Verifying Featured Transition Systems using Variability Parity Games

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1 Introduction

Model verification techniques can be used to improve the quality of software. These techniques require the behaviour of the software to be modelled, we then want to check if the model behaves conforming to some requirement. Different languages are proposed and well studied to express these requirements, examples are LTL, CTL, CTL* and μ-calculus. Once the behaviour is modelled and the requirement is expressed in some language we can use modal checking techniques to determine if the model satisfies the requirement.

These techniques are well suited to model and verify the behaviour of a single software product. However software systems can be designed to have certain parts enabled or disabled. This gives rise to many software products that all behave very similar but not identical, such a collection is often called a *product family*. The differences between the products in a product family is called the *variability* of the family. A family can be verified by using the above mentioned techniques to verify every single product independently. However this approach does not use the similarities in behaviour of these different products, an approach that would make use of the similarities could potentially be a lot more efficient.

Labelled transition systems (LTSs) are often used to model the behaviour of a system, while it can model behaviour well it can't model variability. Efforts to also model variability include I/O automata, modal transition systems and *featured transition systems* (FTSs). Specifically the latter is well suited to model all the different behaviours of the software products as well as the variability of the entire system in a single model.

Efforts have been made to verify requirements for entire FTSs, as well as to be able to reason about features. Notable contributions are fLTL, fCTL and fNuSMV. However, as far as we know, there is no technique to verify an FTS against a μ-calculus formula. The modal μ-calculus is very expressive and subsumes other temporal logics like LTL, CTL and CTL*. In this thesis we will introduce a technique to do this. We first look at LTSs, the modal μ-calculus and FTSs. Next we will look at an existing technique to verify an LTS, namely solving *parity games*, as well as show how this technique can be used to verify an FTS by verifying every software product it describes independently. An extension to this technique is than proposed, namely solving *variability parity games*. We will formally define variability parity games and prove that solving a them can be used to verify FTSs.

2 Verifying transition systems

We first look at labelled transition systems (LTSs) and the modal μ-calculus and what it means to verify an LTS. The definition below are derived from [1].

Definition 2.1. *A labelled transition system (LTS) is a tuple $M = (S, Act, trans, s_0)$, where:*

- *S is a set of states,*
- *Act a set of actions,*
- *$trans \subseteq S \times Act \times S$ is the transition relation with $(s, a, s') \in trans$ denoted by $s \xrightarrow{a} s'$,*
- *$s_0 \in S$ is the initial state.*

Consider the example in figure 1 (directly taken from [2]) of a coffee machine where we have two actions: ins (insert coin) and std (get standard sized coffee).

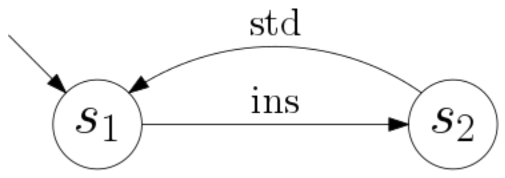


Figure 1: Coffee machine LTS C

Definition 2.2. *A modal μ-calculus formula over the set of actions Act and a set of variables $\mathcal{X}$ is defined by*

$$\varphi = \top \mid \bot \mid X \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \langle a \rangle \varphi \mid [a]\varphi \mid \mu X.\varphi \mid \nu X.\varphi$$

with $a \in Act$ and $X \in \mathcal{X}$.

No negations in the language because negations can be pushed inside to the propositions, ie. the $\top$ and $\bot$ elements.

The modal μ-calculus contains boolean constants $\top$ and $\bot$, propositional operators $\vee$ and $\wedge$, modal operators $\langle\,\rangle$ and $[\,]$ and fixpoint operators μ and ν. A formula is closed when variables only occur in the scope of a fixpoint operator for that variable.

A modal μ-calculus formula can be interpreted with an LTS, this results in a set of states for which the formula holds.

Definition 2.3. *For LTS $(S, Act, trans, s_0)$ we inductively define the interpretation of a modal μ-calculus formula φ, notation $[\![\varphi]\!]^\eta$, where $\eta : \mathcal{X} \to \mathcal{P}(S)$ is a logical variable valuation, as a set of states where φ is valid, by:*

$$\begin{aligned}
[\![\top]\!]^\eta &= S \\
[\![\bot]\!]^\eta &= \emptyset \\
[\![\varphi_1 \wedge \varphi_2]\!]^\eta &= [\![\varphi_1]\!]^\eta \cap [\![\varphi_2]\!]^\eta \\
[\![\varphi_1 \vee \varphi_2]\!]^\eta &= [\![\varphi_1]\!]^\eta \cup [\![\varphi_2]\!]^\eta \\
[\![\langle a \rangle \varphi]\!]^\eta &= \{s \in S \mid \exists_{s' \in S}\, s \xrightarrow{a} s' \wedge s' \in [\![\varphi]\!]^\eta\} \\
[\![[a]\varphi]\!]^\eta &= \{s \in S \mid \forall_{s' \in S}\, s \xrightarrow{a} s' \implies s' \in [\![\varphi]\!]^\eta\} \\
[\![\mu X.\varphi]\!]^\eta &= \bigcap_{f \subseteq S} \{f \mid f = [\![\varphi]\!]^{\eta[X:=f]}\} \\
[\![\nu X.\varphi]\!]^\eta &= \bigcup_{f \subseteq S} \{f \mid f = [\![\varphi]\!]^{\eta[X:=f]}\} \\
[\![X]\!]^\eta &= \eta(X)
\end{aligned}$$

Given closed formula φ, LTS $M = (S, Act, trans, s_0)$ and $s \in S$ we write $(M, s) \models \varphi$ iff $s \in [\![\varphi]\!]^\eta$ for M, we say that formula φ holds for M in state s. If formula φ holds for M in the initial state we say that formula φ holds for M and write $M \models \varphi$.

Again consider the coffee machine example (figure 1) and formula $\varphi = \nu X.\mu Y.([ins]Y \wedge [std]X)$ (taken from [2]) which states that action std must occur infinitely often over all runs. Obviously this holds for the coffee machine, therefore we have $C \models \varphi$.

2.1 Featured transition systems

A featured transition system extends the LTS definition to express variability. It does so by introducing *features* and *products* into the definition. Features are options that can be enabled or disabled for the system. A product is a feature assignments, ie. a set of features that is enabled for that product. Not a products are valid, some features might be mutually exclusive and some features might always be required. To express products one can use feature diagrams as explained in [3]. Feature diagrams offer a nice way of expressing which feature assignments are valid, but since they offer just that we simply represent the valid products with a set of feature assignments. Finally FTSs guards every transition with a boolean expression over the set of features. We have the following definition, based on [3]:

Definition 2.4. *A featured transition system (FTS) is a tuple $M = (S, Act, trans, s_0, N, P, \gamma)$, where:*

- *$S, Act, trans, s_0$ are defined as in an LTS,*
- *N is a non-empty set of features,*
- *$P \subseteq \mathcal{P}(N)$ is a set of products, ie. feature assignments, that are valid,*
- *$\gamma : trans \to \mathbb{B}(N)$ is a total function, labelling each transition with a boolean expression over the features. A product $p \in \mathcal{P}(N)$ satisfying the boolean expression of transition t is denoted by $p \models \gamma(t)$, $\gamma(t)(p) = 1$ or $p \in [\![\gamma(t)]\!]$. The boolean expression that is satisfied by any feature assignment is denoted by $\top$, ie $p \models \top$ for any p.*

  A transition $s \xrightarrow{a} s'$ and $\gamma((s, a, s')) = f$ is denoted by $s \xrightarrow{a|f} s'$.

