1 Methodology

In this part, we use the publically available DeepCore data sample [1] which is an updated version of what was used by the IceCube collaboration in an ν_μ disappearance analysis [2].

The detector systematics include ice absorption and scattering, and overall, lateral, and head-on optical efficiencies of the DOMs. They are applied as correction factors using the best-fit points from the DeepCore 2019 ν_τ appearance analysis [3].

The data events include 14901 track-live events and 26001 cascade-like events, both divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins.

The neutrino flux at the detector is calculated by propagating atmospheric neutrino flux [4] through the Earth by solving the Schrödinger equation for varying density. The Earth density profile is taken from the PREM [5]. The oscillation parameters are from the best-fit in the global analysis in [6]: $\theta_{12} = 33.44°$, $\theta_{13} = 8.57°$, $\Delta m_{21}^2 = 7.42\,\mathrm{eV}^2$, and we marginalize over Δm_{31}^2 and θ_{23}. The oscillation probability $P_{\alpha\beta}$ then acts as a weight, yielding the propagated flux at detector level for flavor β as

$$\Phi_\beta = \sum_\alpha \frac{\mathrm{d}^2\phi_\alpha}{\mathrm{d}E^t \mathrm{d}\cos\theta_z^t} P_{\alpha\beta}\,, \tag{1}$$

where we sum over the initial lepton flavors $\alpha \in e, \mu, \bar{e}, \bar{\mu}$. The event rate for each bin reads

$$N_{ij} = T \int_{(\cos\theta_z^r)_i}^{(\cos\theta_z^r)_{i+1}} \mathrm{d}\cos\theta_z^r \int_{E_j^r}^{E_{j+1}^r} \mathrm{d}E^r \int_0^\pi R(\theta^r, \theta^t)\mathrm{d}\cos\theta^t \int_0^\infty R(E^r, E^t)\mathrm{d}E^t \\ \times \left[\sum_\beta \Phi_\beta A_\beta^{\mathrm{eff}}\right]\,, \tag{2}$$

where T is the live time of the detector, and A_β^{eff} its effective area for flavor β. $R(x^r, x^t)$ is a Gaussian resolution function, which is responsible for the smearing between the reconstructed and true parameters x^r and x^t, respectively. It takes the form

$$R(x^r, x^t) = \frac{1}{\sqrt{2\pi}\sigma_{x^r}} \exp\left[\frac{(x^r - \mu(x^t))^2}{2\sigma_{x^r}^2}\right]\,. \tag{3}$$

Assuming no bias in the reconstruction, the mean of the Gaussian can be taken as $\mu(x^t) = x^t$. As seen in Fig. 1, the distribution of simulated events is skewed. Instead, we assume a log-normal distribution between E^{true} and E^{reco}, and train a Gaussian Process Regressor on the dataset, from which we can extract a predicted mean and standard deviation for each E^{reco}. We then sample values from this distribution to yield the points of E^{true} at which to integrate over.

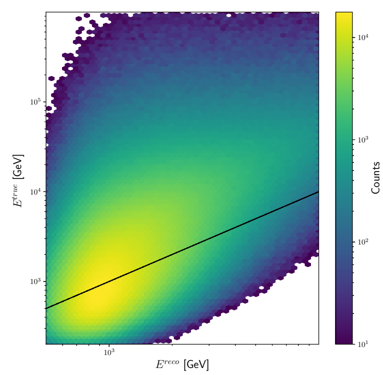

Figure 1: Relationship between the true and reconstructed muon energy in the IceCube MC sample

In Icecube, the zenith angle resolution for track-like events is less than 2°, making $\cos(\theta_z^{true})$ coincide with $\cos(\theta_z^{reco})$ for our study [7].

Given a Monte Carlo simulation with weights $w_{k\beta}$, we can construct the event count as

$$N_{ijk} = C_{ijk} \sum_{\beta} w_{ijk,\beta} \Phi_{\beta} \,, \tag{4}$$

where $C_{k\beta}$ is the correction factor from the detector systematic uncertainty and Φ_β is defined as Eq. 1. We have now substituted the effect of the Gaussian smearing by treating the reconstructed and true quantities as a migration matrix.

We define our χ^2 as

$$\chi^2(\hat{\theta}, \alpha, \beta) = \sum_{ijk} \frac{\left(N^{\rm th} + N^{\mu_{atm}} - N^{\rm data}\right)^2_{ijk}}{\left(\sigma^{\rm data}\right)^2_{ijk} + \left(\sigma^{\rm uncor}_{\nu+\mu_{\rm atm}}\right)^2_{ijk}} + \frac{(1-\alpha)^2}{\sigma_\alpha^2} + \frac{\beta^2}{\sigma_\beta^2} \tag{5}$$

where we minimize over the model parameters $\hat{\theta} \in \{\Delta m^2_{31}, \theta_{23}, \varepsilon', \varepsilon_{\mu\tau}\}$, and the penalty terms α and β. $N^{\rm th}_{ijk}$ is the expected number of events from theory, and $N^{\rm data}_{ijk}$ is the observed number of events in that bin. The event count takes the form

$$N^{\rm th}_{ijk} = \alpha \left[1 + \beta \cos(\theta_z^{reco})_i\right] N_{ijk}(\hat{\theta}) \,, \tag{6}$$

with $N_{ijk}(\hat{\theta})$ from Eq. 4. $N^{\mu_{atm}}_{ijk}$ is the muon background, which is left to float freely in the analysis. We use $\sigma_\alpha = 0.25$ as the atmospheric flux normalization error, and $\sigma_\beta = 0.04$ as the zenith angle slope error [4]. The observed event number has an associated Poissonian uncertainty $\sigma_i^{\mathrm{data}} = \sqrt{N_i^{\mathrm{data}}}$. The term $\sigma_{\nu+\mu_{\mathrm{atm}}}^{\mathrm{uncor}}$ accounts for both uncertanties in the finite MC statistics and in the data-driven muon background estimate [1].

References

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