Fast Channel Estimation for IRS-Assisted OFDM

Beixiong Zheng, *Member, IEEE*, Changsheng You, *Member, IEEE*, and Rui Zhang, *Fellow, IEEE*

***Abstract*—In this letter, we study efficient channel estimation for an intelligent reflecting surface (IRS)-assisted orthogonal frequency division multiplexing (OFDM) system to achieve minimum training time. First, a fast channel estimation scheme with reduced OFDM symbol duration is proposed for arbitrary frequency-selective fading channels. Next, under the typical condition that the IRS-user channel is line-of-sight (LoS) dominant, another fast channel estimation scheme based on the novel concept of *sampling-wise IRS reflection variation* is proposed. Moreover, the pilot signal and IRS training reflection pattern are jointly optimized for both proposed schemes. Finally, the proposed schemes are compared in terms of training time and channel estimation performance via simulations, as well as against benchmark schemes.**

***Index Terms*—Intelligent reflecting surface (IRS), orthogonal frequency division multiplexing (OFDM), channel estimation, IRS training reflection design, pilot design.**

I. Introduction

As an enabling technology for smart and reconfigurable wireless communication environment, intelligent reflecting surface (IRS) has recently drawn a great deal of attention. By leveraging a large number of low-cost passive elements that are able to reflect signals with adjustable amplitudes and/or phase shifts, IRS is capable of significantly enhancing the wireless communication system throughput in an energy-efficient and cost-effective manner [1], [2].

However, the promising gains brought by IRS critically depend on the channel state information (CSI) that is practically difficult to acquire, due to the large number of channel coefficients associated with massive IRS reflecting elements. This issue becomes more challenging for orthogonal frequency division multiplexing (OFDM) systems with frequency-selective fading channels, which incur even more channel coefficients due to the multi-path delay spread. Some prior works [3]–[5] have addressed this problem for IRS-assisted OFDM systems by estimating the cascaded IRS channels via different IRS training reflection designs (e.g., the ON/OFF-based design [3] and the discrete Fourier transform (DFT) matrix-based design [4], [5]). Moreover, a novel element-grouping strategy was proposed by properly grouping adjacent IRS elements into a sub-surface, which provides a flexible system trade-off between training overhead/design complexity and passive beamforming performance by adjusting the size of each sub-surface [3], [4]. However, in the above works as well as others for IRS channel estimation under flat-fading channels (see, e.g., [6]–[8]), the number of (OFDM) pilot symbols should be no less than the number of all links including the direct link and the cascaded IRS links (whose number equals to the number of IRS reflecting elements/sub-surfaces), which can still be prohibitively high and thus is inapplicable to practical systems with insufficient pilot symbols/training time.

Motivated by the above, this letter studies more efficient channel estimation design for an IRS-assisted OFDM system to achieve minimum training time. First, for arbitrary frequency-selective fading channels, we propose a fast channel estimation scheme by shortening the duration of OFDM symbols, which achieves much less training time as compared to the schemes given in [3]–[5]. Next, under the typical scenario with single-path IRS-user channel (or line-of-sight (LoS) dominant channel in practice) due to their nearby deployment, we propose another fast channel estimation scheme based on the novel concept of *sampling-wise IRS reflection variation*, which creates artificial linear and time-variant (ALTV) cascaded channels within one OFDM symbol to facilitate fast IRS channel estimation. It is shown that the latter proposed scheme in general achieves much better channel estimation performance with less training time as compared to the former one, both with their corresponding jointly optimized IRS training reflection pattern and pilot signal design.

The authors are with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583, email: {elezbe, eleyouc, elezhang}@nus.edu.sg.

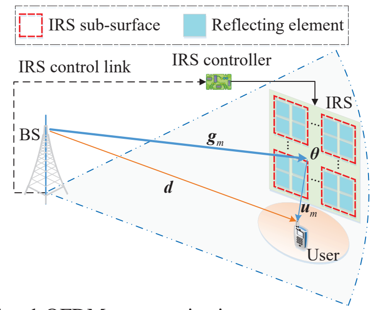

Fig. 1. IRS-assisted OFDM communication.

