Transmission Line and Voltage Standing Wave Ratio

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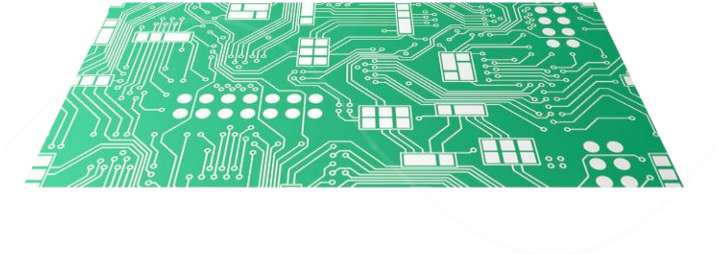
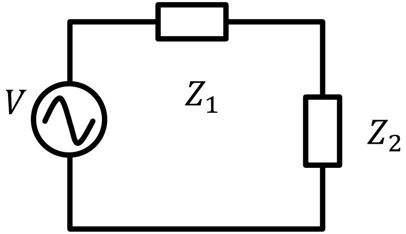
Circuit and transmission line theory

Low frequency: Wavelength is much larger than the circuit size

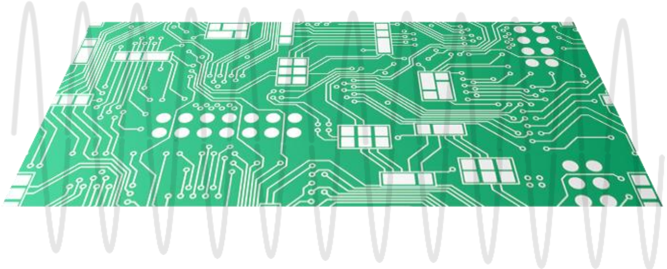


Circuit theory

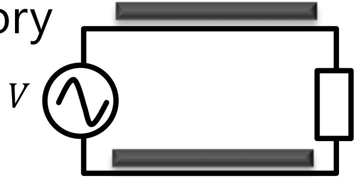
Quasi-static



High frequency: Wavelength is much smaller than circuit size



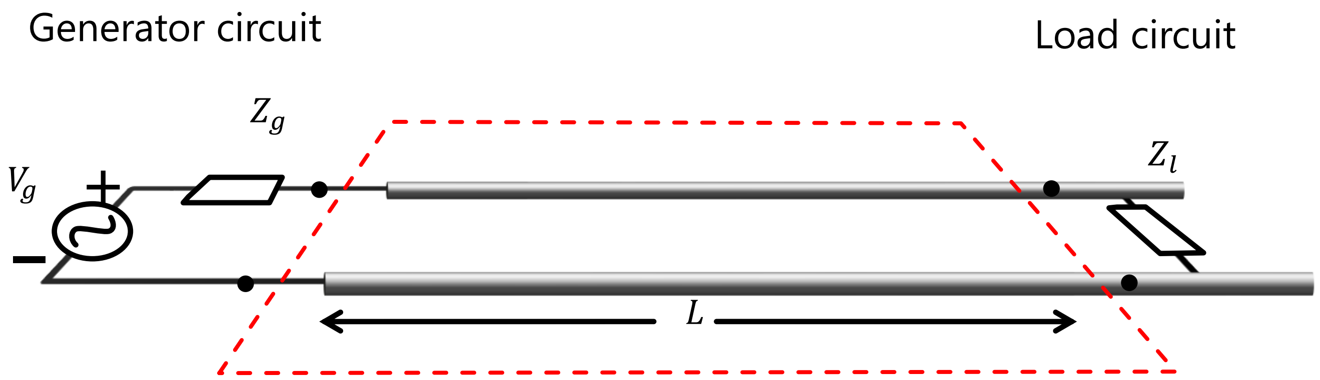
Transmission line theory



Different phases at different locations in the circuit

Unlike in circuit theory, the **length** of a transmission line is important to determine voltage and current

Transmission lines



A transmission line is a two-port network connecting a generator circuit at the sending end to a load at the receiving end.

