System of linear equations

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1) Using Gauss elimination method, show that the following system of equations are consistent and unique solution

a) $x+2y=5$, $5x-3y=-1$.

Solution:

Given system of equations are

$$x+2y=5 \quad \text{--- (i)}$$
$$5x-3y=-1 \quad \text{--- (ii)}$$

Multiply eqn (i) by 5 we get

$\cancel{2x+10y=10}$ $\quad 5x+10 = 25$

$\cancel{2x+4y=10}$

Now, for eliminating we need to subtract from eqn

$\cancel{2x+4y=10}$ $\quad 5x+10y=25$

$$\underline{-5x \mp 3y = \mp 1}$$
$$13y = 26$$

We have equations.

$$x+2y=5 \quad \text{--- (iii)}$$
$$13y=26 \quad \text{--- (iv)}$$

So, from (iv) $y=2$

Put $y=2$ in eqn (iii) we get

$$x = 5-2\times2$$
$$= 1$$

Hence, value of $x=1$ and $y=2$.

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1-b) **Solution**

Given system of equations,

$3x - 2y = 5 \quad - (i)$

$4x - y = 10 \quad - (ii)$

Writting the system of equations in matrix form we,

$$\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

Now,

Apply, $R_2 \rightarrow 3R_2 - 4R_1$

$$\sim \begin{pmatrix} 3 & -2 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

we have ~~three~~ these equation.

$3x - 2y = 5 \quad - (iii)$

$5y = 10 \quad - (iv)$

From (iv) we get $y = 2$.

substituting y in (iii) we get, $x = 3$

Hence, the values $(x, y) = (3, 2)$

c) Solution

Given equation are

$x+3y-2z=0$ — (i)

$2x-3y+z=1$ — (ii)

$4x-3y+z=3$ — (iii)

now, representing the system of equation in matrix form

$$\begin{pmatrix} 1 & 3 & -2 \\ 2 & -3 & 1 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

Now, Apply $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\sim \begin{pmatrix} 1 & 3 & -2 \\ 0 & -9 & 5 \\ 0 & -15 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

Apply $R_3 \to 3R_3 - 5R_2$

$$\sim \begin{pmatrix} 1 & 3 & -2 \\ 0 & -9 & 5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

So, we get following equations.

$x+3y-2z=0$ — (iv)

$-9y+5z=1$ — (v)

$2z=4$ — (vi)

from (vi) we get, $z=2$.

from (v) we get, $y=1$

from (iv) we get, $x=1$

Hence $(x,y,z)=(1,1,2)$

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1.d) $x+3y-z=-2$, $3x+2y-z=3$, $-6x-4y-2z=18$

Solution.

Given equations are,

$x+3y-z=-2$ — (I)

$3x+2y-z=3$ — (II)

$-6x-4y-2z=18$ — (III)

Writing in matrix form we get,

$$\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & -1 \\ -6 & -4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 18 \end{pmatrix}$$

Now, Apply, $R_2 \to R_2 - 3R_1$
$R_3 \to R_3 + 6R_1$

$$\sim \begin{pmatrix} 1 & 3 & -1 \\ 0 & -7 & 2 \\ 0 & 14 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ 6 \end{pmatrix}$$

Apply, $R_3 \to R_3 + 2R_2$

$$\sim \begin{pmatrix} 1 & 3 & -1 \\ 0 & -7 & 2 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 9 \\ 24 \end{pmatrix}$$

We get the following equations-

$x+3y-z=-2$ — (IV)

$-7y+2z=9$ — (V)

$-4z=24$ — (VI)

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Solving we get, $z = -6$ from (iv)
from (v) we get, $y = -3$
from (iii) we get, $x = 1$.
thus, $(x, y, z) = (1, -3, -6)$
Hence, the system of equations are consistent and unique

e) $x_1 - 2x_2 + 3x_3 = 10$, $2x_1 + 3x_2 - 2x_3 = 1$, $-x_1 - 2x_2 + 4x_3 = 13$

Solution

Given, system of equation

$$x_1 - 2x_2 + 3x_3 = 10 \quad \text{--- (i)}$$
$$2x_1 + 3x_2 - 2x_3 = 1 \quad \text{--- (ii)}$$
$$-x_1 - 2x_2 + 4x_3 = 13 \quad \text{--- (iii)}$$

