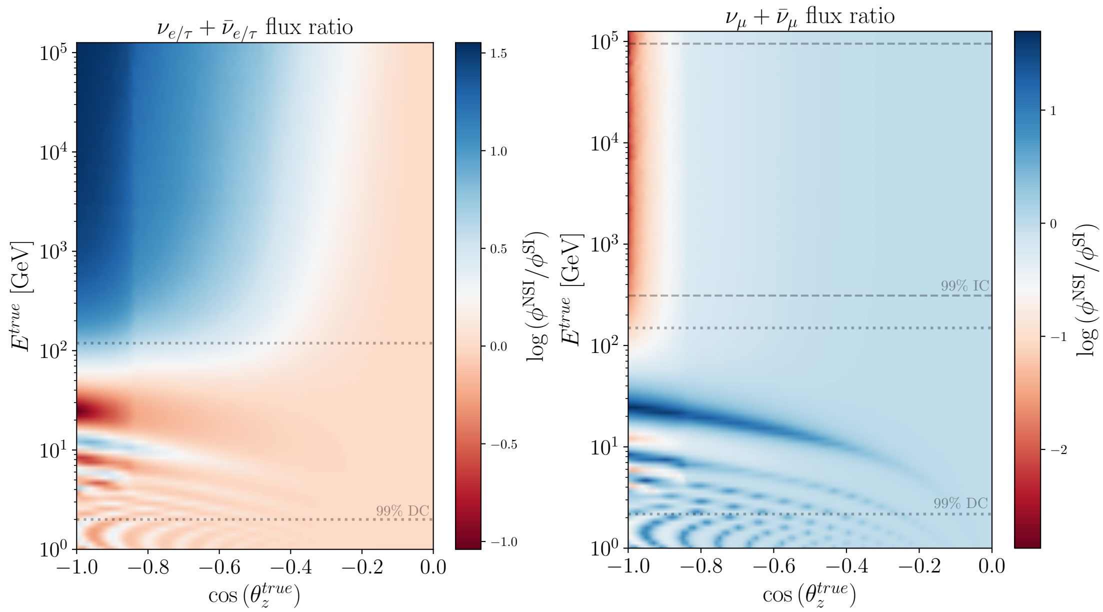

FIG. 1. Ratio of propagated NSI to SI atmospheric fluxes at detector level. Dotted (dashed) lines show the region in which 99% of the DeepCore (IceCube) MC events are contained. *Left panel:* Propagated fluxes of ν_e and ν_τ neutrinos and anti-neutrinos. *Right panel:* Propagated fluxes of ν_μ neutrinos and anti-neutrinos

I. METHODOLOGY

The neutrino flux at the detector is calculated by propagating the atmospheric neutrino flux [**?**] through the Earth by solving the Schrödinger equation for varying density. The Earth density profile is taken from the PREM [**?**]. The baseline for a given trajectory is determined using an average neutrino production height of 15 km and an Earth radius of 6371 km.

The oscillation probability $P_{\alpha\beta}$ acts as a weight, yielding the propagated flux at detector level for flavor β as

$$\phi_\beta^{\mathrm{det}} = \sum_\alpha P_{\alpha\beta}\phi_\alpha^{\mathrm{atm}}\,, \tag{1}$$

where we sum over the initial lepton flavors $\alpha \in \{e, \mu, \bar{e}, \bar{\mu}\}$. To illustrate the impact of NSI on both probability and flux level, we plot the oscillograms in Fig. **??**. In the left panel, we have marked the region in which 99% of the DeepCore cascade events originating from ν_e and ν_τ fluxes are contained. In the right panel, we show the two regions in which 99% of the IceCube and DeepCore track events originating from ν_μ fluxes are contained.

