TMA4140 - Homework Set 3

Basic structures: Sets, Functions, Sequences and Sums

RETTES

Henry S. Sjøen

September 17, 2018

Contents

1 Chapter 2.6 Matrices

1.1 TODO: Exercise 27c

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find $A \cdot B$.

2 Chapter 3.1 Algorithms

2.1 TODO: Exercise 53

Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for:
a) 51 cents
b) 69 cents
c) 76 cents
d) 60 cents

2.2 TODO: Exercise 55

Use the greedy algorithm to make change using quarters, dimes, and pennies (but no nickels) for each of the amounts given in Exercise 53. For which of these amounts does the greedy algoritm use the fewest coins of these denominations possible?
a) 51 cents
b) 69 cents
c) 76 cents
d) 60 cents

2.3 TODO: Exercise 56

Show that if there were a coin worth 12 cents, the greedy algoritm using quarters, 12-cent coins, dimes, nickels, and pennies would not always produce change using the fewest coins possible.

3 Chapter 3.2 The Growth of Functions

3.1 TODO: Exercise 27a, 27b

Give a big-O estimate for each of these functions. For the function g in your estimate that $f(x)$ is $O(g(x))$, use a simple function g of the smallest order.
a)$nlog(n^2+1)+n^2logn$
b)$(nlogn+1)^2+(logn+1)(n^2+1)$

3.2 TODO: Exercise 30c, 30e

Show that each of these pairs of functions are of the same order.
c)$\lfloor x+1/2 \rfloor, x$
e)$log_{10}x, log_2 x$

3.3 TODO: Exercise 34

Show that $3x^2+x+1$ is $\theta(3x^2)$ by directly finding the constants k, C_1 and C_2 in Exercise 33.

Exercise 33: Show that if $f(x)$ and $g(x)$ are functions from the set of real numbers to the set of real numbers, then $f(x)$ is $\theta(g(x))$ if and only if there are positive constants k, C_1 and C_2 such that $C_1|g(x) \leq |f(x)| \leq C_2|g(x)|$ whenever $x > k$.

$$3x^2+x+1 = \theta(3x^2) \tag{1}$$

3.4 TODO: Exercise 42

Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x)}$ is $O(2^{g(x)})$?

4 Chapter 4.1 Divisibility and Modular Arithmetic

4.1 TODO: Exercise 11

What time does a 12-hour clock read:
a) 80 hours after it reads 11:00?
b) 40 hours before it reads 12:00?
c) 100 hours after it reads 6:00?