FIG. 1. Relationship between the true and reconstructed muon energy in the IceCube MC sample [4]
. Shaded area shows the 99.9th percentile limits predicted by the regressor trained on this set.

I. METHODOLOGY

The neutrino flux at the detector is calculated by propagating the atmospheric neutrino flux [1] through the Earth by solving the Schrödinger equation for varying density. The Earth density profile is taken from the PREM [2]. The oscillation probability $P_{\alpha\beta}$ then acts as a weight, yielding the propagated flux at detector level for flavor β as

$$\phi_\beta^{\text{det}} = \sum_\alpha \frac{\mathrm{d}^2\phi_\alpha^{\text{atm}}}{\mathrm{d}E^t \mathrm{d}\cos\theta_z^t} P_{\alpha\beta} \,, \tag{1}$$

where we sum over the initial lepton flavors $\alpha \in \{e, \mu, \bar{e}, \bar{\mu}\}$.

A. IceCube

The event rate for each bin reads

$$N_{ij} = T \int_{(\cos\theta_z^r)_i}^{(\cos\theta_z^r)_{i+1}} \mathrm{d}\cos\theta_z^r \int_{E_j^r}^{E_{j+1}^r} \mathrm{d}E^r \int_0^\pi R(\theta^r, \theta^t)\mathrm{d}\cos\theta^t \int_0^\infty R(E^r, E^t)\mathrm{d}E^t \times \left[\sum_\beta \phi_\beta^{\text{det}} A_\beta^{\text{eff}}\right] \,, \tag{2}$$

where T is the live time of the detector, and A_β^{eff} its effective area for flavor β. We use the effective area of the 86 string configuration made public by the IceCube collaboration [3]. $R(x^r, x^t)$ is a Gaussian resolution function, which is responsible for the smearing between the reconstructed and true parameters x^r and x^t, respectively. It takes the form

$$R(x^r, x^t) = \frac{1}{\sqrt{2\pi}\sigma_{x^r}} \exp\left[\frac{(x^r - \mu(x^t))^2}{2\sigma_{x^r}^2}\right] \,. \tag{3}$$

Assuming no bias in the reconstruction, the mean of the Gaussian can be taken as $\mu(x^t) = x^t$. As seen in Fig. 1, the distribution of simulated events is skewed. Instead, we assume a log-normal distribution between E^{true} and E^{reco}, and train a Gaussian Process Regressor on the dataset [4], from which we can extract a predicted mean and standard deviation for each E^{reco}. We then sample values from this distribution to yield the points of E^{true} at which to integrate over.

In Icecube, the zenith angle resolution for track-like events is less than 2°, making $\cos(\theta_z^{true})$ coincide with $\cos(\theta_z^{reco})$ for our study [5]. The data is from the IC86 sterile analysis [5].

B. DeepCore

In this part, we use the publically available DeepCore data sample [6] which is an updated version of what was used by the IceCube collaboration in an ν_μ disapprearance analysis [7].

The detector systematics include ice absorption and scattering, and overall, lateral, and head-on optical efficiencies of the DOMs. They are applied as correction factors using the best-fit points from the DeepCore 2019 ν_τ appearance analysis [8].

The data include 14901 track-live events and 26001 cascade-like events, both divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins.

The oscillation parameters are from the best-fit in the global analysis in [9]: $\theta_{12} = 33.44°$, $\theta_{13} = 8.57°$, $\Delta m_{21}^2 = 7.42\,\text{eV}^2$, and we marginalize over Δm_{31}^2 and θ_{23}.

Given a Monte Carlo simulation with weights $w_{k\beta}$, we can construct the event count as

$$N_{ijk} = C_{ijk} \sum_\beta w_{ijk,\beta} \phi_\beta^{\text{det}}, \tag{4}$$

where $C_{k\beta}$ is the correction factor from the detector systematic uncertainty and ϕ_β^{det} is defined as Eq. 1. We have now substituted the effect of the Gaussian smearing by treating the reconstructed and true quantities as a migration matrix.