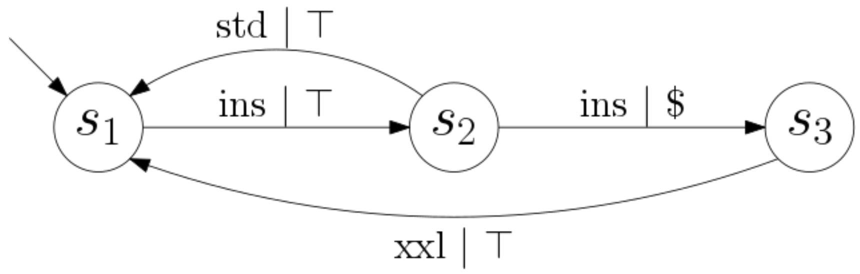


Figure 2: Coffee machine FTS C

Consider the example in figure 2 (directly taken from [2]) which shows an FTS for a coffee machine For this example we have two features $N = \{\$, €\}$ and two valid products $P = \{\{\$\}, \{€\}\}$.

An FTS expresses they behaviour of multiple products, we can derive the behaviour of a single product by simply removing all the transitions from the FTS for which the product doesn't satisfy the feature expression guarding the transition. We call this a *projection* ([3]).

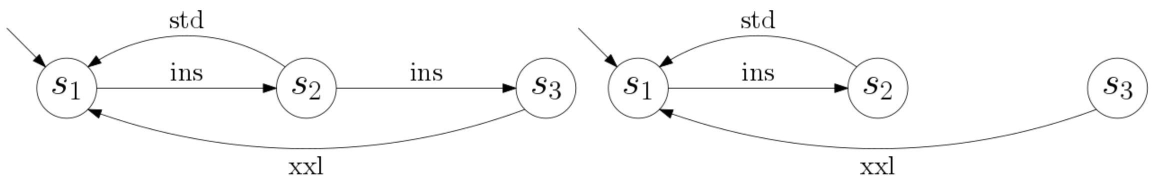
Definition 2.5. *The projection of an FTS $M = (S, Act, trans, s_0, N, P, \gamma)$ to a product $p \in P$, noted $M_{|p}$, is the LTS $M' = (S, Act, trans', s_0)$, where $trans' = \{t \in trans \mid p \models \gamma(t)\}$.*

The coffee machine example can be projected to its two products, which results in the LTSs in figure 3.

We want to verify the FTS against a modal μ-calculus formula φ. That is, we want to find out for which products in the FTS its projection satisfies φ. Formally, given FTS $M = (S, Act, trans, s_0, N, P, \gamma)$ and modal μ-calculus formula φ we want to find $P_s \subseteq P$ such that:

- for every $p \in P_s$ we have $M_{|p} \models \varphi$ and
- for every $p \in P \backslash P_s$ we have $M_{|p} \not\models \varphi$.

Furthermore a counterexample for every $p \in P \backslash P_s$ is preferred.



(a) C projected to the dollar product: $C_{|\{\$\}}$ (b) C projected to the euro product: $C_{|\{€\}}$

Figure 3: Projections of the coffee machine FTS

3 Verification using parity games

Verifying LTSs against a modal μ-calculus formula can be done by solving a parity game. This is done by translating an LTS in combination with a formula to a parity game, the solution of the parity game provides the information needed to conclude if the model satisfies the formula. This relation is depicted in figure 4. This technique is well known and well studied, in this section we will first look at parity games, the translation from LTS and formula to a parity game and finally what we can do with this technique to verify FTS.

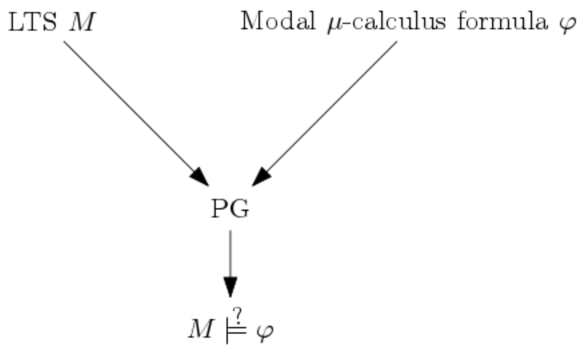

LTS M
Modal μ-calculus formula φ
PG
$M \overset{?}{\models} \varphi$

Figure 4: LTS verification using PG

3.1 Parity games

Definition 3.1. *[4] A parity game (PG) is a tuple* (V, V_0, V_1, E, ρ)*, where:*

- $V = V_0 \cup V_1$ *and* $V_0 \cap V_1 = \emptyset$*,*
- V_0 *is the set of vertices owned by player* 0*,*
- V_1 *is the set of vertices owned by player* 1*,*
- $E \subseteq V \times V$ *is the edge relation,*
- $\rho : V \to \mathbb{N}$ *is a priority assignment.*

We write $\alpha \in \{0, 1\}$ to denote an arbitrary player. We write $\overline{\alpha}$ to denote α's opponent, ie. $\overline{0} = 1$ and $\overline{1} = 0$.

A parity game is played by players 0 and 1. A play starts with placing a token on vertex $v \in V$. Player α moves the token if the token is on a vertex owned by α, ie. $v \in V_\alpha$. The token can be moved to $w \in V$, with $(v, w) \in E$. A series of moves results in a sequence of vertices, called path. For path π we write π_i to denote the i^{th} vertex in path π. A play ends when the token is on vertex $v \in V_\alpha$ and α can't move the token anywhere, in this case player $\overline{\alpha}$ wins the play. If the play results in an infinite path π then we determine the highest priority that occurs infinitely often in this path, formally

$$\max\{p \mid \forall_j \exists_i j < i \wedge p = \rho(\pi_i)\}$$

If the highest priority is odd then player 1 wins, if it is even player 0 wins.

