## 0.1 How NSIs alter the matter potential

Up until now, we have only considered neutrino interactions with electrons, protons, and neutrons. Now we extend this to include we can extend this to include the up and down quarks which are present in the Earth as the fundamental components of neutrons and protons. Consider interactions beyond the Standard Model through the following Lagrangians,

$$\begin{aligned}\mathcal{L}_{\mathrm{CC}} &= -2\sqrt{2}G_F\epsilon_{\alpha\beta}^{ff'X}\left(\bar{\nu}_\alpha\gamma^\mu P_L\ell_\beta\right)\left(\bar{f}'\gamma_\mu P_X f\right)\\ \mathcal{L}_{\mathrm{NC}} &= -2\sqrt{2}G_F\epsilon_{\alpha\beta}^{fX}\left(\bar{\nu}_\alpha\gamma^\mu P_L\nu_\beta\right)\left(\bar{f}\gamma_\mu P_X f\right)\,,\end{aligned}$$

where CC denotes the charged current interaction with the matter field $f \neq f' \in \{u, d\}$, and NC denotes the neutral current interaction with $f \in \{e, u, d\}$.

We have no independent sensitivity for the neither chirality nor flavor type of $\epsilon^X$, so we sum over these and study the effective matter NSI parameter $\epsilon_{\alpha\beta}$:

$$\epsilon_{\alpha\beta} = \sum_{X\in\{L,R\}}\sum_{f\in\{e,u,d\}}\frac{N_f}{N_e}\epsilon_{\alpha\beta}^{fX}\,. \tag{0.1}$$

Our matter study will be wholly confined to the interior of the Earth, where we assume electrical neutrality and equal distribution of neutrons and protons, we get $N_u/N_e \simeq U_d/N_e \simeq 3$. Also we assume the components $\epsilon_{\alpha\beta}$ to be real. Thus,

$$\epsilon_{\alpha\beta} = \sum_X[\epsilon_{\alpha\beta}^{eX} + 3(\epsilon_{\alpha\beta}^{uX} + \epsilon_{\alpha\beta}^{dX})] \tag{0.2}$$

Now, $\epsilon_{\alpha\beta}$ enters the Hamiltonian as entries of a potential-like matrix. In Eq. 0.3, $A_{CC}\mathrm{diag}(1, 0, 0)$ is our familiar matter potential from the Standard Model. There is also our new term, $A_{CC}\epsilon$, which contains the components $\epsilon_{\alpha\beta}$:

$$\begin{aligned}H &= \frac{1}{2E}\left[UM^2U^\dagger + A_{CC}\mathrm{diag}(1,0,0) + A_{CC}\epsilon\right]\\ &= \frac{1}{2E}\left[UM^2U^\dagger + A_{CC}\begin{pmatrix}1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{\mu e} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{\tau e} & \epsilon_{\tau\mu} & \epsilon_{\tau\tau}\end{pmatrix}\right]\,.\end{aligned} \tag{0.3}$$

In the limit $\epsilon_{\alpha\beta} \to 0$, we recover the standard interaction Hamiltonian from Eq. **??**. We can draw several conclusions from this form of the Hamiltonian. Any nonzero off-diagonal element $\epsilon_{\alpha\beta}, \alpha \neq \beta$ will make the NSI violate lepton flavor, just as the off-diagonal elements of $U$ does in the SM. Moreover, since the SM potential has the same order in $A_{CC}$ as the NSIs, any $\epsilon_{\alpha\beta} \sim 1$ will make the new matter effect be the same order as the SM effect. Regarding the source of these new parameters, $\epsilon_{\alpha\beta}$ have the scale

$$\epsilon_{\alpha\beta} \propto \frac{m_W^2}{m_\epsilon^2} \sim \frac{10^{-2}}{m_\epsilon^2} \tag{0.4}$$

in TeV, so the new interactions generated at a mass scale of $m_\epsilon = 1\,\mathrm{TeV}$ will produce NSI parameters in the order of $10^{-2}$, two magnitudes below the SM matter effect. Thus, if we assume the new interactions to arise from a higher-energy theory at or above TeV scale, we then predict that the NSI parameters contribute at most $10^{-2}$ to the SM matter effect, decreasing quadratically.