C. PINGU

The methodology behind the PINGU simulations are the same as with our DeepCore study I B. We use the public MC [10], which allows us to construct the event count as in Eq. 4. However, since no detector systematics is yet modelled for PINGU, the correction factors C_{ijk} are all unity. As with the DeepCore data, the PINGU MC is divided into eight $\log_{10} E^{reco} \in [0.75, 1.75]$ bins, and eight $\cos(\theta_z^{reco}) \in [-1, 1]$ bins for both track- and cascade-like events. We generate "data" by predicting the event rates at PINGU with the following best-fit parameters from [9], except for the CP-violating phase which is set to zero for simplicity.

$$\Delta m_{21}^2 = 7.42 \times 10^{-5}\,\text{eV}^2, \quad \Delta m_{31}^2 = 2.517 \times 10^{-3}\,\text{eV}^2,$$
$$\theta_{12} = 33.44°, \quad \theta_{13} = 8.57°, \quad \theta_{23} = 49.2°, \quad \delta_{\text{CP}} = 0. \tag{5}$$

II. RESULTS

For our analyses, we define our χ^2 as

$$\chi^2(\hat{\theta}, \alpha, \beta, \kappa) = \sum_{ijk} \frac{\left(N^{\text{th}} - N^{\text{data}}\right)_{ijk}^2}{\left(\sigma_{ijk}^{\text{data}}\right)^2 + \left(\sigma_{ijk}^{\text{syst}}\right)^2} + \frac{(1-\alpha)^2}{\sigma_\alpha^2} + \frac{\beta^2}{\sigma_\beta^2} \tag{6}$$

where we minimize over the model parameters $\hat{\theta} \in \{\Delta m_{31}^2, \theta_{23}, \epsilon', \epsilon_{\mu\tau}\}$, the penalty terms α and β, and the free parameter κ. N_{ijk}^{th} is the expected number of events from theory, and N_{ijk}^{data} is the observed number of events in that bin. We set $\sigma_\alpha = 0.25$ as the atmospheric flux normalization error, and $\sigma_\beta = 0.04$ as the zenith angle slope error [1]. The observed event number has an associated Poissonian uncertainty $\sigma_{ijk}^{\text{data}} = \sqrt{N_{ijk}^{\text{data}}}$.

For IceCube, the event count takes the form

$$N_{ijk}^{\text{th}} = \alpha \left[1 + \beta(0.5 + \cos(\theta_z^{reco})_i)\right] N_{ijk}(\hat{\theta}), \tag{7}$$

with $N_{ijk}(\hat{\theta})$ from Eq. 2 Here, we allow the event distribution to rotate around the median zenith of -0.5.

For DeepCore and PINGU, the event count takes the form

$$N_{ijk}^{\text{th}} = \alpha \left[1 + \beta \cos(\theta_z^{reco})_i\right] N_{ijk}(\hat{\theta}) + \kappa N_{ijk}^{\mu_{atm}}, \tag{8}$$