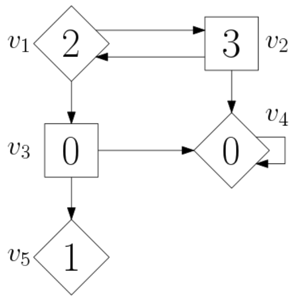


Figure 5: Parity game example

Figure 5 shows an example of a parity game. We usually depict the vertices owned by player 0 by diamonds and vertices owned by player 1 by boxes, the priority is depicted inside the vertices. If the game starts by placing a token on v_1 we can consider the following exemplary paths:

- $\pi = v_1 v_3 v_5$ is won by player 1 since player 0 can't move at v_5.
- $\pi = (v_1 v_2)^\omega$ is won by player 1 since the highest priority occurring infinitely often is 3.
- $\pi = v_1 v_3 (v_4)^\omega$ is won by player 0 since the highest priority occurring infinitely often is 0.

A strategy for player α is a function $\sigma : V^* V_\alpha \to V$ that maps a path ending in a vertex owned by player α to the next vertex. Parity games are positionally determined [4], therefore a strategy $\sigma : V_\alpha \to V$ that maps the current vertex to the next vertex is sufficient.

A strategy σ for player α is winning from vertex v if and only if any play that results from following σ results in a win for player α. The graph can be divided in two partitions $W_0 \subseteq V$ and $W_1 \subseteq V$, called winning sets. If and only if $v \in W_\alpha$ then player α has a winnings strategy from v. Every vertex in the graph is either in W_0 or W_1 [4]. Furthermore finite parity games are decidable [4].

3.2 Creating parity games

A parity game can be created from a combination of an LTS and a modal μ-calculus formula. To do this we first examine some of the properties of the modal μ-calculus.

First we introduce the notion of unfolding, a fixpoint formula $\mu X.\varphi$ can be unfolded resulting in formula φ where every occurrence of X is replaced by $\mu X.\varphi$, denoted by $\varphi[X := \mu X.\varphi]$. A fixpoint formula is equivalent to its unfolding ([4]), ie. for some LTS $[\![\mu X.\varphi]\!]^\eta = [\![\varphi[X := \mu X.\varphi]]\!]^\eta$. The same holds for the fixpoint operator ν.

Next we define alternating depth, this definition is literally taken from [4]:

Definition 3.2. *The dependency order on bound variables of φ is the smallest partial order such that $X \leq_\varphi Y$ if X occurs free in $\sigma Y.\psi$. The alternation depth of a μ-variable X in formula φ is the maximal length of a chain $X_1 \leq_\varphi \cdots \leq_\varphi X_n$ where $X = X_1$, variables $X_1, X_3, \ldots$ are μ-variables and variables $X_2, X_4, \ldots$ are ν-variables. The alternation depth of a ν-variable is defined similarly. The alternation depth of formula φ, denoted adepth(φ), is the maximum of the alternation depths of the variables bound in φ, or zero if there are no fixpoints.*

The example formula $\varphi = \nu X.\mu Y.([ins]Y \wedge [std]X)$ which states that for an LTS with $Act = \{ins, std\}$ the action std must occur infinitely often over all runs. Since X occurs free in $\mu Y.([ins]Y \wedge [std]X)$ we have $adepth(Y) = 1$ and $adepth(X) = 2$. As shown in [4] we have find that formula $\mu X.\psi$ has the same alternation depth as its unfolding $\psi[X := \mu X.\psi]$. Similarly for the greatest fixpoint.

We can now define the transformation from an LTS and a formula to a parity game. This relation is shown in the following diagram:

$$\begin{tikzcd} \text{LTS} \arrow[r, "\varphi"] & \text{PG} \end{tikzcd}$$

Definition 3.3. *[4] LTS2PG(M, φ) converts LTS $M = (S, Act, trans, s_0)$ and closed formula φ to a PG (V, V_0, V_1, E, ρ).*

A vertex in the parity game is represented by a pair (s, ψ) where $s \in S$ and ψ is a modal μ-calculus formula. We will create a vertex for every state with every subformula of φ plus the unfoldings of fixpoint operators. Formally we define the set of vertices by first defining the set of subformula's:

$$F = \{\psi \mid \psi \text{ is a subformula of } \varphi\}, \text{ note that } \varphi \text{ is a subformula of } \varphi,$$

and the set of unfolded fixpoint subformula's:

$$F_\sigma = \{\psi[X := \sigma X.\psi] \mid \sigma X.\psi \in F \text{ with } \sigma \in \{\mu, \nu\}\}$$

Having these sets we can define the set of vertices by:

$$V = S \times (F \cup F_\sigma)$$

We create the parity game with the smallest set E such that:

- $V = V_0 \cup V_1$,
- $V_0 \cap V_1 = \emptyset$ *and*
- *for every* $v = (s, \psi) \in V$ *we have:*
  - *If* $\psi = \top$ *then* $v \in V_1$.

- *If* $\psi = \bot$ *then* $v \in V_0$.
- *If* $\psi = \psi_1 \vee \psi_2$ *then:*
  $v \in V_0$,
  $(v, (s, \psi_1)) \in E$ *and*
  $(v, (s, \psi_2)) \in E$.
- *If* $\psi = \psi_1 \wedge \psi_2$ *then:*
  $v \in V_1$,
  $(v, (s, \psi_1)) \in E$ *and*
  $(v, (s, \psi_2)) \in E$.
- *If* $\psi = \langle a \rangle \psi'$ *then* $v \in V_0$ *and for every* $s \xrightarrow{a} s'$ *we have* $(v, (s', \psi')) \in E$.
- *If* $\psi = [a]\psi'$ *then* $v \in V_1$ *and for every* $s \xrightarrow{a} s'$ *we have* $(v, (s', \psi')) \in E$.
- *If* $\psi = \mu X.\psi'$ *then* $(v, (s, \psi')) \in E$.
- *If* $\psi = \nu X.\psi'$ *then* $(v, (s, \psi')) \in E$.
- *If* $\psi = X$ *then* $(v, \sigma X.\psi') \in E$ *where* $\sigma X.\psi' \in F$.

Note that directing formula $\sigma X.\psi'$ *to* ψ' *and directing* X *to* $\sigma X.\psi'$ *is equivalent to using unfolding which would direct* $\sigma X.\psi'$ *to* $\psi'[X := \sigma X.\psi']$.

$$\textit{Finally we have } \rho(s, \psi) = \begin{cases} 2\lfloor adepth(X)/2 \rfloor & \textit{if } \psi = \nu X.\psi' \\ 2\lfloor adepth(X)/2 \rfloor + 1 & \textit{if } \psi = \mu X.\psi' \\ 0 & \textit{otherwise} \end{cases}$$

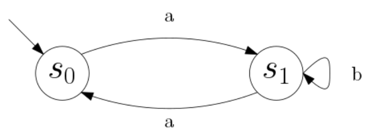


Figure 6: LTS M

Consider the LTS in figure 6 and formula $\varphi = \mu X.[a]X \vee \langle b \rangle \top$ expressing that on any path reached by a's can eventually do a b action. We will use this as a working example in the next few sections. The resulting parity game is depicted in figure 7. Solving this parity game results in the following winning sets:

$$\begin{aligned}
W_0 = \{&(s_1, \mu X.[a]X \vee \langle b \rangle \top)\\
&(s_1, [a]X \vee \langle b \rangle \top)\\
&(s_1, [a]X)\\
&(s_1, X)\\
&(s_1, \top)\\
&(s_2, \mu X.[a]X \vee \langle b \rangle \top)\\
&(s_2, [a]X \vee \langle b \rangle \top)\\
&(s_2, [a]X)\\
&(s_2, X)\\
&(s_2, \langle b \rangle \top)\\
&(s_2, \top)\\
&\}\\
W_1 = \{&(s_1, \langle b \rangle \top)\}
\end{aligned}$$

