S4



Ehang 184

Winged eVTOLs

Kittyhawk Cora



Kitty Hawk, a
California-based
corporation,
operated by
Zephyr Airworks
in New Zealand.

Vahana



A³ technological development arm of Airbus, located in Santa Clara, CA.

Joby S2 & S4



Joby Aviation, a Santa Cruz, CA, USA based company.



Wingless eVTOLs

Ehang 184



Ehang, based in Guangzhou, China.



Volocopter, based in Bruchsal, Germany.

Boeing Cargo Air Vehicle (CAV)



The Boeing Company, based in Chicago, IL, USA.



Wingless eVTOLs Characteristics

Ehang 184



Technical Details

Flight Endurance: 25min

Aircraft Gross Weight: 260kg

Flight Speed: 100km/h

Cruising altitude: 500m (AGL)

Charging Time: 1 hrRated Payload: 100kg

Materials

 Main frame: Reinforced composite material with carbon fiber and epoxy

• Other components: aerial aluminum alloy

Geometry

Length: 3.99mHeight: 1.45m

Width: 4m

Interior

Seat Depth: 395mm

• Seatback Height: 860mm

• Seatback angle: 95 degrees

Storage capacity: 18-inch backpack

Volocopter 2X

Technical Details

• Passenger Capacity: 2 pax

Max. take off mass (MTOM): 450 kg

Max. payload: 160 kg

• Operating weight empty (OWE): 290 kg

Max. range (@ MTOM): 27 km (17 mi) at an optimal range cruise speed of 70 km/h (43 mph)

Max. airspeed (limited time): 100 km/h

Rate of climb (@ MTOM): 3 m/s

Structure/Geometry

Fibre composites; lightweight construction

• Overall height: 2.15 m

Diameter of the rotor rim incl. propellers: 9.15 m

Diameter of a single propeller: 1.80 m

Cockpit: 3.20 m length / 1.25 m width / 1.21 m height

Skids: 3.02 m length / 2.06 m width

Power supply & battery

Number of battery packs: 9 independent battery systems with quick release

Battery type: Lithium-ion battery

Motors & rotors

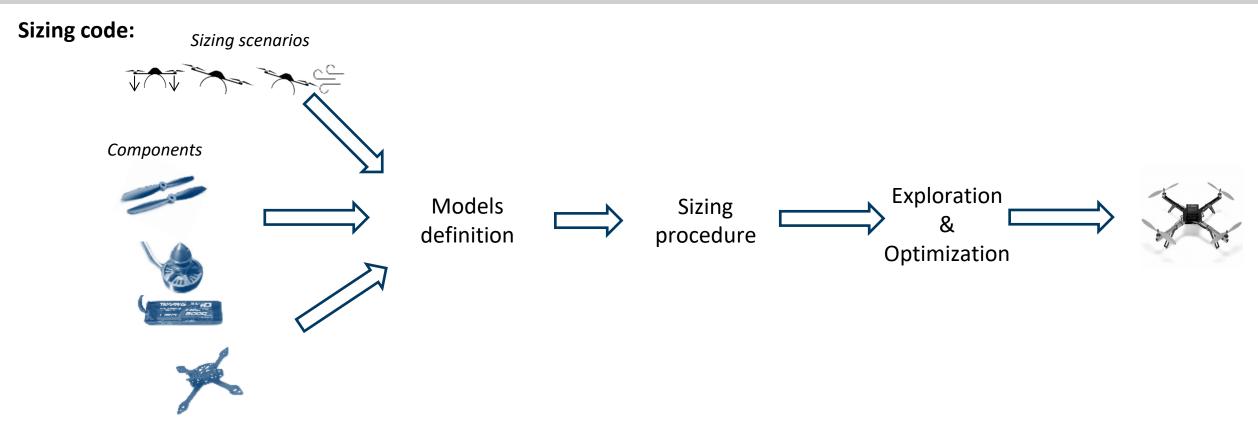
Number of motors & rotors: 18 each

Engine type: 3-phase PM synchronous motor, brushless DC electric motor (BLDC)





Introduction: Objectives



Evaluation of the sizing code on multiple specifications:









Sizing Scenarios and Design Drivers



	System Components	Design Drivers (Function, Fast degradation, Slow degradation, Imperfection)	Hover	Take-off	Vertical & Horizontal flight
	Propeller				
	Motor				
THE REAL PROPERTY OF THE PERSON OF THE PERSO	ESC (Electronic Speed Controller)				
TROUGHAN AND AND AND AND AND AND AND AND AND A	Battery				
X	Frame				

Sizing scenarios equations: Hover and Take-Off

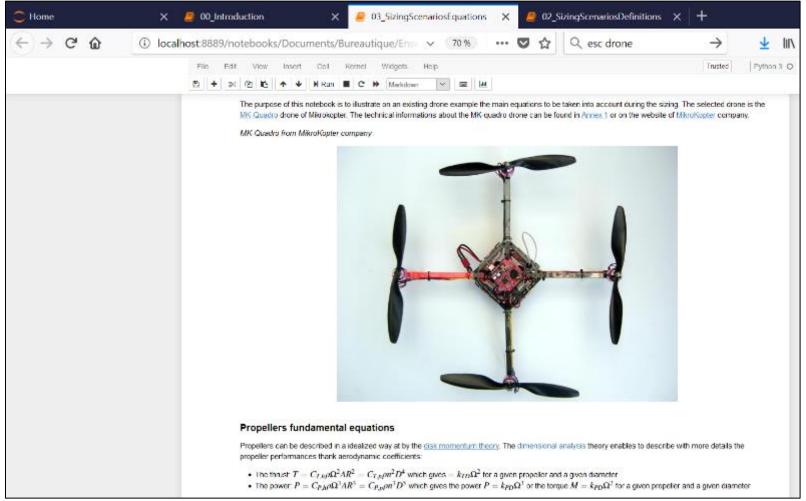


Project (activity+report):





A set of Jupyter Notebooks with active and usable python codes

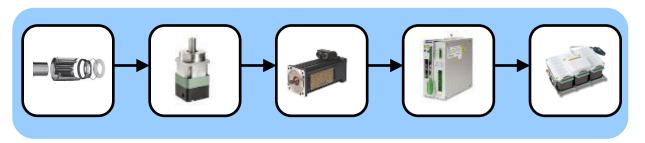




Preliminary design and sizing of mechatronic systems Estimation models - Scaling laws & Linear regression

Components parameters estimation

During the preliminary design:



What are: Volume, Mass, Inertia, Stiffness, Operational limits? ...