II. System Model and Problem Description

As shown in Fig. 1, we consider the basic IRS-assisted point-to-point communication system, where an IRS is deployed to assist the transmission from a base station (BS) to a user, both of which are equipped with a single antenna.[1] As in [3]–[5], the IRS composed of M_0 reflecting elements is divided into M sub-surfaces, denoted by the set $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$, each consisting of $\mu = M_0/M$ (assumed to be an integer) adjacent elements that share a common reflection coefficient for reducing the implementation complexity. Moreover, the IRS is connected to a smart controller that adjusts its reflection coefficients and exchanges information with the BS via a separate reliable wireless link [1]. In this letter, the quasi-static block fading channel model is assumed for all the channels involved, which remain approximately constant within the channel coherence time.

Let $\boldsymbol{d} \in \mathbb{C}^{L_d \times 1}$, $\boldsymbol{g}_m \in \mathbb{C}^{L_1 \times 1}$, and $\boldsymbol{u}_m \in \mathbb{C}^{L_2 \times 1}$ denote the baseband equivalent channels in the time domain for the BS$\rightarrow$user, BS$\rightarrow$sub-surface m, and sub-surface $m\rightarrow$user links, respectively, where L_d, L_1, and L_2 denote the maximum multi-path delay spreads (normalized by the sampling period $1/B$ with B denoting the system bandwidth) of these links. Let $\boldsymbol{\theta} \triangleq [\theta_1, \theta_2, \ldots, \theta_M]^T = [e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_M}]^T$ denote the equivalent reflection coefficients of IRS sub-surfaces, where

[1]The proposed channel estimation can be applied to the multi-user downlink communication where the users estimate their individual channels in parallel as well as the uplink communication with multi-antenna BS by treating each BS antenna/user as an equivalent user/BS antenna in the downlink case.

$$\underbrace{\begin{bmatrix} r_{m,0} \\ r_{m,1} \\ \vdots \\ r_{m,N-1} \end{bmatrix}}_{\boldsymbol{r}_m} = \begin{bmatrix} \theta_m^{(0)} q_{m,0} & 0 & \cdots & 0 & \theta_m^{(0)} q_{m,L-1} & \cdots & \theta_m^{(0)} q_{m,1} \\ \theta_m^{(1)} q_{m,1} & \theta_m^{(1)} q_{m,0} & & 0 & 0 & & \theta_m^{(1)} q_{m,2} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \theta_m^{(L-1)} q_{m,L-1} & \theta_m^{(L-1)} q_{m,L-2} & & 0 & 0 & & 0 \\ 0 & \theta_m^{(L)} q_{m,L-1} & & 0 & 0 & & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \theta_m^{(N-1)} q_{m,L-1} & \theta_m^{(N-1)} q_{m,L-2} & \cdots & \theta_m^{(N-1)} q_{m,0} \end{bmatrix} \underbrace{\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} \theta_m^{(0)} & & \\ & \ddots & \\ & & \theta_m^{(N-1)} \end{bmatrix}}_{\boldsymbol{\Theta}_m} \underbrace{\begin{bmatrix} x_0 & x_{N-1} & \cdots & x_{N-L+1} \\ x_1 & x_0 & \cdots & x_{N-L+2} \\ x_2 & x_1 & \cdots & x_{N-L+3} \\ \vdots & \vdots & \ddots & \vdots \\ x_{L-1} & x_{L-2} & \cdots & x_0 \\ \vdots & \vdots & & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-L} \end{bmatrix}}_{\boldsymbol{X}} \underbrace{\begin{bmatrix} q_{m,0} \\ q_{m,1} \\ \vdots \\ q_{m,L-1} \end{bmatrix}}_{\boldsymbol{q}_m} \tag{8}$$

