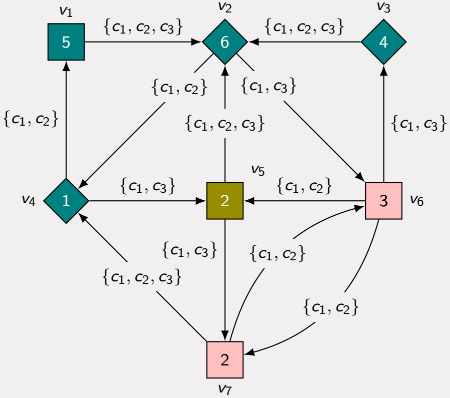
Verifying SPLs using parity games expressing variability



Sjef van Loo

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Msc Thesis
Computer Science and Engineering

Supervised by T.A.C. Willemse

Outline

- Verification & SPLs
- Problem statement
- Variability Parity Games & algorithms
- Experimental results
- Conclusions

Verification

- Creating correct software is difficult
- Even when testing is done rigorously errors can slip in
- Mathematically *model the behaviour* of software (LTS)
- Mathematically *specify a requirement* (modal μ-calculus)
- Check if the model satisfies the requirement

LTS M — Modal μ-calculus formula φ

M satisfies φ?

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Software product lines

- Software product lines are configurable systems
- Many variants of the same system, i.e. *software products*
- *e.g.* an elevator that can be configured to detect overload
- FTSs can be used to model the entire system using *features*
- An FTS can be transformed to an LTS given a specific feature assignment

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Problem statement

- Find all the products in an SPL that satisfy a requirement

FTS M Modal μ-calculus formula φ

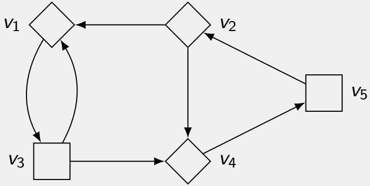
$\downarrow$

Which products in M satisfy φ?

- Do so more efficiently than verifying every product independently

Variability parity game

Parity game: (V, V_0, V_1, E, Ω)
Players 0 (even, $\Diamond$) and 1 (odd, $\Box$)



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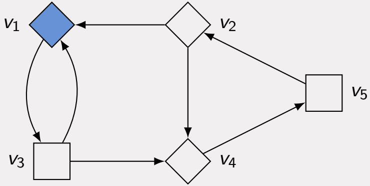

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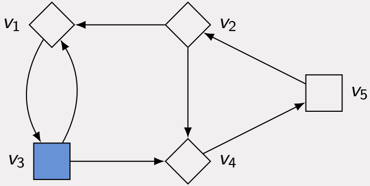

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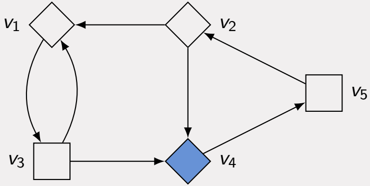

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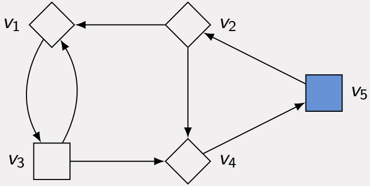

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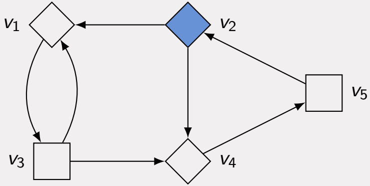

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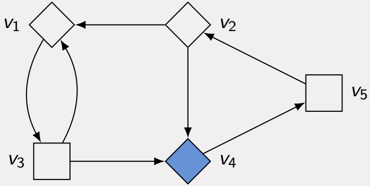

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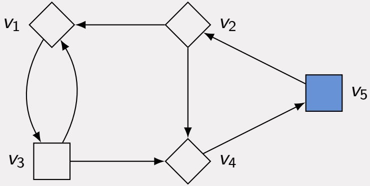