Now, this can be written in matrix form as:

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -2 \\ -1 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 1 \\ 13 \end{pmatrix}$$

Apply, $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$

$$\sim \begin{pmatrix} 1 & -2 & 3 \\ 0 & 7 & -8 \\ 0 & -4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -19 \\ 23 \end{pmatrix}$$

Apply, $R_3 \to 7R_3 + 4R_2$

$$\sim \begin{pmatrix} 1 & -2 & 3 \\ 0 & 7 & -8 \\ 0 & 0 & 17 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -19 \\ 85 \end{pmatrix}$$

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we get the following equations.

$$x_1 - 2x_2 + 3x_3 = 10 \quad \text{—(IV)}$$
$$7x_2 - 8x_3 = -19 \quad \text{—(V)}$$
$$17x_3 = 85 \quad \text{—(VI)}$$

Solving eqn (VI) we get, $x_3 = 5$.
from eqn (V) we get, $x_2 = 3$.
from eqn (IV) we get, $x_1 = 1$.

thus, the solution of system of equation is $(1, 3, 5)$.
Hence, the system of equations are consistent and unique.

Q.No.2 Examine whether the following system of equations are consistent. If so, solve the following system of equation using Gaussian elimination method if possible.

a) $x_1 - x_2 + x_3 = 1$, $3x_1 + x_2 + 5x_3 = 11$, $4x_1 + 2x_2 + 7x_3 = 16$.

Solution,

Here, the given system of equation is

$$x_1 - x_2 + x_3 = 1 \quad \text{—(I)}$$
$$3x_1 + x_2 + 5x_3 = 11 \quad \text{—(II)}$$
$$4x_1 + 2x_2 + 7x_3 = 16 \quad \text{—(III)}$$

writing in matrix form.

$$\begin{pmatrix} 1 & -1 & 1 \\ 3 & 1 & 5 \\ 4 & 2 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \\ 16 \end{pmatrix}$$

Now,

Apply, $R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 4R_1$.

$$\sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 12 \end{pmatrix}$$

Apply, $R_3 \to 4R_3 - 6R_2$.

$$\sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix}$$

So, we get the following equations,

$$x_1 - x_2 + x_3 = 1 \quad \text{—(IV)}$$

$$4x_2 + 2x_3 = 8 \quad \text{—(V)}$$

Here out of three equation only two exists after elimination and it satisfies $0 \cdot x_3 = 0$. Hence, the system is consistent and has infinite solution.

So, put $x_3 = K$ then,

from eq^n (V) $x_2 = 2 - K/2$

from eq^n (IV) we get, $x_1 = 3 - 3K/2$

Hence, the solution of equation is

$$(x_1, x_2, x_3) = \left(\frac{6-3K}{2}, \frac{4-K}{2}, K\right), \quad K \in \mathbb{R}$$

2.b $x + 3y + 4z = 8$, $2x + y + 2z = 5$, $5x + 2z = 7$

Solution,

Given, the system of linear equation

$$x + 3y + 4z = 8 \quad \text{—(I)}$$

$$2x + y + 2z = 5 \quad \text{—(IV)}$$

$$5x + 2z = 7 \quad \text{—(III)}$$

Now, writing in matrix form we get,

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$$\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 7 \end{pmatrix}$$

Apply, $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 5R_1$.

$$\sim \begin{pmatrix} 1 & 3 & 4 \\ 0 & -5 & -6 \\ 0 & -15 & -18 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -11 \\ -33 \end{pmatrix}$$

Apply, $R_3 \to R_3 - 3R_2$

$$\sim \begin{pmatrix} 1 & 3 & 4 \\ 0 & -5 & -6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -11 \\ 0 \end{pmatrix}$$

Here, number of equation is 2 and no of unknown variables is 3. so, it is consistent and has infinite solutions.