with $N_{ijk}(\hat{\theta})$ from Eq. 4. $N_{ijk}^{\mu_{atm}}$ is the muon background, which is left to float freely in the DeepCore analysis. The background at PINGU can be considered neglible to first order [10], and we thus put $\kappa = 0$ when calculating the
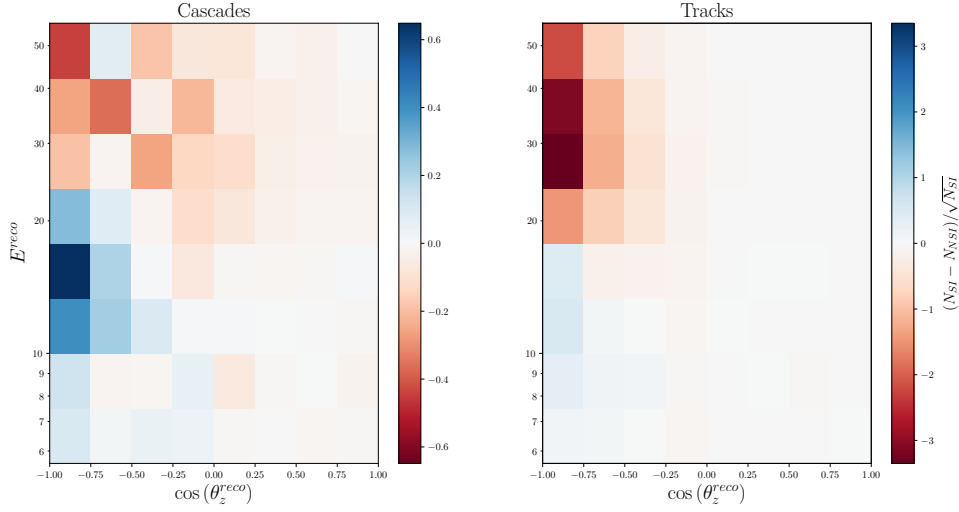
FIG. 2. Expected pulls of predicted event numbers for DeepCore. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count
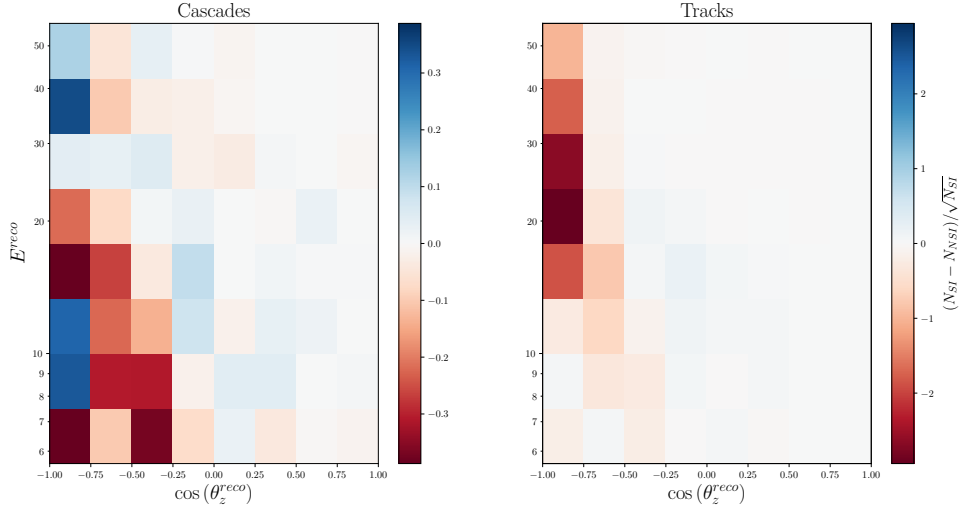

FIG. 3. Expected pulls of predicted event numbers for PINGU. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count

PINGU χ^2. For IceCube, we set $\sigma_{ijk}^{\text{syst}} = f\sqrt{N_{ijk}^{\text{data}}}$. For DeepCore, we use the provided systematic error distribution which accounts for both uncertanties in the finite MC statistics and in the data-driven muon background estimate [6].

We plot the event pull $(N_{NSI} - N_{SI})/\sqrt{N_{SI}}$ where $N_{(N)SI}$ are the numbers of expected events assuming (non-)standard interactions. This gives the normalized difference in the number of expected events at the detector, and illustrates the expected sensitivity for the NSI parameters.

For the joint analysis, we follow the parameter goodness-of-fit prescription [12] and construct the joint χ^2 as

$$\chi^2_{joint} = \chi^2_{IC} + \chi^2_{DC} + \chi^2_P - \chi^2_{IC,min} - \chi^2_{DC,min} - \chi^2_{P,min} \tag{9}$$

with test statistic $\chi^2_{joint,min}$.

