TMA4140 - Homework Set 3
Basic structures: Sets, Functions, Sequences and Sums
RETTES

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Contents

1   Chapter 2.6 Matrices

1.1   Exercise 27c

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find $A \odot B$.

$$
\begin{aligned}
A \odot B &= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 1) \\ (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 1) \\ (0 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 1) \end{bmatrix} \\
&= \begin{bmatrix} 0 \vee 0 \vee 1 & 1 \vee 0 \vee 0 & 1 \vee 0 \vee 1 \\ 0 \vee 1 \vee 0 & 1 \vee 0 \vee 0 & 1 \vee 1 \vee 0 \\ 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 & 0 \vee 0 \vee 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\end{aligned}
\tag{1}
$$

2 Chapter 3.1 Algorithms

2.1 Exercise 53

Use the greedy algorithm to make change using quarters (25), dimes (10), nickels (5), and pennies (1) for:
a) 51 cents = 2 quarters (25) + 1 penny (1) $\Rightarrow$ 3 coins
b) 69 cents = 2 quarters (25) + 1 dime (10) + 1 nickel (5) + 4 pennies (1) $\Rightarrow$ 8 coins
c) 76 cents = 3 quarters (25) + 1 penny (1) $\Rightarrow$ 4 coins
d) 60 cents = 2 quarters (25) + 1 dime (10) $\Rightarrow$ 3 coins

2.2 Exercise 55

Use the greedy algorithm to make change using quarters, dimes, and pennies (but no nickels) for each of the amounts given in Exercise 53. For which of these amounts does the greedy algoritm use the fewest coins of these denominations possible?
a) 51 cents $\Rightarrow$ 2 quarters (25) + 1 penny (1) $\Rightarrow$ 3 coins
Exercise 53 and 55 provides the same answer.
b) 69 cents = 2 quarters (25) + 1 dimes (10) + 9 pennies (1) $\Rightarrow$ 12 coins
Exercise 53 uses the fewest amount of coins of the two.
c) 76 cents = 3 quarters (25) + 1 penny (1) $\Rightarrow$ 4 coins
Exercise 53 and 55 provides the same answer.
d) 60 cents = 2 quarters (25) + 1 dime (10) $\Rightarrow$ 3 coins
Exercise 53 and 55 provides the same answer.

2.3 Exercise 56

Show that if there were a coin worth 12 cents, the greedy algoritm using quarters, 12-cent coins, dimes, nickels, and pennies would not always produce change using the fewest coins possible.

If we wanted change for 15 cents then the greedy algorithm would give us 1 (12) + 3 pennies (1) = 15 cents $\Rightarrow$ 4 coins. But a more fitting change would be 1 dime + 1 nickel $\Rightarrow$ 2 coins.

3   Chapter 3.2 The Growth of Functions

3.1   Exercise 27a, 27b

Give a big-O estimate for each of these functions. For the function g in your estimate that $f(x)$ is $O(g(x))$, use a simple function g of the smallest order.
a)$nlog(n^2 + 1) + n^2 logn = O(n^2 logn)$
b)$(nlogn + 1)^2 + (logn + 1)(n^2 + 1) = O(n^2(logn)^2)$

3.2   Exercise 30c, 30e

Show that each of these pairs of functions are of the same order.
c)$\lfloor x + 1/2 \rfloor, x$
Let $f(x) = \lfloor x + 1/2 \rfloor$ and $g(x) = x$. $x < 2\lfloor x + 1/2 \rfloor$ and for $x > 2$: $|\lfloor x + 1/2 \rfloor| > \frac{1}{2} \cdot |x|$
That means $\lfloor x + 1/2 \rfloor$ is $\Omega(x)$.
$\lfloor x + 1/2 \rfloor < 2x \, for \, x > 2$ it follows that $|\lfloor x + 1/2 \rfloor| < 2cdot|x|$
Thus, $\lfloor x + 1/2 \rfloor$ is $O(x)$.
And we conclude that $\lfloor x + 1/2 \rfloor$ and x are of the same order.
e)$log_{10}x, log_2x$
Let $f(x) = log_{10}x$ and $g(x) = log_2x$.

$$f(x) = log_{10}x$$
$$= \frac{logx}{log10}$$
$$= \frac{log2}{log2} \frac{logx}{log10} \tag{2}$$
$$= \frac{log2}{log10} \frac{logx}{log2}$$
$$= \frac{1}{log_2 10} log_2x$$

Thus $|log_{10}x| \le \frac{1}{log_2 10}x|$ and it follows that $log_{10}x$ is $O(log_2x)$

3.3   Exercise 34

Show that $3x^2 + x + 1$ is $\theta(3x^2)$ by directly finding the constants k, C_1 and C_2 in Exercise 33.

Exercise 33: Show that if $f(x)$ and $g(x)$ are functions from the set of real numbers to the set of real numbers, then $f(x)$ is $\theta(g(x))$ if and only if there are positive constants k, C_1 and C_2 such that $C_1|g(x) \leq |f(x)| \leq C_2|g(x)|$ whenever $x > k$.

$$3x^2 + x + 1 = \theta(3x^2) \tag{3}$$

3.4   Exercise 42

Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x)}$ is $O(2^{g(x)})$?
No it does not. For example $f(x) = 2x$ and $g(x) = x$. Then $f(x)$ is $O(g(x))$. But $2f(x)$ is not $O(2^{g(x)})$.

4 Chapter 4.1 Divisibility and Modular Arithmetic

4.1 Exercise 11

What time does a 12-hour clock read:
a) 80 hours after it reads 11:00?
Answer: 7:00
b) 40 hours before it reads 12:00?
Answer: 8:00
c) 100 hours after it reads 6:00?
Answer: 10:00