We have two more modifications to the matrix $\epsilon$. First, all terms of the Hamiltonian must of course be Hermitian, thus

$$\begin{pmatrix}\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{\mu e} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{\tau e} & \epsilon_{\tau\mu} & \epsilon_{\tau\tau}\end{pmatrix} = \begin{pmatrix}\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix}\,. \tag{0.5}$$

Now we have reduced the possible number of NSI parameters from 9 down to 6. Moreover, it is common to subtract $\epsilon_{\mu\mu}$ from the diagonal, which we are free to do since any multiple added or subtracted from diagonal does not affect the eigenvectors of the Hamiltonian and thus not the probabilities. So just as we subtracted away $V_{NC}$ back in Eq. **??**, we can subtract $\epsilon_{\mu\mu}$ and rotate it away. Our NSI potential matrix now takes the form

$$\begin{aligned}\begin{pmatrix}\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix} - \epsilon_{\mu\mu}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} &= \begin{pmatrix}\epsilon_{ee}-\epsilon_{\mu\mu} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{e\mu} & 0 & \epsilon_{\mu\tau}\\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau}-\epsilon_{\mu\mu}\end{pmatrix}\\ &= \begin{pmatrix}\epsilon_{ee}-\epsilon_{\mu\mu} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{e\mu} & 0 & \epsilon_{\mu\tau}\\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon'\end{pmatrix}\,,\end{aligned} \tag{0.6}$$

where $\epsilon' = \epsilon_{\tau\tau} - \epsilon_{\mu\mu}$.

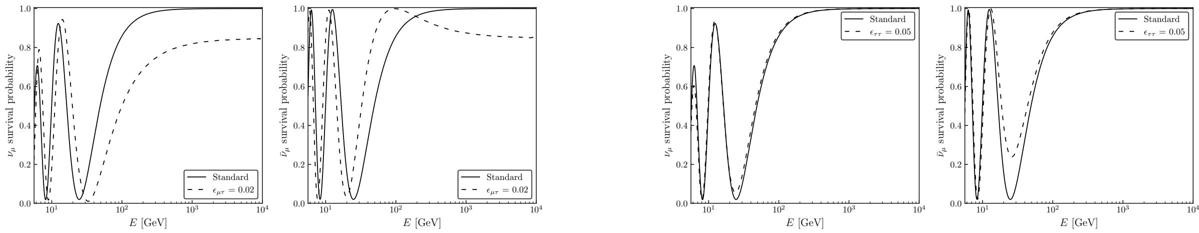


*Fig. 0.1:* *Left panel:* Muon neutrino and antineutrino survival probabilities for $\cos(\theta_z^{true}) = -1$ when $\epsilon_{\mu\tau} = 0.02$. All other NSI parameters are fixed to zero. *Right panel:* Muon neutrino and antineutrino survival probabilities for $\cos(\theta_z^{true}) = -1$ when $\epsilon_{\tau\tau} = 0.05$. All other NSI parameters are fixed to zero.

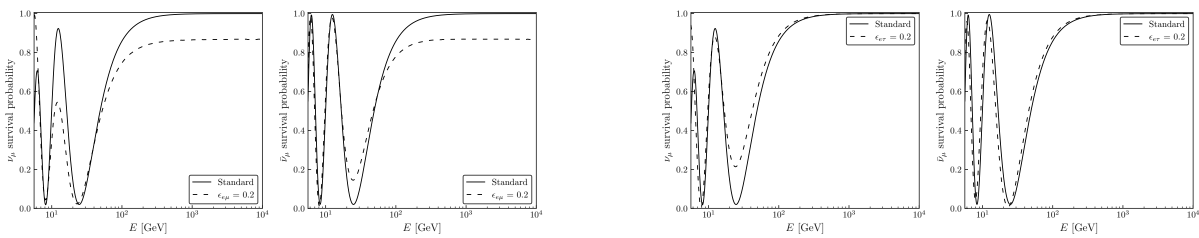


*Fig. 0.2:* Muon neutrino and antineutrino survival probabilities for $\cos(\theta_z^{true}) = -1$*Left panel:* $\epsilon_{e\mu} = 0.2$. All other NSI parameters are fixed to zero. *Right panel:* $\epsilon_{e\tau} = 0.2$. All other NSI parameters are fixed to zero.

