Generalization of Algorithms for the Orienteering Problem

Etienne Palanga

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Introduction

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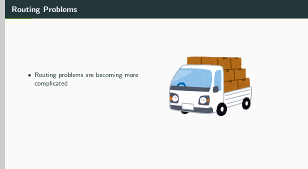
Routing Problems

- Routing problems are becoming more complicated



- Routing problems not only more prevalent and more complicated
  - Navigation of vehicles or people between a predetermined set of points (like a set of customers)
  - Some constraint or goal
  - $\Rightarrow$ Want optimal path or paths
- Navigation in general
  - Want to save time/fuel (shortest path)
  - Want a beautiful trail to walk (highest quality path)
- Online Shopping $\rightarrow$ shipping packages to customers
  - Want to deliver the most packages in the smallest amount of time

Routing Problems

- Routing problems are becoming more complicated
  - Amazon with ≈ 4.75 billion shipped packages [8]

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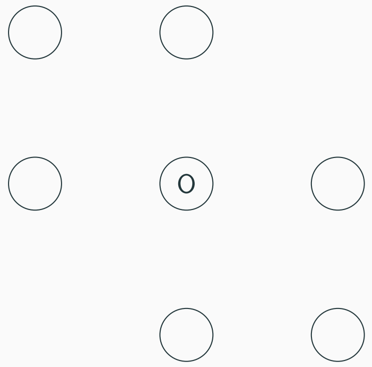
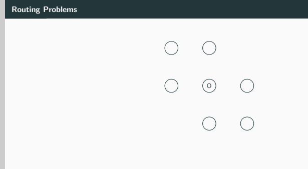
- Example: Amazon
  - 4.75 billion shipped packages
  - Need a way to manage this
- Frequent Task: Optimize Routes in some respect
  - Reduce fuel consumption/travel time
  - Maximize profit gained
  - How do I deliver the most goods while on a time budget?
  - ... (you can think of a lot of things)

Routing Problems

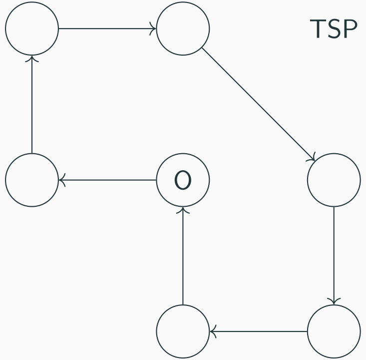


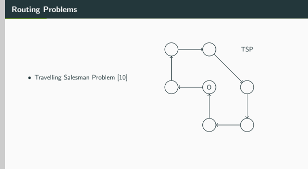


- Such problems are no stranger to computer science

Routing Problems

- Travelling Salesman Problem [10]

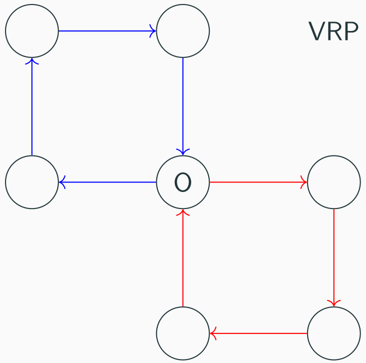


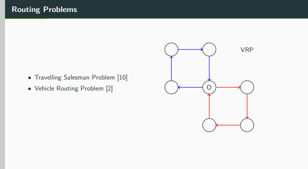


- Most famous: Travelling Salesman Problem
  - Graph
  - Visit all nodes exactly once and return to the origin using the least distance
- But there are also others

Routing Problems

- Travelling Salesman Problem [10]
- Vehicle Routing Problem [2]

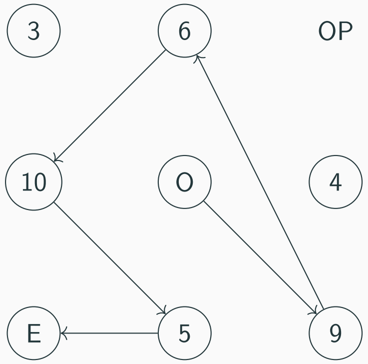


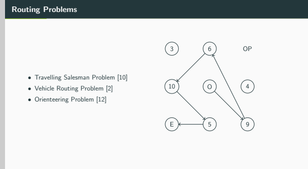


- Vehicle Routing Problem
  - Generalization of the TSP
  - Multiple vehicles starting at the origin
  - ⇒ multiple routes minimizing the travel cost

Routing Problems

- Travelling Salesman Problem [10]
- Vehicle Routing Problem [2]
- Orienteering Problem [12]





- **Orienteering Problem**
  - No need to visit all nodes
  - Nodes have scores/profit, edges have weights (omitted here for clarity)
  - Time/distance limit
  - Find a path (not necessarily cycle) with maximum profit that does not violate time/distance limit
  - Now defined more accurately

What is the issue?