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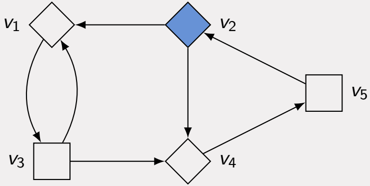

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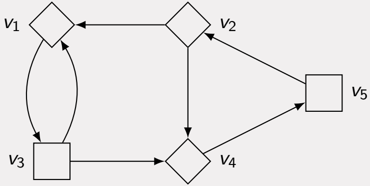

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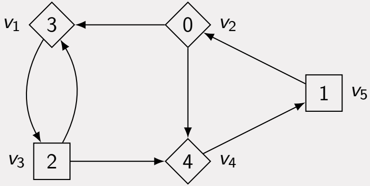

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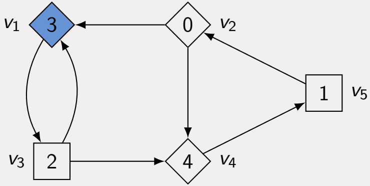

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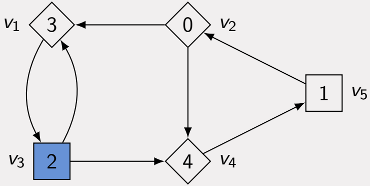

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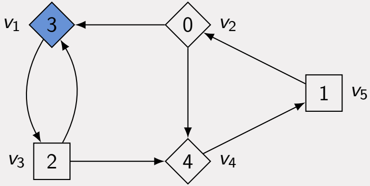

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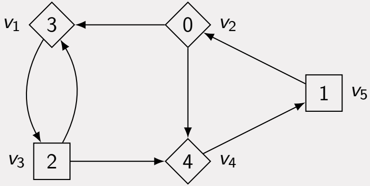

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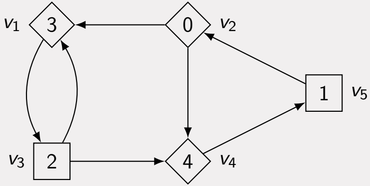

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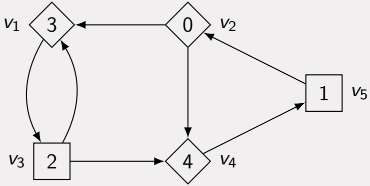

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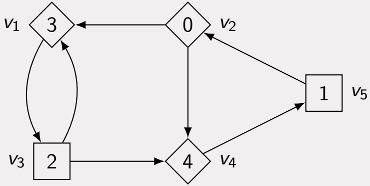

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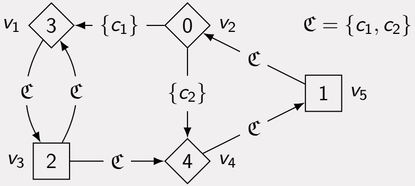
Variability parity game:
$(V, V_0, V_1, E, \Omega, \mathfrak{C}, \theta)$



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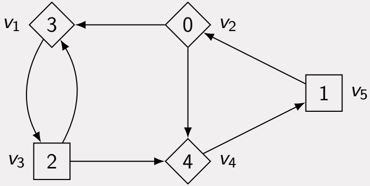
Variability parity game

Parity game: (V, V_0, V_1, E, Ω)
Players 0 (even,$\Diamond$) and 1 (odd,$\Box$)



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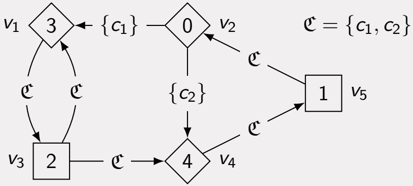

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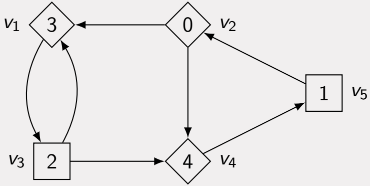

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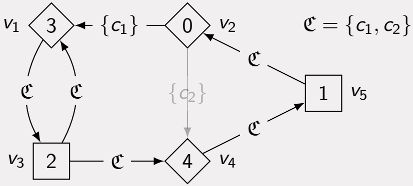