## *0.2 Non-standard interactions in IceCube*

In our analysis of IceCube, we are constrained to muon track events. Thus, we are not able to test any theory which does not modify $P_{\alpha\mu}$. Moreover, the IceCube data is avaibable in the range 500 GeV to 10 TeV range, where any rapid oscillations have settled down.

Since all standard matter potentials are diagonal, the elements $\epsilon_{\alpha\beta}, \alpha = \beta$ will directly adjust the matter potential felt by flavor $\alpha$. The off-diagonal terms have a more interesting theoretical implication as the open up matter interactions across flavors. Remember, in the Standard model, we are restriced to weak interactions that conserve lepton number. However, a off-diagonal NSI parameter allows flavor transitions during matter interactions. Moreover, $\epsilon_{\alpha\beta}$ modifies the $\nu_\alpha \to \nu_\beta$ transition independently of the mixing matrix. Thus, we can in theory boost a flavor transition which is supressed by the mixing matrix. Moreover, since we have 6 NSI parameters, and the PMNS matrix is parametrized with only 3 parameters, we have more degrees of freedom when adjusting the probabilities using the NSI matrix. Each term in the PMNS matrix consists of at least two of the three mixing angles, making them much more dependent on each other.

As discussed in Eq. **??**, the atmospheric $\nu_\mu \to \nu_\tau$ transition will be the most abundant, making $\epsilon_{\mu\tau}$, $\epsilon_{\mu\mu}$, $\epsilon_{\tau\tau}$ the most suitable NSI parameters to constrain from muon events. As we will see, $\epsilon_{e\mu}$ is also a candidate, alebit a weaker one.

In Fig. 0.2, we see how the introduction of $\epsilon_{\mu\tau} = 0.02$ alters the $\nu_\mu$ and $\bar{\nu}_\mu$ survival probabilities for neutrinos that traverse the entire Earth diameter (i.e. $\cos(\theta_z^{true}) = -1$). $\epsilon_{\mu\tau}$ does not dramatically change neither amplitude nor frequency of the probabilities. Instead, it seems to strech or compress the oscillations. Since the only difference between the way neutrinos and antineutrinos interact with matter is the sign of the potential, the probability for $\nu_\mu$ with positive $\epsilon_{\alpha\beta}$ is identical to the probability for $\bar{\nu}_\mu$ with negative $\epsilon_{\alpha\beta}$. Thus, the dashed line in the right panel not only shows the survival probability for $\bar{\nu}_\mu$ with $\epsilon_{\mu\tau} = 0.02$, but also the survival probability for $\nu_\mu$ with $\epsilon_{\mu\tau} = -0.02$. Hence, we note that $\epsilon_{\mu\tau} > 0$ stretches (compresses) $P_{\mu\mu}$ for neutrinos (antineutrinos), while $\epsilon_{\mu\tau} < 0$ compresses (stretches) $P_{\mu\mu}$ for neutrinos (antineutrinos).

The value of $\epsilon_{\tau\tau}$ does not affect neither $P_{\mu\mu}$ nor $P_{\bar{\mu}\bar{\mu}}$, in the IceCube region above 500 GeV. Hence, we will not be able to say anything about $\epsilon_{\tau\tau}$ in our IceCube study. Comparing the probabilities in Fig. 0.2 with $\epsilon_{\tau\tau} = 0.05$ with the ones for $\epsilon_{\mu\tau} = 0.02$ in Fig. 0.2, we see that even though we let $\epsilon_{\tau\tau}$ take 2.5 times the value of $\epsilon_{\mu\tau}$, its effect on $P_{\mu\mu}$ is smaller. The weakening of the $P_{\bar{\mu}\bar{\mu}}$ resonance will be visible in DeepCore, but we should expect a less stringent constraint due to the weakness of the effect compared to $\epsilon_{\mu\tau}$.