With the strategies σ_0 for player 0 and σ_1 for player 1 being (vertices with one outgoing edge are omitted):

$$\begin{aligned}
\sigma_0 &= \{(s_1, [a]X \vee \langle b \rangle \top) \mapsto (s_1, [a]X),\\
&\quad (s_2, [a]X \vee \langle b \rangle \top) \mapsto (s_2, \langle b \rangle \top)\}\\
\sigma_1 &= \{\}
\end{aligned}$$

State s in LTS M only satisfies φ iff player 0 has a winning strategy from vertex (s, φ). This is formally stated in the following theorem which is proven in [4].

Theorem 3.1. *Given LTS $M = (S, Act, trans, s_0)$, modal μ-calculus formula φ and state $s \in S$ it holds that $M, s \models \varphi$ iff $s \in W_0$ for the game $LTS2PG(M, \varphi)$.*

3.3 FTSs and parity games

Using the theory we have seen thus far we can verify FTSs. Namely by verifying every projection of the FTS to a valid product. This relation is depicted in the following diagram where Π indicates a projection:

$$\begin{tikzcd}
\text{FTS} \arrow[d, Rightarrow, "\Pi"'] & \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}$$

As mentioned before verifying products dependently is potentially more efficient. In the next two sections we define an extension to parity games, namely *variability parity games* (VPG) which can

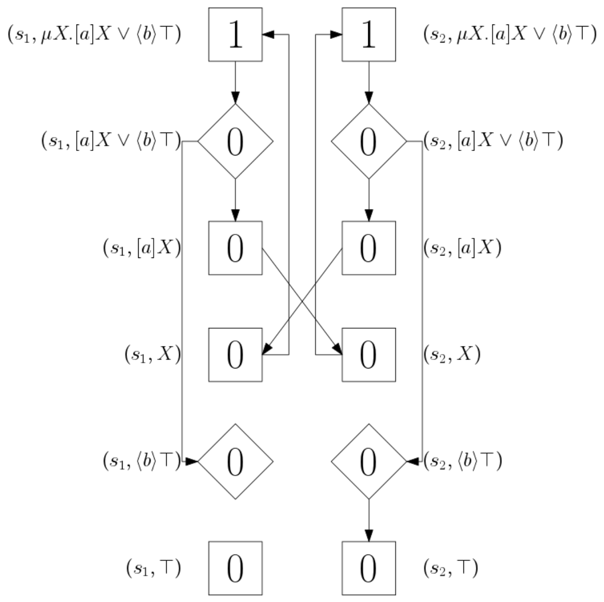

$(s_1, \mu X.[a]X \vee \langle b\rangle\top)$
1
$(s_2, \mu X.[a]X \vee \langle b\rangle\top)$
1
$(s_1, [a]X \vee \langle b\rangle\top)$
0
$(s_2, [a]X \vee \langle b\rangle\top)$
0
$(s_1, [a]X)$
0
$(s_2, [a]X)$
0
(s_1, X)
0
(s_2, X)
0
$(s_1, \langle b\rangle\top)$
0
$(s_2, \langle b\rangle\top)$
0
$(s_1, \top)$
0
$(s_2, \top)$
0

Figure 7: Parity game resulting from M and φ

be used to verify an FTS. We will translate an FTS and a formula into a VPG which solution will provide the information needed to conclude for which products the FTS satisfies the formula. This relation is depicted in figure 8.

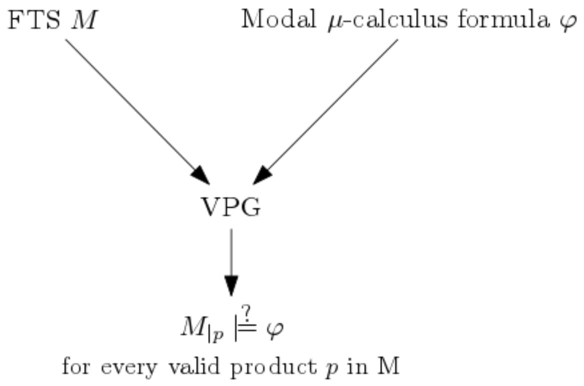

FTS M
Modal μ-calculus formula φ
VPG
$M_{|p} \overset{?}{\models} \varphi$
for every valid product p in M

Figure 8: FTS verification using VPG

4 Featured parity games

Before we can define variability parity games we first define *featured parity games* (FPG), featured parity games extend the definition of parity games to capture the variability represented in an FTS.

It uses the same concepts as FTSs, like features, products and a function that guards edges. In this section we will introduce the definition of FPGs and show that solving them answers the verification questions for FTS: For which products in the FTS does its projection satisfy φ?

First we introduce the definition of an FPG:

Definition 4.1. *A featured parity game (FPG) is a tuple* $(V, V_0, V_1, E, \rho, N, P, \gamma)$*, where:*

- $V = V_0 \cup V_1$ *and* $V_0 \cap V_1 = \emptyset$*,*
- V_0 *is the set of vertices owned by player* 0*,*
- V_1 *is the set of vertices owned by player* 1*,*
- $E \subseteq V \times V$ *is the edge relation,*
- $\rho : V \to \mathbb{N}$ *is a priority assignment,*
- N *is a set of features,*
- $P \subseteq \mathcal{P}(N)$ *is a set of products, ie. feature assignments, for which the game can be played,*
- $\gamma : E \to \mathbb{B}(N)$ *is a total function, labelling each edge with a Boolean expression over the features.*

An FPG is played similarly to a PG, however the game is played for a specific product $p \in P$. Player α can only move the token from $v \in V_\alpha$ to $w \in V$ if $(v, w) \in E$ and $p \models \gamma(v, w)$.

A game played for product $p \in P$ results in winnings sets W_0^p and W_1^p, which are defined similar to the W_0 and W_1 winning sets for parity games.

An FPG can simply be projected to a product p by removing the edges that are not satisfied by p.

Definition 4.2. *The projection from FPG* $G = (V, V_0, V_1, E, \rho, N, P, \gamma)$ *to a product* $p \in P$*, noted* $G_{|p}$*, is the parity game* (V, V_0, V_1, E', ρ) *where* $E' = \{e \in E \mid p \models \gamma(e)\}$*.*

Playing FPG G for a specific product $p \in P$ is the same as playing the PG $G_{|p}$. Any path that is valid in G for p is also valid in $G_{|p}$ and vice versa. Therefore the strategies are also interchangeable, furthermore the winning sets W_α for $G_{|p}$ and W_α^p for G are identical. Since parity games are positionally determined so are FPGs. Similarly, since finite parity games are decidable, so are finite FPGs.

