1 Neutrino Masses and Oscillations

As we saw in Eq. 3 , the neutrino fields ν_α only couple to the associated lepton fields ℓ_α, conserving the lepton number L_α. We will now introduce two separate extensions to this part of the Standard Model.

We introduce a right-handed neutrino field, ν_R. It has the usual properties of the conventional left-handed neutrino such as hypercharge and color zero. Moreover, since the electroweak gauge group $\mathrm{SU}(2)_L$ only couple to left-handed particles and right-handed antiparticles, it transforms as a singlet under the SM symmetry group $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. This neutrino is *sterile* since it doesn't participate any of the SM interactions.

We extend the SM by adding a right-handed component to the Higgs-lepton Yukawa Lagrangian from Eq. 1 with neutrino Yukawa couplings $Y'^{\nu}_{\alpha\beta}$,

$$\mathcal{L}_H = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell'_{\alpha L}Y'^{\ell}_{\alpha\beta}\ell'_{\beta R} + \nu'_{\alpha L}Y'^{\nu}_{\alpha\beta}\nu'_{\beta R}\right] \quad (1)$$

Similar to how we diagonalized the lepton Yukawa couplings $Y'^{\ell}_{\alpha\beta}$ in Eq. 2, we diagonalize $Y'^{\nu}_{\alpha\beta}$ as

$$V^{\nu\dagger}_{\alpha kL}Y'^{\nu}_{\alpha\beta}V^{\nu}_{\beta jR} = Y^{\nu}_{kj}\,. \quad (2)$$

Now we introduce a crucial difference between the properties of the charged lepton and the neutrino fields. While the charged lepton flavor eigenstate was uniquely determined by its mass eigenstate, the neutrino flavor is a superposition of mass eigenstates. This is because neutrinos are indirectly detected via the observation of its associated charged lepton, so there is no requirement of neutrino flavor eigenstates to have a definite mass. The flavor of a neutrino is then, by definition, the flavor of the associated charged lepton. This is commonly introduced as giving the mass eigenstates Latin numerals and letters, while the flavor eigenstates stay as Greek letters.

So, let the neutrino field with chriality X be denoted ν_X, with components having Latin numerals to distinguish them from the flavour components, i.e

$$\nu_{kX} = V^{\nu\dagger}_{k\beta X}\nu'_{\beta X}\,. \quad (3)$$

The diagonalized Lagrangian now takes the form

$$\begin{aligned}\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell'_{\alpha L}Y'^{\ell}_{\alpha\beta}\ell'_{\beta R} + \nu'_{\alpha L}Y'^{\nu}_{\alpha\beta}\nu'_{\beta R}\right]\\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\Big[\ell'_{\alpha L}V^{\ell}_{\alpha\beta L}Y^{\ell}_{\alpha\beta}V^{\ell\dagger}_{\alpha\beta R}\ell'_{\beta R}\\ &\qquad + \nu'_{\alpha L}V^{\nu}_{\alpha kL}Y^{\nu}_{kj}V^{\nu\dagger}_{\beta jR}\nu'_{\beta R}\Big]\\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell^\dagger_{\alpha L}Y^{\ell}_{\alpha\beta}\ell_{\beta R} + \nu^\dagger_{kL}Y^{\nu}_{kj}\nu_{jR}\right]\\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}Y^{\ell}_{\alpha\beta}\ell_{\beta R} + \bar\nu_{kL}Y^{\nu}_{kj}\nu_{jR}\right] \quad (4)\end{aligned}$$

By construction, $Y^{\ell}_{\alpha\beta}$ and Y^{ν}_{kj} are diagonal, so we write them as $y^{\ell}_{\alpha}\delta_{\alpha\beta}$ and $y^{\nu}_{k}\delta_{kj}$ respectively, leaving the Lagrangian as

$$\begin{aligned}\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}y^{\ell}_{\alpha}\delta_{\alpha\beta}\ell_{\beta R} + \bar\nu_{kL}y^{\nu}_{k}\delta_{kj}\nu_{jR}\right]\\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}y^{\ell}_{\alpha}\ell_{\alpha R} + \bar\nu_{kL}y^{\nu}_{k}\nu_{kR}\right]\\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[y^{\ell}_{\alpha}\bar\ell_{\alpha L}\ell_{\alpha R} + y^{\nu}_{k}\bar\nu_{kL}\nu_{kR}\right] \quad (5)\end{aligned}$$

Now, the Dirac neutrino field is

$$\nu_k = \nu_{kL} + \nu_{kR}\,. \quad (6)$$

Multiplying ν_k with its conjugate $\bar\nu_k$, we get

$$\begin{aligned}\bar\nu_k\nu_k &= \bar\nu_{kL}\nu_{kL} + \bar\nu_{kR}\nu_{kL} + \bar\nu_{kL}\nu_{kR} + \bar\nu_{kR}\nu_{kR}\\ &= \bar\nu_{kL}\nu_{kR} + \bar\nu_{kR}\nu_{kL}\\ &= \bar\nu_{kL}\nu_{kR} + \text{h.c.} \quad (7)\end{aligned}$$

The same calculation for the charged lepton field yields the same result for ℓ_k. Substituting this result and expanding the Higgs VEV into the fields gives us

$$\begin{aligned}\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[y^{\ell}_{\alpha}\bar\ell_{\alpha}\ell_{\alpha} + y^{\nu}_{k}\bar\nu_{k}\nu_{k}\right]\\ &= -\frac{y^{\ell}_{\alpha}v}{\sqrt{2}}\bar\ell_{\alpha}\ell_{\alpha} - \frac{y^{\nu}_{k}v}{\sqrt{2}}\bar\nu_k\nu_k - \frac{y^{\ell}_{\alpha}}{\sqrt{2}}\bar\ell_{\alpha}\ell_{\alpha}H - \frac{y^{\nu}_{k}}{\sqrt{2}}\bar\nu_k\nu_k H\,.\end{aligned} \quad (8)$$

Thus, this extension to the SM generates neutrino masses by the Higgs mechanism, in the same fashion as with the charged leptons and the quarks:

$$m_k = \frac{y_k^\nu v}{\sqrt{2}} \tag{9}$$

Substituting the new transformation from Eq. 3 into the weak charged current, we get

$$\begin{aligned} j_L^\rho &= 2\bar{\nu}'_{\alpha L}\gamma^\rho \ell'_{\alpha L} \\ &= 2\bar{\nu}_{kL} V_{k\alpha}^{\prime\nu\dagger} V_{\alpha\alpha}^{\prime\ell} \gamma^\rho \ell_{\alpha L} \end{aligned} \tag{10}$$

Call $V_{k\alpha}^{\prime\nu\dagger} V_{\alpha\alpha}^{\prime\ell} = U_{k\alpha}^\dagger$ Now, the current in Eq. 10 conserves lepton number, since the neutrino field with flavor α only couples to the lepton field with flavor α. However, the Higgs-lepton Yukawa Lagrangian in Eq. 5 violates lepton number conservation since it couples the charged lepton flavor α to the neutrino mass eigenstate k, which is a superposition of flavors. There is no transformation that leaves both the interaction and kinetic Lagrangian invariant. We now have

$$j_L^\rho = 2U_{\alpha k}^* \bar{\nu}_{kL}\gamma^\rho \ell_{\alpha L} \tag{11}$$

Derivation of neutrino oscillation formula