We can obtain the matrix equation as:

$x + 3y + 4z = 8$ — (IV)

$-5y - 6z = -11$ — (V)

Let, $z = K$ and then from eqn (V) we get, $y = \frac{11-6K}{5}$

from eqn (IV) we get, $x = 8 - \frac{3(11-6K)}{5} - 4K$

$= \frac{7-2K}{5}$

Hence the solution of system of equation is $\left(\frac{7-2K}{5}, \frac{11-6K}{5}, K\right)$

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c) $x_1+x_2+x_3=-3$, $3x_1+x_2-2x_3=-2$, $2x_1+4x_2+7x_3=7$.

Solution,

the given system of equation is

$$x_1+x_2+x_3=-3 \quad \text{— (i)}$$
$$3x_1+x_2-2x_3=-2 \quad \text{— (ii)}$$
$$2x_1+4x_2+7x_3=7 \quad \text{— (iii)}$$

matrix form of the system of equation is,

$$\begin{pmatrix}1 & 1 & 1\\ 3 & 1 & -2\\ 2 & 4 & 7\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}-3\\ -2\\ 7\end{pmatrix}$$

Now, Apply, $R_2 \to R_2-3R_1$
$R_3 \to R_3-2R_1$

$$\sim\begin{pmatrix}1 & 1 & 1\\ 0 & -2 & -5\\ 0 & 2 & 5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}-3\\ 7\\ 13\end{pmatrix}$$

Apply, $R_3 \to R_3+R_2$

$$\sim\begin{pmatrix}1 & 1 & 1\\ 0 & -2 & -5\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}-3\\ 7\\ 20\end{pmatrix}$$

Here, we get no of equation is 2 but no of unknown is 3 and $0\cdot x_3 \neq 0$ so, this is inconsistent and has no solution.

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d) $x_1 + 2x_2 + 3x_3 = 4$, $4x_1 + 5x_2 + 6x_3 = -7$, $7x_1 + 8x_2 + 9x_3 = 10$

⇒ Solution,

Given system of equation is,

$x_1 + 2x_2 + 3x_3 = 4$ — (i)

$4x_1 + 5x_2 + 6x_3 = -7$ — (ii)

$7x_1 + 8x_2 + 9x_3 = 10$ — (iii)

writing in matrix form as,

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ 10 \end{pmatrix}$$

Apply, $R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 7R_1$

$$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -23 \\ -6 \end{pmatrix}$$

Apply, $R_3 \to R_3 - 2R_2$

$$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -23 \\ 40 \end{pmatrix}$$

Here,

$0 \cdot x_3 \neq 40$ for all $x \in \mathbb{R}$

Hence, the system is inconsistent and has no solution.

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Exercise B

1) Find the inverse of each of the following matrices using Gauss - Jordan method

b) $M = \begin{pmatrix} 2 & -3 \\ -2 & 5 \end{pmatrix}$.,

Solution,

Given, $M = \begin{pmatrix} 2 & -3 \\ -2 & 5 \end{pmatrix}$

the augumented of given matrix with I is;

$$[M:I] = \left(\begin{array}{cc:cc} 2 & -3 & 1 & 0 \\ -2 & 5 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/2$
$R_2 \to R_2 + R_1$

$$\sim \left(\begin{array}{cc:cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right)$$

Apply, $R_2 \to R_2/2$

$$\sim \left(\begin{array}{cc:cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right)$$

Apply, $R_1 \to R_1 + 3/2 R_2$

$$\sim \left(\begin{array}{cc:cc} 1 & 0 & 5/4 & 3/4 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right)$$

$$\therefore M^{-1} = \begin{pmatrix} 5/4 & 3/4 \\ 1/2 & 1/2 \end{pmatrix}$$

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1.c) $B = \begin{pmatrix} 2 & -3 \\ -2 & 5 \end{pmatrix}$

Solution.