4.1 Creating featured parity games

A featured parity game can be created from an FTS in combination with a model μ-calculus formula. This relation is shown in the following diagram:

$$\begin{tikzcd}\text{FTS} \arrow[r, "\varphi"] & \text{FPG}\end{tikzcd}$$

We translate an FTS to an FPG by first creating a PG from the diagram as if there were no transition guards, next we apply the same guards to the FPG as are present in the FTS for edges that originate from transitions. The features and valid products in the FPG are identical to those in the FTS.

Definition 4.3. *$FTS2FPG(M,\varphi)$ converts FTS $M = (S, Act, trans, s_0, N, P, \gamma)$ and closed formula φ to FPG $(V, V_0, V_1, E, \rho, N, P, \gamma')$.*

We have (V, V_0, V_1, E, ρ) = $LTS2PG((S, Act, trans, s_0), \varphi)$ and

$$\gamma'((s,\psi),(s',\psi')) = \begin{cases} \gamma(s,a,s') & \textit{if } \psi = \langle a\rangle\psi' \textit{ or } \psi = [a]\psi' \\ \top & \textit{otherwise} \end{cases}$$

Consider our working example which we extend to an FTS depicted in figure 9, for this example we have features $N = \{f\}$ and products $P = \{\emptyset, \{f\}\}$.

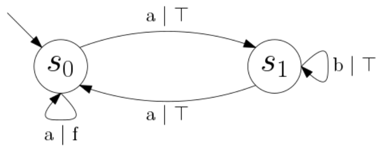


Figure 9: FTS M

We can translate this FTS with formula $\varphi = \mu X.[a]X \vee \langle b\rangle\top$ to an FPG depicted in figure 10. As we can see from the FTS if feature f is enabled then we have an infinite path of a's where b is never enabled, therefore φ doesn't hold for $M_{|\{f\}}$ however as we have seen φ does hold for $M_{|\emptyset}$. For the product $\emptyset$ we have the same winning set as before:

$$\begin{aligned} W_0^{\emptyset} = \{&(s_1, \mu X.[a]X \vee \langle b\rangle\top) \\ &(s_1, [a]X \vee \langle b\rangle\top) \\ &(s_1, [a]X) \\ &(s_1, X) \\ &(s_1, \top) \\ &(s_2, \mu X.[a]X \vee \langle b\rangle\top) \\ &(s_2, [a]X \vee \langle b\rangle\top) \\ &(s_2, [a]X) \\ &(s_2, X) \\ &(s_2, \langle b\rangle\top) \\ &(s_2, \top) \\ \} \\ W_1^{\emptyset} = \{&(s_1, \langle b\rangle\top)\} \end{aligned}$$

In the FPG we can see that if f is enabled then player 1 can move the token from $(s_1, [a]X)$ to (s_1, X). This results in player 0 either moving the token to $(s_1, \langle b\rangle\top)$ and losing or an infinite path where 1 occurs infinitely often which is also player 1 wins. For product $\{f\}$ we have winning sets:

$$\begin{aligned}
W_0^{\{f\}} = \{&(s_1, \top)\\
&(s_2, \mu X.[a]X \vee \langle b\rangle\top)\\
&(s_2, [a]X \vee \langle b\rangle\top)\\
&(s_2, X)\\
&(s_2, \langle b\rangle\top)\\
&(s_2, \top)\\
\}&\\
W_1^{\{f\}} = \{&(s_1, \mu X.[a]X \vee \langle b\rangle\top)\\
&(s_1, [a]X \vee \langle b\rangle\top)\\
&(s_1, [a]X)\\
&(s_1, X)\\
&(s_1, \langle b\rangle\top)\\
&(s_2, [a]X)\}
\end{aligned}$$

In the next section we will show how the winning sets relate to the model verification problem.

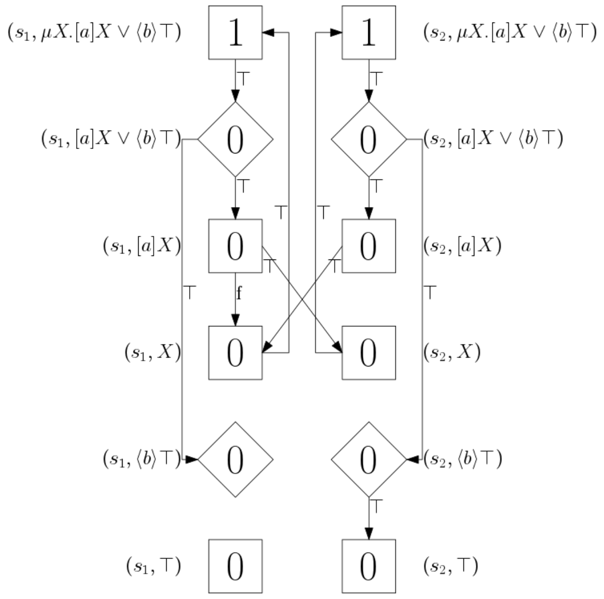

$(s_1, \mu X.[a]X \vee \langle b\rangle\top)$
1
$(s_2, \mu X.[a]X \vee \langle b\rangle\top)$
1
$\top$
$(s_1, [a]X \vee \langle b\rangle\top)$
0
$(s_2, [a]X \vee \langle b\rangle\top)$
0
$(s_1, [a]X)$
0
$(s_2, [a]X)$
0
f
(s_1, X)
0
(s_2, X)
0
$(s_1, \langle b\rangle\top)$
0
$(s_2, \langle b\rangle\top)$
0
$(s_1, \top)$
0
$(s_2, \top)$
0

Figure 10: FPG for M and φ

4.2 FTS verification using FPG

We can create an FPG from an FTS and project it to an PG, this is shown in the following diagram:

$$\begin{tikzcd}
\text{FTS} \arrow[r, "\varphi"] & \text{FPG} \arrow[d, Rightarrow, "\Pi"] \\
 & \text{PG}
\end{tikzcd}$$

Earlier we saw that we could also derive a PG by projecting the FTS to a product and then translation the resulting LTS to a PG, depicted by the following diagram:

$$\begin{tikzcd}
\text{FTS} \arrow[d, Rightarrow, "\Pi"'] & \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}$$

We will now show that the resulting parity games are identical.

Theorem 4.1. *Given:*

- *FTS $M = (S, Act, trans, s_0, N, P, \gamma)$,*
- *a closed modal mu-calculus formula φ,*
- *a product $p \in P$*

it holds that the parity games LTS2PG($M_{|p}, \varphi$) and FTS2FPG(M, φ)$_{|p}$ are identical.