A. IceCube

The event rate for each bin reads

$$N_{ij} = T\int_{(\cos\theta_z^r)_i}^{(\cos\theta_z^r)_{i+1}} \mathrm{d}\cos\theta_z^r \int_{E_j^r}^{E_{j+1}^r} \mathrm{d}E^r \int_0^\pi R(\theta^r,\theta^t)\mathrm{d}\cos\theta^t \int_0^\infty R(E^r,E^t)\mathrm{d}E^t \times \left[\sum_\beta \phi_\beta^{\mathrm{det}} A_\beta^{\mathrm{eff}}\right]\,, \tag{2}$$

where T is the live time of the detector, and A_β^{eff} its effective area for flavor β. We use the effective area of the 86 string configuration made public by the IceCube collaboration [**?**]. $R(x^r, x^t)$ is a resolution function, which is responsible for the smearing between the reconstructed and true parameters x^r and x^t, respectively. We assume a log-normal distribution, giving it the form

$$R(x^r, x^t) = \frac{1}{\sqrt{2\pi}\sigma_{x^r} x^r} \exp\left[-\frac{(\log x^r - \mu(x^t))^2}{2\sigma_{x^r}^2}\right]\,. \tag{3}$$

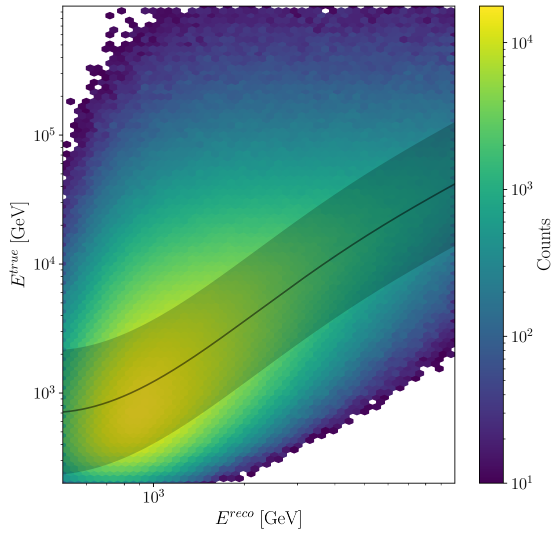

FIG. 2. Relationship between the true and reconstructed muon energy in the IceCube MC sample [?]
. Shaded area shows the 99.9th percentile limits predicted by the regressor trained on this set.

Assuming no bias in the reconstruction, the mean of the Gaussian can be taken as $\mu(x^t) = x^t$. As seen in Fig. **??**, the distribution of simulated events is skewed in energy. To model this bias between E^{true} and E^{reco}, we train a Gaussian process regressor on the dataset [?], from which we can extract a predicted mean and standard deviation for a given E^{reco}. We then take the $E^{\rm true}$ points of the 99th percentile of each distribution to obtain the limits of $E^{\rm true}$ at which to integrate over. We have no angular resolution function since the angle resolution in Icecube for track-like events is less than 2°, making $\cos(\theta_z^{true})$ coincide with $\cos(\theta_z^{reco})$ for our study [?]. The data is from the IC86 sterile analysis [?].

B. DeepCore

In this part, we use the publically available DeepCore data sample [?] which is an updated version of what was used by the IceCube collaboration in a ν_μ disapprearance analysis [?].

The detector systematics include ice absorption and scattering, and overall, lateral, and head-on optical efficiencies of the DOMs. They are applied as correction factors using the best-fit points from the DeepCore 2019 ν_τ appearance analysis [?].

The data include 14901 track-like events and 26001 cascade-like events, both divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins. Each event has a Monte Carlo weight $w_{ijk,\beta}$, from which we can construct the event count as

$$N_{ijk} = C_{ijk} \sum_\beta w_{ijk,\beta} \phi_\beta^{\rm det}\,, \tag{4}$$

where $C_{k\beta}$ is the correction factor from the detector systematic uncertainty and $\phi_\beta^{\rm det}$ is defined as Eq. **??**. We have now substituted the effect of the Gaussian smearing by treating the reconstructed and true quantities as a migration matrix.

