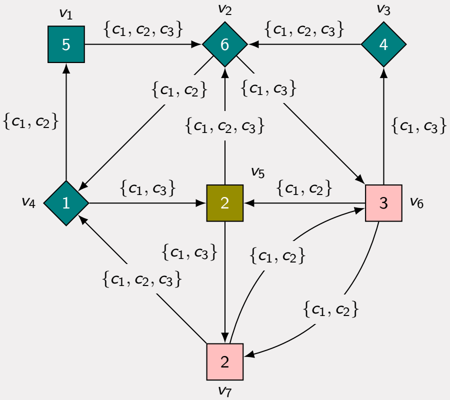
Verifying SPLs using parity games expressing variability



Sjef van Loo

6 November, 2019

Msc Thesis
Computer Science and Engineering

Supervised by T.A.C. Willemse

TU/e

Outline

- Verification & SPLs
- Problem statement
- Variability Parity Games & algorithms
- Experimental results
- Conclusions

Verification

- Creating correct software is difficult
- Even when testing is done rigorously errors can slip in

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- Mathematically *model the behaviour* of software (LTS)
- Mathematically *specify a requirement* (modal μ-calculus)

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- Even when testing is done rigorously errors can slip in
- Mathematically *model the behaviour* of software (LTS)
- Mathematically *specify a requirement* (modal μ-calculus)
- Check if the model satisfies the requirement

LTS M Modal μ-calculus formula φ

M satisfies φ?

Software product lines

- Software product lines are configurable systems
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- *e.g.* an elevator that can be configured to detect overload

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- *e.g.* an elevator that can be configured to detect overload
- FTSs can be used to model the entire system using *features*
- An FTS can be transformed to an LTS given a specific feature assignment

Problem statement

- Find all the products in an SPL that satisfy a requirement

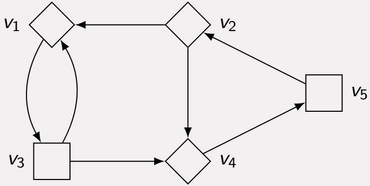
FTS M Modal μ-calculus formula φ

Which products in M satisfy φ?

- Do so more efficiently than verifying every product independently

Variability parity game

Parity game: (V, V_0, V_1, E, Ω)
Players 0 (even,$\Diamond$) and 1 (odd,$\Box$)



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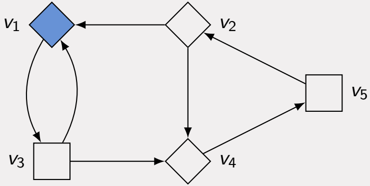

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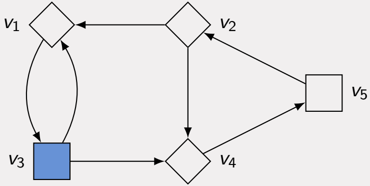

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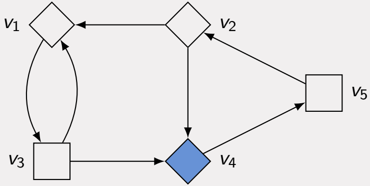

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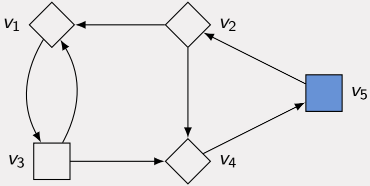

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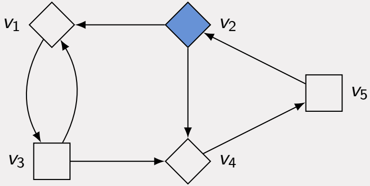

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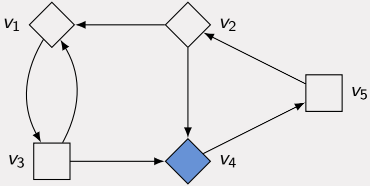