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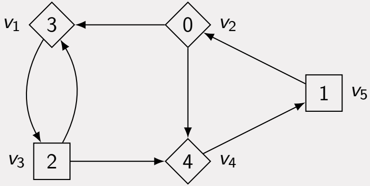

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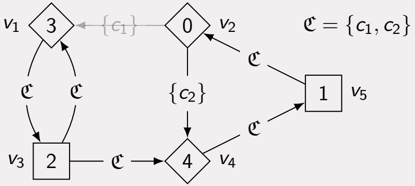

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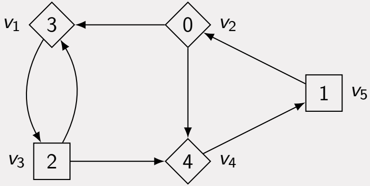

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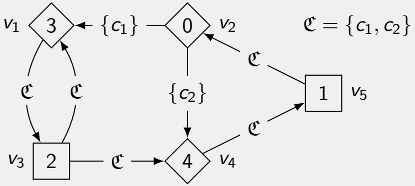

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Variability parity game

LTS M

Modal μ-calculus formula φ

M satisfies φ?

Theorem

A parity game can be constructed from an LTS and a modal μ-calculus formula φ such that M satisfies φ iff special vertex v_0 is won by player 0 in the resulting parity game.

Theorem

A VPG can be constructed from an FTS and a modal μ-calculus formula φ such that M satisfies φ for product p iff special vertex v_0 is won by player 0 in the resulting VPG played for p.

Variability parity game

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FTS M Modal μ-calculus formula φ

Which products in M satisfy φ?

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$$\begin{tikzcd}
\text{FTS} \arrow[d, dashed, "\Pi_p"'] & \\
\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}$$

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\text{LTS} \arrow[r, "\varphi"] & \text{PG}
\end{tikzcd}
$$

VPG algorithms

- Solve VPGs *independently*; solve every parity game expressed by the VPG
- Solve VPGs *collectively*; solve the VPG as a whole
- Introduced two collective algorithms
  - Recursive algorithm
  - Incremental pre-solve algorithm
- Evaluate performance of independent approach vs collective approach

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VPG algorithms - Recursive algorithm

Attractor calculation: Find all vertices from where player α can force the play to a vertex in U.

Example: $\alpha = 0, U = \{v_4\}$

$$A_0 = U, A = \cup_{i \geq 0} A_i$$

$$A_{i+1} = A_i \cup \{v \in V_\alpha \mid \exists_w \; : (v, w) \in E \wedge w \in A_i\}$$
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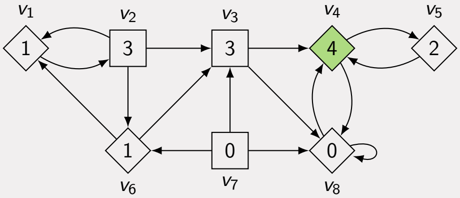

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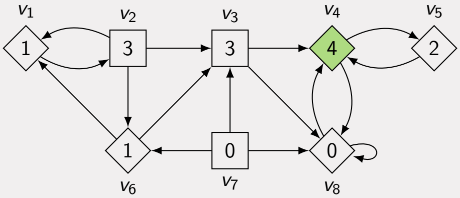

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A				$\checkmark$				

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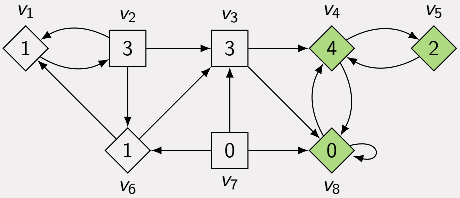

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A				✓	✓			✓

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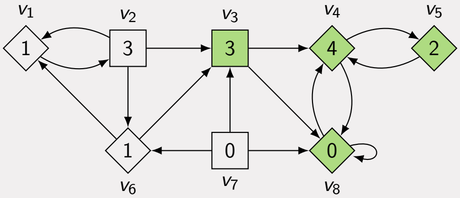