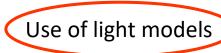


Use of calculation sheets



Exchanges with experts or subcontractors





Scaling laws



Dimensional analysis & Estimation models

$$f(y, L, d_1, d_2, ..., p_1, p_2, ...) = 0$$

$$\pi_y = g(\pi_1, \pi_2, ..., \pi_{p1}, ...)$$

$$\pi_{y} = g(\pi_{1}, \pi_{2}, \dots, \pi_{p1}, \dots)$$

$$p=n-k-1$$

Recall on the Buckingham theorem: if there is a physically meaningful equation involving a certain number n of physical variables, then the original equation can be rewritten in terms of a set of p = n - k dimensionless parameters $\pi_1, \pi_2, ..., \pi_n$ where k is the number of independent physical dimensions involved

$$\pi_y = yL^{a_L} \prod p_i^{a_i}$$

$$\pi_i = \frac{d_i}{L}$$

$$\pi_{pi} = L^{a_{Lpi}} \prod p_j^{a_j}$$

$$\pi_i \& \pi_{pi}$$
 constants \rightarrow Scaling laws $y = kL^a$

Budinger, M., Liscouët, J., Hospital, F., & Maré, J. C. (2012). Estimation models for the preliminary design of electromechanical actuators. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 226(3), 243-259.

$\pi_i \& \pi_{pi}$ variables \rightarrow Regression models

Sanchez, F., Budinger, M., & Hazyuk, I. (2017). Dimensional analysis and surrogate models for the thermal modeling of Multiphysics systems. Applied Thermal Engineering, 110, 758-771.



Scaling laws: principle and notation



Principle:

$$2r \downarrow \bigcirc$$

$$l^* = \frac{l'}{l}$$
Scaling ratio

Scaling ratio:

$$l^* = \frac{l'}{l} \leftarrow$$
Studied component Reference component

Example:

Winding resistance (Résistance d'un bobinage) :

$$R = \frac{\rho l}{S} = \rho \frac{l_{co} N}{S_{co}/N} \quad \Longrightarrow \quad R^* = \frac{N^{*2}}{l^*}$$

with: l_{co} length of one turn (spire) of copper wire (fil de cuivre) S_{co} total section of copper *N* number of turns

Geometric similitary

Example: Scaling law for the volume of a cylinder

What becomes of this relationship in the case of geometric similarity on all sizes : $r^* = l^* = d^*$?

$$r^* = l^*$$
 $\qquad \qquad \qquad \qquad \qquad \qquad V^* = \frac{V'}{V} = \frac{\pi r'^2 l'}{\pi r^2 l} = r^{*2} l^* = l^{*3}$

Determining scaling laws for the surface S, the mass M, the inertia J:

$$\begin{cases} M = \int \rho_m dV \Rightarrow M^* = l^{*3} \\ J = \int r^2 dM \Rightarrow J^* = l^{*5} \end{cases}$$



Different ways of finding the scaling laws

Example: Thermal time constant of an electric motor. It is assumed here that the motor temperature rise is uniform and is mainly due to convection between the stator and the air.

Direct approach:

$$au_{th} = R_{th}.C_{th}$$

$$R_{th} = 1/hS \Rightarrow R_{th}* = l*-2$$

$$C_{th} = c_m.M \Rightarrow C_{th}* = l*3$$

$$\tau_{th}^{*} = R_{th}^{*} C_{th}^{*} = l^{*1}$$

Buckingham theorem approach

$$f(\tau_{th}, d, l, h, c_{v}, \rho) = 0$$

$$F\left(\frac{c_{v}\rho l}{h\tau_{th}}, \frac{l}{d}\right) = 0$$

$$F\left(\frac{c_{v}\rho l}{h\tau_{th}}\right) = 0$$
constant

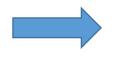
$$\tau_{th}^{*} = l^{*1}$$

Scaling law adaptation

Which parameter:

• Limits motor torque?



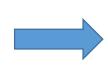


Maximal temperature of winding

 T_{max}

Gives mechanical structure or component size ?





Maximal stress

 σ_{max}

The scale changes have to be done with $X = Constant \text{ or } X^*=1$



$$T_{max}^*=1$$

$$\sigma_{max}^*$$
=1

Scaling with constant constraint (1)

Predicting the diameter of the leg of an elephant from a mouse :

<u>Hypothesis</u>:

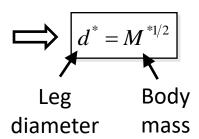
• During scale change, mechanical constraints in the leg remain constant.

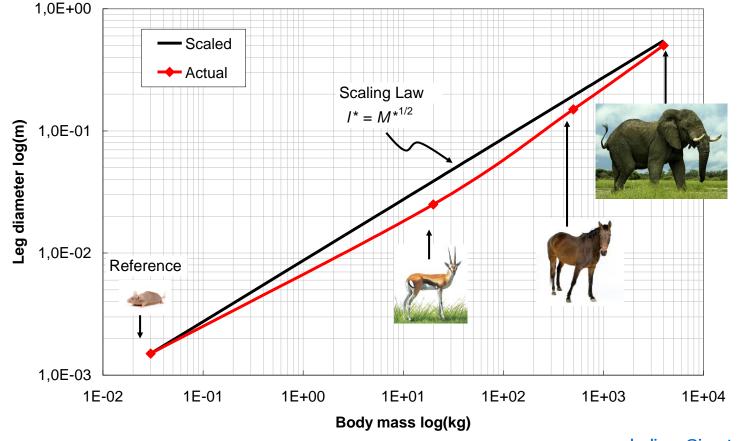
- ⚠ No linear approximation!
- ⚠ No data fitting!
- → Reflects the physics of the sizing phenomena!

$$\sigma_{\max}^* = 1 = F^*/S^*$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$
Mech. Weight Leg constraint section

$$\begin{cases} F^* = M^* \\ S^* = d^{*2} \end{cases}$$







Scaling with constant constraint (2)

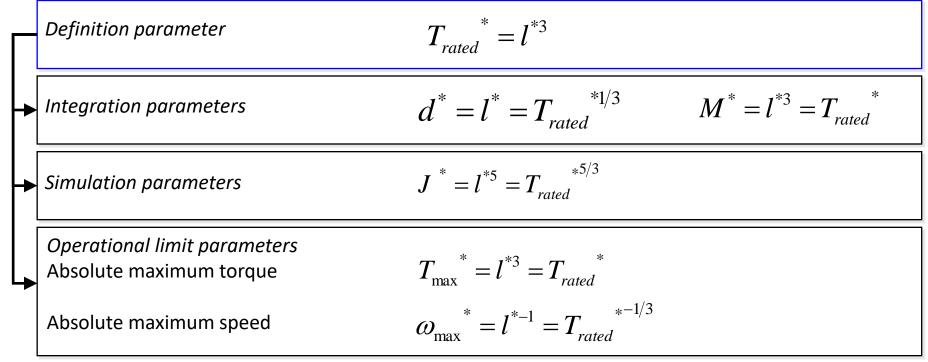
Scaling laws for the cyclo reducer:

Hypothesis:

- remain constant.