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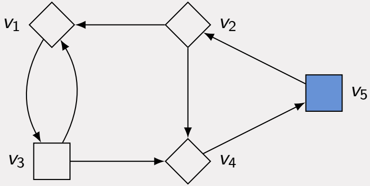

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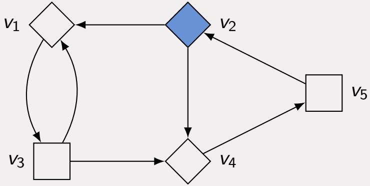

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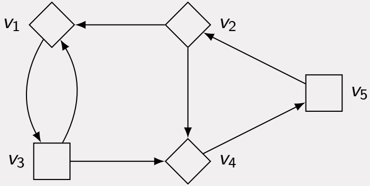

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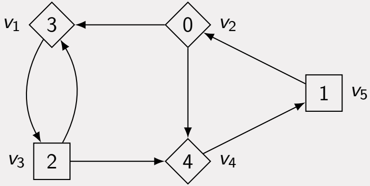

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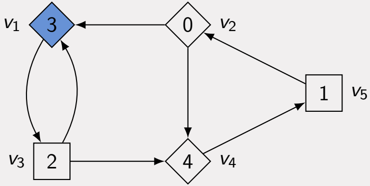

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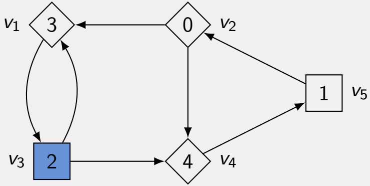

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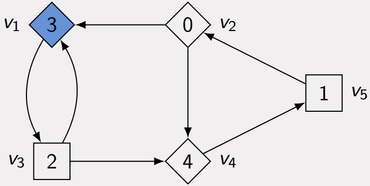

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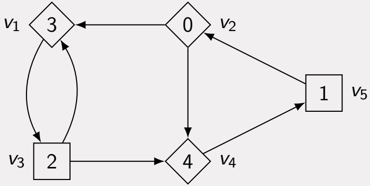

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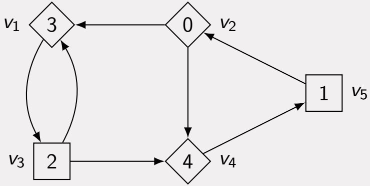

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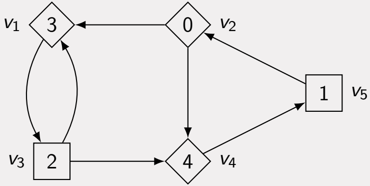

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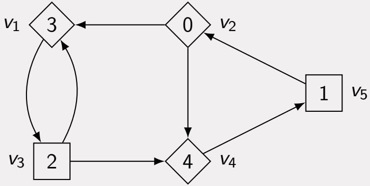

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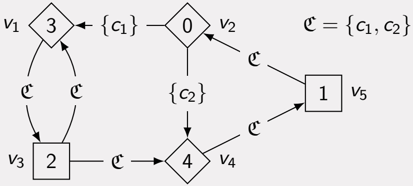

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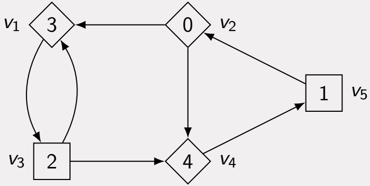

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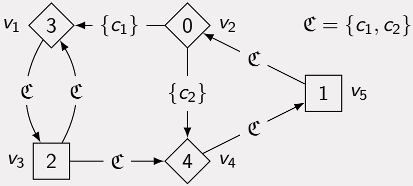

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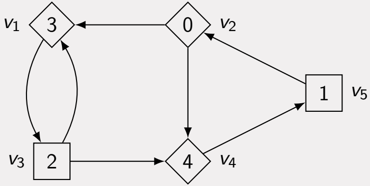

