I. METHODOLOGY

The neutrino flux at the detector is calculated by propagating the atmospheric neutrino flux [1] through the Earth by solving the Schrödinger equation for varying density. The Earth density profile is taken from the PREM [2]. The baseline for a given trajectory is determined using an average neutrino production height of 15 km and an Earth radius of 6371 km. The oscillation probability $P_{\alpha\beta}$ then acts as a weight, yielding the propagated flux at detector level for flavor β as

$$\phi_\beta^{\mathrm{det}} = \sum_\alpha P_{\alpha\beta}\phi_\alpha^{\mathrm{atm}} \,, \tag{1}$$

where we sum over the initial lepton flavors $\alpha \in \{e, \mu, \bar{e}, \bar{\mu}\}$.

A. IceCube

The event rate for each bin reads

$$N_{ij} = T\int_{(\cos\theta_z^r)_i}^{(\cos\theta_z^r)_{i+1}} \mathrm{d}\cos\theta_z^r \int_{E_j^r}^{E_{j+1}^r} \mathrm{d}E^r \int_0^\pi R(\theta^r,\theta^t)\mathrm{d}\cos\theta^t \int_0^\infty R(E^r,E^t)\mathrm{d}E^t \times \left[\sum_\beta \phi_\beta^{\mathrm{det}} A_\beta^{\mathrm{eff}}\right] \,, \tag{2}$$

where T is the live time of the detector, and A_β^{eff} its effective area for flavor β. We use the effective area of the 86 string configuration made public by the IceCube collaboration [3]. $R(x^r, x^t)$ is a Gaussian resolution function, which is responsible for the smearing between the reconstructed and true parameters x^r and x^t, respectively. It takes the form

$$R(x^r, x^t) = \frac{1}{\sqrt{2\pi}\sigma_{x^r}} \exp\left[\frac{(x^r - \mu(x^t))^2}{2\sigma_{x^r}^2}\right] \,. \tag{3}$$

Assuming no bias in the reconstruction, the mean of the Gaussian can be taken as $\mu(x^t) = x^t$. As seen in Fig. 1, the distribution of simulated events is skewed. Instead, we assume a log-normal distribution between E^{true} and E^{reco}, and train a Gaussian Process Regressor on the dataset [4], from which we can extract a predicted mean and standard deviation for each E^{reco}. We then sample values from this distribution to yield the points of E^{true} at which to integrate over.

In Icecube, the zenith angle resolution for track-like events is less than 2°, making $\cos(\theta_z^{true})$ coincide with $\cos(\theta_z^{reco})$ for our study [5]. The data is from the IC86 sterile analysis [5].

B. DeepCore

In this part, we use the publically available DeepCore data sample [6] which is an updated version of what was used by the IceCube collaboration in an ν_μ disapprearance analysis [7].

The detector systematics include ice absorption and scattering, and overall, lateral, and head-on optical efficiencies of the DOMs. They are applied as correction factors using the best-fit points from the DeepCore 2019 ν_τ appearance analysis [8].

The data include 14901 track-live events and 26001 cascade-like events, both divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins.

The oscillation parameters are from the best-fit in the global analysis in [9]: $\theta_{12} = 33.44^\circ$, $\theta_{13} = 8.57^\circ$, $\Delta m_{21}^2 = 7.42\,\mathrm{eV}^2$, and we marginalize over Δm_{31}^2 and θ_{23}.

Given a Monte Carlo simulation with weights $w_{k\beta}$, we can construct the event count as

$$N_{ijk} = C_{ijk}\sum_\beta w_{ijk,\beta}\phi_\beta^{\mathrm{det}} \,, \tag{4}$$

where $C_{k\beta}$ is the correction factor from the detector systematic uncertainty and $\phi_\beta^{\mathrm{det}}$ is defined as Eq. 1. We have now substituted the effect of the Gaussian smearing by treating the reconstructed and true quantities as a migration matrix.