Geometrical similitude.
$$\Rightarrow S^* = l^{*2}$$
During scale change, mechanical constraints $\Rightarrow \sigma_{\max}^* = 1 = F^*/S^*$





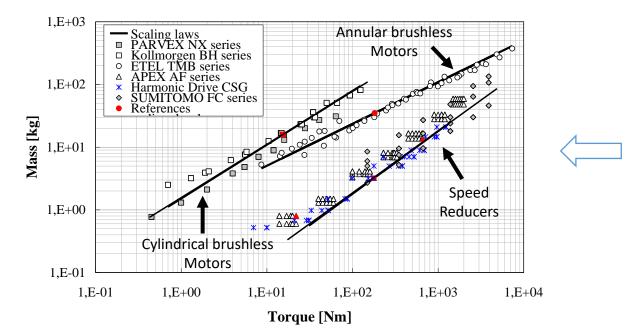


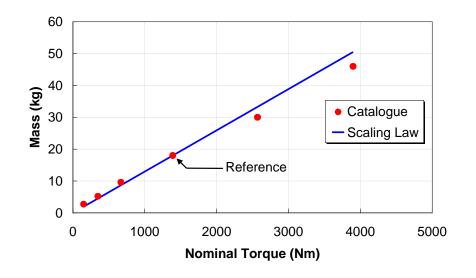
Comparison with catalogue data

Estimated and real (catalogue data) mass as function of nominal torque for a given type of cycloidal reducers (FA series of Sumitomo).









Brushless motor and speed reducer masses as a function of the nominal torque.

Log-Log graphs: $Y = a.X^{\alpha} \longrightarrow \log(Y) = \log(a) + \alpha.\log(X)$



Least square linear regression principle

Model form:

dependent (response) parameter $y = a + b.x + c.x^2 + \varepsilon$ Non random Random or systematic component component

Matrix representation:

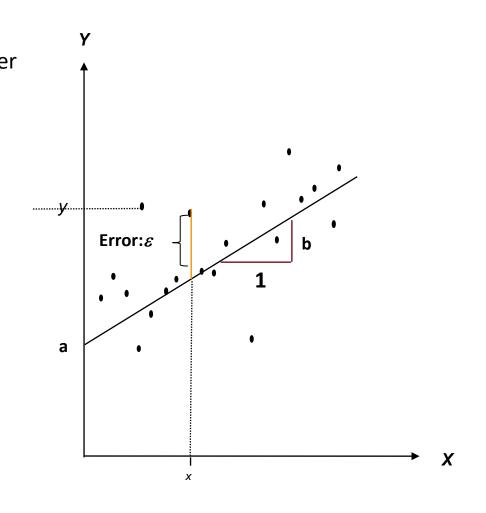
$$\begin{pmatrix} y_1 \\ \dots \\ y_p \end{pmatrix} = \begin{pmatrix} 1 & x_1 & x_1^2 \\ \dots & \dots \\ 1 & x_p & x_p^2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \dots \\ \varepsilon_p \end{pmatrix}$$

$$Y = X \cdot \widetilde{\beta} + \widetilde{\varepsilon}$$

Square error:

$$L(\beta) = \widetilde{\varepsilon}^t . \widetilde{\varepsilon} = (Y - X . \widetilde{\beta})^t (Y - X . \widetilde{\beta})$$

minimum for
$$\frac{\partial L(\beta)}{\partial \beta} = 0$$
 and $\widetilde{\beta} = (X^t X)^{-1} X^t Y$

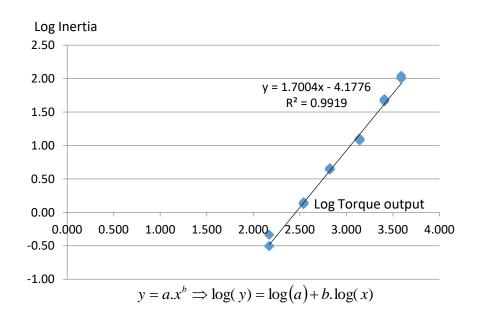


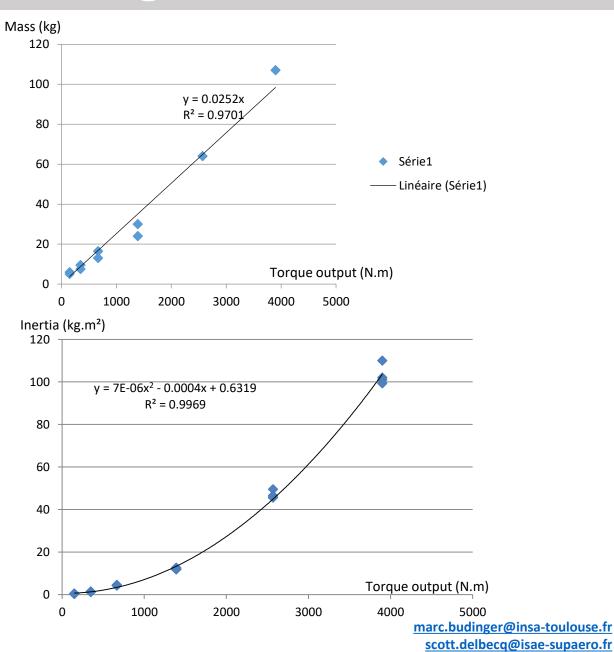
Gear reducer: Simple linear regression

Mass and inertia with respect to output torque:



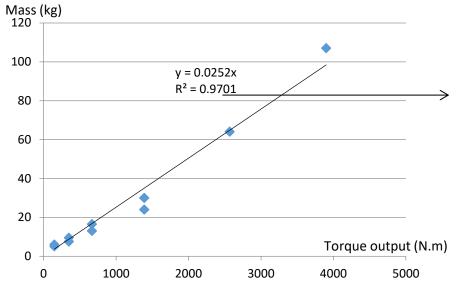
cyclo reducer



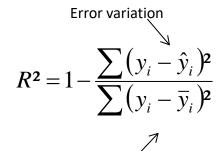




Verification

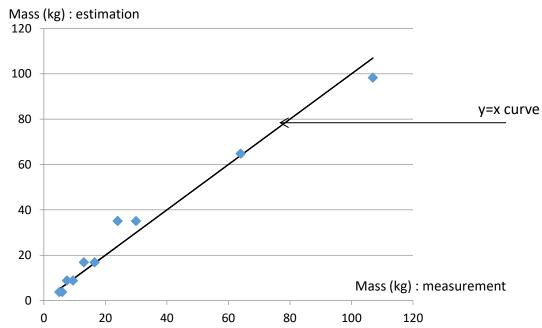


Coefficient of determination R²:



Total variation

Percentage of total variation explained by the regression.