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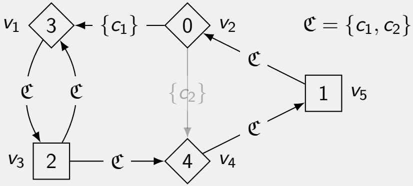

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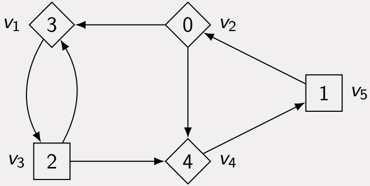

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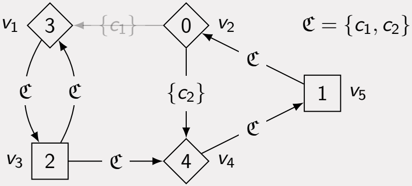

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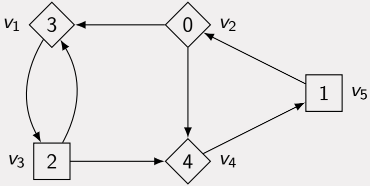

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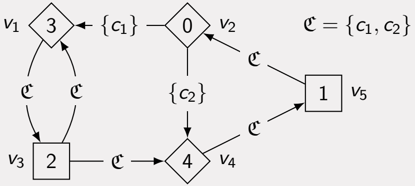

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- *Solving*: Partition the vertices in W_0^c, W_1^c, for every $c \in \mathfrak{C}$

Variability parity game

LTS M

Modal μ-calculus formula φ

M satisfies φ?

Variability parity game

$$\begin{tikzcd}
\text{LTS } M \arrow[dr, dash] & & \text{Modal } \mu\text{-calculus formula } \varphi \arrow[dl, dash] \\
& \text{PG} \arrow[d] & \\
& M \text{ satisfies } \varphi? &
\end{tikzcd}$$

Theorem

A parity game can be constructed from an LTS and a modal μ-calculus formula φ such that M satisfies φ iff special vertex v_0 is won by player 0 in the resulting parity game.

Variability parity game

LTS M Modal μ-calculus formula φ

PG

$\downarrow$

M satisfies φ?

FTS M Modal μ-calculus formula φ

$\downarrow$

Which products in M satisfy φ?

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$$\begin{array}{ccc} \text{LTS } M & & \text{Modal } \mu\text{-calculus formula } \varphi \\ & \searrow \quad \swarrow & \\ & \text{PG} & \\ & \downarrow & \\ & M \text{ satisfies } \varphi? & \end{array}$$

$$\begin{array}{ccc} \text{FTS } M & & \text{Modal } \mu\text{-calculus formula } \varphi \\ & \searrow \quad \swarrow & \\ & \text{VPG} & \\ & \downarrow & \\ & \text{Which products in } M \text{ satisfy } \varphi? & \end{array}$$

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$$
\begin{tikzcd}
\text{FTS} \arrow[d, dashed, "\Pi_p"'] & \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}
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$$\begin{tikzcd}
\mathrm{FTS} \arrow[r, "\varphi"] \arrow[d, dashed, "\Pi_p"'] & \mathrm{VPG} \arrow[d, dashed, "\Pi_p"] \\
\mathrm{LTS} \arrow[r, "\varphi"] & \mathrm{PG}
\end{tikzcd}$$

VPG algorithms

- Solve VPGs *independently*; solve every parity game expressed by the VPG
- Solve VPGs *collectively*; solve the VPG as a whole

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- Solve VPGs *independently*; solve every parity game expressed by the VPG
- Solve VPGs *collectively*; solve the VPG as a whole
- Introduced two collective algorithms
  - Recursive algorithm
  - Incremental pre-solve algorithm
- Evaluate performance of independent approach vs collective approach

VPG algorithms - Recursive algorithm

- Existing parity game algorithm
- Spends most time performing the *attractor* calculation
- Introduce an efficient attractor calculation for VPGs

VPG algorithms - Recursive algorithm

Attractor calculation: Given player α and vertex set U: Find all vertices from where player α can force the play to U.