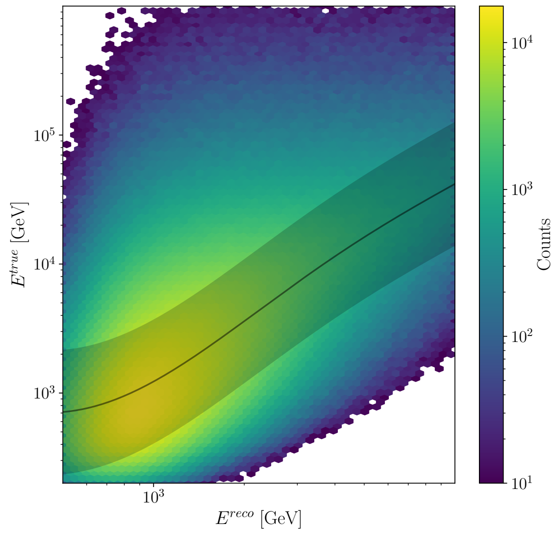

FIG. 1. Relationship between the true and reconstructed muon energy in the IceCube MC sample [4]
. Shaded area shows the 99.9th percentile limits predicted by the regressor trained on this set.

C. PINGU

The methodology behind the PINGU simulations are the same as with our DeepCore study I B. We use the public MC [10], which allows us to construct the event count as in Eq. 4. However, since no detector systematics is yet modelled for PINGU, the correction factors C_{ijk} are all unity. As with the DeepCore data, the PINGU MC is divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins for both track- and cascade-like events. We generate "data" by predicting the event rates at PINGU with the following best-fit parameters from [9], except for the CP-violating phase which is set to zero for simplicity.

$$\begin{aligned} &\Delta m_{21}^2 = 7.42 \times 10^{-5}\,\mathrm{eV}^2, \;\; \Delta m_{31}^2 = 2.517 \times 10^{-3}\,\mathrm{eV}^2, \\ &\theta_{12} = 33.44^\circ, \quad \theta_{13} = 8.57^\circ, \quad \theta_{23} = 49.2^\circ, \quad \delta_{\mathrm{CP}} = 0\,. \end{aligned} \tag{5}$$

II. RESULTS

For our analyses, we define our χ^2 as

$$\chi^2(\hat{\theta}, \alpha, \beta, \kappa) = \sum_{ijk} \frac{\left(N^{\mathrm{th}} - N^{\mathrm{data}}\right)^2_{ijk}}{\left(\sigma^{\mathrm{data}}_{ijk}\right)^2 + \left(\sigma^{\mathrm{syst}}_{ijk}\right)^2} + \frac{(1-\alpha)^2}{\sigma_\alpha^2} + \frac{\beta^2}{\sigma_\beta^2} \tag{6}$$

where we minimize over the model parameters $\hat{\theta} \in \{\Delta m_{31}^2, \theta_{23}, \epsilon', \epsilon_{\mu\tau}\}$, the penalty terms α and β, and the free parameter κ. N^{th}_{ijk} is the expected number of events from theory, and N^{data}_{ijk} is the observed number of events in that bin. We set $\sigma_\alpha = 0.25$ as the atmospheric flux normalization error, and $\sigma_\beta = 0.04$ as the zenith angle slope error [1]. The observed event number has an associated Poissonian uncertainty $\sigma^{\mathrm{data}}_{ijk} = \sqrt{N^{\mathrm{data}}_{ijk}}$.

For IceCube, the event count takes the form

$$N^{\mathrm{th}}_{ijk} = \alpha \left[1 + \beta(0.5 + \cos{(\theta_z^{reco})_i})\right] N_{ijk}(\hat{\theta})\,, \tag{7}$$

with $N_{ijk}(\hat{\theta})$ from Eq. 2 Here, we allow the event distribution to rotate around the median zenith of -0.5.