$\phi_m \in [0, 2\pi)$ represents the phase shift of sub-surface m and the reflection amplitudes of all elements are set to one or the maximum value for simplicity. Thus, the effective cascaded channel from the BS to the user via sub-surface m can be expressed as the convolution of the BS→IRS channel, the IRS reflection coefficient, and the IRS→user channel, which is given by

$$\boldsymbol{g}_m * \theta_m * \boldsymbol{u}_m = \theta_m \boldsymbol{g}_m * \boldsymbol{u}_m = \theta_m \boldsymbol{q}_m \tag{1}$$

where $\boldsymbol{q}_m \triangleq \boldsymbol{g}_m * \boldsymbol{u}_m \in \mathbb{C}^{L_r \times 1}$ denotes the cascaded BS→IRS→user channel (without IRS phase shifts) associated with sub-surface m and $L_r = L_1 + L_2 - 1$ is the maximum delay spread of the cascaded BS→IRS→user channel. Furthermore, we take the maximum delay spread of the direct BS→user channel and the cascaded BS→IRS→user channel, i.e., $L = \max\{L_d, L_r\}$ and apply zero-padding [5].

As a result, the superimposed channel impulse response (CIR) from the BS to the user by combining the direct BS→user channel and the cascaded BS→IRS→user channel, denoted by $\boldsymbol{h} \in \mathbb{C}^{L \times 1}$, is given by

$$\boldsymbol{h} = \boldsymbol{d} + \boldsymbol{Q}\boldsymbol{\theta} \tag{2}$$

where $\boldsymbol{Q} = [\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_M] \in \mathbb{C}^{L \times M}$ denotes the cascaded BS→IRS→user channel matrix by stacking $\boldsymbol{q}_m$ with $m = 1, \ldots, M$. According to (2), it is required to estimate the direct channel $\boldsymbol{d}$ and the cascaded channels $\{\boldsymbol{q}\}_{m=1}^{M}$ for the IRS-assisted OFDM system.[2] Thus, the total number of channel coefficients to be estimated is $L(M+1)$.

Consider the OFDM transmission with N sub-carriers, where each OFDM symbol of length N is appended by a cyclic prefix (CP) of length $L_{cp} \geq L$ to mitigate the inter-symbol interference and we usually have $N \gg L$ in practice. In the existing literature on IRS channel estimation (see, e.g., [3]–[8]), at least $M+1$ pilot symbols are required to estimate the total $L(M+1)$ channel coefficients. As a result, it requires $\eta_0 = (M+1)(N+L_{cp})$ sampling periods for channel training in the case of OFDM, which is practically high when M is large. To reduce channel training time for the IRS-assisted OFDM, we present two new fast channel estimation schemes by exploiting the fact that $N \gg L$ under two different channel setups in the following two sections, respectively.

III. Channel Estimation Based on Short-OFDM-Symbol-Wise Varying IRS Reflection

In this section, we propose a new fast channel estimation scheme (referred to as Scheme 1) for arbitrary frequency-selective fading channels. Specifically, the length of each OFDM symbol (without CP) is shortened to N_0 with $N \gg N_0 \geq L$ for channel training. Let $\boldsymbol{s} \triangleq [S_0, S_1, \ldots, S_{N_0-1}]^T$ denote the short-OFDM symbol in the frequency domain, which is first transformed into the time domain via an N_0-point inverse DFT (IDFT) and then appended by a CP of length $L_{cp} \geq L$. After CP removal at the user side, the i-th received short-OFDM symbol in the frequency domain, denoted by $\boldsymbol{z}^{(i)}$, can be expressed as

$$\boldsymbol{z}^{(i)} = \boldsymbol{S}\boldsymbol{F}\left(\boldsymbol{d} + \boldsymbol{Q}\boldsymbol{\theta}^{(i)}\right) + \boldsymbol{v}^{(i)} \tag{3}$$