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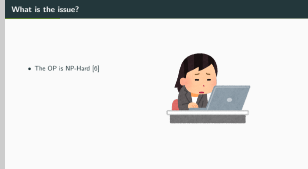
- So what are we here to talk about?

What is the issue?

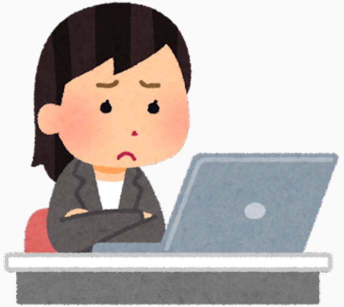
- The OP is NP-Hard [6]

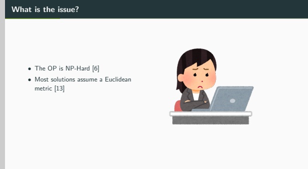


- Unsurprisingly, OP is NP-Hard
  - Okay, so there are are some approximate solutions, right?

What is the issue?

- The OP is NP-Hard [6]
- Most solutions assume a Euclidean metric [13]

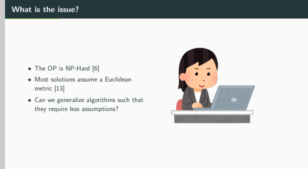




- Yes, but many solutions in the literature require Euclidean metric
  - Nodes are points in euclidean plane
  - Distances are euclidean distance
  - Most assume at least some amount of restrictions
- Not good if trying to solve general instances

What is the issue?

- The OP is NP-Hard [6]
- Most solutions assume a Euclidean metric [13]
- Can we generalize algorithms such that they require less assumptions?



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The Orienteering Problem

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Definition (Orienteering Problem [13])
Let $G = (V = \{v_1, \ldots, v_n\}, E)$ be an undirected graph with a cost function $t : E \to \mathbb{R}_+$ and a profit function $s : V \to \mathbb{R}$. Also, let $T_{max} \in \mathbb{R}_+$ be a cost limit.

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- Proper definition of the OP
  - Start with an undirected graph
  - Cost function t for the edges
    - Also “distance”, “travel time”, “weight”
  - Profit function s for the nodes
    - Also “score”
  - Maximum cost T_{max}

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The $\textsc{OrienteeringProblem}$ aims at finding a path $P = [p_1, \dots, p_k], p_i \in V$ with $p_1 = v_1$ and ending at $p_k = v_n$ which maximizes the total profit $S(P) := \sum_{p_i \in P} s(p_i)$ while respecting the cost limit, that is

$$T(P) := \sum_{i=1}^{k-1} t(v_i, v_{i+1}) \leq T_{max}$$

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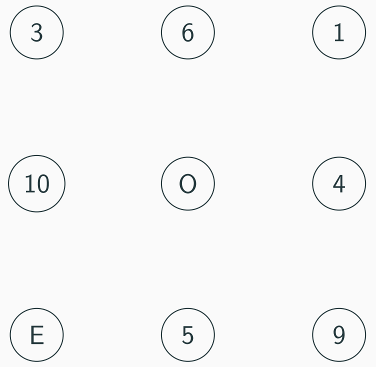
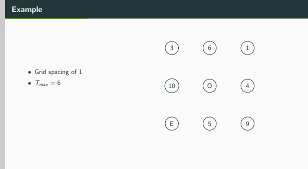
- Orienteering Problem
  - Want a path from v_1 to v_n
    - Need not be fixed but often are
    - Named like this for convenience
  - Path should maximize the total profit...
  - ...while respecting the cost limit T_{max}
- Questions?