Proof. Let $G^F = FTS2FPG(M, \varphi) = (V^F, V_0^F, V_1^F, E^F, \rho^F, N, P, \gamma')$, using definition 5.5, and $G^F_{|p} = (V^F, V_0^F, V_1^F, E^{F\prime}, \rho^F)$, using definition 4.2. Furthermore we have $M_{|p} = (S, Act, trans', s_0)$ and we let $G = LTS2PG(M_{|p}, \varphi) = (V, V_0, V_1, E, \rho)$. We depict the different transition systems and games in the following diagram.

$$\begin{tikzcd}
\text{FTS } M \arrow[rr, "\varphi"] \arrow[d, Rightarrow, "\Pi"'] & & \text{FPG } G^F \arrow[d, Rightarrow, "\Pi"] \\
\text{LTS } M_{|p} \arrow[r, "\varphi"] & \text{PG } G & \text{PG } G^F_{|p}
\end{tikzcd}$$

We will prove that $G = G^F_{|p}$. We first note that game G is created by

$$(V, V_0, V_1, E, \rho) = LTS2PG((S, Act, trans', s_0), \varphi)$$

and the vertices, edges and priorities of game G^F are created by

$$(V^F, V_0^F, V_1^F, E^F, \rho^F) = LTS2PG((S, Act, trans, s_0), \varphi)$$

Using the definition of LTS2PG (5.4) we find that the vertices and the priorities only depend on the states in S and the formula φ, since these are identical in the above two statements we immediately get $V = V^F, V_0 = V_0^F, V_1 = V_1^F$ and $\rho = \rho^F$. The vertices and priorities don't change when an FTS is projected, therefore $G^F_{|p}$ has the same vertices and priorities as G^F.

Now we are left with showing that $E = E^{F'}$ in order to conclude that that $G = G^F_{|p}$. We will do this by showing $E \subseteq E^{F'}$ and $E \supseteq E^{F'}$.

First let $e \in E$. Note that a vertex in the parity game is represented by a pair of a state and a formula. So we can write $e = ((s,\psi),(s',\psi'))$. To show that $e \in E^{F'}$ we distinguish two cases:

- If $\psi = \langle a\rangle\psi_1$ or $\psi = [a]\psi_1$ then there exists an $a \in Act$ such that $(s,a,s') \in trans'$. Using definition the FTS projection definition (2.5) we get $(s,a,s') \in trans$ and $p \models \gamma(s,a,s')$. Using the $FTS2FPG$ definition (5.5) we find that $\gamma'((s,\psi),(s',\psi')) = \gamma(s,a,s')$ and therefore $p \models \gamma'((s,\psi),(s',\psi'))$. Now using the FPG projection definition (4.2) we find $((s,\psi),(s',\psi')) \in E^{F'}$.

- Otherwise the existence of the edge does not depend on the $trans$ parameter and therefore $((s,\psi),(s',\psi')) \in E^{F'}$ if $(s,\psi) \in V^F$, since $V^F = V$ we have $(s,\psi) \in V^F$.

We can conclude that $E \subseteq E^{F'}$, next we will show $E \supseteq E^{F'}$. Let $e = ((s,\psi),(s',\psi')) \in E^{F'}$. We distinguish two cases:

- If $\psi = \langle a\rangle\psi_1$ or $\psi = [a]\psi_1$ then there exists an $a \in Act$ such that $(s,a,s') \in trans$. Using the FPG projection definition (4.2) we get $p \models \gamma'(s,a,s')$. Using the $FTS2FPG$ definition (5.5) we get $p \models \gamma(s,a,s')$. Using the projection definition (5.5) we get $(s,a,s') \in trans'$ and therefore $((s,\psi),(s',\psi')) \in E$.

- Otherwise the existence of the edge does not depend on the $trans$ parameter and therefore $((s,\psi),(s',\psi')) \in E$ if $(s,\psi) \in V$, since $V^F = V$ we have $(s,\psi) \in V$.

□

Having proven this we can put the relation between the different games and transition systems in the following diagram:

$$\begin{tikzcd}
\text{FTS} \arrow[r, "\varphi"] \arrow[d, Rightarrow, "\Pi"'] & \text{FPG} \arrow[d, Rightarrow, "\Pi"] \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}$$

Finally we will prove that solving an FTS answers the verification question.

Theorem 4.2. *Given:*

- *FTS* $M = (S, Act, trans, s_0, N, P, \gamma)$,
- *closed modal mu-calculus formula* φ,
- *product* $p \in P$ *and*

- *state $s \in S$*

it holds that $M_{|p}, s \models \varphi$ if and only if $(s, \varphi) \in W_0^p$ in FTS2FPG(M, φ).

Proof. The winning set W_α^p is equal to winning set W_α in $FTS2FPG(M, \varphi)_{|p}$, for any $\alpha \in \{0, 1\}$, using the FPG definition (4.1). Using theorem 4.1 we find that the game $FTS2FPG(M, \varphi)_{|p}$ is equal to the game $LTS2PG(M_{|p}, \varphi)$, obviously their winning sets are also equal. Using the well studied relation between parity games and LTS verification, stated in theorem 3.1, we know that $M_{|p}, s \models \varphi$ if and only if $(s, \varphi) \in W_0$ in game $LTS2PG(M_{|p}, \varphi)$. Winning set W_α^p is equal to W_α, therefore the theorem holds. □

Revisiting our prior example we can see the theorem in action by noting that $M_{|\emptyset} \models \varphi$ and $M_{|\{f\}} \not\models \varphi$. This is reflected in the winning sets: $(s_1, \mu X.[a]X \vee \langle b \rangle \top) \in W_0^\emptyset$ and $(s_1, \mu X.[a]X \vee \langle b \rangle \top) \notin W_0^{\{f\}}$.

5 Variability parity games

Next we will introduce *variability parity games* (VPGs). VPGs are very similar to FPGs, they do however abstract from the notion of features and instead uses configuration for a syntactically more pleasant representation that is not solely tailored for FTSs. Furthermore in VPGs deadlocks are removed and therefore only having to reason about infinite paths and no longer about finite paths. Finally we will show the relation between VPGs and FTS verification, which is almost identical to the relation between FPGs and FTS verification.

First we introduce VPGs:

Definition 5.1. *A variability parity game (VPG) is a tuple $(V, V_0, V_1, E, \rho, \mathfrak{C}, \theta)$, where:*

- *$V = V_0 \cup V_1$ and $V_0 \cap V_1 = \emptyset$,*
- *V_0 is the set of vertices owned by player 0,*
- *V_1 is the set of vertices owned by player 1,*
- *$E \subseteq V \times V$ is the edge relation; we assume that E is total, i.e. for all $v \in V$ there is some $w \in V$ such that $(v, w) \in E$,*
- *$\rho : V \to \mathbb{N}$ is a priority assignment,*
- *$\mathfrak{C}$ is a finite set of configurations,*
- *$\theta : E \to \mathcal{P}(\mathfrak{C}) \setminus \{0\}$ is the configuration mapping, satisfying for all $v \in V$, $\bigcup\{\theta(v, w) | (v, w) \in E\} = \mathfrak{C}$.*

A VPG is played similarly to a PG, however the game is played for a specific configuration $c \in \mathfrak{C}$. Player α can only move the token from $v \in V_\alpha$ to $w \in V$ if $(v, w) \in E$ and $c \in \theta(v, w)$. Furthermore VPGs don't have deadlocks, therefore every play results in an infinite path.