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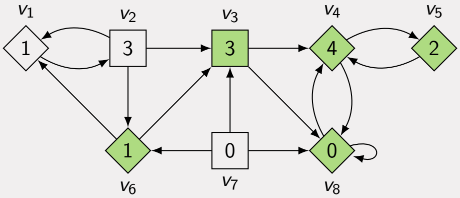

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A			✓	✓	✓	✓		✓

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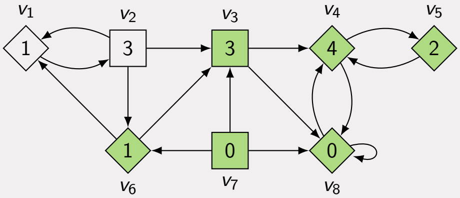

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

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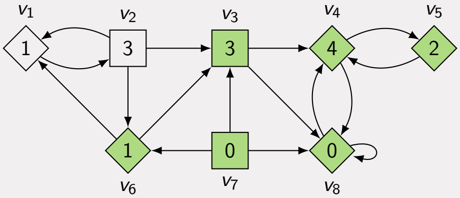

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A			✓	✓	✓	✓	✓	✓

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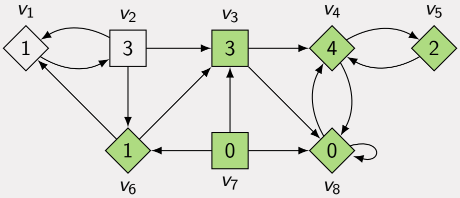

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				✓				
A_1				✓	✓			✓
A_2			✓	✓	✓			✓
A_3			✓	✓	✓	✓		✓
A_4			✓	✓	✓	✓	✓	✓
A_5			✓	✓	✓	✓	✓	✓

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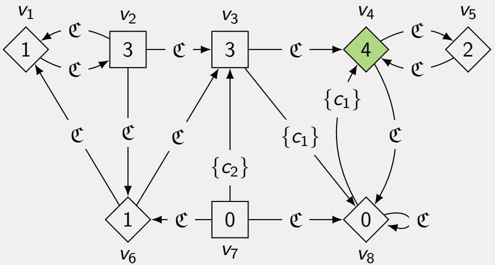
VPG algorithms - Recursive algorithm

Find a set of configurations for every vertex.

$\mathfrak{C} = \{c_1, c_2\}$



$A : V \to 2^{\mathfrak{C}}$

$$A_{i+1}(v) = A_i(v) \cup \begin{cases} \bigcup_{(v,w)\in E} (\theta(v,w) \cap A_i(w)) & \text{if } v \in V_\alpha \\ \bigcap_{(v,w)\in E} ((\mathfrak{C}\backslash\theta(v,w)) \cup A_i(w)) & \text{if } v \notin V_\alpha \end{cases}^{*}$$

*: Simplified version of the attractor definition presented in the report

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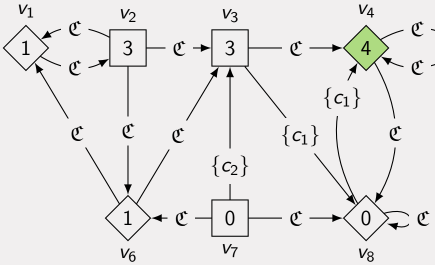

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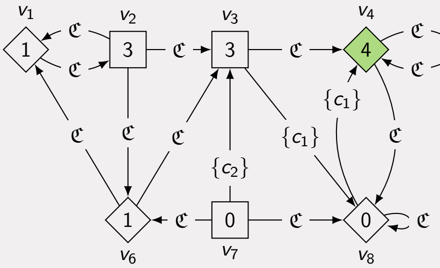

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A_0				$\mathfrak{C}$				

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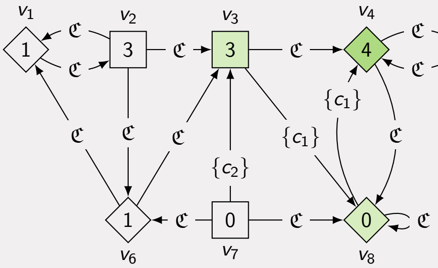