Example

- Grid spacing of 1
- $T_{max} = 6$

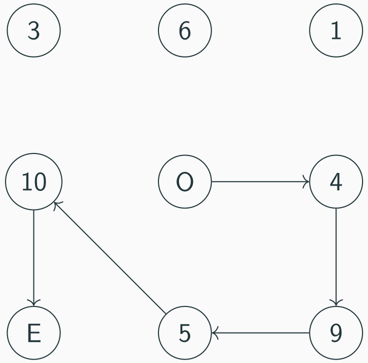


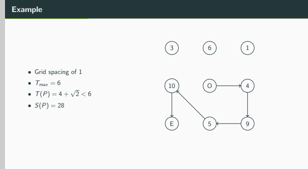


- Example OP instance
  - $T_{max} = 6$
  - Edges between all nodes (omitted for clarity)
  - Horizontal/Vertical distance between neighboring nodes: 1
  - Diagonal movement: $\sqrt{2} \approx 1.4$
- Ask audience: What is the optimal path?

Example

- Grid spacing of 1
- $T_{max} = 6$
- $T(P) = 4 + \sqrt{2} < 6$
- $S(P) = 28$





- Best path as far as I can see

Complete Graphs

Complete Graphs

- Getting into the problem
  - Restrictions on allowed input graphs
  - Introduce common ones in order of strictness
  - After: introduce algorithms and try to reduce the restrictions they require

Complete Graphs

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A graph $G = (V, E)$ is *complete* if for any two nodes $v_i \neq v_j \in V$ there exists a corresponding edge $\{v_i, v_j\} \in E$.

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- First: Complete Graph
- Probably known to everyone
  - But for completeness' sake
- Almost unanimous
- Simple to transform any graph into complete graph. Insert missing edges with weight
  - Weight of the shortest path between nodes
  - ∞ if the missing edges should not be taken
    - Might not work for all algorithms though (example later)

Triangle Inequality

- First actual restriction
  - Similarly defined as in geometry

Triangle Inequality

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Definition (Triangle Inequality [1])
A complete, weighted graph $G = (V, E)$ satisfies the *triangle inequation* if any three nodes $u \neq v \neq w \in V$ statisfy the following inequality:

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  - Similarly defined as in geometry
- Any three nodes' weights satisfy the triangle inequality
- $\Rightarrow$ The direct edge is always the shortest path between nodes.
  - No detour will ever be faster
- Less often explicitly stated in literature
- Usually implied as part of Euclidean Metric
  - Defined shortly

Triangle Inequality

- What do we get out of this?

Triangle Inequality

Triangle Inequality

- Many problems have better approximations on such graphs [1]
- Easier to reason about the rest of the graph.

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Triangle Inequality

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- Simple greedy algorithm

- Nodes fulfilling the former property are called *available*

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- Many problems have better approximations on such graphs [1]
- Easier to reason about the rest of the graph.
- Simple greedy algorithm
  - Let $P = [p_1, \dots, p_k]$ is the current path
  - Always pick the node $v \in V \setminus P$ such that

$$T(P \cup [v, v_n]) \leq T_{max}$$

  and

$$\frac{s(v)}{t(p_k, v)}$$

  is maximized.
- Nodes fulfilling the former property are called *available*
- If no such node exists, go to the end node.

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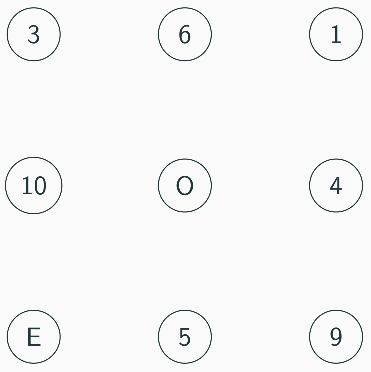
- Many problems: better approximations on more restricted graphs
- Easier to reason about rest of graph
  - example following now
- Simple greedy algorithm
- We always want to pick a node per step
  - Navigating to that node and then to the end may not violate T_{max}
  - Maximize the ratio between score gained and distance traveled
- If no such node, go to the end

Example



$T_{max} = 7$

$T(P) = 0$

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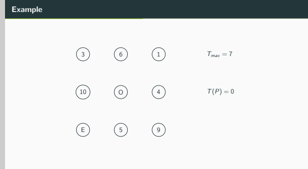