A game played for configuration $c \in \mathfrak{C}$ results in winning sets W_0^c and W_1^c, which are defined similar to the W_0 and W_1 winning sets for parity games.

Definition 5.2. *The projection from VPG $G = (V, V_0, V_1, E, \rho, \mathfrak{C}, \theta)$ to a configuration $c \in \mathfrak{C}$, noted $G_{|c}$, is the parity game (V, V_0, V_1, E', ρ) where $E' = \{e \in E | c \in \theta(e)\}$.*

Playing VPG G for a specific configuration $c \in \mathfrak{C}$ is the same as playing the PG $G_{|c}$. Any path that is valid in G for c is also valid in $G_{|c}$ and vice versa. Therefore the strategies are also interchangeable, furthermore the winning sets W_α for $G_{|c}$ and W_α^c for G are identical. Since parity games are positionally determined so are VPGs. Similarly, since finite parity games are decidable, so are finite VPGs.

5.1 Creating variability parity games

We will define a translation from an FPG to a VPG. This relation is shown in the following diagram:

$$\begin{tikzcd} \text{FPG} \arrow[r] & \text{VPG} \end{tikzcd}$$

To do so we have to translate products to configuration, which is trivial, furthermore we have to make the FPG deadlock free. This is done by creating two losing vertices l_0 and l_1 such that player α loses when the token is in vertex l_α. Any vertex that can not move for a configuration will get an edge that is admissible for that configuration towards one of the losing vertices.

Definition 5.3. *FPG2VPG(G^F) converts FPG $G^F = (V^F, V_0^F, V_1^F, E^F, \rho^F, N, P, \gamma)$ to VPG $G = (V, V_0, V_1, E, \rho, \mathfrak{C}, \theta)$.*

Let P be defined as $\{p_0, p_1, \ldots, p_m\}$, we define $\mathfrak{C} = \{c_0, c_1, \ldots, c_m\}$.

We create vertices l_0 and l_1 and define $V_0 = V_0^F \cup \{l_0\}$, $V_1 = V_1^F \cup \{l_1\}$ and $V = V_0 \cup V_1$.

We construct E by first making $E = E^F$ and adding edges (l_0, l_0) and (l_1, l_1) to E. Simultaneously we construct θ by first making $\theta(e) = \{c_i \in \mathfrak{C} | p_i \models \gamma(e)\}$ for every $e \in E^F$. Furthermore $\theta(l_0, l_0) = \theta(l_1, l_1) = \mathfrak{C}$.

Next, for every vertex $v \in V_\alpha$ with $\alpha = \{0, 1\}$, we have $C = \mathfrak{C} \backslash \bigcup \{\theta(v, w) | (v, w) \in E\}$. If $C \neq \emptyset$ then we add (v, l_α) to E and make $\theta(v, l_\alpha) = C$. Finally we have

$$\rho(v) = \begin{cases} 1 & \text{if } v = l_0 \\ 0 & \text{if } v = l_1 \\ \rho^F(v) & \text{otherwise} \end{cases}$$

Again considering our previous working example we can translate the FPG shown in figure 10 to the VPG shown in figure........

5.2 Variability parity games

5.3 Creating parity games

Originating from an FTS and a modal μ-calculus we can create an FPG (from which we can create a PG by projection) and from the FPG we can create a VPG. For a specific product we can project the FTS to an LTS, from which we can create a PG. The relation between the transition systems and games is displayed in the following diagram.

$$\begin{tikzcd}
\text{FTS} \arrow[r, "\varphi"] \arrow[d, Rightarrow, "\Pi"'] & \text{FPG} \arrow[r] \arrow[d, Rightarrow, "\Pi"] & \text{VPG} \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG} &
\end{tikzcd}$$

The projections are defined in the previous section. In this section we will define the horizontal arrows in the diagram. First we show how to create a PG from an LTS and a modal μ-calculus formula, this part is well studied and the approach is based on [4].

Definition 5.4. *[4] LTS2PG(M, φ) converts LTS $M = (S, Act, trans, s_0)$ and closed formula φ to a PG (V, V_0, V_1, E, ρ).*

A vertex in the parity game is represented by a pair (s, ψ) where $s \in S$ and ψ is a modal μ-calculus formula. We will create a vertex for every state with every subformula of φ except subformula's of the form X. Furthermore we create a vertex for every state with the unfolding of every fixpoint formula. Formally we define the set of vertices by first defining the set of subformula's:

$$F = \{\psi \mid \psi \text{ is a subformula of } \varphi\}, \text{ note that } \varphi \text{ is a subformula of } \varphi,$$

the set of subformula's that are of the form X:

$$F_X = \{\psi \in F \mid \psi = X\}$$

and the set of unfolded fixedpoint subformula's:

$$F_\sigma = \{\psi[X := \sigma X.\psi] \mid \sigma X.\psi \in F \text{ with } \sigma \in \{\mu, \nu\}\}$$

Having these sets we can define the set of vertices by:

$$V = S \times (F \backslash F_X \cup F_\sigma)$$

We create the parity game with the smallest set E such that:

- *$V = V_0 \cup V_1$,*
- *$V_0 \cap V_1 = \emptyset$ and*
- *for every $v = (s, \psi) \in V$ we have:*
  - *If $\psi = \top$ then $v \in V_1$.*
  - *If $\psi = \bot$ then $v \in V_0$.*
  - *If $\psi = \psi_1 \vee \psi_2$ then:*
    $v \in V_0$,
    $(v, (s, \psi_1)) \in E$ and
    $(v, (s, \psi_2)) \in E$.
  - *If $\psi = \psi_1 \wedge \psi_2$ then:*
    $v \in V_1$,
    $(v, (s, \psi_1)) \in E$ and
    $(v, (s, \psi_2)) \in E$.

- *If $\psi = \langle a\rangle\psi_1$ then $v \in V_0$ and for every $s \xrightarrow{a} s'$ we have $(v, (s', \psi_1)) \in E$.*
- *If $\psi = [a]\psi_1$ then $v \in V_1$ and for every $s \xrightarrow{a} s'$ we have $(v, (s', \psi_1)) \in E$.*
- *If $\psi = \mu X.\psi_1$ then $(v, (s, \psi_1[X := \mu X.\psi_1])) \in E$.*
- *If $\psi = \nu X.\psi_1$ then $(v, (s, \psi_1[X := \nu X.\psi_1])) \in E$.*

Note that since φ is closed and we use unfolding there will never be an edge $(v, (s, X)) \in E$.

