0.1 χ^2 minimization

For our analyses, we define our χ^2 as

$$\chi^2(\hat{\theta},\alpha,\beta,\kappa)=\sum_{ijk}\frac{\left(N^{\rm th}-N^{\rm data}\right)^2_{ijk}}{\left(\sigma^{\rm data}_{ijk}\right)^2+\left(\sigma^{\rm syst}_{ijk}\right)^2}+\frac{(1-\alpha)^2}{\sigma^2_\alpha}+\frac{\beta^2}{\sigma^2_\beta} \tag{0.1}$$

where we minimize over the model parameters $\hat{\theta}\in\{\Delta m^2,\theta_{23},\epsilon',\epsilon_{\mu\tau}\}$, the penalty terms α and β, and the free parameter κ. $N^{\rm th}_{ijk}$ is the expected number of events from theory, and $N^{\rm data}_{ijk}$ is the observed number of events in that bin. We set $\sigma_\alpha=0.25$ as the atmospheric flux normalization error, and $\sigma_\beta=0.04$ as the zenith angle slope error [**?**]. The observed event number has an associated Poissonian uncertainty $\sigma^{\rm data}_{ijk}=\sqrt{N^{\rm data}_{ijk}}$.

For IceCube, the event count takes the form

$$N^{\rm th}_{ijk}=\alpha\left[1+\beta(0.5+\cos{(\theta_z^{reco})_i})\right]N_{ijk}(\hat{\theta})\,, \tag{0.2}$$

with $N_{ijk}(\hat{\theta})$ from Eq. **??** Here, we allow the event distribution to rotate around the median zenith of -0.5.

For DeepCore and PINGU, the event count takes the form

$$N^{\rm th}_{ijk}=\alpha\left[1+\beta\cos{(\theta_z^{reco})_i}\right]N_{ijk}(\hat{\theta})+\kappa N^{\mu_{atm}}_{ijk}\,, \tag{0.3}$$

with $N_{ijk}(\hat{\theta})$ from Eq. **??**. $N^{\mu_{atm}}_{ijk}$ is the muon background, which is left to float freely in the DeepCore analysis. The background at PINGU can be considered neglible to first order [**?**], and we thus put $\kappa=0$ when calculating the PINGU χ^2. For IceCube, we set $\sigma^{\rm syst}_{ijk}=f\sqrt{N^{\rm data}_{ijk}}$. For DeepCore, we use the provided systematic error distribution which accounts for both uncertanties in the finite MC statistics and in the data-driven muon background estimate [**?**].