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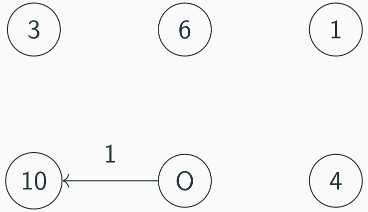
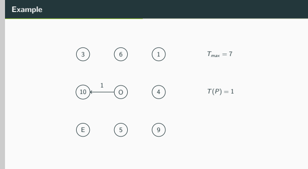


$T_{max} = 7$

$T(P) = 1$



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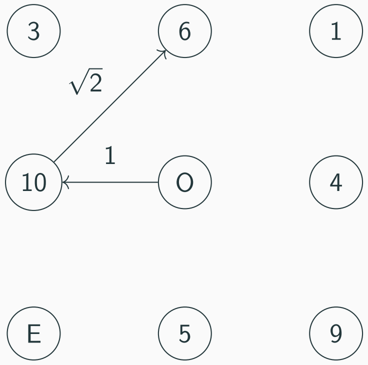
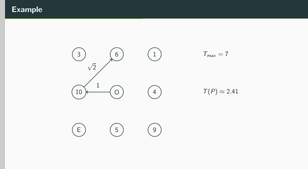


$T_{max} = 7$

$T(P) \approx 2.41$



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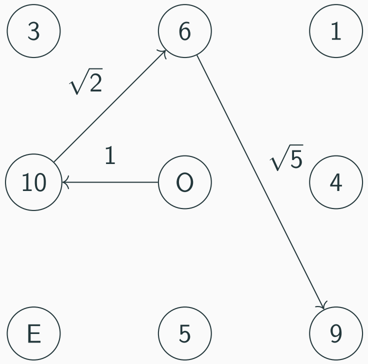
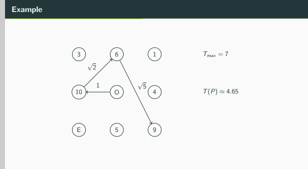


$T_{max} = 7$

$T(P) \approx 4.65$



Z.B. jetzt

- Remaining capacity of around 2.35.
- To which node can we go (and then to the end) without volating T_{max}?

$\Rightarrow$ Only the node 5

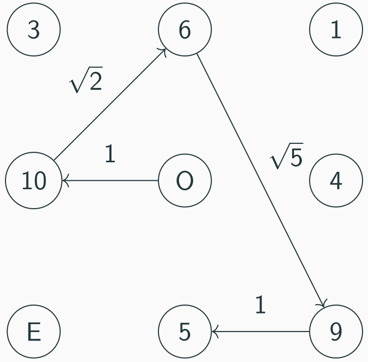
Technically we could travel to node 4 instead without immediately violating T_{max}

- But travelling to the end then would violate it.
- Since we know this: Triangle Inequality $\Rightarrow$ There is no shorter way

$\Rightarrow$ Travelling to 4 does not allow reaching the end

Both algorithms presented today, rely on this fact

Example



$T_{max} = 7$

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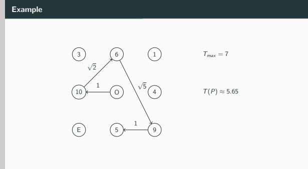

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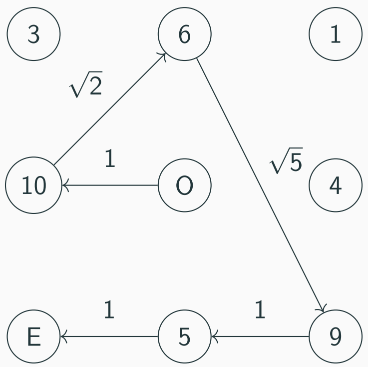

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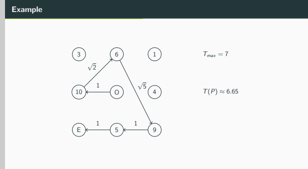

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For every node $v_i \in V$, there are coordinates $(x_i, y_i)^\intercal \in \mathbb{R}^2$ and for each pair of nodes $v_i \neq v_j \in V$ the edge weights are defined as:

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- Entails the previous two restrictions
- Very common in the literature [5, 6, 11, 12]

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- Edge weights are distances between them

Note, that this entails completeness and the triangle inequality

$\Rightarrow$ Entails the previous two restrictions

Commonly seen in the literature

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Will discuss the S algorithm and RI algorithm