Given, matrix $B = \begin{pmatrix} 2 & -3 \\ -2 & 5 \end{pmatrix}$

Now, Augmented of given matrix B with Unit matrix I is

$$[B : I] = \left(\begin{array}{cc:cc} 2 & -3 & 1 & 0 \\ -2 & 5 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/2$
$R_2 \to R_2 + R_1$

$$\sim \left(\begin{array}{cc:cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right)$$

Apply, $R_2 \to R_2/2$

$$\sim \left(\begin{array}{cc:cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right)$$

Apply, $R_1 \to R_1 + 3/2 R_2$

$$\sim \left(\begin{array}{cc:cc} 1 & 0 & 5/4 & 3/4 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right)$$

Hence, $B^{-1} = \begin{pmatrix} 5/4 & 3/4 \\ 1/2 & 1/2 \end{pmatrix}$

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2) Find the inverse of matrix using Gauss-Jordan method.

b) $R = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{pmatrix}$

Solution,

Given matrix $R = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{pmatrix}$.

Now, Augmented matrix R with identity matrix is

$$[R:I] = \left(\begin{array}{ccc|ccc} 3 & 2 & 6 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/3$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right)$$

Apply, $R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1 & -1/3 & 1 & 0 \\ 0 & 5/3 & 0 & -2/3 & 0 & 1 \end{array}\right)$$

Apply $R_3 \to R_3 - 5R_2$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1 & -1/3 & 1 & 0 \\ 0 & 0 & -5 & 1 & -5 & 1 \end{array}\right)$$

Apply $R_3 \to R_3/(-5)$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1 & -1/3 & 1 & 0 \\ 0 & 0 & 1 & -1/5 & 1 & -1/5 \end{array}\right)$$

Apply, $R_2 \to R_2 \times 3$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & 3 & -1 & 3 & 0 \\ 0 & 0 & 1 & -1/5 & 1 & -1/5 \end{array}\right)$$

Apply, $R_2 \to R_2 \cancel{-3R_3} -3R_3$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2/3 & 2 & 1/3 & 0 & 0 \\ 0 & 1 & 0 & -2/5 & 0 & 3/5 \\ 0 & 0 & 1 & -1/5 & 1 & -1/5 \end{array}\right)$$

Apply, $R_1 \to R_1 - 2/3 R_2$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & 2 & 3/5 & 0 & \overset{-2/5}{\cancel{3/5}} \\ 0 & 1 & 0 & -2/5 & 0 & 3/5 \\ 0 & 0 & 1 & -1/5 & 1 & -1/5 \end{array}\right)$$

Apply, $R_1 \to R_1 - 2R_3$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -2 & [illegible] \\ 0 & 1 & 0 & -2/5 & 0 & 3/5 \\ 0 & 0 & 1 & -1/5 & 1 & -1/5 \end{array}\right)$$

Hence, $R^{-1} = \begin{pmatrix} 1 & -2 & 0 \\ -2/5 & 0 & 3/5 \\ -1/5 & 1 & -1/5 \end{pmatrix}$

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3) Solve the following system of equations using Gauss-Jordan inversion method.

b) $3x_1 - x_2 = 5$, $8x_1 - 3x_2 = 14$

Solution,

the system of equation given to us

$$3x_1 - x_2 = 5$$
$$8x_1 - 3x_2 = 14$$

Now, augmented matrix with identity can be written as.

$$[A:I] = \left(\begin{array}{cc:cc} 3 & -1 & 1 & 0 \\ 8 & -3 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/3$
$R_2 \to 3R_2 - 8R_1$

$$\sim \left(\begin{array}{cc:cc} 1 & -1/3 & 1/3 & 0 \\ 0 & -1 & -8 & 3 \end{array}\right)$$

Apply $R_2 \to R_2 \times (-1)$

$$\sim \left(\begin{array}{cc:cc} 1 & -1/3 & 1/3 & 0 \\ 0 & 1 & 8 & -3 \end{array}\right)$$

Apply, $R_1 \to R_1 + R_2/3$

$$\sim \left(\begin{array}{cc:cc} 1 & 0 & 3 & -1 \\ 0 & 1 & 8 & -3 \end{array}\right)$$

Hence, $A^{-1} = \begin{pmatrix} 3 & -1 \\ 8 & -3 \end{pmatrix}$

Now,

$$Y = A^{-1}X$$

$$= \begin{pmatrix} 3 & -1 \\ 8 & -3 \end{pmatrix} \begin{pmatrix} 5 \\ 14 \end{pmatrix} = \begin{pmatrix} 3\times5 - 1\times14 \\ 8\times5 - 3\times14 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

c) $4x - 3y = -17, \quad -6x + 5y = 27$

Soln:-

let coefficient matrix be $A = \begin{pmatrix} 4 & -3 \\ -6 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} -17 \\ 27 \end{pmatrix}$