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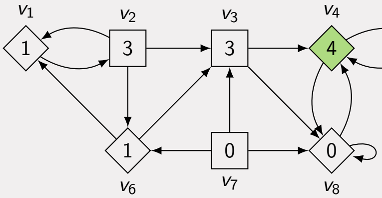

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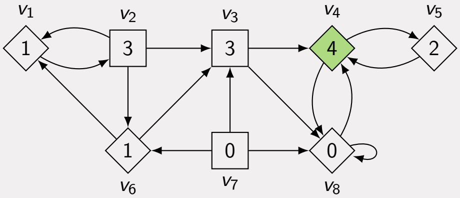

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A				$\checkmark$				

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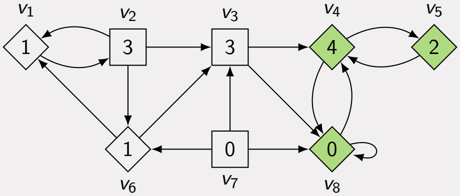

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A				✓	✓			✓

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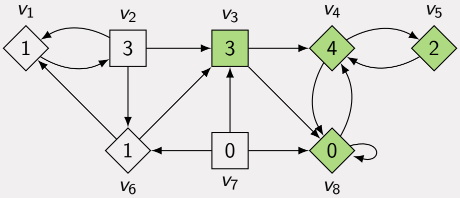

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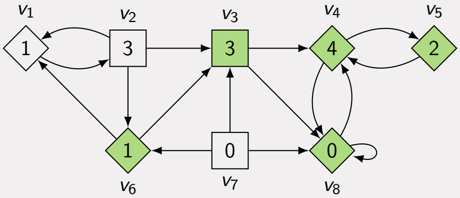

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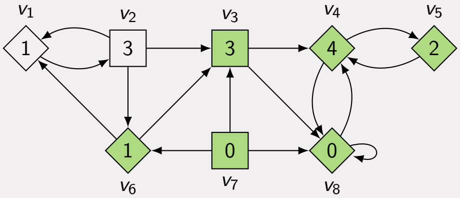

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A			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

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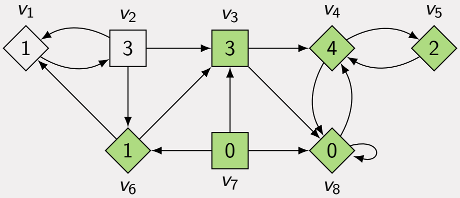

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A			✓	✓	✓	✓	✓	✓

$$A_0 = U, A = \cup_{i \geq 0} A_i$$

$$A_{i+1} = A_i \cup \{v \in V_\alpha \mid \exists_w : (v, w) \in E \land w \in A_i\}$$
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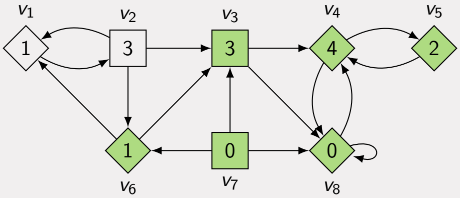

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				✓				
A_1				✓	✓			✓
A_2			✓	✓	✓			✓
A_3			✓	✓	✓	✓		✓
A_4			✓	✓	✓	✓	✓	✓
A_5			✓	✓	✓	✓	✓	✓

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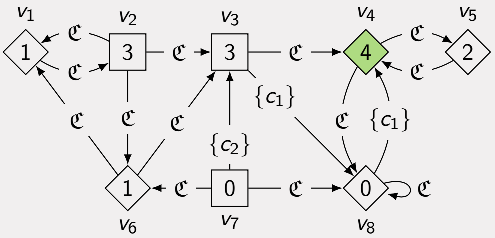