- It is a structure to carry alternating current and electromagnetic waves from one place to another

- E.g.: connecting radio transmitters and receivers with their antennas, distributing cable television signals, and computer network connections, etc.

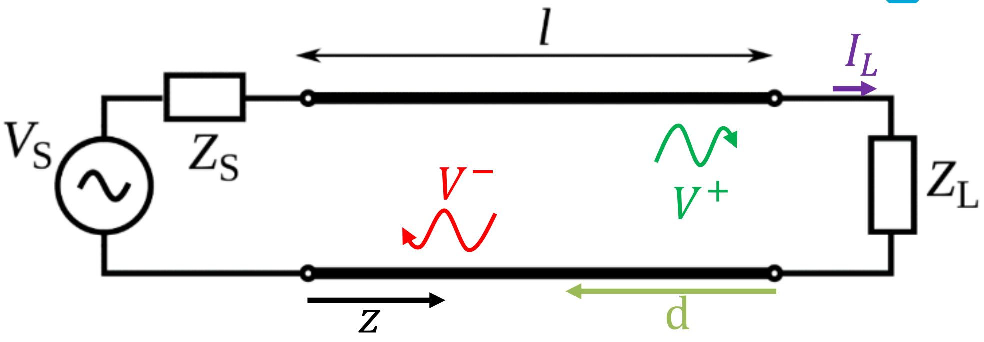
Outline

1. Basics of travelling wave
    1. Current and Voltage waves in Transmission Lines
    2. Introduction on Characteristic impedance, load and mismatch terms

2. Background information
    1. Intro to Transmission Line (TL)
    2. Intro to Standing wave

3. How can we measure mismatch and find load?
    1. Voltage Standing Wave Ratio
    2. Load Calculation

1. Basics of travelling wave



$$V(z) = V^+e^{-\gamma z} + V^-e^{+\gamma z}$$ Voltage wave

We can write voltage at z=l or d=0

$$V_L = \underbrace{V^+e^{-\gamma l}}_{V_L^+} + \underbrace{V^-e^{+\gamma l}}_{V_L^-}$$

$$V(d) = V_L^+e^{+\gamma d} + V_L^-e^{-\gamma d}$$

$$I_L = I_L^+ + I_L^- = \frac{V_L^+}{Z_{ch}} - \frac{V_L^-}{Z_{ch}}$$

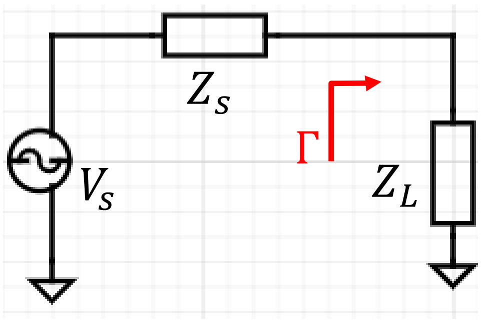
$$I(d) = \frac{V_L^+e^{+\gamma d} + V_L^-e^{-\gamma d}}{Z_{ch}}$$ Current wave

- One of the mathematical ways to represent waves are the complex exponentials. In our notation $e^{-\gamma z}$ denotes forward propagating wave.
- $\gamma = \alpha + j\beta$ where α quantifies loss terms while β determines propagation properties
- Waves in TL can be decomposed as incident (forward going), and reflected (backward going)
- TL's have a characteristic impedance (Z_{ch}) determined by dimensions of the TL and the dielectric material (independent of length). The voltage and current waves within the TL scales with Z_{ch}

Introduction on Characteristic impedance, load and mismatch terms

Resistance against applied voltage (R) + Storage due to change in applied voltage (X) → Impedance or Load Z_L=(R+jX)