Now, writing coefficient matrix as augmented matrix with I.

$$[A : I] = \left(\begin{array}{cc:cc} 4 & -3 & 1 & 0 \\ -6 & 5 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/4$

$R_2 \to 4R_2 + 6R_1$

$$\sim \left(\begin{array}{cc:cc} 1 & -3/4 & 1/4 & 0 \\ 0 & 2 & 6 & 4 \end{array}\right)$$

Apply, $R_2 \to R_2/2$

$R_1 \to R_1 + 3/8 R_2$

$$\sim \left(\begin{array}{cc:cc} 1 & 0 & 5/2 & 3/2 \\ 0 & 1 & 3 & 2 \end{array}\right)$$

Hence, $A^{-1} = \begin{pmatrix} 5/2 & 3/2 \\ 3 & 2 \end{pmatrix}$

Now,

$$X = A^{-1}C = \begin{pmatrix} 5/2 & 3/2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} -17 \\ 27 \end{pmatrix}$$

$$= \begin{pmatrix} 5/2 \times (-17) + 3/2 \times 27 \\ 3 \times (-17) + 2 \times 27 \end{pmatrix}$$

Hence $\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

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4 Solve the following system of equations using matrix inversion method:

a) $x_1 - 2x_2 - x_3 = 1$, $x_1 - x_2 + 2x_3 = 9$, $2x_1 - 3x_2 - x_3 = 4$.

<u>solution</u>:

the given system of matrix is,

$$x_1 - 2x_2 - x_3 = 1 \quad \text{— (i)}$$
$$x_1 - x_2 + 2x_3 = 9 \quad \text{— (ii)}$$
$$2x_1 - 3x_2 - x_3 = 4 \quad \text{— (iii)}$$

Now,

let A be the coefficient matrix then,

$$A = \begin{pmatrix} 1 & -2 & -1 \\ 1 & -1 & 2 \\ 2 & -3 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix}.$$

Now, $|A| = 1\begin{vmatrix} -1 & 2 \\ -3 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + -1\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}$

$= (1+6) + 2(-1-4) - 1(-3+2) = 7 - 10 + 1$

$= -2$

Here, $|A| \neq 0$ hence inverse exists

Now,

$$\text{Adj}(A) = \begin{pmatrix} 1+6 & 1-4 & -3+2 \\ +2-3 & -1+2 & -3+4 \\ -4-1 & 2+1 & -1+2 \end{pmatrix}^T = \begin{pmatrix} 7 & 5 & -1 \\ 1 & +1 & -1 \\ -5 & -3 & 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 7 & 1 & -5 \\ 5 & 1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$$

Now, $X = A^{-1}C = \dfrac{\text{Adj}(A)}{|A|} C$

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$$\begin{pmatrix} 7 & 1 & -5 \\ 5 & 1 & -3 \\ -1 & -1 & 1 \end{pmatrix} \times \frac{1}{-2} \times \begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix}$$

$$\cancel{= \frac{1}{-2} \begin{pmatrix} 7\times 1 + 9\times(1) + 4\times(-5) \\ -3\times 1 + 1\times 9 + 3\times 4 \\ -1\times 1 + 1\times 9 + 1\times 4 \end{pmatrix}}$$

$$\cancel{= \frac{-1}{2} \begin{pmatrix} -58 \\ 18 \\ 8 \end{pmatrix} = \begin{pmatrix} 29 \\ -9 \\ -3 \end{pmatrix}}$$

$$= \frac{-1}{2} \begin{pmatrix} 7\times 1 + 1\times 9 - 5\times 4 \\ 5\times 1 + 1\times 9 - 3\times 4 \\ -1\times 1 - 1\times 9 + 1\times 4 \end{pmatrix}$$

$$= \frac{-1}{2} \begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Hence, $C = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$3x + y + z = 15, \quad x + y + z = 3, \quad y - z = 1$

Solution,

Let, A be coefficient matrix then, we can write coefficient of equations as,

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \text{ and } X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 15 \\ 3 \\ -1 \end{pmatrix}$$

Now, $|A| = 3\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$= 3(-1-1) - 1(-1-0) + 1(1-0) = -4$

Here, $|A| \neq 0$ hence inverse exists.