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Let $P := [p_1, \dots, p_l]$ be an unfinished path and $r \in \mathbb{R}_+$ a constant. If $A \subseteq V \setminus P$ is the set of available nodes, then the *desirability value* $D(v_i)$ for every $v_i \in A$ is

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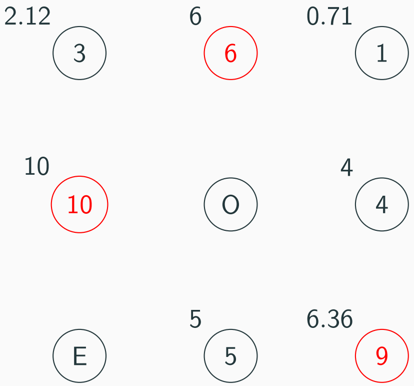
Example

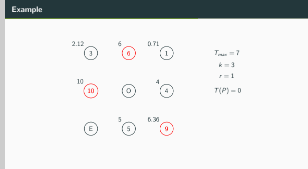


$T_{max} = 7$

$k = 3$

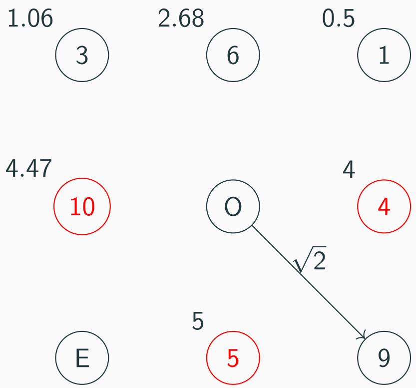
$r = 1$

$T(P) = 0$



- Start at the origin
  - Nodes on a grid with horiz/vert. distance of 1
  - $\Rightarrow$ Diagonal $= \sqrt{2} \approx 1.41$, long diagonal $= \sqrt{5} \approx 2.24$
  - Always consider the 3 best nodes
  - We set $r = 1$ to simplify calculations
  - Score inside nodes, Desirability on the sides
- First step: $10, 6$ and 9 are the best.
  - We randomly pick 9

Example



$T_{max} = 7$

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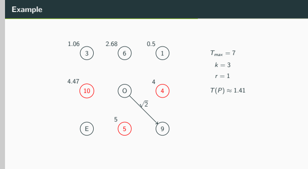
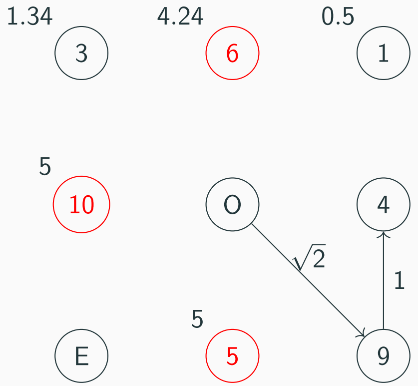
$T(P) \approx 1.41$



- Now we continue like this

Example



$T_{max} = 7$

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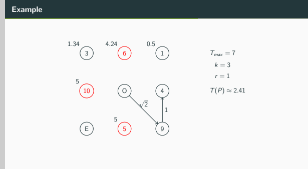

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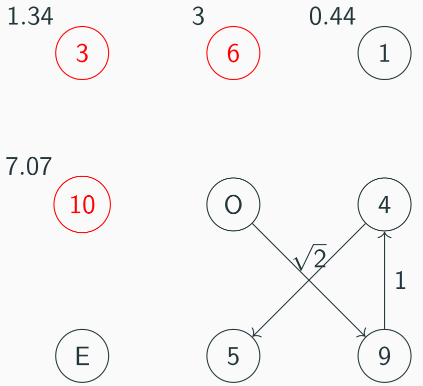
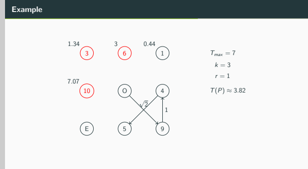