Case study application: Electrical components



Scaling laws:

Battery

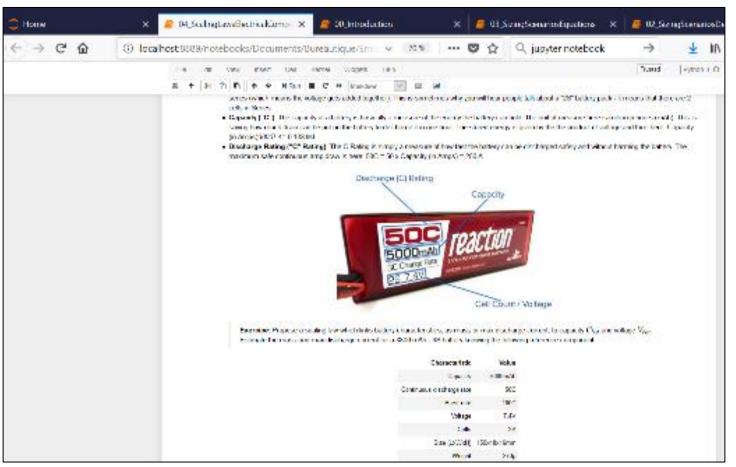


Brushless motor



ESC



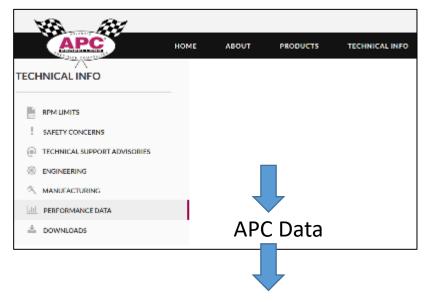


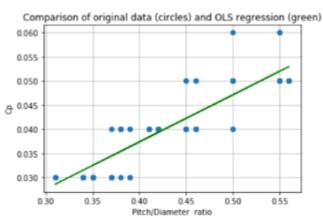


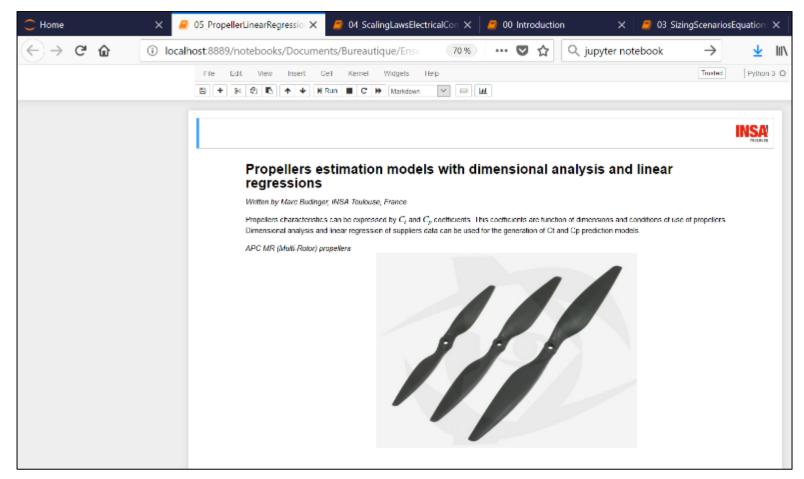
Case study application: Multi-Rotor propellers



Linear regression:









Preliminary design and sizing of mechatronic systems Sizing procedures and optimization problems definition

Sizing procedure and optimization

The preliminary design is an inverse problem:

What we want Design **Systems Components** specifications specifications model

What we can have thanks modeling

Simulation System **Components** characteristics model performances **Inverse problem**

Two main solutions for this problem:

Sizing or selection procedure:

Example:

Harmonic Drive reducer

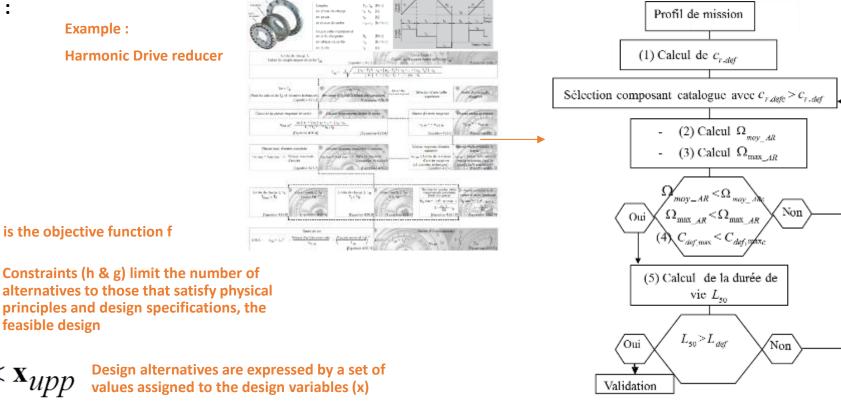
Optimization algorithms:

Minimize The goal is the objective function f

Subject to $\mathbf{h}(\mathbf{x}) = \mathbf{0}$

alternatives to those that satisfy physical principles and design specifications, the g(x) < 0feasible design

Design alternatives are expressed by a set of values assigned to the design variables (x)



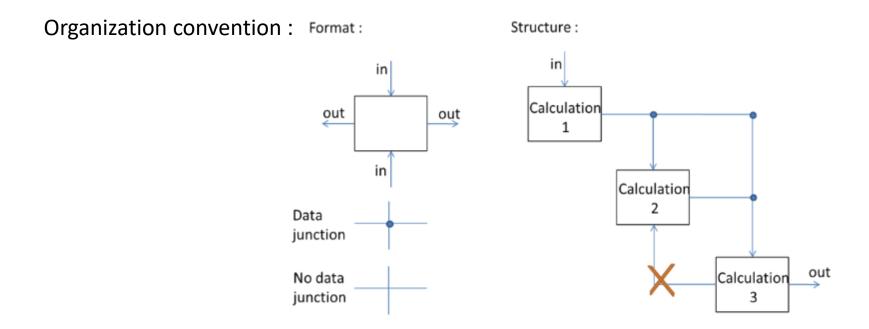


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Sizing procedures / N² diagrams

The N² diagram:

- Causal diagram where the graphical entities (blocks) and interactions (arrows) are dictated by an organization convention
- The inputs of the blocks are represented by vertical arrows while the outputs must be horizontal arrows.