The oscillation parameters used on our DeepCore simulations are from the best-fit in the global analysis in [?]: $\theta_{12} = 33.44^\circ$, $\theta_{13} = 8.57^\circ$, $\Delta m^2_{21} = 7.42\,{\rm eV}^2$, and we marginalize over Δm^2_{31} and θ_{23}.

We plot the event pull $(N_{NSI} - N_{SI})/\sqrt{N_{SI}}$ where $N_{(N)SI}$ are the numbers of expected events assuming (non-)standard interactions in Fig. **??**. This gives the normalized difference in the number of expected events at the detector, and illustrates the expected sensitivity of DeepCore for the NSI parameters.

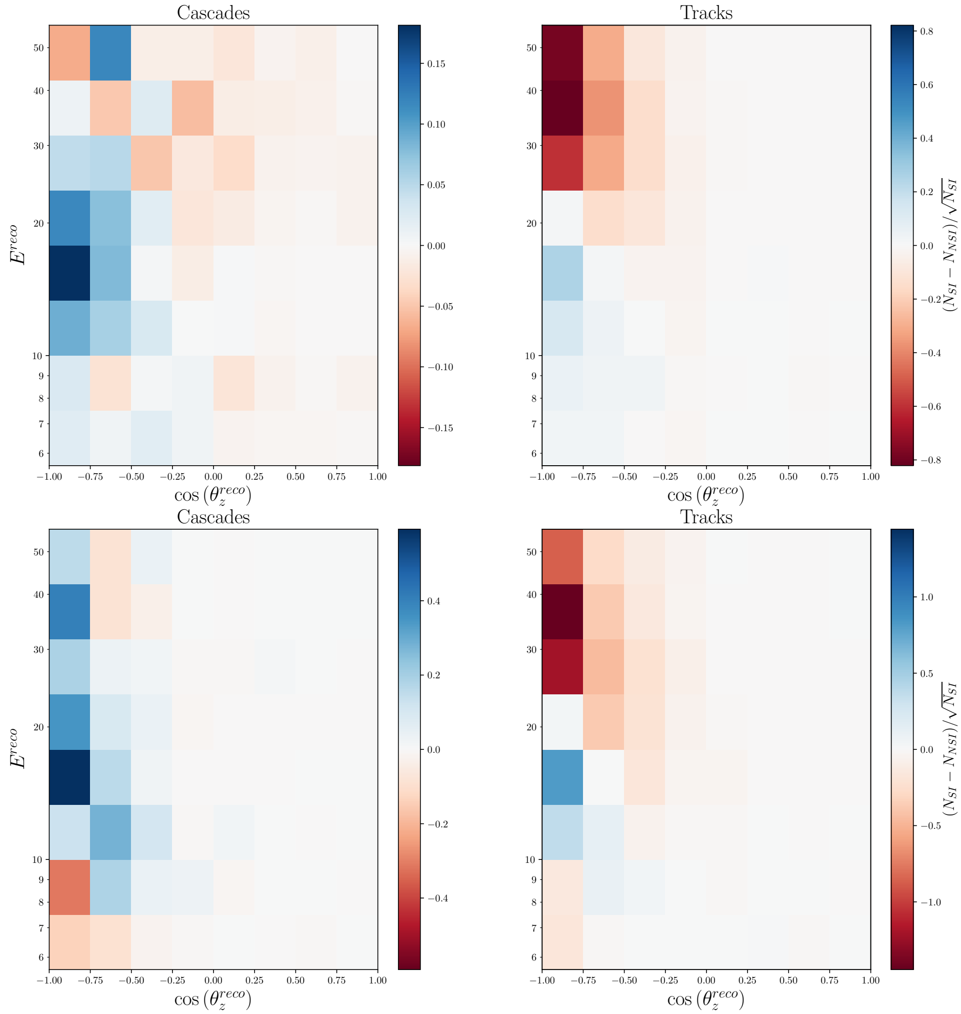

FIG. 3. Expected pulls of predicted event numbers for DeepCore and PINGU after 3 years. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count