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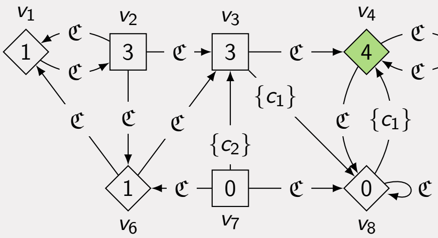

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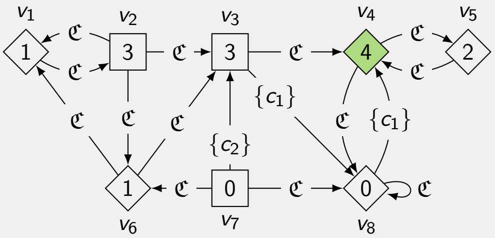

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A_0				$\mathfrak{C}$				

$$A_{i+1}(v) = A_i(v) \cup \begin{cases} \bigcup_{(v,w)\in E} \left(\theta(v,w) \cap A_i(w)\right) & \text{if } v \in V_\alpha \\ \bigcap_{(v,w)\in E} \left((\mathfrak{C} \backslash \theta(v,w)) \cup A_i(w)\right) & \text{if } v \notin V_\alpha \end{cases}^{*}$$

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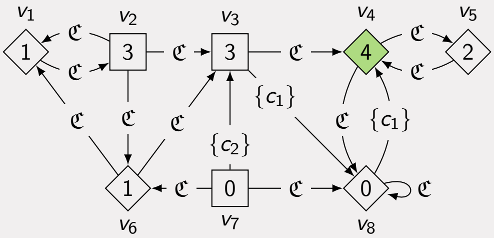

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If $v \in V_\alpha$: Find for every outgoing edge which configurations play to A_i, add them all to $A_i(v)$

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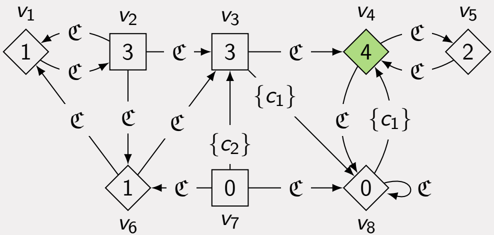

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	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				$\mathfrak{C}$				

If $v \notin V_\alpha$: Find for every outgoing edge through which configurations player $1-\alpha$ cannot escape A_i, add the intersection to $A_i(v)$

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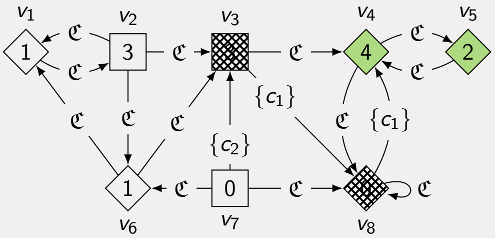

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	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$A_1(v_5) = A_0(v_5) \cup (\theta(v_5, v_4) \cap A_0(v_4))$$
$$= \emptyset \cup (\mathfrak{C} \cap \mathfrak{C}) = \mathfrak{C}$$

$$A_{i+1}(v) = A_i(v) \cup \begin{cases} \bigcup_{(v,w)\in E} (\theta(v,w) \cap A_i(w)) & \text{if } v \in V_\alpha \\ \bigcap_{(v,w)\in E} ((\mathfrak{C} \backslash \theta(v,w)) \cup A_i(w)) & \text{if } v \notin V_\alpha \end{cases}^{*}$$

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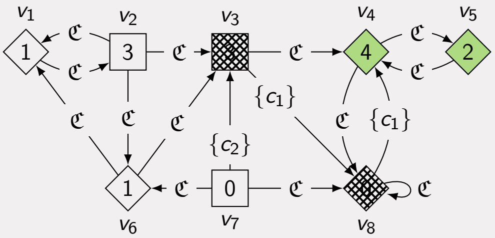