- quick visual summary of the organization of the studied system
 - couplings and algebraic loops between blocks are easily identified



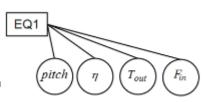
Sizing procedures / Graphs and dependency matrixes

- Sizing problem = a constraint network where sizing parameters represent variables and equalities/inequalities represent constraints.
- Constraint networks can be represented by graph and dependency matrixes

Graph representations of equation :

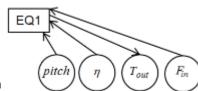
 $T_{out} - F_{in} \cdot \frac{pitch}{n} = 0$

Non-oriented graph and equation

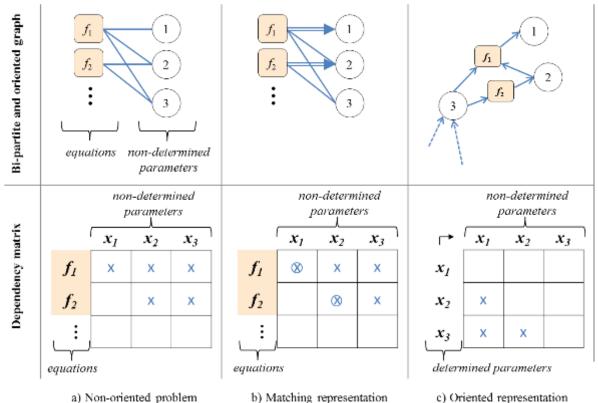


 $T_{out} = F_{in}.\frac{pitch}{\eta}$

Oriented graph and equation



Matching of a 2x2 problem :





Optimization / Pseudo code representation

Pseudo-code skeleton of an optimization problem:

Constants: Optimization variables and their limits:

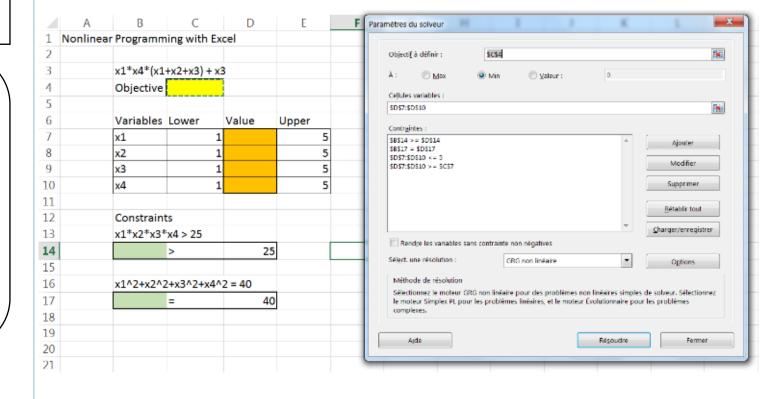
Sizing procedure:

Constraints:

Objective:

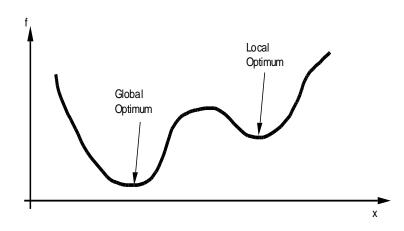
Excel solver:

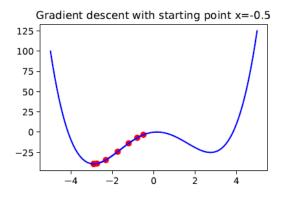
- Add-in for Excel : /File/Options/Add-in
- Access : /Data/Solver

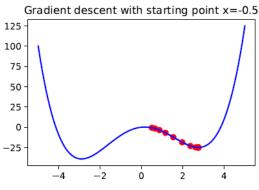


Optimization / Design Optimization Concepts

Local and global optima



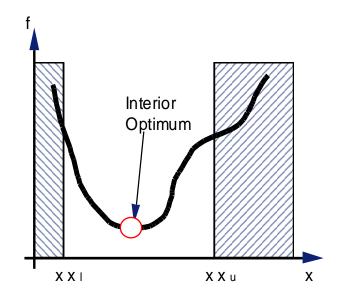


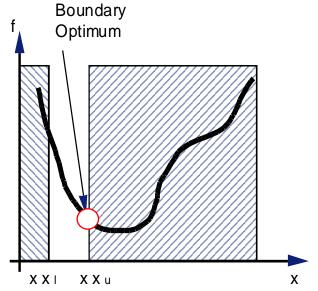


Gradient descent example

Interior and Boundary Optima

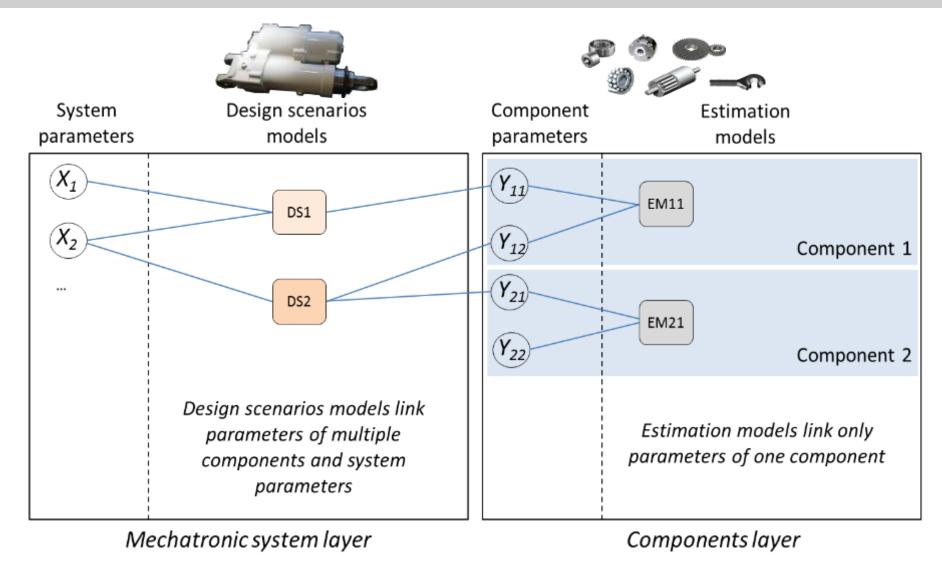
- An optimum is interior if all the constraints are inactive
- An optimum that is not interior is a boundary optimum







Design graphs: representation of a set of sizing equations p31



Bipartite graph for each design layer

Relationships between the parameters and equations are represented as nonoriented edges



Design graphs: orientation and explicit formulation

Objectives:

- to order flows of information into a sizing procedure.
- a sizing procedure well posed and easy to use in any computing environment → this procedure must be explicit without internal numerical solver.