At 90% CL: PINGU: $-0.046 < \epsilon_{\mu\mu} < 0.041$ DC: $-0.045 < \epsilon_{\mu\mu} < 0.041$ joint: $-0.038 < \epsilon_{\mu\mu} < 0.033$
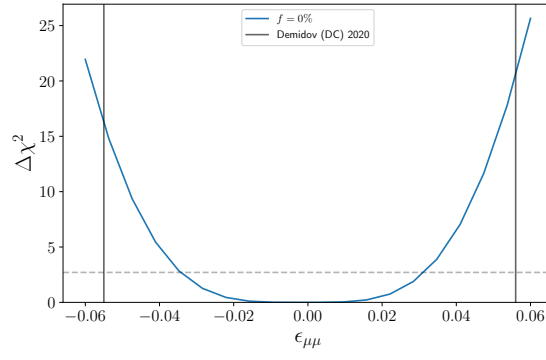
FIG. 4. Expected pulls of predicted event numbers for PINGU. We compare the NSI event count with $\epsilon_{\mu\tau} = -0.05$ to the standard interaction count
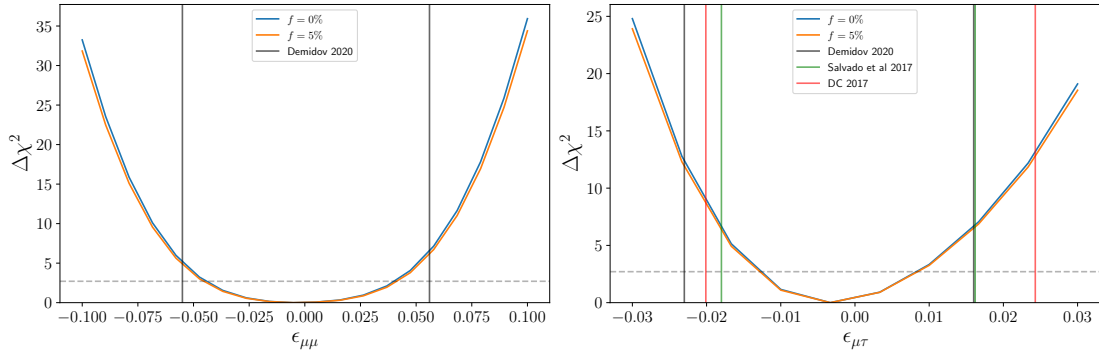

