problem3 → d) Hessian → $f(\beta)=\beta^T C\beta$ ($C \to$ symmetric & $\in R^{p\times p}$)

$$\nabla^2 f(\beta) = \begin{bmatrix} \frac{\partial}{\partial\beta_1\partial\beta_2} f(\beta) & \frac{\partial}{\partial\beta_1\partial\beta_2} f(\beta) & \cdots & \frac{\partial}{\partial\beta_1\beta_p} f(\beta) \\ \vdots & & & \\ \frac{\partial}{\partial\beta_p\partial\beta_1} f(\beta) & \cdots & & \frac{\partial}{\partial\beta_p\partial\beta_p} f(\beta) \end{bmatrix}$$

$$= \begin{bmatrix} c_{11}+c_{11} & \cdots & c_{p1}+c_{1p} \\ \vdots & & \\ c_{1p}+c_{p1} & \cdots & c_{pp}+c_{pp} \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{p1} \\ \vdots & & \\ c_{1p} & \cdots & c_{pp} \end{bmatrix} + \begin{bmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & & \\ c_{p1} & \cdots & c_{pp} \end{bmatrix}$$

$$= C + C^T \underset{\text{(symmetric)}}{=\!=} 2C$$

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