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	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$\begin{aligned} A_1(v_8) &= A_0(v_8) \cup (\theta(v_8, v_8) \cap A_0(v_8)) \\ &\cup (\theta(v_8, v_4) \cap A_0(v_4)) \\ &= \emptyset \cup (\mathfrak{C} \cap \emptyset) \cup (\{c_1\} \cap \mathfrak{C}) = \{c_1\} \end{aligned}$$

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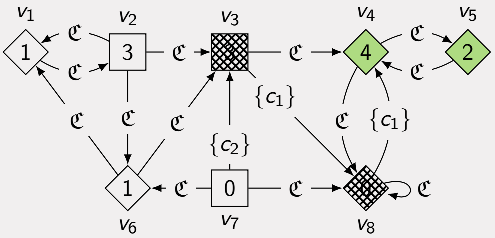

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A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$
\begin{aligned}
A_1(v_3) = A_0(v_3) \cup (\\
&((\mathfrak{C} \backslash \theta(v_3, v_4)) \cup A_0(v_4)) \cap \\
&((\mathfrak{C} \backslash \theta(v_3, v_8)) \cup A_0(v_8))) \\
&= \emptyset \cup ((\emptyset \cup \mathfrak{C}) \cap (\{c_2\} \cup \emptyset)) = \{c_2\}
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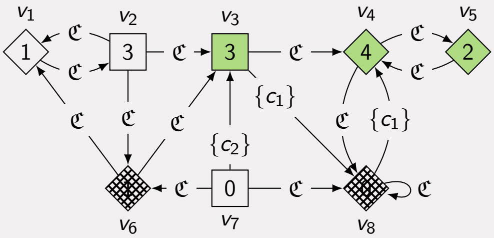

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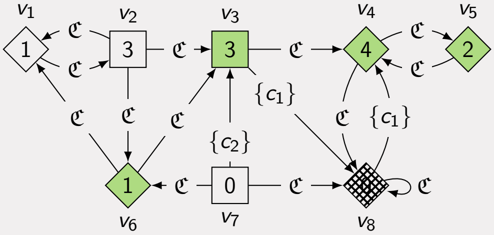

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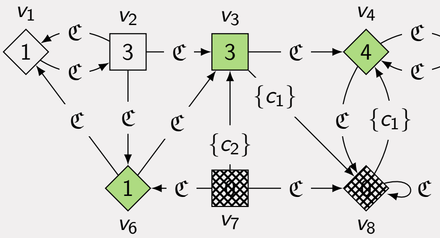

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- Solve VPG by solving every parity game independently using the original recursive algorithm
- Solve VPG collectively using the modified attractor calculation
- Modified attractor calculation relies on set operations betweens sets of configurations
- Can we perform these operations efficiently?

VPG algorithms - Recursive algorithm - Symbolically representing sets

- Represent sets simply as a collection of all its elements (*explicit*)
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Boolean formula:

$$F(x_2, x_1, x_0) = (\neg x_2 \wedge x_1 \wedge \neg x_0) \vee (x_2 \wedge (x_1 \vee \neg x_0))$$

$\mathbf{x_2x_1x_0}$	$\mathbf{F(x_2, x_1, x_0)}$
000	0
001	0
010	1
011	0
100	1
101	0
110	1
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- Boolean operators $\vee, \wedge$ coincide with set operators $\cup, \cap$

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- FTSs use features
- FTSs use boolean formulas to enable/disable parts of the system
- VPGs are constructed such that every edge either:
  - admits all configurations, or
  - is guarded by a set that coincides with a formula from the FTS

VPG algorithms - Recursive algorithm - Time complexities

n: # vertices, e: # edges, d: # distinct priorities, c # configurations

Set operations	
Explicit sets	
Symbolic sets	

Algorithms	
Original recursive algorithm	
Independent approach	
Collective algorithm (explicit)	
Collective algorithm (symbolic)	