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A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$A_1(v_5) = A_0(v_5) \cup (\theta(v_5, v_4) \cap A_0(v_4))$$
$$= \emptyset \cup (\mathfrak{C} \cap \mathfrak{C}) = \mathfrak{C}$$

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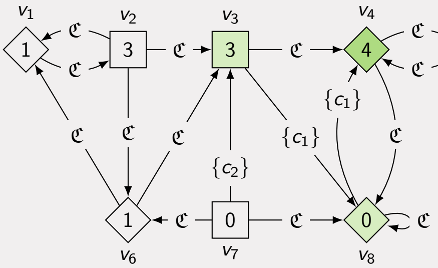

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	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$\begin{aligned} A_1(v_8) &= A_0(v_8) \cup (\theta(v_8, v_8) \cap A_0(v_8)) \\ &\cup (\theta(v_8, v_4) \cap A_0(v_4)) \\ &= \emptyset \cup (\mathfrak{C} \cap \emptyset) \cup (\{c_1\} \cap \mathfrak{C}) = \{c_1\} \end{aligned}$$

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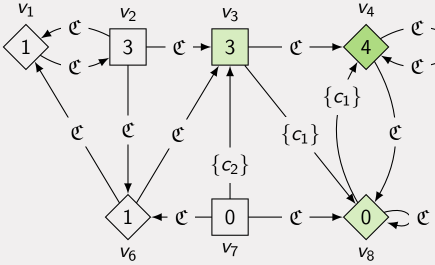

$A : V \to 2^{\mathfrak{C}}$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
A_0				$\mathfrak{C}$				
A_1			$\{c_2\}$	$\mathfrak{C}$	$\mathfrak{C}$			$\{c_1\}$

$$\begin{aligned}
A_1(v_3) = A_0(v_3) \cup (&\\
&((\mathfrak{C} \backslash \theta(v_3, v_4)) \cup A_0(v_4)) \cap\\
&((\mathfrak{C} \backslash \theta(v_3, v_8)) \cup A_0(v_8)))\\
&= \emptyset \cup ((\emptyset \cup \mathfrak{C}) \cap (\{c_2\} \cup \emptyset)) = \{c_2\}
\end{aligned}$$

$$A_{i+1}(v) = A_i(v) \cup \begin{cases} \bigcup_{(v,w) \in E} (\theta(v,w) \cap A_i(w)) & \text{if } v \in V_\alpha \\ \bigcap_{(v,w) \in E} ((\mathfrak{C} \backslash \theta(v,w)) \cup A_i(w)) & \text{if } v \notin V_\alpha \end{cases}{}^{*}$$

*: Simplified version of the attractor definition presented in the report

VPG algorithms - Recursive algorithm

Find a set of configurations for every vertex.

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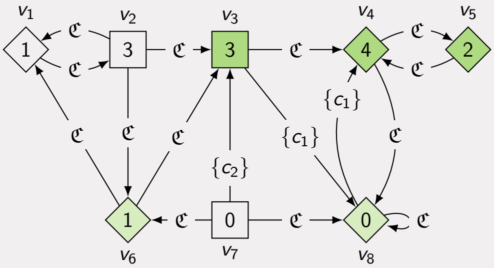

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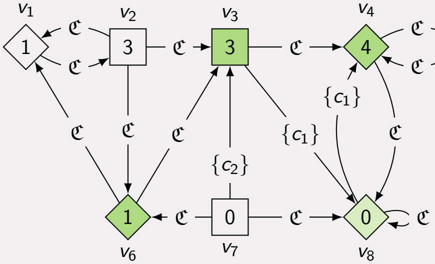