For DeepCore and PINGU, the event count takes the form

$$N^{\mathrm{th}}_{ijk} = \alpha \left[1 + \beta \cos{(\theta_z^{reco})_i}\right] N_{ijk}(\hat{\theta}) + \kappa N^{\mu_{atm}}_{ijk}\,, \tag{8}$$

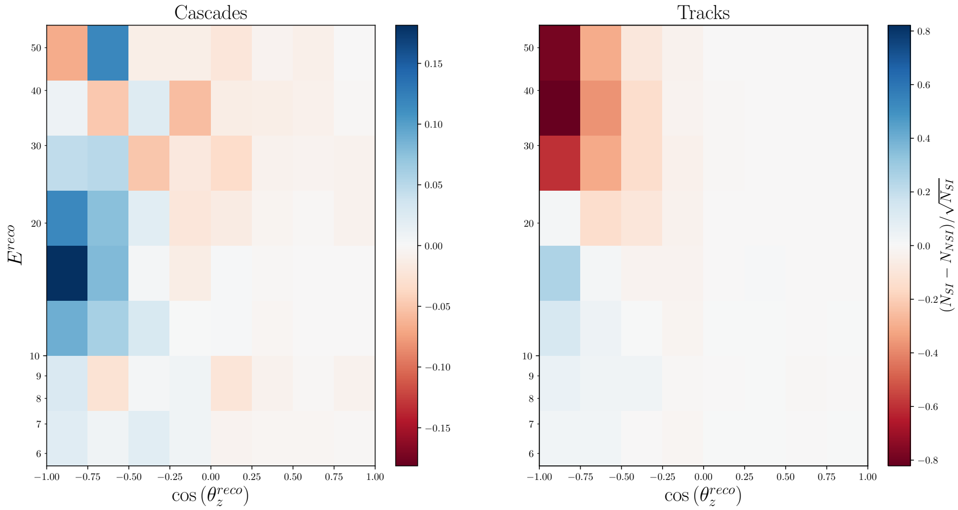

FIG. 2. Expected pulls of predicted event numbers for DeepCore. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count

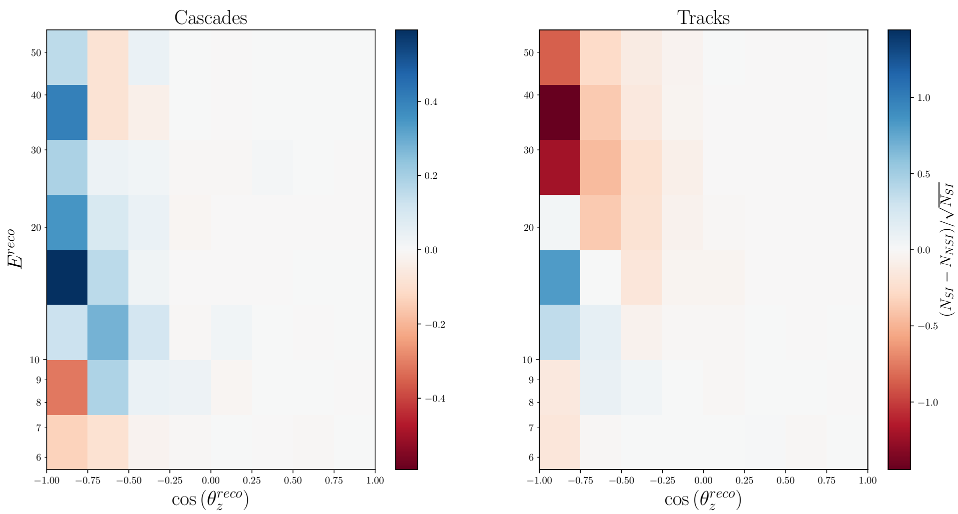

FIG. 3. Expected pulls of predicted event numbers for PINGU. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count

with $N_{ijk}(\hat{\theta})$ from Eq. 4. $N_{ijk}^{\mu_{atm}}$ is the muon background, which is left to float freely in the DeepCore analysis. The background at PINGU can be considered neglible to first order [10], and we thus put $\kappa = 0$ when calculating the PINGU χ^2 . For IceCube, we set $\sigma_{ijk}^{\text{syst}} = f\sqrt{N_{ijk}^{\text{data}}}$. For DeepCore, we use the provided systematic error distribution which accounts for both uncertanties in the finite MC statistics and in the data-driven muon background estimate [6].

We plot the event pull $(N_{NSI} - N_{SI})/\sqrt{N_{SI}}$ where $N_{(N)SI}$ are the numbers of expected events assuming (non-)standard interactions. This gives the normalized difference in the number of expected events at the detector, and illustrates the expected sensitivity for the NSI parameters.