Finally we have $\rho(s, \psi) = \begin{cases} 2\lfloor adepth(X)/2\rfloor & \text{if } \psi = \nu X.\psi' \\ 2\lfloor adepth(X)/2\rfloor + 1 & \text{if } \psi = \mu X.\psi' \\ 0 & \text{otherwise} \end{cases}$

Next we define the transformation from FTS to FPG.

Definition 5.5. *FTS2FPG(M, φ) converts FTS $M = (S, Act, trans, s_0, N, P, \gamma)$ and closed formula φ to FPG $(V, V_0, V_1, E, \rho, N, P, \gamma')$.*

We have (V, V_0, V_1, E, ρ) = LTS2PG($(S, Act, trans, s_0), \varphi$) and

$$\gamma'((s, \psi), (s', \psi')) = \begin{cases} \gamma(s, a, s') & \text{if } \psi = \langle a\rangle\psi' \text{ or } \psi = [a]\psi' \\ \top & \text{otherwise} \end{cases}$$

Finally we define how to create a VPG from an FPG. This transformation abstracts from the notion of products and uses configurations for a syntactically more pleasant representation. Furthermore in VPGs deadlocks are removed, this is done by creating two losing vertices l_0 and l_1 such that player α loses when the token is in vertex l_α. Any vertex that can not move for a configuration will get an edge that is admissible for that configuration towards one of the losing vertices.

Definition 5.6. *FPG2VPG(G^F) converts FPG $G^F = (V^F, V^F_0, V^F_1, E^F, \rho^F, N, P, \gamma)$ to VPG $G = (V, V_0, V_1, E, \rho, \mathfrak{C}, \theta)$.*

Let P be defined as $\{p_0, p_1, \ldots, p_m\}$, we define $\mathfrak{C} = \{c_0, c_1, \ldots, c_m\}$.

We create vertices l_0 and l_1 and define $V_0 = V^F_0 \cup \{l_0\}$, $V_1 = V^F_1 \cup \{l_1\}$ and $V = V_0 \cup V_1$.

We construct E by first making $E = E^F$ and adding edges (l_0, l_0) and (l_1, l_1) to E. Simultaneously we construct θ by first making $\theta(e) = \{c_i \in \mathfrak{C} | p_i \models \gamma(e)\}$ for every $e \in E^F$. Furthermore $\theta(l_0, l_0) = \theta(l_1, l_1) = \mathfrak{C}$.

Next, for every vertex $v \in V_\alpha$ with $\alpha = \{0, 1\}$, we have $C = \mathfrak{C} \backslash \bigcup\{\theta(v, w) | (v, w) \in E\}$. If $C \neq \emptyset$ then we add (v, l_α) to E and make $\theta(v, l_\alpha) = C$. Finally we have

$$\rho(v) = \begin{cases} 1 & \text{if } v = l_0 \\ 0 & \text{if } v = l_1 \\ \rho^F(v) & \text{otherwise} \end{cases}$$

5.4 Correctness

Theorem 5.1. *Given:*

- *FTS $M = (S, Act, trans, s_0, N, P, \gamma)$,*

- *closed modal mu-calculus formula φ,*
- *product $p \in P$ and*
- *state $s \in S$*

it holds that $M_{|p}, s \models \varphi$ if and only if $(s, \varphi) \in W_0^p$ in FTS2FPG(M, φ).

Proof. The winning set W_α^p is equal to winning set W_α in FTS2FPG$(M, \varphi)_{|p}$ using definition 4.1. Using theorem 4.1 we find that the game FTS2FPG$(M, \varphi)_{|p}$ is equal to the game LTS2PG$(M_{|p}, \varphi)$, obviously their winning sets are also equal. Using the modal verification proof from [4] we know that $M_{|p}, s \models \varphi$ if and only if $(s, \varphi) \in W_0$. Winning set W_α^p is equal to W_α, therefore the theorem holds. □

Theorem 5.2. *Given:*

- *FPG $G^F = (V^F, V_0^F, V_1^F, E^F, \rho^F, N, \{p_0, p_1, \ldots, p_m\}, \gamma)$,*
- *product p_i,*
- *player $\alpha \in \{0, 1\}$*

we have for winning sets $W_\alpha^{p_i}$ in G and $W_\alpha^{c_i}$ in FPG2VPG(G^F) that $W_\alpha^{p_i} \subseteq W_\alpha^{c_i}$.

Proof. Let $G = (V, V_0, V_1, E, \rho, \mathfrak{C}, \theta) =$ FPG2VPG(G^F). Consider finite play π that is valid in game G^F for product p_i. We have for every (π_i, π_{i+1}) in π that $(\pi_i, \pi_{i+1}) \in E^F$ and $p_i \models \gamma((\pi_i, \pi_{i+1}))$. From definition 5.6 it follows that $(\pi_i, \pi_{i+1}) \in E$ and $c_i \in \theta(\pi_i, \pi_{i+1})$. So we can conclude that path π is also valid in game G for configuration c_i. Since the play is finite the winner is determined by the last vertex v in π, player α wins such that $v \in V_{\overline{\alpha}}$. Furthermore we know, because the play is finite, that there exists no $(v, w) \in E^F$ with $p \models \gamma(v, w)$. From this we can conclude that $(v, l_{\overline{\alpha}}) \in E$ and $c_i \in \theta(v, l_{\overline{\alpha}})$. Vertex $l_{\overline{\alpha}}$ has one outgoing edge, namely to itself. So finite play π will in game G^F results in an infinite play $\pi(l_{\overline{\alpha}})^\omega$. Vertex $l_{\overline{\alpha}}$ has a priority with the same parity as player α, so player α wins the infinite play in G for configuration c_i.

Consider infinite play π that is valid in game G^F for product p_i. As shown above this play is also valid in game G for configuration c_i. Since the win conditions of both games are the same the play will result in the same winner.

Consider infinite play π that is valid in game G for configuration c_i. We distinguish two cases:

- If l_α doesn't occur in π then the path is also valid for game G^F with product p_i and has the same winner.
- If $\pi = \pi'(l_\alpha)^\omega$ then the winner is player $\overline{\alpha}$. The path π' is valid for game G^F with product p_i. Let vertex v be the last vertex of π'. Since $(v, l_\alpha) \in E$ and $c_i \in \theta(v, l_\alpha)$ we know that there is no $(v, w) \in E^F$ with $p_i \models \gamma(v, w)$ and that vertex v is owned by player α. So in game G^F player α can't move at vertex v and therefore loses the game (in which case the winner is also $\overline{\alpha}$.

We have shown that every path (finite or infinite) in game G^F with product p_i can be played in game G with configuration c_i and that they have the same winner. Furthermore every infinite path in game G with configuration c_i can be either played as an infinite path or the first part of the path can be played in G^F with product p_i and they have the same winner. From this we can conclude that the theorem holds. □

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