Now, $\text{Adj}(A) = \begin{pmatrix} (-1-1) & -(-1-0) & (1\times1-0) \\ -(1\times-1-1) & (3\times-1) & -3\times1 \\ (1\times1-1\times1) & -(3-1) & (3-1) \end{pmatrix}^T$

$= \begin{pmatrix} -2 & 1 & 1 \\ 2 & -3 & -3 \\ 0 & -2 & 2 \end{pmatrix}^T = \begin{pmatrix} -2 & 2 & 0 \\ 1 & -3 & -2 \\ 0 & -3 & 2 \end{pmatrix}$

Now, $X = \dfrac{\text{Adj}(A)}{|A|} \times C$

$= \dfrac{1}{-4} \times \begin{pmatrix} -2 & 2 & 0 \\ 1 & -3 & -2 \\ 0 & -3 & 2 \end{pmatrix} \times \begin{pmatrix} 15 \\ 3 \\ -1 \end{pmatrix}$

$= \dfrac{-1}{4} \times \begin{pmatrix} -2\times15+2\times3 \\ 1\times15-3\times3-2\times(-1) \\ 0\times15-3\times3+2\times(-1) \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix}$

Hence, $C = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix}$

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5) Solve the following system of equations using Gauss Jordan matrix inversion method:

b) $4x_1+3x_2-5x_3=1$, $3x_1+7x_2-x_3=8$, $x_1+x_2-x_3=0$

Solution

Given system of equation is:

$$4x_1+3x_2-5x_3=1$$
$$3x_1+7x_2-x_3=8$$
$$x_1+x_2-x_3=0$$

writing the coefficient of equation as augmented matrix with identity matrix

$$\left(\begin{array}{ccc:ccc} 4 & 3 & -5 & 1 & 0 & 0 \\ 3 & 7 & -1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

Apply, $R_1 \to R_1/4$
$R_2 \to 4R_2 - 3R_1$
$R_3 \to 4R_3 - R_1$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 3/4 & -5/4 & 1/4 & 0 & 0 \\ 0 & 19 & 11 & -3 & 4 & 0 \\ 0 & 1 & 1 & -1 & 0 & 4 \end{array}\right)$$

Apply, $R_3 \to 19R_3 - R_2$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 3/4 & -5/4 & 1/4 & 0 & 0 \\ 0 & 19 & 11 & -3 & 4 & 0 \\ 0 & 0 & 8 & -16 & -4 & 76 \end{array}\right)$$

Apply, $R_3 \to R_3/8$
$R_2 \to R_2/19$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 3/4 & -5/4 & 1/4 & 0 & 0 \\ 0 & 1 & 11/19 & -3/19 & 4/19 & 0 \\ 0 & 0 & 1 & -2 & -4/8 & 36/8 \end{array}\right)$$

Apply, $R_2 \to \frac{-11}{19} R_3 + R_2$.

$$\sim \left(\begin{array}{ccc:ccc} 1 & 3/4 & -5/4 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1/2 & -11/2 \\ 0 & 0 & 1 & -2 & -1/2 & 19/2 \end{array}\right)$$

Apply, $R_1 \to R_1 - 3/4 R_2$

$R_1 \to R_1 + 5/4 R_3$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -3 & -1 & 16 \\ 0 & 1 & 0 & 1 & 1/2 & -11/2 \\ 0 & 0 & 1 & -2 & -1/2 & 19/2 \end{array}\right)$$

$$\therefore A^{-1} = \begin{pmatrix} -3 & -1 & 16 \\ 1 & 1/2 & -11/2 \\ -2 & -1/2 & 19/2 \end{pmatrix}$$

Now,

$$X = A^{-1} C = \begin{pmatrix} -3 & -1 & 16 \\ 1 & 1/2 & -11/2 \\ -2 & -1/2 & 19/2 \end{pmatrix} \begin{pmatrix} -1 \\ 8 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \times -1 - 1 \times 8 \\ 1 \times -1 + 8/2 \\ -2 \times -1 - 8/2 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ -2 \end{pmatrix}$$

Q.No c) $x+y+z=0, \quad x-2y+z=-3, \quad x+2y-z=5.$

Solution.

