TMA4140 - Homework Exercise Set 6

Henry S. Sjøen & Toralf Tokheim

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1 Chapter 6.3

1.1 Exercise 13

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate? Total number of people in the group is n-men and n-women therefore $2n$ in total. For men we have: $n! \times n! = (n!)^2$ and it's the same for women. Therefore when combined we get the total number of arrangements, which are $(n!)^2 + (n!)^2 = 2(n!)^2$ if the row alternates between men and women.

1.2 Exercise 34

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

3 possible type of ways to form a committee:
Possibility number 1: 6 women and 0 men gives us 5005 ways.

$$C(15,6) \times C(10,0) = 5005 \tag{1}$$

Possibility number 2: 5 women and 1 man gives us 30030 ways.

$$C(15,5) \times C(10,1) = 30030 \tag{2}$$

Possibility number 3: 4 women and 2 men gives us 61425 ways.

$$C(15,4) \times C(10,2) = 61425 \tag{3}$$

So to combine all the possible ways to form a committee we get:

$$\begin{aligned} C(15,6) \times C(10,0) \times C(15,5) \times C(10,1) \times C(15,4) \times C(10,2) \\ = 5005 + 30030 + 61425 = 96460 \end{aligned} \tag{4}$$

2 Chapter 6.4

2.1 Exercise 4

Find the coefficient of x^5y^8 in $(x+y)^{13}$.

From the binomial theorem it follows that this coefficient is

$$\binom{13}{8} = \frac{13!}{8!(13-8)!} = \frac{13!}{8!5!} = 1\ 287 \tag{6}$$

$$\begin{aligned}(x+y)^n &= \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j \\ &= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + ... + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n\end{aligned} \tag{5}$$

2.2 Exercise 9

What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x-3y)^{200}$?

$$\begin{aligned}\binom{200}{99} * 2^{101} * (-3)^{99} &= -\,(2^{101} * 3^{99} * \binom{200}{99}) \\ &= -\,(2^{101} * 2^{99} \frac{200!}{99!(200-99)!}) \\ &= -\,(2^{101} * 2^{99} \frac{200!}{99!101!})\end{aligned} \tag{7}$$

2.3 Exercise 12

The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}, 0 \leq k \leq 10$, is: $1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1$.

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k} \tag{8}$$

Setting $= 10$ and $k = 1$.

$$\begin{aligned}\binom{10+1}{1} &= \binom{10}{1-1} + \binom{10}{1} \\ \frac{11!}{1!(10)!} &= \frac{10!}{0!(10)!} + \frac{10!}{1!(9)!} \\ 11 &= 1 + 10 \\ 11 &= 11\end{aligned} \tag{9}$$

Pascal's identity is verified.
Using the binomial coefficients $\binom{n+1}{k}$ to find the immediate following row in Pascal's triangle.

Substituting $n = 10$ and $k = 0$ in $\binom{n+1}{k}$

$$\binom{10+1}{0} = \binom{11}{0} = 1 \tag{10}$$

Substituting $n = 10$ and $k = 1$ in $\binom{n+1}{k}$ to find the immediately following number in the row... and so on.

$$\binom{10+1}{1} = \binom{11}{1} = 11 \tag{11}$$

Substituting $n = 10$ and $k = 2$

$$\binom{10+1}{2} = \binom{11}{2} = 55 \tag{12}$$

Substituting $n = 10$ and $k = 3$

$$\binom{10+1}{3} = \binom{11}{3} = 165 \tag{13}$$

Substituting $n = 10$ and $k = 4$

$$\binom{10+1}{4} = \binom{11}{4} = 330 \tag{14}$$

Substituting $n = 10$ and $k = 5$

$$\binom{10+1}{5} = \binom{11}{5} = 462 \tag{15}$$

Substituting $n = 10$ and $k = 6$

$$\binom{10+1}{6} = \binom{11}{6} = 462 \tag{16}$$

Substituting $n = 10$ and $k = 7$

$$\binom{10+1}{7} = \binom{11}{7} = 330 \tag{17}$$

Substituting $n = 10$ and $k = 8$

$$\binom{10+1}{8} = \binom{11}{8} = 165 \tag{18}$$

Substituting $n = 10$ and $k = 9$

$$\binom{10+1}{9} = \binom{11}{9} = 55 \tag{19}$$

Substituting $n = 10$ and $k = 10$

$$\binom{10+1}{10} = \binom{11}{10} = 11 \tag{20}$$

Substituting $n = 10$ and $k = 11$

$$\binom{10+1}{11} = \binom{11}{11} = 1 \tag{21}$$

Which gives us the row

$$\begin{aligned}[\binom{11}{0}, \binom{11}{1}, \binom{11}{2}, \binom{11}{3}, \binom{11}{4}, \binom{11}{5}, \binom{11}{6}, \binom{11}{7}, \binom{11}{8}, \binom{11}{9}, \binom{11}{10}, \binom{11}{11}] \\ = [1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1]\end{aligned} \tag{22}$$

3 Chapter 6.5

3.1 Exercise 6

How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

There are $C(n+r-1, r) = C(n+r-1, n-1) r$-combinations from a set with n elements when repetition is allowed. Setting $n = 3$ and $r = 5$.

$$C(3+5-1, 5) = C(7, 5) \tag{23}$$

Since $C(n, r) = C(n, n-r)$, we get...

$$C(7, 2) = \frac{7 \times 6}{2} = \frac{42}{2} = 21 \tag{24}$$

There are therefore 21 different ways to select 5 unordered elements from a set of 3 elements when repetition is allowed.

3.2 Exercise 14

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$, where x_1, x_2, x_3, and x_4 are nonnegative integers?

There are $C(n+r-1, r) = C(n+r-1, n-1)r$-combinations from a set with n elements when repetition is allowed.

Select 17 items from a set with four elements... Set $n = 4$ and $r = 17$.

$$\begin{aligned} C(4+17-1, 17) &= C(4+17-1, 3) \\ &= C(20, 3) \\ &= 1140 \end{aligned} \tag{25}$$

There are 1140 Solutions.

3.3 TODO: Exercise 30

How many different strings can be made from the letters in *MISSISSIPPI*, using all the letters?

There are 11 number of letters, so... $n = 11$. There are 4 unique letters, therefore...$k = 4$.

$$\frac{n!}{n_1!n_2!...n_k!} = \frac{11!}{1!4!4!2!} = 34650 \tag{26}$$

We can form 34650 different strings from the letters.

3.4 TODO: Exercise 54

How many ways are there to distribute five indistinguishable objects into three indistinguishable boxes?

Assuming we can have empty boxes, Then we can have the following distinct arrangements:

$$\begin{gathered} [0][0][5] \\ [0][1][4] \\ [0][2][3] \\ [1][1][3] \\ [1][2][2] \end{gathered} \tag{27}$$

So there are 5 distinct orderings / total number of partitions.

4 Chapter 6.6

4.1 TODO: Exercise 5a

Find the next larger permutation in lexicographic order after the permutation of 1432. Next order is 2134.

4.2 TODO: Exercise 5c

Find the next larger permutation in lexicographic order after the permutation of 12453. Next order is 12534.