C. PINGU

The methodology behind the PINGU simulations are the same as with our DeepCore study **??**. We use the public MC [**?**], which allows us to construct the event count as in Eq. **??**. However, since no detector systematics is yet modelled for PINGU, the correction factors C_{ijk} are all unity. As with the DeepCore data, the PINGU MC is divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins for both track- and cascade-like events. We generate "data" by predicting the event rates at PINGU with the following best-fit parameters from [**?**], except for the CP-violating phase which is set to zero for simplicity.

$$\begin{aligned} &\Delta m_{21}^2 = 7.42 \times 10^{-5}\,\mathrm{eV}^2, \;\; \Delta m_{31}^2 = 2.517 \times 10^{-3}\,\mathrm{eV}^2, \\ &\theta_{12} = 33.44^\circ, \quad \theta_{13} = 8.57^\circ, \quad \theta_{23} = 49.2^\circ, \quad \delta_{\mathrm{CP}} = 0\,. \end{aligned} \tag{5}$$

II. RESULTS

For our analyses, we define our χ^2 as

$$\chi^2(\hat{\theta}, \alpha, \beta, \kappa) = \sum_{ijk} \frac{\left(N^{\rm th} - N^{\rm data}\right)^2_{ijk}}{\left(\sigma^{\rm data}_{ijk}\right)^2 + \left(\sigma^{\rm syst}_{ijk}\right)^2} + \frac{(1-\alpha)^2}{\sigma_\alpha^2} + \frac{\beta^2}{\sigma_\beta^2} \tag{6}$$

where we minimize over the model parameters $\hat{\theta} \in \{\Delta m^2_{31}, \theta_{23}, \epsilon', \epsilon_{\mu\tau}\}$, the penalty terms α and β, and the free parameter κ. $N^{\rm th}_{ijk}$ is the expected number of events from theory, and $N^{\rm data}_{ijk}$ is the observed number of events in that bin. We set $\sigma_\alpha = 0.25$ as the atmospheric flux normalization error, and $\sigma_\beta = 0.04$ as the zenith angle slope error [**?**]. The observed event number has an associated Poissonian uncertainty $\sigma^{\rm data}_{ijk} = \sqrt{N^{\rm data}_{ijk}}$.

For IceCube, the event count takes the form

$$N^{\rm th}_{ijk} = \alpha\left[1 + \beta(0.5 + \cos\left(\theta_z^{reco}\right)_i)\right] N_{ijk}(\hat{\theta})\,, \tag{7}$$

with $N_{ijk}(\hat{\theta})$ from Eq. **??** Here, we allow the event distribution to rotate around the median zenith of -0.5.

For DeepCore and PINGU, the event count takes the form

$$N^{\rm th}_{ijk} = \alpha\left[1 + \beta\cos\left(\theta_z^{reco}\right)_i\right] N_{ijk}(\hat{\theta}) + \kappa N^{\mu_{atm}}_{ijk}\,, \tag{8}$$

with $N_{ijk}(\hat{\theta})$ from Eq. **??**. $N^{\mu_{atm}}_{ijk}$ is the muon background, which is left to float freely in the DeepCore analysis. The background at PINGU can be considered neglible to first order [**?**], and we thus put $\kappa = 0$ when calculating the PINGU χ^2 . For IceCube, we set $\sigma^{\rm syst}_{ijk} = f\sqrt{N^{\rm data}_{ijk}}$. For DeepCore, we use the provided systematic error distribution which accounts for both uncertanties in the finite MC statistics and in the data-driven muon background estimate [**?**].

First, we set all standard oscillation parameters to their current best-fit values of Eq. **??**, except for Δm^2_{31} and θ_{23}, which we marginalize over their 3σ ranges of 2.435×10^{-3} to $2.598 \times 10^{-3}\,{\rm eV}^2$ and 40.1 to 51.7° respectively.