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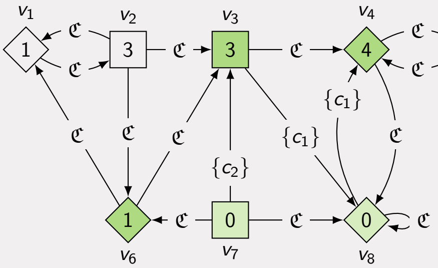

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VPG algorithms - Recursive algorithm - Symbolically representing sets

- Represent sets simply as a collection of all its elements (*explicit*)
- Alternatively, represent sets as boolean formulas (*symbolic*)
- Example: $S = \{s_0, \ldots, s_7\}$, $T = \{s_2, s_4, s_6, s_7\}$
- Boolean variables: x_2, x_1, x_0

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$$F(x_2, x_1, x_0) = (\neg x_2 \wedge x_1 \wedge \neg x_0) \vee (x_2 \wedge (x_1 \vee \neg x_0))$$

$\mathbf{x_2 x_1 x_0}$	$\mathbf{F(x_2, x_1, x_0)}$
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- Boolean operators $\vee, \wedge$ coincide with set operators $\cup, \cap$

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VPG algorithms - Recursive algorithm - Symbolically representing sets

- Boolean formulas can be expressed as BDDs
- Simple formulas $\implies$ small BDDs $\implies$ quick set operators
- FTSs use features
- FTSs use boolean formulas to enable/disable parts of the system
- VPGs are constructed such that every edge either:
  - admits all configurations, or
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VPG algorithms - Recursive algorithm - Set operations

- Explicitly: $O(c)$
- Symbolically: $O(c^2)$
- In practice, if the BDDs are small then a symbolic representation outperforms an explicit representation

VPG algorithms - Recursive algorithm - Time complexities

n: # vertices, e: # edges, d: # distinct priorities, c # configurations

- Original recursive algorithm: $O(e * n^d)$
- Independently solving a VPG, *i.e.* solve c parity games: $O(c * e * n^d)$
- Collective recursive algorithm:
  - with explicit configuration sets: $O(n * c^2 * e * n^d)$
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Experimental results - SPL games

Minepump SPL

- Keep a mine shaft free from water
- 10 features that change the sensor/actor setup
- 128 valid feature assignments
- 600 states and 1400 transitions
- 9 requirements
- 9 VPGs ranging from 3000 to 9200 vertices and 2 to 4 distinct priorities

Elevator SPL

- Elevator travelling between five floor
- 5 features, including overload detection and parking
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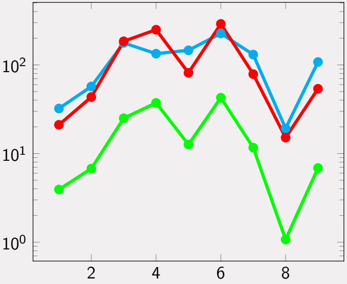
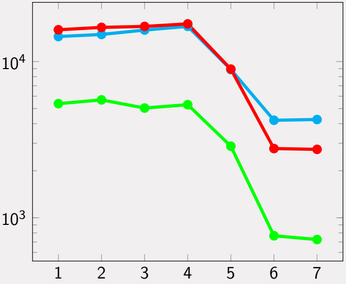


Figure: Running times, in ms, on the minepump games.



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Independent recursive algorithm
Collective recursive algorithm with a symbolic representation of configurations
Collective recursive algorithm with an explicit representation of configurations

Discussion

Collective recursive algorithm:

- Symbolic variant increases performance 3-18 times (SPL games)
- For certain random games the symbolic performance drops rapidly, explicit performance is steady

Incremental pre-solve algorithm:

- Slightly increases performance of SPL games and random games
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Locally solving (V)PGs

- Parity games: Terminate when special vertex v_0 is solved
- VPGs: Terminate when special vertex v_0 is solved for all configurations
- Introduced local variants of existing algorithms and of the novel VPG algorithms
- Relative performance: How much quicker are the collective algorithms?
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Results:

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Conclusions

- VPGs can be used to verify SPLs
- Collective approaches outperform independent approaches
- Locally solving VPGs can increase performance (more so than locally solving parity games does)