For the joint analysis, we follow the parameter goodness-of-fit prescription [12] and construct the joint χ^2 as

$$\chi^2_{joint} = \chi^2_{IC} + \chi^2_{DC} + \chi^2_{P} - \chi^2_{IC,min} - \chi^2_{DC,min} - \chi^2_{P,min} \tag{9}$$

with test statistic $\chi^2_{joint,min}$.

At 90% CL: PINGU: $-0.046 < \epsilon_{\mu\mu} < 0.041$ DC: $-0.045 < \epsilon_{\mu\mu} < 0.041$ joint: $-0.038 < \epsilon_{\mu\mu} < 0.033$

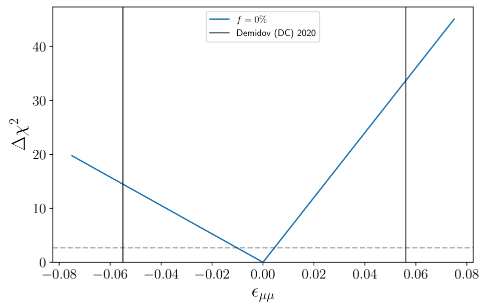

FIG. 4. Expected pulls of predicted event numbers for PINGU. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count

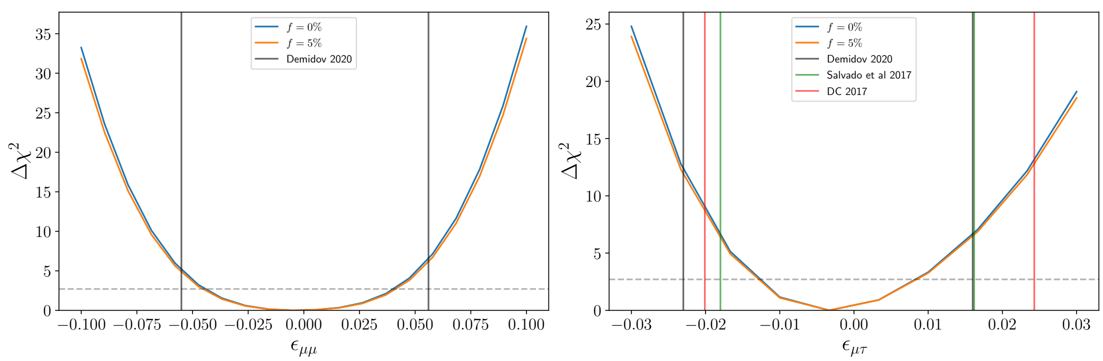

FIG. 5. Confidence levels from this analysis on the NSI parameters for systematic error and without. The black lines show the 90% credibility region from [11].

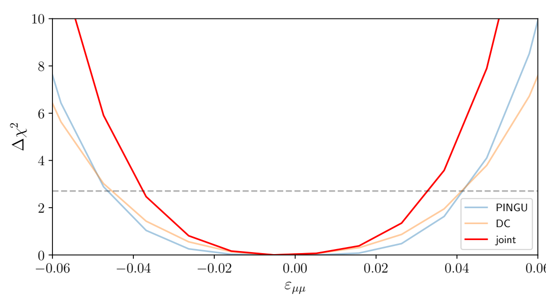

FIG. 6.

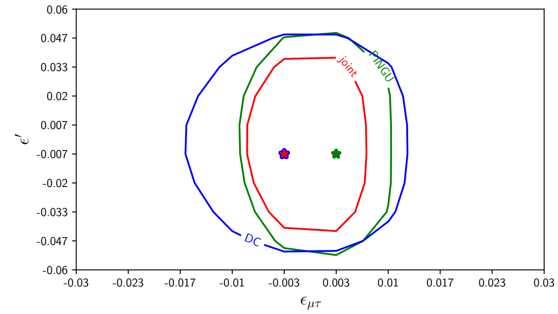

FIG. 7.