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Set operations	
Explicit sets	$O(c)$
Symbolic sets	$O(c^2)$

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Experimental results - SPL games

Minepump SPL

- Keep a mine shaft free from water
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- Elevator travelling between five floor
- 5 features, including overload detection and parking

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Elevator SPL

- Elevator travelling between five floor
- 5 features, including overload detection and parking
- 64 valid feature assignments
- 34k states and 200k
- 7 requirements
- 7 VPGs ranging from 440k and 1.85m vertices and 2 to 3 distinct priorities

Experimental results - SPL games

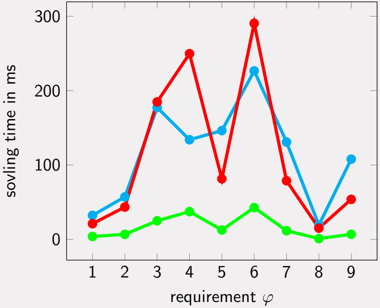

Figure: Running times, in ms, on the minepump games.

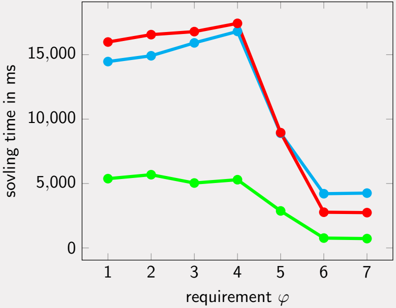

Figure: Running times, in ms, on the elevator games.

Independent recursive algorithm
Collective recursive algorithm with a symbolic representation of configurations
Collective recursive algorithm with an explicit representation of configurations

Discussion

Collective recursive algorithm:

- Symbolic variant increases performance 3-18 times (SPL games)
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Incremental pre-solve algorithm:

- Same time complexity as independent approach ($O(c * e * n^d)$)
- Slightly increases performance of SPL games and random games
- Not consistent or very significant

Locally solving (V)PGs

- Parity games: Terminate when special vertex v_0 is solved
- VPGs: Terminate when special vertex v_0 is solved for all configurations

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- VPGs: Terminate when special vertex v_0 is solved for all configurations
- Introduced local variants of existing algorithms and of the novel VPG algorithms
- Relative performance: How much quicker are the collective algorithms?
- Is the relative local performance greater than the relative global performance?

Locally solving (V)PGs

- Parity games: Terminate when special vertex v_0 is solved
- VPGs: Terminate when special vertex v_0 is solved for all configurations
- Introduced local variants of existing algorithms and of the novel VPG algorithms
- Relative performance: How much quicker are the collective algorithms?
- Is the relative local performance greater than the relative global performance?

Results:

- Recursive algorithms: no significant increase in relative performance
- Incremental pre-solve algorithm: increases relative performance for random games

Conclusions

- VPGs can be used to verify SPLs
- Collective approaches outperform independent approaches
- Locally solving VPGs can increase performance (more so than locally solving parity games does)

Extra - Incremental pre-solve

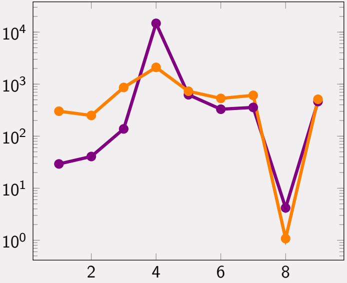

Figure: Running times, in ms, on the minepump games.

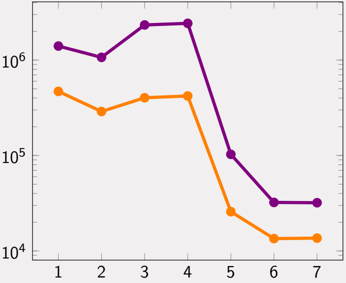

Figure: Running times, in ms, on the elevator games.