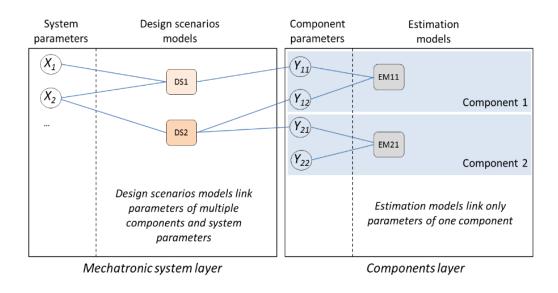
Proposed process:

	Process (« WHATs »)		Methods (« HOWs »)	Tools (implement the «HOWs ») : Design Graphs
1	Problem definition	1 1	Gather all the	System Design scenarios Component Estimation
1	Problem definition	1.1	equations/inequalities describing the problem.	parameters models X_1
2	2 Orientate the problem		Match equations/parameters	DS1 >
		2.2	Identify and highlight over- constrained and under- constrained singularities	X_1 X_2 Y_{11} Y_{12}
		2.3	ı	designer manually modifying parameters status (to inequalities to constraint while introducing safety factors.
3	Break problem calculation loops.	3.1	Identify and highlight algebraic loops	X ₁
		3.2	Identify and highlight the minimum set of parameters to be fixed to suppress loop.	X ₂
		3.3	Loops are displaced to upper	ayer manually by the designer.

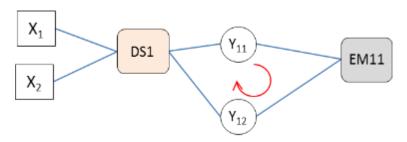


Design graphs: main steps and problems

1/ Problem definition

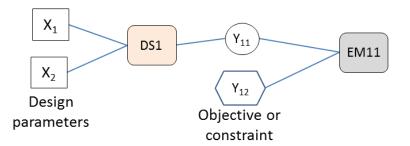


3/ Algebraic loops

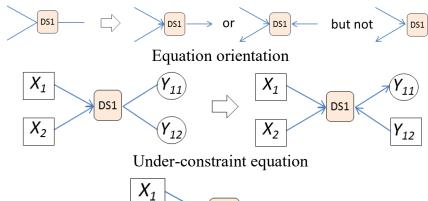


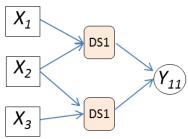
→ The optimization problem is defined (inputs, constraints, objective)

2/ Equations orientation



Inputs (design parameters) and outputs (objectives or constraint) representation





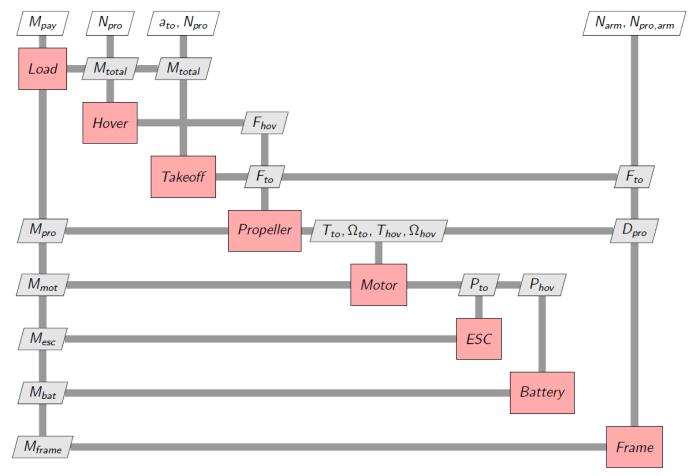
Over-constraint equations



Case study, global sizing process



Multirotor sizing example



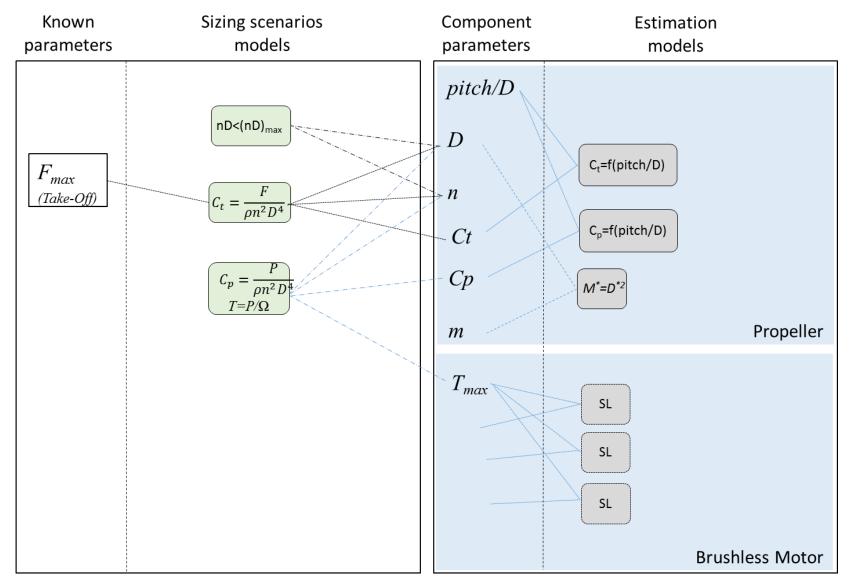
Question:

- Explain notation of N2 diagrams.
- Which classical sizing problem can be highlighted by this notation?
- Do you see a problem here? If yes, give a solution.



