1 Sterile and Massive Neutrinos

As we saw in Eq. 3 , the neutrino fields ν_α only couple to the associated lepton fields ℓ_α, conserving the lepton number L_α. We will now introduce two separate extensions to this part of the Standard Model.

We introduce a right-handed neutrino field, ν_R. It has the usual properties of the conventional left-handed neutrino such as hypercharge and color zero. Moreover, since the electroweak gauge group $\mathrm{SU}(2)_L$ only couple to left-handed particles and right-handed antiparticles, it transforms as a singlet under the SM symmetry group $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. This neutrino is *sterile* since it doesn't participate any of the SM interactions.

We extend the SM by adding a right-handed component to the Higgs-lepton Yukawa Lagrangian from Eq. 1 with neutrino Yukawa couplings $Y'^{\nu}_{\alpha\beta}$,

$$\mathcal{L}_H = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell'_{\alpha L}Y'^{\ell}_{\alpha\beta}\ell'_{\beta R} + \nu'_{\alpha L}Y'^{\nu}_{\alpha\beta}\nu'_{\beta R}\right] \quad (1)$$

Similar to how we diagonalized the lepton Yukawa couplings $Y'^{\ell}_{\alpha\beta}$ in Eq. 2, we diagonalize $Y'^{\nu}_{\alpha\beta}$ as

$$V^{\nu\dagger}_{\alpha kL}Y'^{\nu}_{\alpha\beta}V^{\nu}_{\beta jR} = Y^{\nu}_{kj}\,. \quad (2)$$

Now, let the neutrino field with chriality X be denoted n_X, with components Latin numerals to distinguish them from the flavour components, i.e

$$\nu_{kX} = V^{\nu\dagger}_{kjX}\nu'_{jX}\,. \quad (3)$$

The diagonalized Lagrangian now takes the form

$$\begin{aligned}
\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell'_{\alpha L}Y'^{\ell}_{\alpha\beta}\ell'_{\beta R} + \nu'_{\alpha L}Y'^{\nu}_{\alpha\beta}\nu'_{\beta R}\right] \\
&= -\left(\frac{v+H}{\sqrt{2}}\right)\Big[\ell'_{\alpha L}V^{\ell}_{\alpha\beta L}Y^{\ell}_{\alpha\beta}V^{\ell\dagger}_{\alpha\beta R}\ell'_{\beta R} \qquad (4) \\
&\qquad + \nu'_{\alpha L}V^{\nu}_{\alpha kL}Y^{\nu}_{kj}V^{\nu\dagger}_{\beta jR}\nu'_{\beta R}\Big] \\
&= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\ell^{\dagger}_{\alpha L}Y^{\ell}_{\alpha\beta}\ell_{\beta R} + \nu^{\dagger}_{kL}Y^{\nu}_{kj}\nu_{jR}\right] \\
&= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}Y^{\ell}_{\alpha\beta}\ell_{\beta R} + \bar\nu_{kL}Y^{\nu}_{kj}\nu_{jR}\right] \qquad (5)
\end{aligned}$$

Now using the fact that Y^{ν}_{kj} is diagonal, we write it as $Y^{\nu}_{kj} = y^{\nu}_{k}\delta_{kj}$, leaving the Lagrangian as

$$\begin{aligned}
\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}y^{\ell}_{\alpha}\delta_{\alpha\beta}\ell_{\beta R} + \bar\nu_{kL}y^{\nu}_{k}\delta_{kj}\nu_{jR}\right] \\
&= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\bar\ell_{\alpha L}y^{\ell}_{\alpha}\ell_{\alpha R} + \bar\nu_{kL}y^{\nu}_{k}\nu_{kR}\right] \\
&= -\left(\frac{v+H}{\sqrt{2}}\right)\left[y^{\ell}_{\alpha}\bar\ell_{\alpha L}\ell_{\alpha R} + y^{\nu}_{k}\bar\nu_{kL}\nu_{kR}\right] \qquad (6)
\end{aligned}$$

Now, the Dirac neutrino field is

$$\nu_k = \nu_{kL} + \nu_{kR}\,. \quad (7)$$

Multiplying ν_k with its conjugate $\bar\nu_k$, we get

$$\begin{aligned}
\bar\nu_k\nu_k &= \bar\nu_{kL}\nu_{kL} + \bar\nu_{kR}\nu_{kL} + \bar\nu_{kL}\nu_{kR} + \bar\nu_{kR}\nu_{kR} \\
&= \bar\nu_{kL}\nu_{kR} + \bar\nu_{kR}\nu_{kL} \\
&= \bar\nu_{kL}\nu_{kR} + \text{h.c.} \qquad (8)
\end{aligned}$$

The same calculation for the charged lepton field yields the same result for ℓ_k. Substituting this result and expanding the Higgs VEV into the fields gives us

$$\begin{aligned}
\mathcal{L}_H &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[y^{\ell}_{\alpha}\bar\ell_{\alpha}\ell_{\alpha} + y^{\nu}_{k}\bar\nu_{k}\nu_{k}\right] \\
&= -\frac{y^{\ell}_{\alpha}v}{\sqrt{2}}\bar\ell_{\alpha}\ell_{\alpha} - \frac{y^{\nu}_{k}v}{\sqrt{2}}\bar\nu_{k}\nu_{k} - \frac{y^{\ell}_{\alpha}}{\sqrt{2}}\bar\ell_{\alpha}\ell_{\alpha}H - \frac{y^{\nu}_{k}}{\sqrt{2}}\bar\nu_{k}\nu_{k}H\,. \qquad (9)
\end{aligned}$$

Thus, this extension to the SM generates neutrino masses by the Higgs mechanism, in the same fashion as with the charged leptons and the quarks:

$$m_k = \frac{y^{\nu}_{k}v}{\sqrt{2}} \quad (10)$$