$T_{max} = 7$

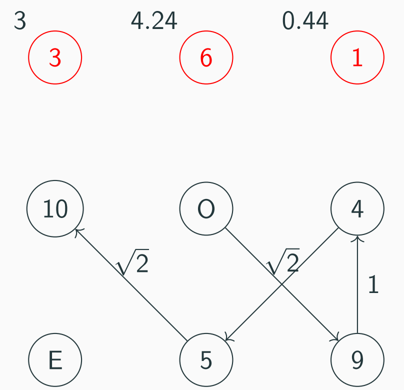
$k = 3$

$r = 1$

$T(P) \approx 3.82$



Example



$T_{max} = 7$

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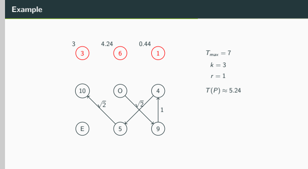
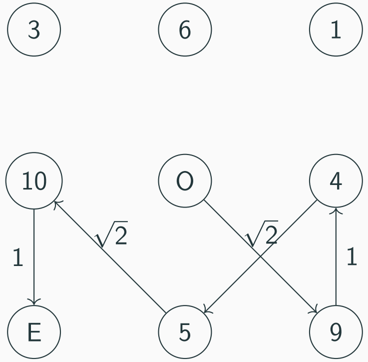
$T(P) \approx 5.24$



- Note in this step:
  - 3 nodes still unvisited
  - But node 3 would require 1 to go there and 2 to the end
  - Other two nodes even further away
  - No choice but to go to the end node
- Note that this algorithm decides this based on the triangle equality like the example algorithm before.

Example



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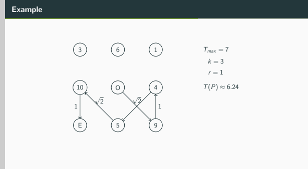
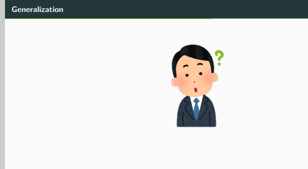


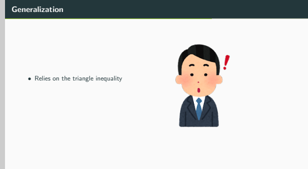
Generalization



- Main point: Trying to drop as many restrictions as feasibly possible
  - So which restrictions does the algorithm rely on

Generalization

- Relies on the triangle inequality



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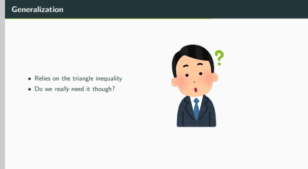
- Nowhere are uniquely Euclidean features ever required
  - Selecting nodes relies on triangle equality though

Generalization

- Relies on the triangle inequality
- Do we *really* need it though?



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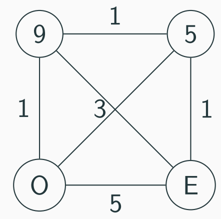
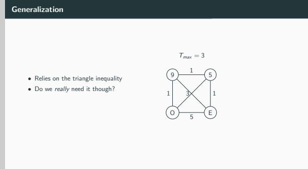
  - So which restrictions does the algorithm rely on
- Nowhere are uniquely Euclidean features ever required
  - Selecting nodes relies on triangle equality though
- Do we really need it?
  - What happens if we drop the triangle inequality

Generalization

$T_{max} = 3$

- Relies on the triangle inequality
- Do we *really* need it though?

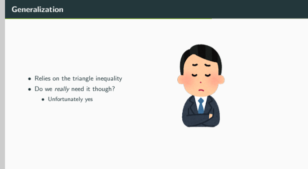




- Starting at the origin, which nodes can we go to? (Algorithm's perspective)
  - 9? No, since going to 9 and then to the goal would cost 4
  - 5? No, since going to 5 and then to the goal would cost 4
  - The goal? No, it would cost 5
- Would return, that there is no path
  - Obviously wrong

Generalization

- Relies on the triangle inequality
- Do we *really* need it though?
  - Unfortunately yes



- Seems like there is no trivial way around it
  - Calculate the shortest path to the end for every node?
    - Might work to an extent
    - ! But shortest paths assume all nodes to be usable
    - If some nodes of the paths are already used, the shortest path lengths might increase

Szwarc-Boryczka Algorithm

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- Let's go to a more recent algorithm
- Slight detour to introduce the algorithm's foundaition

Harmony Search [3, 4, 5]

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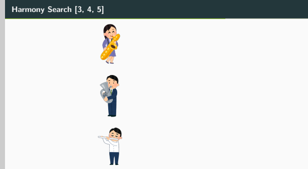
- Harmony Search
  - Optimization Technique proposed by Geem in 2000
  - Name stems from analogy it is inspired by
    - Maybe a little weird at first but makes more sense by the end (imo)