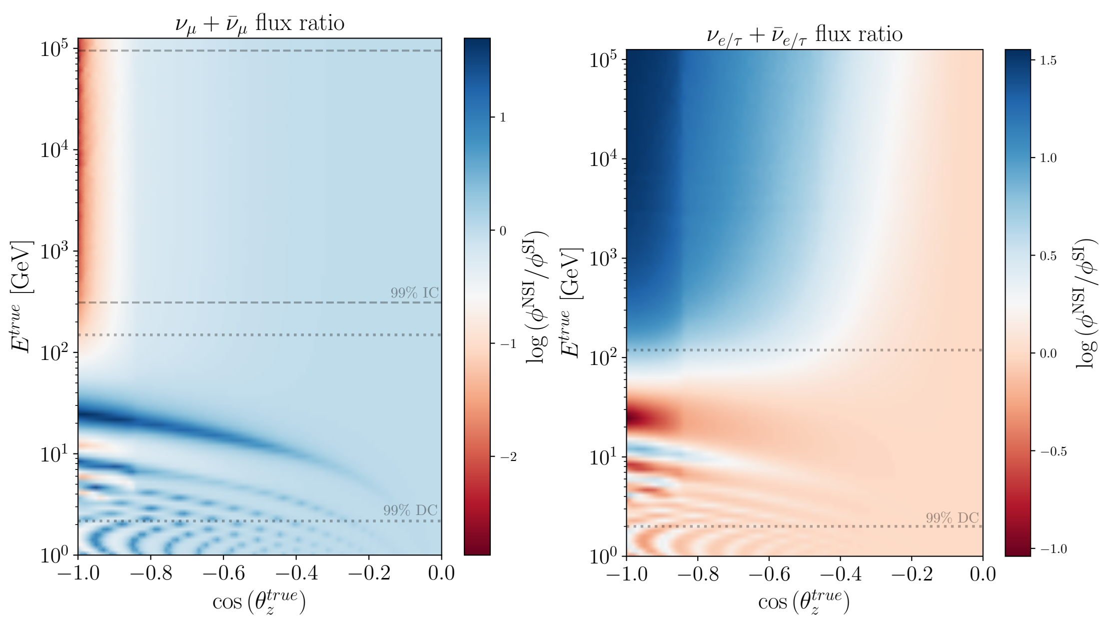

FIG. 8. Ratio of NSI to SI atmospheric fluxes at detector level. Dotted (dashed) lines show the region in which 99% of the DeepCore (IceCube) MC events are contained.

DeepCore (2017)

- ✓ Honda atmospheric fluxes
- × Only look at tracks and $\epsilon_{\mu\tau}$
- × DC Monte Carlo from an older dataset
- × 8 E bins from $6.3\,\mathrm{eV}^2$ to $56\,\mathrm{eV}^2$
- × 8 z bins from -1 to 0
- × Use "Overall" and "relative ν_e to ν_μ" normalization
- × Prior on spectral index
- × No zenith angle normalization
- ✓ No priors on $\Delta m^2_{31}, \theta_{23}, \theta_{13}$

Demidov (2020) DC analysis

- ✓ Honda atmospheric fluxes
- ✓ Looks at tracks + cascades for $\epsilon_{\mu\tau}$ and $\epsilon_{\tau\tau}$
- ✓ Data and Monte Carlo from DC 2018
- ✓ 8 E bins from $5.6\,\mathrm{eV}^2$ to $56\,\mathrm{eV}^2$
- ✓ 8 z bins from -1 to 1
- × Use "Overall" and "relative ν_e to ν_μ" normalization
- × Prior on spectral index
- × No zenith angle normalization
- ✓ No priors on $\Delta m^2_{31}, \theta_{23}$
- ✓ Fixes $\Delta m^2_{21}, \theta_{12}, \theta_{13}$
- × Uncertainty on hadron production in atmosphere
- × Uncertainty on neutrino nucleon cross section

Choubey et. al. PINGU analysis

- ✓ Honda atmospheric fluxes
- × Only consider ν_μ events
- × A_{eff} and resolution from PINGU Letter of intent
- × 27 E bins from 2 to $101\,\mathrm{eV}^2$
- × 10 z bins from -1 to 0
- ✓ Flux normalization uncertainty of 20%
- × Uncertanties in cross-section and spectral index
- ✓ Zenith angle uncertainty of 5%
- × Priors on $\Delta m^2_{31}, \theta_{23}, \theta_{13}$

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