TMA4140 - Homework Exercise Set 8

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Contents

1 Section 5.2

1.1 Todo: Exercise 4

Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is *true* for $n \geq 18$. **a)** Show statements $P(18), P(19), P(20)$ and $P(21)$ are *true*, completing the basis step of the proof. **b)** What is the inductive hypothesis of the proof? **c)** What do you need to prove in the inductive step? **d)** Complete the inductive step for $k \geq 21$. **e)** Explain why these steps show that this statement is *true* whenever $n \geq 18$.

1.2 Todo: Exercise 14

Suppose you begin with a pile of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute rs. Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n-1)/2$.

2 Section 5.3

2.1 Todo: Exercise 12

Prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.

2.2 Todo: Exercise 18

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, Show that $A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$ when n is a positive integer.

3 Section 5.4

3.1 Todo: Exercise 3

Trace Algorithm 3 when it finds $gcd(8, 13)$. That is, show all the steps used by Algorithm 3 to find $gcd(8, 13)$.

4 Section 9.1

4.1 Todo: Exercise 7

Determine whether the relation R on the set of all integers is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*, where $(x, y) \in R$ if and only if
a) $x \neq y$.
b) $xy \geq 1$.
c) $x = y + 1$ or $x = y - 1$.
d) $x \equiv y(mod 7)$.
e) x is a multiple of y.
f) x and y are both negative or both nonnegative.
g) $x = y^2$.
h) $x \geq y^2$.

4.2 Todo: Exercise 40

Let R_1 and R_2 be the "divides" and "is a multiple of" relations on the set of all positive integers, respectively.
That is, $R_1 = \{(a, b)|a \text{ divides } b\}$ and $R_2 = \{(a, b)|a \text{ is a multiple of } b\}$. Find...

4.2.1 Todo: Exercise 40.a

$R_1 \cup R_2$.

4.2.2 Todo: Exercise 40.c

$R_1 - R_2$.

5 Section 9.3

5.1 Todo: Exercise 10

How many nonzero entries does the matrix representing the relation R on $A = \{1, 2, 3, ..., 1000\}$ consisting of the first 1000 positive integers have if R is...
a) $\{(a, b)|A \leq b\}$?
b) $\{(a, b)|a = b \pm 1\}$?
c) $\{(a, b)|a + b = 1000\}$?
d) $\{(a, b)|a + b \leq 1001\}$?
e) $\{(a, b)|a \neq 0\}$?

5.2 Todo: Exercise 14

Let R_1 and R_2 be the relations represented by the matrices...
$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Find the matrices that represent...

5.2.1 Todo: Exercise 14.a

$R_1 \cup R_2$

$$R_1 \cup R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

5.2.2 Todo: Exercise 14.b

$R_1 \cap R_2$.

$$R_1 \cap R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{2}$$

5.2.3 Todo: Exercise 14.c

$R_2 \circ R_1$.

$$R_2 \circ R_1 \tag{3}$$