Case study, propeller selection



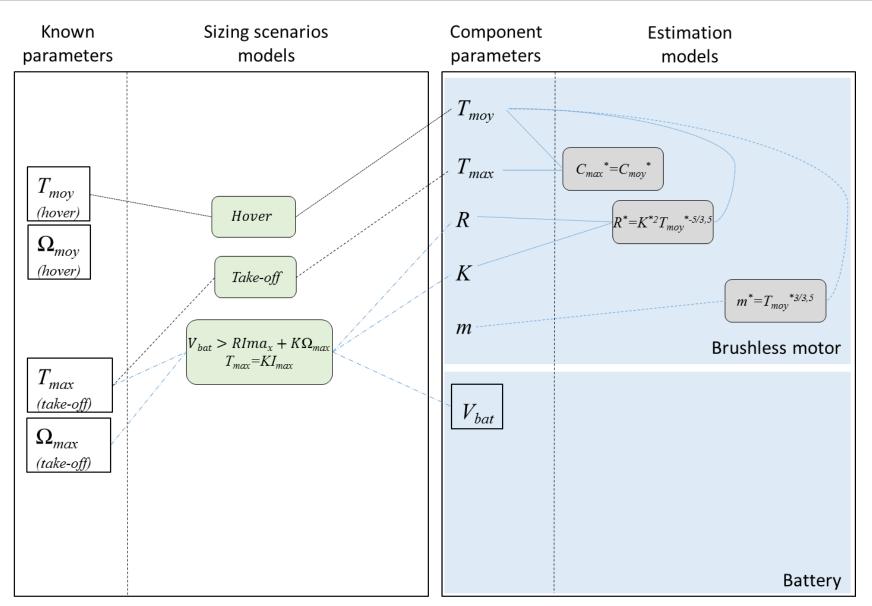


Questions:

- Give the main sizing problems you are able to detect.
- Propose one or multiple solutions (which can request equation manipulation, addition of design variables, addition of constraints)
- Orientate the arrows
- Give equations order, inputs/outputs at each step of this part of sizing procedure



Case study, motor selection



The following diagram represents the design graph of the propeller's selection. The mean speed/thrust (Ω moy & Tmoy), the max speed/thrust (Ω max & Tmax) and the battery voltage are assumed to be known here.

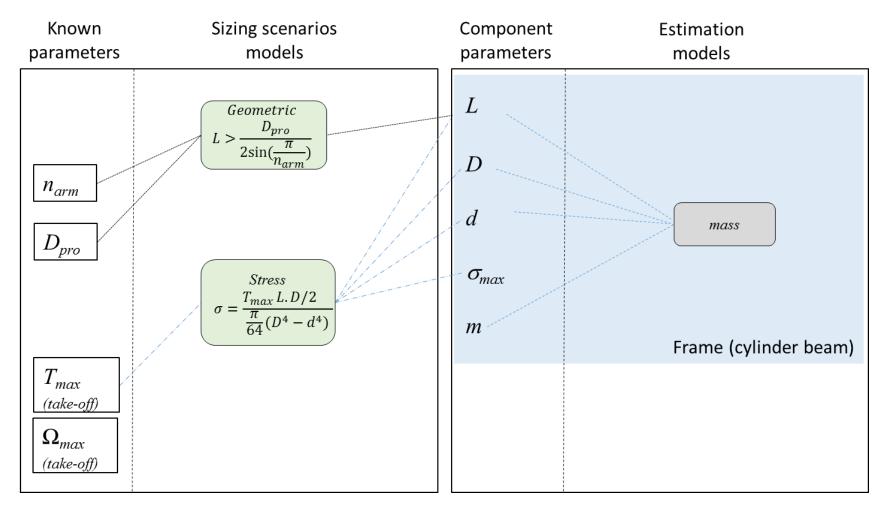
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Questions:

- Give the 2 main sizing problems you are able to detect here.
- Propose one or multiple solutions (which can request equation manipulation, addition of design variables, addition of constraints)
- Orientate the arrows and write equations order, inputs/outputs at each step of this part of sizing procedure, additional constraints

Case study, frame sizing





The following diagram represents the design graph of the frame sizing (round tube).

Questions:

- Give the main sizing problems you are able to detect here.
- Propose one or multiple solutions (which can request equation manipulation, addition of design variables, addition of constraints)
- Orientate the arrows and write equations order, inputs/outputs at each step of this part of sizing procedure, additional constraints



Notebooks

Battery and ESC Sizing



Overall Multirotor Sizing



Preliminary design and sizing of multirotor drones Discussions and exchanges

Increase the numbers of sizing criteria

Sizing scenarios



Vertical flight: calculation of drag force

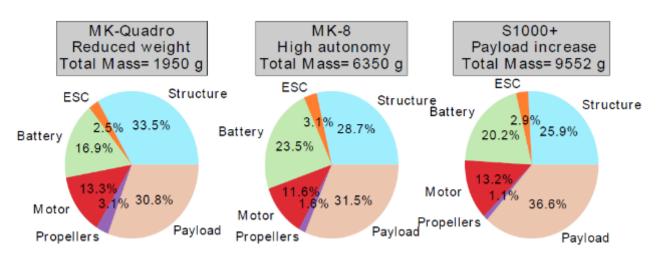


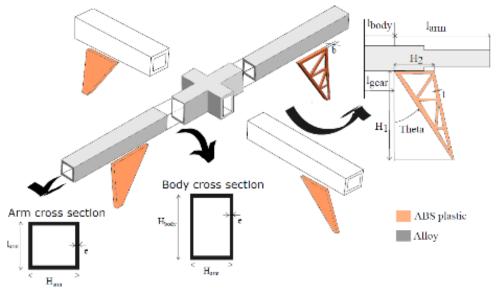
Forward speed and : aerodynamic forces (lift ?, drag ?), propeller



Max forward speed and gust

Components

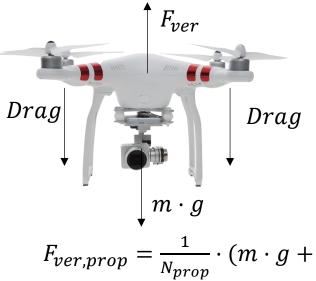






Maximum vertical speed

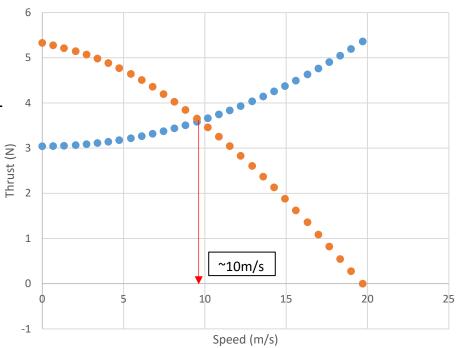
Vertical movement



Required thrust increases with the square of the speed

Propeller performance for speeds

Propeller thrust decreases as speed increases.