For the joint analysis, we follow the parameter goodness-of-fit prescription [**?**] and construct the joint χ^2 as

$$\chi^2_{\rm joint} = \chi^2_{\rm IC} + \chi^2_{\rm DC} + \chi^2_{\rm P} - \chi^2_{IC,min} - \chi^2_{\rm DC,min} - \chi^2_{\rm P,min} \tag{9}$$

with test statistic $\chi^2_{\rm joint,min}$.

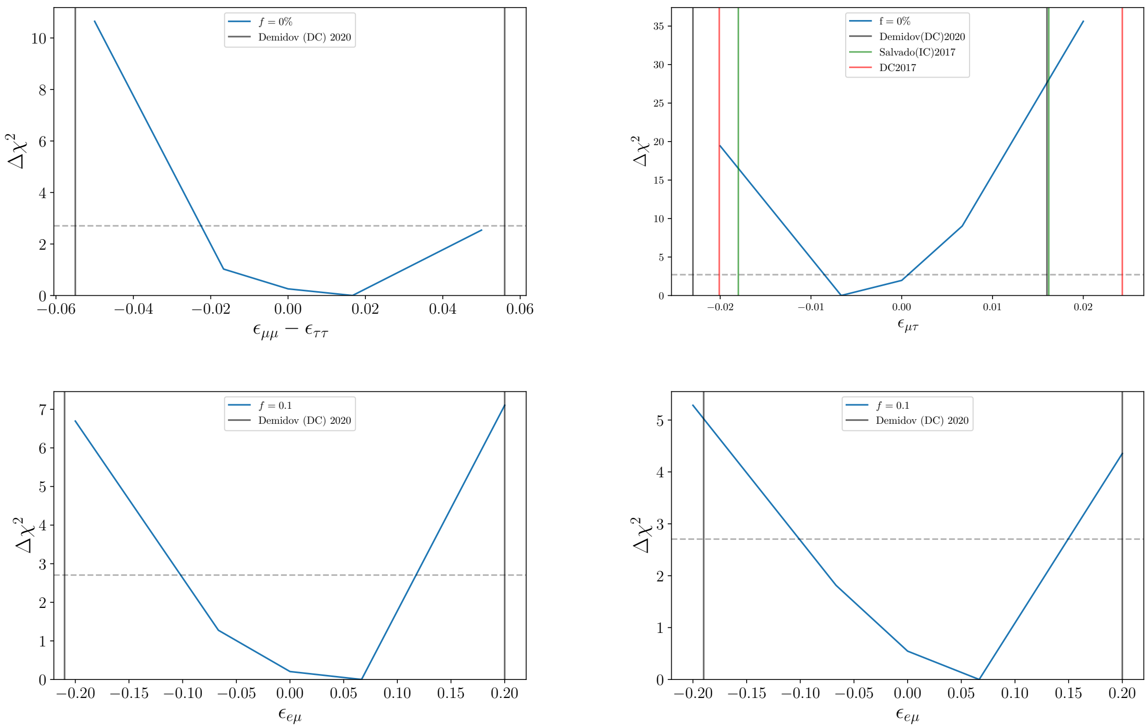

FIG. 4. The expected values of $\Delta\chi^2$ after three years of PINGU data. Δm^2_{31} and θ_{23} and have been marginalized out, and all NSI parameters not shown in each plot are set to zero.