Harmony Search [3, 4, 5]

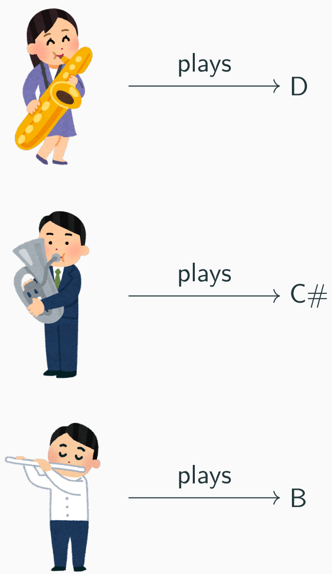
- Imagine band during an improvisation session

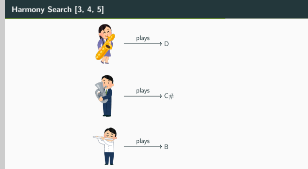
Harmony Search [3, 4, 5]



- Several individual players playing notes (or pitches as called here)
  - Resulting “collection” of pitches results in a *harmony*

Harmony Search [3, 4, 5]





- Several individual players playing notes (or pitches as called here)
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- At first kind of poorly coordinated
  - Resulting harmonies might be sort of random

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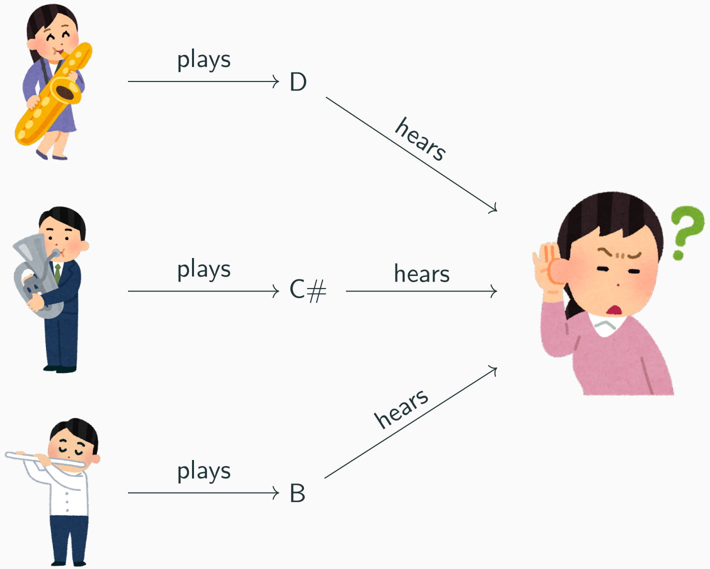

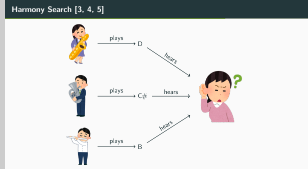

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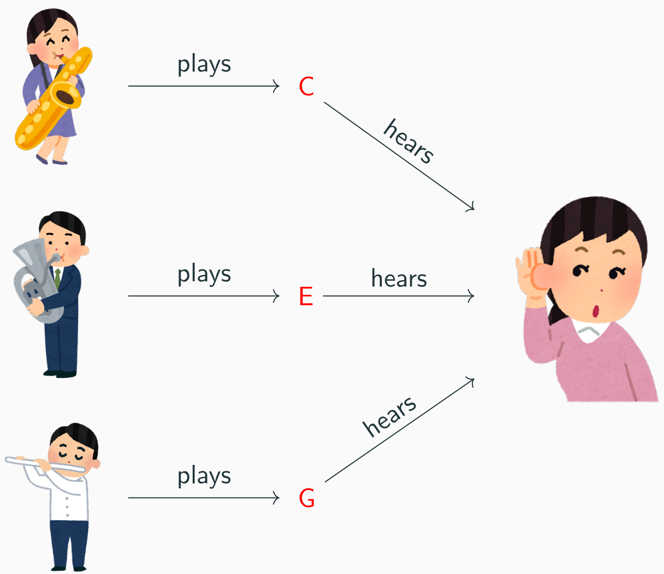

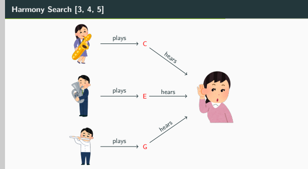

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