Multiresolution Analysis

Basis of translates

$$V_0 = \text{span}\{\varphi(x-k)\}_{k \in Z} \quad W_0 = \text{span}\{\psi(x-k)\}_{k \in Z}$$

Translation and Dilation
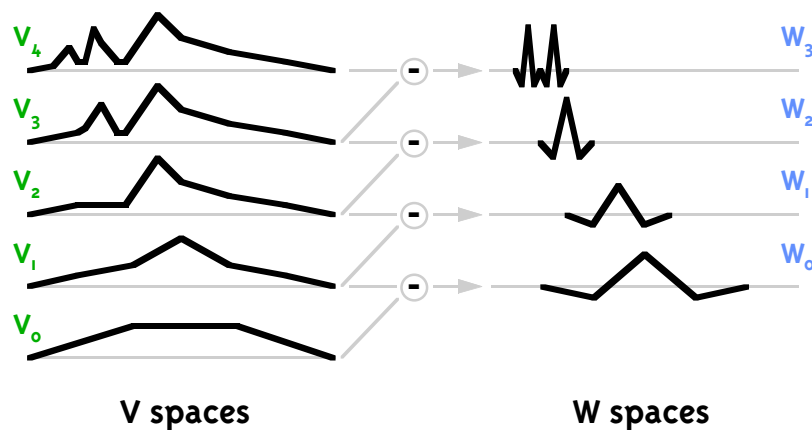
$$f(x) \in V_j \Rightarrow f(x+1) \in V_j \quad f(x) \in V_j \Rightarrow f(2x) \in V_{j+1}$$

Ladder of spaces

$$V_j \subset V_{j+1}$$

$$V_{j+1} = V_j \oplus W_j$$

$$\bigoplus_{j \in Z} W_j = L_2(R)$$

1

Multiresolution Analysis

V spaces   W spaces

2

Other Classic Constructions

Daubechies
- orthogonal wavelets of arbitrary smoothness

Tons of others...
Lifting
- any finite filter

Other Properties

Orthogonality
- preserves energy
- bi-orthogonality more flexible

Smoothness
- localization towards high frequencies

Vanishing moments
- localization towards low frequencies

Stability

Web Resources

Wavelet digest

- http://www.math.sc.edu/~wavelet
  - newsletter, search engine, bibs, software

Amara's wavelet page

- http://www.amara.com/current/wavelet.html
  - short intro to wavelets
  - overview of many software packages

Web Resources

Mathsoft wavelet resources

- http://www.mathsoft.com/wavelets.html
  - links to many online papers

Course page

- http://www.cs.caltech.edu/~ps/waveletcourse/
  - online course materials and these slides
  - liftpack

Books on Wavelets

Wavelets and Subband Coding

- Vetterli/Kovacevic

- signal processing point of view

  - http://cm.bell-labs.com/math/people/jelena/book.html

Wavelets for Computer Graphics: Theory and Applications

- Stollnitz/DeRose/Salesin

  - http://www.amath.washington.edu/~stoll/
    pub.html#WaveletBook

Books on Wavelets

Wavelets and Filter Banks

- Strang/Nguyen

- combines math and signal processing: very accessible with many exercices and Matlab toolbox

- http://saigon.ece.wisc.edu/~waveweb/
  Tutorials/book.html