TMA4140 - Homework Set 2

Basic structures: Sets, Functions, Sequences and Sums

RETTES

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September 14, 2018

Contents

1 Chapter 2.1 - Sets

1.1 Exercise 5

Determine whether each of these pairs are equal.
a) $\{1, 3, 3, 3, 5, 5, 5, 5\}, \{5, 3, 1\} \Rightarrow True.$
b) $\{\{1\}\}, \{1, \{1\}\} \Rightarrow False.$
c) $\emptyset, \{\emptyset\} \Rightarrow False$

1.2 TODO: Exercise 24

Determine wether each of these sets is the power set of a set, where a and b are distinct elements.
a) $\emptyset$
b) $\{\emptyset, \{a\}\}$
c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

2 Chapter 2.2 - Set Operations

2.1 TODO: Exercise 18c

Let A, B and C be sets. Show that: $A \cap B) \subseteq (A \cup B \cup C)$

$$A \cap B) \subseteq (A \cup B \cup C) \tag{1}$$

2.2 TODO: Exercise 18d

Let A, B and C be sets. Show that: $(A - B) - C \subseteq A - C$

$$(A - B) - C \subseteq A - C \tag{2}$$

2.3 TODO: Exercise 46

Show that if A, B, and C are sets, then: [1]

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \tag{3}$$

$$\begin{aligned} |A \cup B \cup C| =& |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ |A \cup B \cup C| =& |A \cup B \cup C| \end{aligned} \tag{4}$$

[1] This is a special case of the inclusion-exclusion principle, which will be studied in Chapter 8.

3 Chapter 2.3 - Functions

3.1 TODO: Exercise 12c

Determine whether each of these functions from Z to Z is one-to-one.

$$f(n) = n^3 \tag{5}$$

3.2 TODO: Exercise 38

Let $f(x) = ax + b$ and $g(x) = cx + d$, where a,b,c, and d are constants. Determine necessary and sufficient conditions on the constants a,b,c, and d so that $f \cdot g = g \cdot f$.

3.3 TODO: Exercise 42

Let f be the function from R to R defined by $f(x) = x^2$. Find:
a) $f^{-1}(\{1\})$
b) $f^{-1}(\{x|0 < x < 1\})$
c) $f^{-1}(\{x|x > 4\})$

4 Chapter 2.4 - Sequences and Summations

4.1 TODO: Exercise 12c

Show that the sequence a_n is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if $a_n = (-4)^n$

$$a_n = -3a_{n-1} + 4a_{n-2} = (-4)^n \tag{6}$$

4.2 Exercise 33d

Compute the double sum

$$\begin{aligned}\sum_{i=0}^{2}\sum_{j=1}^{3} ij &= (0*1+0*2+0*3)+(1*1+1*2+1*3)+(2*1+2*2+2*3)\\ &= (0+0+0)+(1+2+3)+(2+4+6)\\ &= 0+6+12\\ &= 18\end{aligned} \tag{7}$$

5 Chapter 2.5 - Cardinality of Sets

5.1 TODO: Exercise 16

Exercise: Show that a subset of a countable set is also countable.
Answer: A set is countable if it is finite or is the same size as N. To show that A is countable, it is sufficient to show that there is an injection from A to N.