Given system of equation

$$x+y+z=0$$
$$x-2y+z=-3$$
$$x+2y-z=5$$

Now, creating augmented coefficient matrix with identity matrix

$$\sim \left(\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 \end{array}\right)$$

Apply $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 0 & -1 & 1 & 0 \\ 0 & 1 & -2 & -1 & 0 & 1 \end{array}\right)$$

Apply, $R_3 \to 3R_3 + R_2$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 0 & -1 & 1 & 0 \\ 0 & 0 & -6 & -4 & 1 & 3 \end{array}\right)$$

Apply, $R_2 \to R_2/(-3)$
$R_3 \to R_3/(-6)$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/3 & -1/3 & 0 \\ 0 & 0 & 1 & 4/6 & -1/6 & -3/6 \end{array}\right)$$

Apply, $R_1 \to R_1 - R_2$
$R_1 \to R_1 - R_3$

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$$\sim \begin{pmatrix} 1 & 0 & 0 & : & 0 & +1/2 & +3/6 \\ 0 & 1 & 0 & : & +1/3 & -1/3 & 0 \\ 0 & 0 & 1 & : & 4/6 & -1/6 & -3/6 \end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix} 0 & +1/2 & +3/6 \\ 1/3 & -1/3 & 0 \\ 4/6 & -1/6 & -3/6 \end{pmatrix}$$

now, $X = A^{-1} C$ $= \begin{pmatrix} 0 & +1/2 & +3/6 \\ 1/3 & -1/3 & 0 \\ 4/6 & -1/6 & -3/6 \end{pmatrix} \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}$

$$= \begin{pmatrix} +1/2 \times (-3) + (+\frac{3}{6}) \times 5 \\ -1/3 \times (-3) \\ -1/6 \times (-3) + (-3/6) \times 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

Hence, the solution of system of linear equation.
$(x, y, z) = (1, 1, -2)$

d.) $x+2y+z=8$, $2x+3y+2z=14$, $3x+2y+2z=13$

⇒ Solution

Given system of equations is

$x+2y+z=8$
$2x+3y+2z=14$
$3x+2y+2z=13$

creating augumented matrix with identity matrix.

$$[A:I] = \begin{pmatrix} 1 & 2 & 1 & : & 8 \\ 2 & 3 & 2 & : & 14 \\ 3 & 2 & 2 & : & 13 \end{pmatrix}$$

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$$[A:I] = \left(\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right)$$

Apply, $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & -4 & -1 & -3 & 0 & 1 \end{array}\right)$$

Apply, $R_3 \to R_3 - 4R_2$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 5 & -4 & 1 \end{array}\right)$$

Apply, $R_2 \to R_2 \times (-1)$

$R_3 \to R_3 \times (-1)$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -5 & 4 & -1 \end{array}\right)$$

Apply, $R_1 \to R_1 - 2R_2$

$R_1 \to R_1 - R_3$

$$\sim \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & 2 & -2 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -5 & 4 & -1 \end{array}\right)$$

Hence $A^{-1} = \begin{pmatrix} 2 & -2 & 1 \\ 2 & -1 & 0 \\ -5 & 4 & -1 \end{pmatrix}$

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Now, $X = A^{-1} C$

or, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 \\ 2 & -1 & 0 \\ -5 & 4 & -1 \end{pmatrix} \begin{pmatrix} 8 \\ 14 \\ 13 \end{pmatrix}$

$= \begin{pmatrix} 2\times8 - 2\times14 + 1\times13 \\ 2\times8 - 1\times14 + 0\times13 \\ -5\times8 + 4\times14 - 1\times13 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Hence, the value of (x, y, z) is $(1, 2, 3)$