Verifying Featured Transition Systems using Variability Parity Games

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1 Definitions

1.1 Transition systems

From [1].

Definition 1.1. An LTS is a tuple $M = (S, Act, trans, I, AP, L)$, where:

- S is a set of states,
- Act a set of actions,
- $trans \subseteq S \times Act \times S$ is the transition relation with $(s, a, s') \in trans$ denoted as $s \xrightarrow{a} s'$,
- $I \subseteq S$ is a set of initial states,
- AP is a set of atomic propositions, and
- $L : S \rightarrow 2^{AP}$ is a labelling function.

Definition 1.2. An FTS is a tuple $M = (S, Act, trans, I, AP, L, N, px, \gamma)$, where:

- $S, Act, trans, I, AP, L$ are defined as in an LTS,
- N is a set of features,
- $px \subseteq \mathcal{P}(N)$ is a set of products, ie. feature assignments, that are valid,
- $\gamma : trans \rightarrow \mathbb{B}(N)$ is a total function, labelling each transition with a Boolean expression over the features. A product $p \in \mathcal{P}(N)$ satisfying the Boolean expression of transition t is denoted as $p \models \gamma(t)$, $\gamma(t)(p) = 1$ or $p \in [\![\gamma(t)]\!]$.

  A transition $s \xrightarrow{a} s'$ and $\gamma((s, a, s')) = f$ is denoted as $s \xrightarrow{a \backslash f} s'$.

Definition 1.3. The projection of an FTS fts to a product $p \in px$, noted $fts_{|p}$, is the LTS $t = (S, Act, trans', I, AP, L)$, where $trans' = \{t \in trans \mid p \models \gamma(t)\}$.

2 Variability Parity Games

2.1 Option 1

Definition 2.1. A Variability Parity Game is a tuple $VG = (V, V_0, V_1, E, \rho, N, \gamma)$, where:

- $V = V_0 \cup V_1$,
- $V_0 \cap V_1 = \emptyset$,

- V_0 is the set of vertices for player 0,
- V_1 is the set of vertices for player 1,
- $E \subseteq V \times V$ is the edge relation,
- $\rho : V \to \mathbb{N}$ is a priority assignment,
- N is a set of features,
- $\gamma : E \to \mathbb{B}(N)$ is a total function, labelling each edge with a Boolean expression over the features.

A VPG is played for a specific $p \subseteq N$. A path π is valid if for all pairs π_i and π_{i+1}) we have $(\pi_i, \pi_{i+1}) \in E$ and $p \models \gamma((\pi_i, \pi_{i+1}))$.

Not deadlock free, so player $\alpha \in \{0, 1\}$ wins iff $\overline{\alpha}$ can't make a move or if the highest priority occurring infinitely often has the same parity as the player.

For a $p \subseteq N$ we have winning sets W_0^p and W_1^p.

Definition 2.2. The projection of a VPG vpg to a product $p \subseteq N$, noted $vpg_{|p}$, is the PG $pg = (V, V_0, V_1, E', \rho)$, where $E' = \{e \in E \mid p \models \gamma(e)\}$.

Definition 2.3. FTS2VPG(fts, φ) converts an FTS and a formula to a VPG.

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Definition 2.4. LTS2PG(lts, φ) converts an LTS and a formula to a PG.

Use some existing definition.

Theorem 2.1. W_α^p for VPG vpg is equal to W_α in $vpg_{|p}$ for any $p \subseteq N$ and $\alpha \in \{0, 1\}$.

From this it theorem follows that the VPG is positionally determined and that the winning sets cover the entire graph.

Lemma 2.2. FTS2VPG$(fts, \varphi)_{|p}$ is equal to LTS2PG$(fts_{|p}, \varphi)$ for any $p \subseteq N$.

Theorem 2.3. Given FTS $fts = (S, Act, trans, I, AP, L, N, px, \gamma)$ and formula φ.
For any product $p \in px$ and state $s \in S$ we have:
fts satisfies φ for product p in state s iff $s \in W_0^p$ in FTS2VPG(fts, φ).

Proof. Winning set W_0^p in FTS2VPG(fts, φ) is equal to winning set W_0 in FTS2VPG$(fts, \varphi)_{|p}$ (using theorem 2.1). Furthermore FTS2VPG$(fts, \varphi)_{|p}$ is equal to LTS2PG$(fts_{|p}, \varphi)$ (using lemma 2.2).

So winning set W_0^p in FTS2VPG(fts, φ) is equal to winning set W_0 in LTS2PG$(fts_{|p}, \varphi)$. Since $fts_{|p}$ satisfies φ in state s iff $s \in W_0$ in LTS2PG$(fts_{|p}, \varphi)$ (existing LTS verification theory) the theorem holds. $\square$

References

[1] A. Classen, M. Cordy, P.-Y. Schobbens, P. Heymans, A. Legay, and J.-F. Raskin, "Featured transition systems: Foundations for verifying variability-intensive systems and their application to ltl model checking," *IEEE Transactions on Software Engineering*, vol. 39, pp. 1069–1089, 2013.