

TMA4140 - Notes

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Contents

1	The foundations: Logic and Proofs	6
1.1	Propositional Logic	6
1.2	Applications of Propositional Logic	6
1.3	Propositional Equivalences	6
1.4	Predicates and Quantifiers	6
1.5	Nested Quantifiers	6
1.6	Rules of Inference	6
1.7	Introduction to Proofs	6
2	Basic Structures: Sets, Functions, Sequences, Sums, and Matrices	6
2.1	Sets	6
2.2	Set Operations	6
2.3	Functions	6
2.4	Sequences and Summations	6
2.5	Cardinality of Sets	6
2.6	Matrices	6
3	Algorithms	6
3.1	Algorithms	6
3.2	The Growth of Functions	6
3.3	Complexity of Algorithms	6
4	Number Theory and Cryptography	6
4.1	Divisibility and Modular Arithmetic	6
4.2	Integer Representations and Algorithms	6
4.3	Primes and Greatest Common Divisor	6
4.4	Solving Congruences	6
4.5	Applications of Congruences	6
4.6	Cryptography	6
5	Induction and Recursion	6
5.1	Mathematical Induction	6
5.2	Strong Induction and Well-Ordering	6
5.3	Recursive Definitions and Structural Induction	6
5.4	Recursive Algorithms	6

6	Counting	6
6.1	The Basics of Counting	6
6.2	The Pigeonhole Principle	6
6.3	Permutations and Combinations	6
6.4	Generalized Permutations and Combinations	6
7	Discrete Probability — Not Pensum...	6
8	Advanced Counting Techniques	6
8.1	Applications of Recurrence Relations	6
8.2	Solving Linear Recurrence Relations	6
9	Relations	6
9.1	Relations and Their Properties	6
9.2	n -ary Relations and Their Applications	6
9.3	Representing Relations	6
9.4	Closures of Relations	6
9.5	Equivalence Relations	6
9.6	Partial Orderings	6
10	Graphs	6
10.1	Graphs and Graph Models	6
10.2	Graph Terminology and Special types of Graphs	6
10.3	Representing Graphs and Graph Isomorphism	6
10.4	Connectivity	6
10.5	Euler and Hamilton Paths	6
10.6	Shortest-Path Problems	6
11	Trees	6
11.1	Introduction to Trees	6
11.2	Applications of Trees	6
11.3	Tree Traversal	6
11.4	Spanning Trees	6
12	Boolean Algebra — Not pensum...	6
13	Modeling Computation	6
13.1	Languages and Grammars	6
13.2	Finite-State Machines with Output	6

13.3 Finite-State Machines with No Output	6
13.4 Language Recognition	6
13.5 Turing Machines	6

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1.5 Nested Quantifiers

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1.7 Introduction to Proofs

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4 Number Theory and Cryptography

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4.4 Solving Congruences