- Frequency dependent due to X
- Complex number in general



Max. Power Transfer

$$Z_s = Z_L^*$$

$$P_{Load} = P_{gen}$$

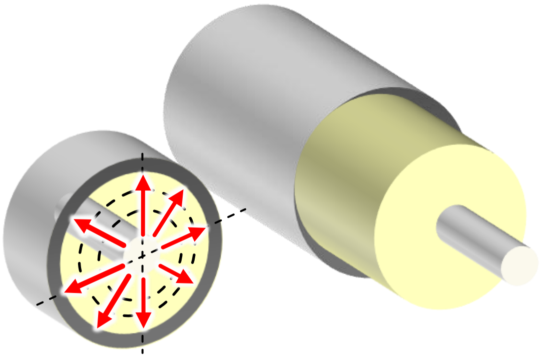
Mismatch

$$Z_s \neq Z_L$$

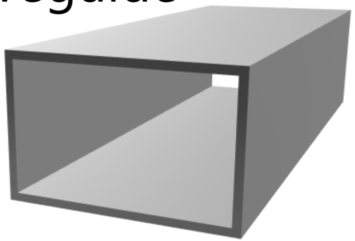
$$\Gamma = \frac{Z_L - Z_S}{Z_L + Z_S} \qquad P_{Load} = (1 - |\Gamma|^2)\, P_{gen}$$

2. Background Information

Coaxial transmission line



Rectangular waveguide



Attenuation constant

$$\gamma = \alpha + j\beta$$

Propagation constant

Transmission Line

$\beta \neq k_0$ due to dielectric

$$\beta = w\sqrt{\mu_0\epsilon_0\mu_r\epsilon_r}$$

$$\lambda_g = \lambda_0/\sqrt{\mu_r\epsilon_r}$$

Waveguide

$\beta \neq k_0$ due to cutoff relation

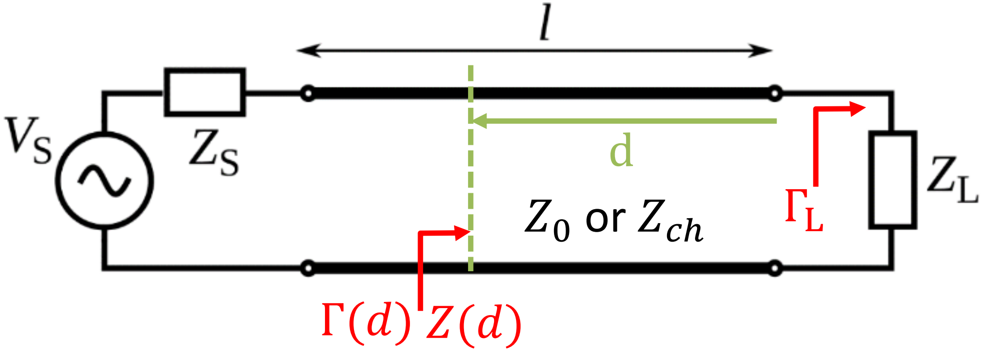
$$\beta = k_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$k_0 = \frac{2\pi}{\lambda_0}$ wave number, f_c cut-off