Need to find the maximum operating point for a certain speed v

HOW?

1. Calculation of vertical force w.r.t. speed (v)

Example: DJI Phantom 2 Vision+:

- Mass: 1,24 kg

- Surface: 0,0306 m²

- *Cd=1,3*

- Number of propellers=4

2. Evolution of thrust force w.r.t. speed (v) using propeller data catalog.

Example: APC MR for 7000 RPM (indicated by the manufacturer)

Total force per heliceAPC Thrust

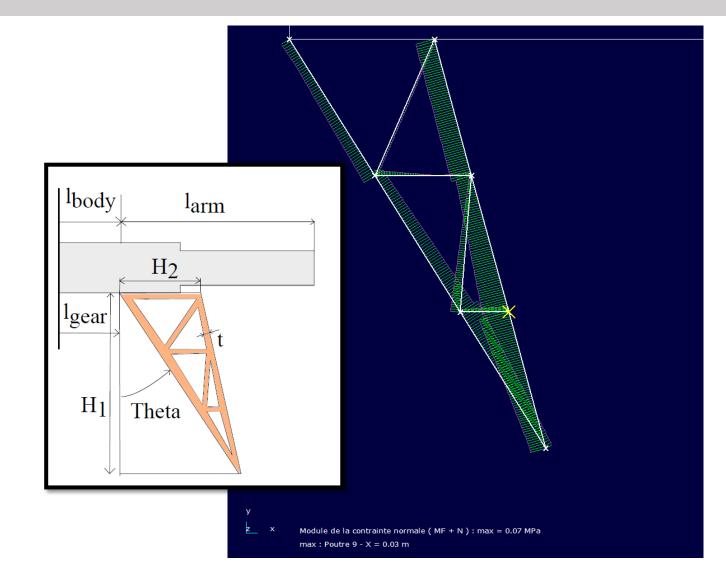


Increase model fidelity

- 1. Sizing landing gears/body on crash scenario
 - a) Evaluate landing gear stiffness
 - b) Evaluate body stiffness
 - c) Calculate max load (energy conversion)

$$F_{max} = \frac{1}{4}(k_{eq}.\delta x + M_{tot}.g) = \frac{1}{4}(V_{impact}.\sqrt{k_{eq}M_{tot}} + M_{tot}.g)$$
$$k_{eq} = 4.\frac{\overset{\sim}{k_1}.\overset{\sim}{k_2}}{\overset{\sim}{k_1}+\overset{\sim}{k_2}}$$

d) Evaluate mechanical constraint

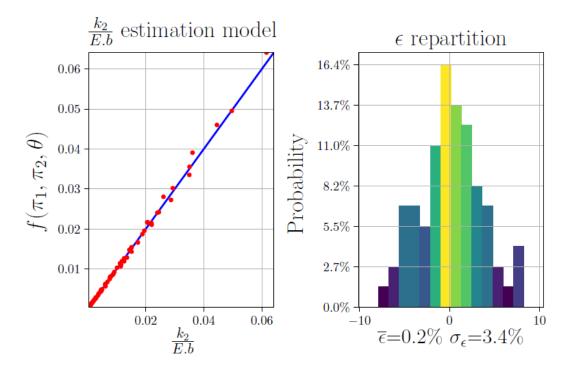




Increase model fidelity

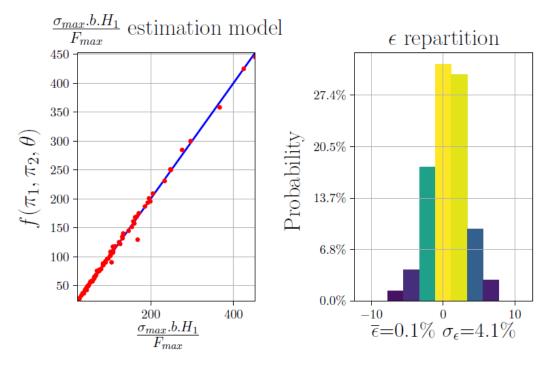
a) Evaluate landing gear stiffness

$$\pi_{k_2} = \frac{k_2}{b.E} \approx f(\frac{H_1}{H_2}, \frac{t}{H_1}, \theta) = f(\pi_1, \pi_2, \theta) = \tilde{\pi}_{k_2}$$
$$\tilde{\pi}_{k_2} = 10^{-0.37053} \cdot \pi_1^{-3.11170} \cdot \pi_2^{1.10205} \cdot \theta^{-6.61617 - 4.86580 \cdot \log(\theta)}$$

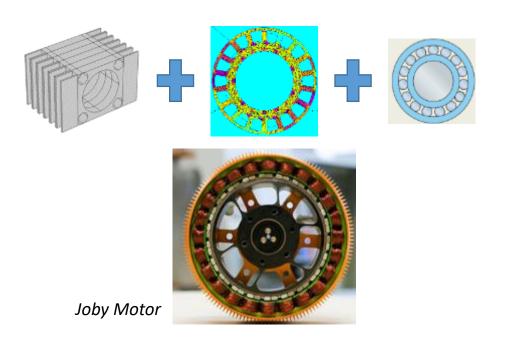


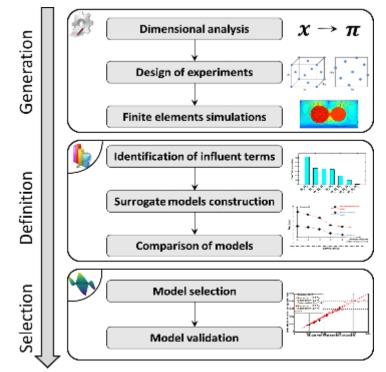
d) Evaluate mechanical constraint

$$\pi_{\sigma_{max}} = \frac{\sigma_{max}.b.H_1}{F} \approx f(\frac{H_1}{H_2}, \frac{t}{H_1}, \theta) = f(\pi_1, \pi_2, \theta) = \tilde{\pi}_{\sigma_{max}}$$
$$\tilde{\pi}_{\sigma_{max}} = 10^{0.14690}.\pi_1^{2.08982}.\pi_2^{-0.98108}.\theta^{3.38363 + 2.66468.log(\theta)}$$



Increase model fidelity





Mass evolution at 30 min. of fixed autonomy