FIG. 5. Confidence levels from this analysis on the NSI parameters for systematic error and without. The black lines show the 90% credibility region from [11].
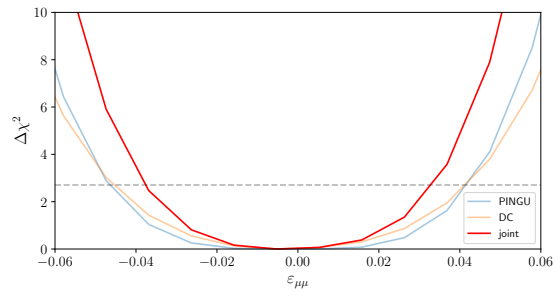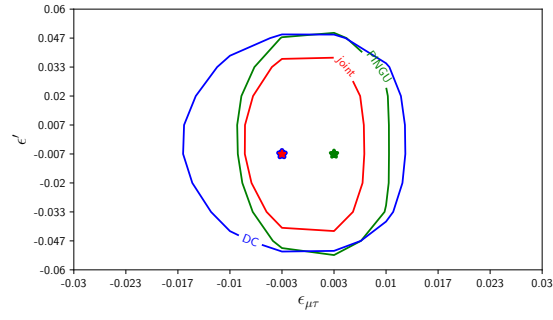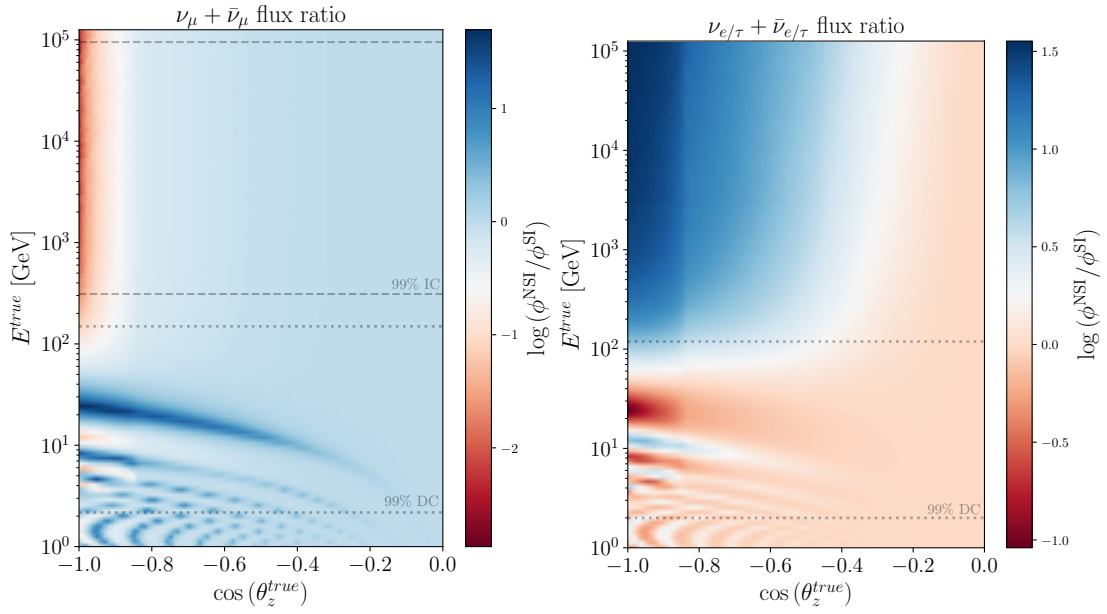

FIG. 6.

FIG. 7.

FIG. 8. Ratio of NSI to SI atmospheric fluxes at detector level. Dotted (dashed) lines show the region in which 99% of the DeepCore (IceCube) MC events are contained.

DeepCore (2017)	Demidov (2020) DC analysis	Choubey et. al. PINGU analysis
✓ Honda atmospheric fluxes	✓ Honda atmospheric fluxes	✓ Honda atmospheric fluxes
✗ Only look at tracks and $\epsilon_{\mu\tau}$	✓ Looks at tracks + cascades for $\epsilon_{\mu\tau}$ and $\epsilon_{\tau\tau}$	✗ Only consider ν_μ events
✗ DC Monte Carlo from an older dataset	✓ Data and Monte Carlo from DC 2018	✗ A_{eff} and resolution from PINGU Letter of intent
✗ 8 E bins from $6.3\,\text{eV}^2$ to $56\,\text{eV}^2$	✓ 8 E bins from $5.6\,\text{eV}^2$ to $56\,\text{eV}^2$	✗ 27 E bins from 2 to $101\,\text{eV}^2$
✗ 8 z bins from -1 to 0	✓ 8 z bins from -1 to 1	✗ 10 z bins from -1 to 0
✗ Use "Overall" and "relative ν_e to ν_μ" normalization	✗ Use "Overall" and "relative ν_e to ν_μ" normalization	✓ Flux normalization uncertainty of 20%
✗ Prior on spectral index	✗ Prior on spectral index	✗ Uncertainties in cross-section and spectral index
✗ No zenith angle normalization	✗ No zenith angle normalization	✓ Zenith angle uncertainty of 5%
✓ No priors on $\Delta m_{31}^2, \theta_{23}, \theta_{13}$	✓ No priors on $\Delta m_{31}^2, \theta_{23}$	✗ Priors on $\Delta m_{31}^2, \theta_{23}, \theta_{13}$
	✓ Fixes $\Delta m_{21}^2, \theta_{12}, \theta_{13}$	
	✗ Uncertainty on hadron production in atmosphere	
	✗ Uncertainty on neutrino nucleon cross section	

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