$$\lambda_w = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}}$$

Phase velocity

$$u_p = \frac{w}{\beta}$$

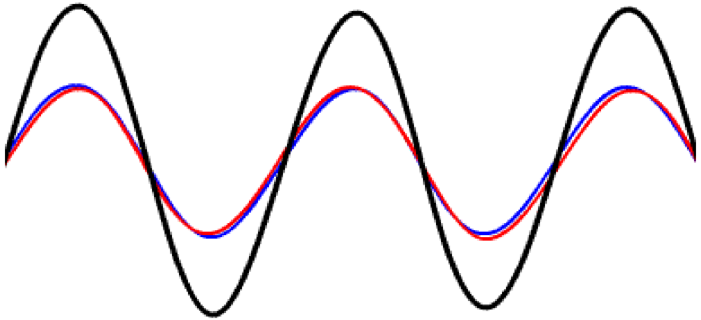


$$\Gamma(d) = \frac{Z(d) - Z_0}{Z(d) + Z_0} = \Gamma_L e^{-2\gamma d}$$

$$\gamma = \alpha + j\beta$$

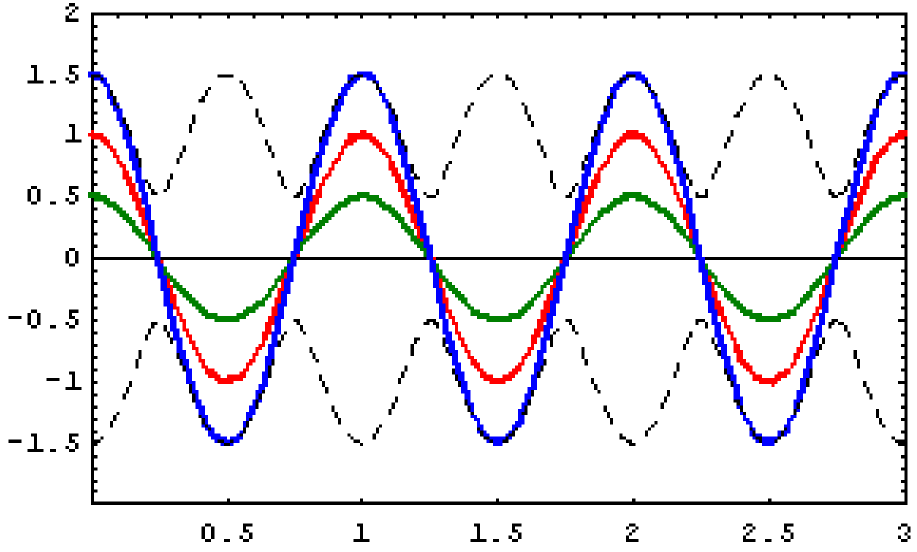
$$Z(d) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)} \xrightarrow[\alpha = 0]{\text{lossless}} Z_{lossless}(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

Standing Wave



Incident, Reflected, Standing

If they don't have the same amplitude
Oscillate between a max and a min



Incident, Reflected, Standing

A measure of how much standing waves are present is the Voltage standing wave ratio (VSWR)

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

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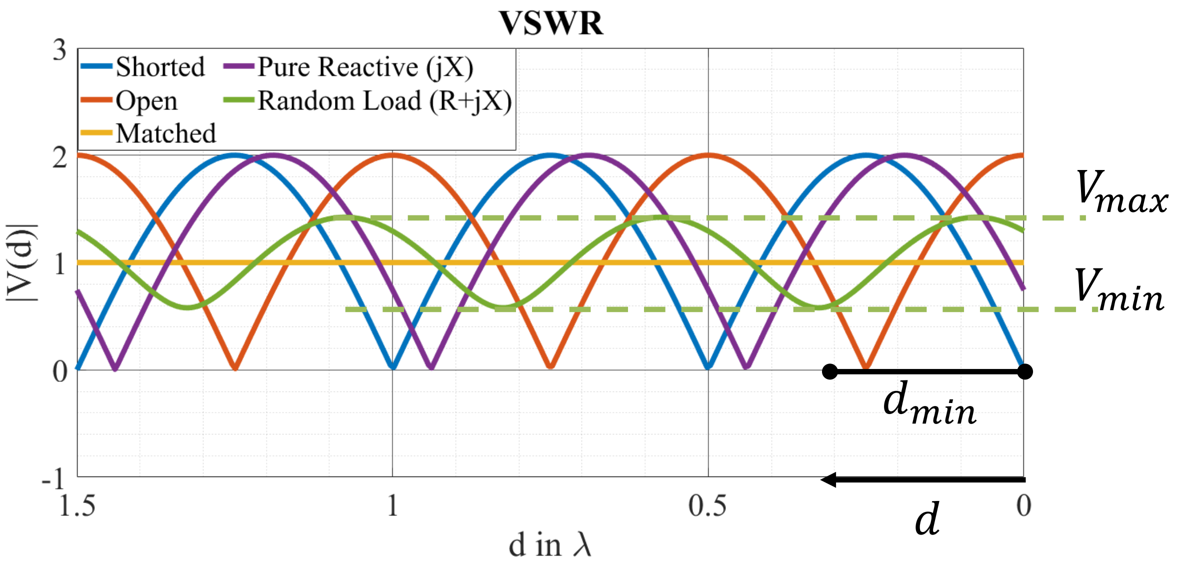
3. How can we measure mismatch and find load?



$$VSWR\ (S) = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$1 \leq S < \infty$$

$|\Gamma_L = 0|$
Matched Load

$|\Gamma_L = 1|$

- $Z_L = 0$
  Shorted Load
- $Z_L = \infty$
  Open Load
- $Z_L = jX$
  Purely Reactive Load

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How to calculate load by VSWR graph?