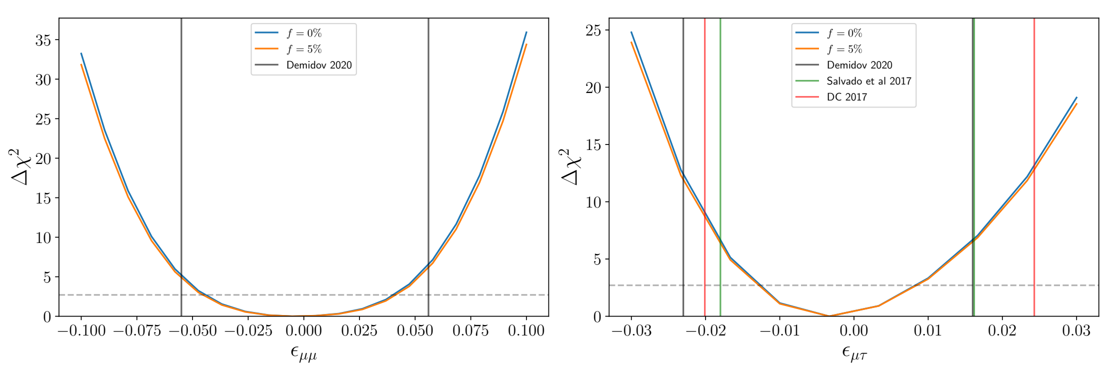

FIG. 5. The expected values of $\Delta\chi^2$ after three years of PINGU data. Δm^2_{31} and θ_{23} and have been marginalized out, and all other NSI parameters are set to zero. The black lines show the 90% credibility region from [**?**]

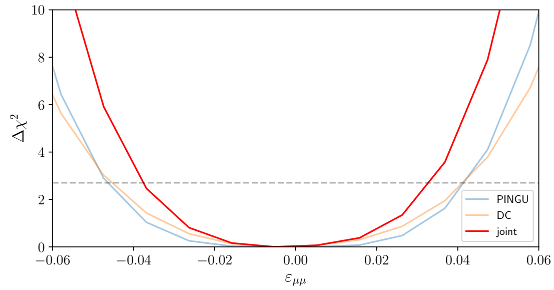

FIG. 6.

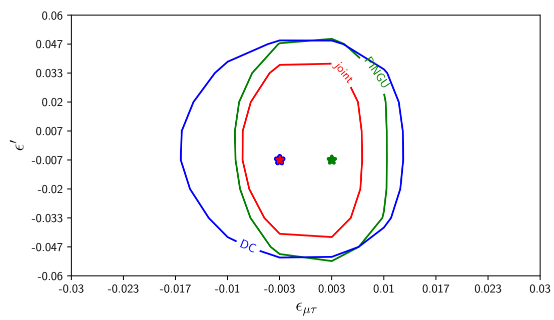

FIG. 7.

DeepCore (2017)

- ✓ Honda atmospheric fluxes
- × Only look at tracks and $\epsilon_{\mu\tau}$
- × DC Monte Carlo from an older dataset
- × 8 E bins from 6.3 eV^2 to 56 eV^2
- × 8 z bins from -1 to 0
- × Use "Overall" and "relative ν_e to ν_μ" normalization
- × Prior on spectral index
- × No zenith angle normalization
- ✓ No priors on $\Delta m^2_{31}, \theta_{23}, \theta_{13}$

Demidov (2020) DC analysis

- ✓ Honda atmospheric fluxes
- ✓ Looks at tracks + cascades for $\epsilon_{\mu\tau}$ and $\epsilon_{\tau\tau}$
- ✓ Data and Monte Carlo from DC 2018
- ✓ 8 E bins from 5.6 eV^2 to 56 eV^2
- ✓ 8 z bins from -1 to 1
- × Use "Overall" and "relative ν_e to ν_μ" normalization
- × Prior on spectral index
- × No zenith angle normalization
- ✓ No priors on $\Delta m^2_{31}, \theta_{23}$
- ✓ Fixes $\Delta m^2_{21}, \theta_{12}, \theta_{13}$
- × Uncertainty on hadron production in atmosphere
- × Uncertainty on neutrino nucleon cross section

Our DC+PINGU analysis

- ✓ Honda atmospheric fluxes
- ✓ Tracks and cascades for all flavors
- ✓ Reco $\rightarrow$ true mapping from Monte Carlo migration matrix
- ✓ 8 E bins from 5.6 eV^2 to 56 eV^2
- ✓ 8 zenith angle bins from -1 to 1
- ✓ Flux normalization uncertainty of 25%
- ✓ Zenith angle uncertainty of 4%
- ✓ No priors on oscillation parameters
- ✓ Marginalize Δm^2_{31} and θ_{23}. All other oscillation parameters are fixed.