Thus, we will use IceCube to constrain $\epsilon_{\mu\tau}$ only.

Moving on to $\epsilon_{e\mu}$ and Fig. 0.2, we see that both probabilities has shifted downwards for $E^{true} > 500$ GeV. In 0.2, we see that the muon channel remains largely unaffected of the value of $\epsilon_{e\tau}$ as we expected. The expection of this lies in the DeepCore region of rapid oscillations, where mixing is more violent.

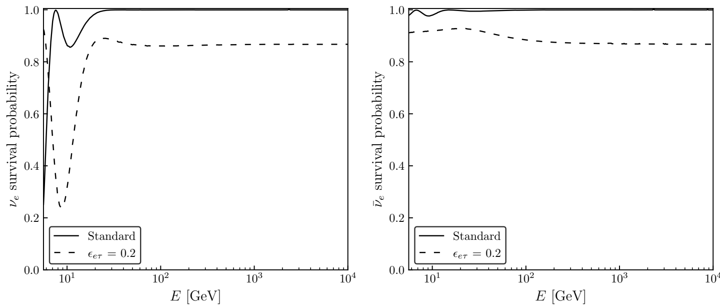


*Fig. 0.3:* Electron neutrino and antineutrino survival probabilities for $\cos(\theta_z^{true}) = -1$ when $\epsilon_{e\tau} = 0.2$. All other NSI parameters are fixed to zero.

| | | Visibility | |
|---|---|---|---|
| Parameter | Range | IC | DC |
| $\epsilon_{\tau\tau}$ | $\pm 0.05$ | × | ✓ |
| $\epsilon_{\mu\tau}$ | $\pm 0.02$ | ✓ | ✓ |
| $\epsilon_{e\mu}$ | $\pm 0.2$ | ✓ | ✓ |
| $\epsilon_{e\tau}$ | $\pm 0.2$ | × | ✓ |

*Tab. 0.1:* Table of NSI parameters and their ranges considered in this study, along with visibility off their signal in IceCube (IC) and DeepCore/PINGU (DC).

## 0.3 Non-standard interactions in DeepCore and PINGU

Now we repeat our proability analysis but for the DeepCore/PINGU region of 5.6 GeV to 56 GeV. As we previously saw, we have rapid oscillations, which means that "indirect" modifications (i.e. $\epsilon_{e\tau}$ will affect the $P_{\mu\mu}$ channel) will be more apparent, since all flavors are involved to a greater degree compared with the more stable region above 500 GeV, where many oscillations have settled down.

Another feature of our DeepCore study includes the fact that we now have access to cascade events, in which $\nu_e$ and $\nu_\tau$ are more abundant. Thus, we are no longer constrained to the $\mu$ channel alone, but we can now find interesting features in the other channels too. However, we remember that the $\nu_\mu$ flux is still the most abundant.

Fig. 0.2 shows that $\epsilon_{\mu\tau}$ affects both $P_{\mu\mu}$ and $P_{\bar{\mu}\bar{\mu}}$ over the whole energy range. Since IceCube also sees this, we hope to be able to boost the constraining of $\epsilon_{\mu\tau}$ by combining the two experiments.

Regarding $\epsilon_{\tau\tau}$ in Fig. 0.2, the signal mainly shows in the $P_{\bar{\mu}\bar{\mu}}$ channel as a weaker resonance dip in the 20 GeV region. Thus, DeepCore/PINGU alone will be used to constrain this parameter.

$\epsilon_{e\mu}$ in Fig. 0.2 shows weak resonance dampening for both $\nu_\mu$ and $\bar{\nu}_\mu$.

For $\epsilon_{e\tau}$, we see a similar resonance dampening in $P_{\mu\mu}$ as we did with $\epsilon_{\tau\tau}$ for $P_{\bar{\mu}\bar{\mu}}$. Hence, we should be able to see the $\epsilon_{e\tau}$ effect in DeepCore/PINGU, but remember that we now have the option to look at the other flavor channels.

Fig. 0.3 shows the electron neutrino and antineutrino survival probabilities, and here we see a clear difference when turning on $\epsilon_{e\tau}$.

We summarize our findings in Table 0.3