Assuming lossless TL we can express reflection coeff. As;

$$Z_L = Z_0 \frac{1-\Gamma_L}{1+\Gamma_L} \quad \text{and} \quad \Gamma_L = |\Gamma_L| e^{j\theta_L}$$

1. Determine $|\Gamma_L|$

$$S = \frac{1+|\Gamma_\mathrm{L}|}{1-|\Gamma_\mathrm{L}|} \quad \text{yields} \quad |\Gamma_\mathrm{L}| = \frac{S-1}{S+1}$$

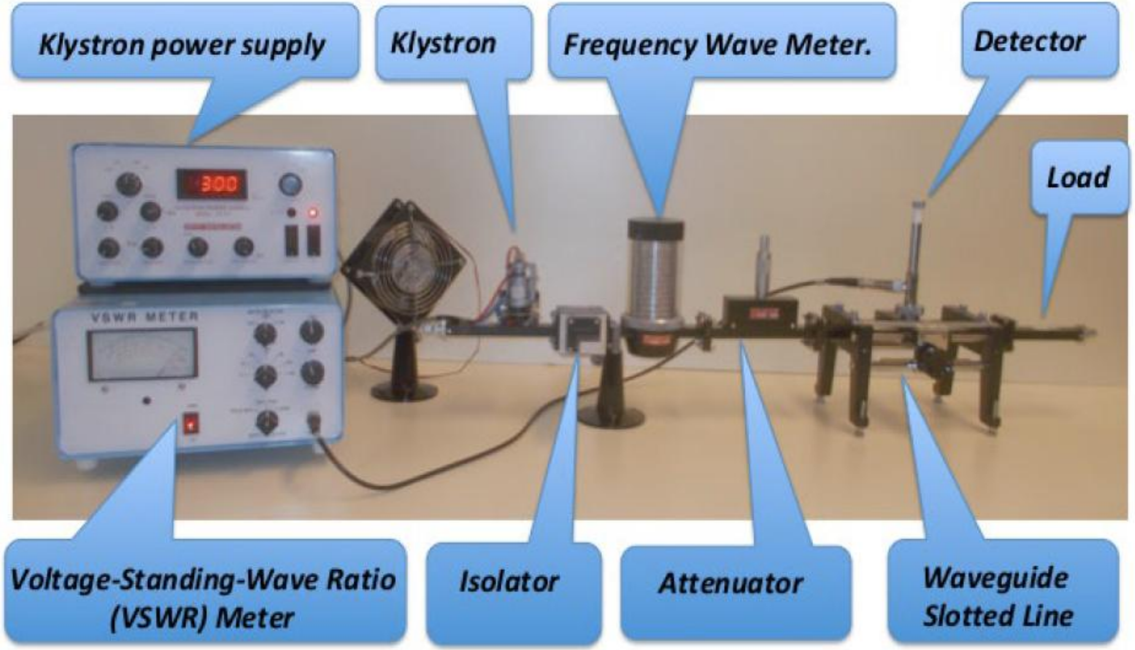
2. Determine θ_L

At $d = d_{min}$ voltage reflected and voltage incident has π phase difference

$$\frac{V_{reflected}}{V_{incident}} = \frac{V_L^- e^{-j\beta d_{min}}}{V_L^+ e^{j\beta d_{min}}} = \Gamma_L e^{-j2\beta d_{min}} = |\Gamma_L| e^{j\theta_L} e^{-j2\beta d_{min}}$$

Phase term should be $-\pi$. So $\theta_L = 2\beta d_{min} - \pi$

Experiment Setup



Summary of experiment procedure;

1. Determine operating frequency of the source using Frequency wave meter
2. Calibrate VSWR meter by sliding probe on slotted line
3. Measure VSWR values of different loads (open, matched load, antenna)
4. Measure VSWR values of short circuit under different piston locations

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