TMA4140: Homework Set 1
RETTES

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Contents

1 1.1 Propositional Logic

Let p,q and r be the propositions:
p: You have the flu.
q: You miss the final examination.
r: You pass the course.

1.1 12c

Proposition: $q \to \neg r$
English sentence: You miss the final examination therefore you do not pass the course.

1.2 12f

Proposition: $(p \wedge q) \vee (\neg q \wedge r)$
English sentence: You have the flu and you miss the final examination or you don't have the flu and you pass the course.

1.3 14

p: You get an A on the final.
q: You do every exercise in this book.
r: you get an A in this class.

1.3.1 14.a

English sentence: You get en A in this class, but you do not do every exercise in this book.
Proposition: $r \wedge \neg q$

1.3.2 14.e

English sentence: Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
Proposition: $(p \wedge q) \rightarrow r$

2 1.3 Propositional Equivalences

2.1 10

Show that each of these conditional statements is a taultology by using thruth tables.

2.1.1 10.a

Conditional statement: $[\neg p \wedge (p \vee q)] \rightarrow q$
Proven: It's a Tautology!

p	q	$\neg p$	$p \wedge q$	$(\neg p \wedge (p \vee q))$	$(\neg p \wedge (p \vee q)) \rightarrow q$
0	0	1	0	0	1
0	1	1	0	0	1
1	0	0	0	0	1
1	1	0	1	0	1

2.1.2 10.b

Conditional statement:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow q \wedge q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$
0	0	0	1	1	1	1	1
1	0	0	0	1	0	0	1
0	1	0	1	0	0	1	1
1	1	0	1	0	0	0	1
0	0	1	1	1	1	1	1
1	0	1	0	1	0	1	1
0	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

Proven: It's a Tautology!

2.1.3 10.c

Conditional statement: $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

Proven: It's a Tautology!

2.1.4 10.d

Conditional statement: $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Proven: It's a Tautology! See table 1 on page 7.

3 1.4 Predicates and Quantifiers

3.1 24

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

3.2 24d

All students in your class can solve quadratic equations.

1. Domain: People in my class.

$\forall x Q(x)$

2. Domain: All People

$P(x)$: x in my class $Q(x)$: x can solve quadratic equations

$\forall x(P(x) \rightarrow Q(x))$

3.3 24e

Some students in your class does not want to be rich.

$R(x)$: x want to be rich.

1. Domain: People in my class.
$\exists x \neg R(x)$
2. Domain: All People
$\exists x(P(x) \land \neg R(x))$

4 1.5 Nested Qupantifiers

4.1 12

Let $C(x, y)$ be the statement *x and y have chatted over the Internet*, where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.

4.2 12b

Rachel has not chatted over the Internet with Chelsea.
Can be expressed as $\neg C(Rachel, Chelsea)$
Where C(x,y) is the relation, *x have chatted with y*.

4.3 12e

Sanjay has chatted with everyone except Joseph.
$\forall y C(Sanjay, y) \land \neg C(Sanjay, Joseph)$

4.4 30

Rewrite each of these statements so that negations appear only within predicates.

4.5 30c

$\neg \exists y[Q(y) \land \forall x \neg R(x, y)]$
$\forall y \neg [Q(y) \land \forall x \neg R(x, y)]$
$\forall y[\neg Q(y) \lor \neg \forall x \neg R(x, y)]$
$\forall y[\neg Q(y) \lor \exists x \neg \neg R(x, y)]$
$\forall y[\neg Q(y) \lor \exists x R(x, y)]$

4.6 30e

$\neg\exists y[\forall x\exists z T(x,y,z) \vee \exists x\forall z U(x,y,z)]$
$\forall y\neg[\forall x\exists z T(x,y,z) \vee \exists x\forall z U(x,y,z)]$
$\forall y[\neg\forall x\exists z T(x,y,z) \wedge \neg\exists x\forall z U(x,y,z)]$
$\forall y[\exists x\neg\exists z T(x,y,z) \wedge \forall x\neg\forall z U x,y,z]$
$\forall y[\exists x\forall z\neg T(x,y,z) \wedge \forall x\exists z\neg U x,y,z]$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Table 1: Table for Exercise 10d