Change Of Variables - Practice Problems

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For problems 1 - 3 compute the Jacobian of each transformation.

1. $x = 4u - 3v^2 \quad y = u^2 - 6v$

2. $x = u^2v^3 \quad y = 4 - 2\sqrt{u}$

3. $x = \dfrac{v}{u} \quad y = u^2 - 4v^2$

4. If R is the region inside $\dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$ determine the region we would get applying the transformation $x = 2u, y = 6v$ to R.

5. If R is the parallelogram with vertices $(1,0), (4,3), (1,6)$ and $(-2,3)$ determine the region we would get applying the transformation $x = \dfrac{1}{2}(v-u)$, $y = \dfrac{1}{2}(v+u)$ to R.

6. If R is the region bounded by $xy = 1, xy = 3, y = 2$ and $y = 6$ determine the region we would get applying the transformation $x = \dfrac{v}{6u}, y = 2u$ to R.

7. Evaluate $\displaystyle\iint_R xy^3\, dA$ where R is the region bounded by $xy = 1, xy = 3, y = 2$ and $y = 6$ using the transformation $x = \dfrac{v}{6u}, y = 2u$.

8. Evaluate $\displaystyle\iint_R (6x-3y)\, dA$ where R is the parallelogram with vertices $(2,0), (5,3), (6,7)$ and $(3,4)$ using the transformation $x = \dfrac{1}{3}(v-u), y = \dfrac{1}{3}(4v-u)$ to R.

9. Evaluate $\displaystyle\iint_R (x+2y)\, dA$ where R is the triangle with vertices $(0,3), (4,1)$ and $(2,6)$ using the transformation $x = \dfrac{1}{2}(u-v), y = \dfrac{1}{4}(3u+v+12)$ to R.

10. Derive the transformation used in problem 8.

11. Derive a transformation that will convert the triangle with vertices $(1,0), (6,0)$ and $(3,8)$ into a right triangle with the right angle occurring at the origin of the uv system.