where $i = 1, \ldots, I_0$ with I_0 denoting the number of short-OFDM symbols for channel training, $\boldsymbol{S} = \text{diag}(\boldsymbol{s})$ is the diagonal matrix of each short-OFDM symbol $\boldsymbol{s}$, $\boldsymbol{F}$ is an $N_0 \times L$ matrix consisting of the first L columns of the $N_0 \times N_0$ unitary DFT matrix, $\boldsymbol{\theta}^{(i)}$ denotes the IRS training reflection coefficients during the transmission of the i-th short-OFDM symbol, and $\boldsymbol{v}^{(i)} \sim \mathcal{N}_c(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{N_0})$ is the additive white Gaussian noise (AWGN) vector at the user with σ^2 being the noise power. By denoting $\tilde{\boldsymbol{S}} = \boldsymbol{S}\boldsymbol{F}$, $\tilde{\boldsymbol{Q}} = [\boldsymbol{d}, \boldsymbol{Q}]$, and $\tilde{\boldsymbol{\theta}}^{(i)} = \begin{bmatrix} 1 \\ \boldsymbol{\theta}^{(i)} \end{bmatrix}$, (3) can be written in a compact form as $\boldsymbol{z}^{(i)} = \tilde{\boldsymbol{S}}\tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\theta}}^{(i)} + \boldsymbol{v}^{(i)}, i = 1, \ldots, I_0$. By stacking the received signal vectors $\left\{\boldsymbol{z}^{(i)}\right\}_{i=1}^{I_0}$ into $\boldsymbol{Z} = [\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}, \ldots, \boldsymbol{z}^{(I_0)}]$, we obtain

$$\boldsymbol{Z} = \tilde{\boldsymbol{S}}\tilde{\boldsymbol{Q}}\boldsymbol{\Psi} + \boldsymbol{V} \tag{4}$$

where $\boldsymbol{\Psi} \triangleq [\tilde{\boldsymbol{\theta}}^{(1)}, \tilde{\boldsymbol{\theta}}^{(2)}, \ldots, \tilde{\boldsymbol{\theta}}^{(I_0)}]$ denotes the IRS training reflection pattern matrix that collects all training reflection coefficients $\{\tilde{\boldsymbol{\theta}}^{(i)}\}_{i=1}^{I_0}$, and $\boldsymbol{V} = [\boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)}, \ldots, \boldsymbol{v}^{(I_0)}]$ denotes the corresponding AWGN matrix. Based on (4), we obtain the least-squares (LS) estimates of $\boldsymbol{d}$ and $\boldsymbol{Q}$ as

$$\left[\hat{\boldsymbol{d}}, \hat{\boldsymbol{Q}}\right] = \tilde{\boldsymbol{S}}^{\dagger}\boldsymbol{Z}\boldsymbol{\Psi}^{\dagger} = \tilde{\boldsymbol{Q}} + \tilde{\boldsymbol{S}}^{\dagger}\boldsymbol{V}\boldsymbol{\Psi}^{\dagger} \tag{5}$$

where $\tilde{\boldsymbol{S}}^{\dagger} = \left(\tilde{\boldsymbol{S}}^H\tilde{\boldsymbol{S}}\right)^{-1}\tilde{\boldsymbol{S}}^H$ and $\boldsymbol{\Psi}^{\dagger} = \boldsymbol{\Psi}^H\left(\boldsymbol{\Psi}\boldsymbol{\Psi}^H\right)^{-1}$. Note that for the LS channel estimation in (5), $I_0 \geq M+1$ is required to ensure the existence of $\boldsymbol{\Psi}^{\dagger}$, implying that at least $M+1$ short-OFDM symbols are required to successfully estimate/distinguish both the direct channel $\boldsymbol{d}$ and the cascaded channel $\boldsymbol{Q}$ associated with the M sub-surfaces. As such, accounting for the CP, it requires $\eta_1 = (M+1)(N_0+L_{cp})$ sampling periods for the channel estimation based on the above short-OFDM-symbol-wise varying IRS training reflections. It is noted that due to $N_0 \ll N$, we have $\eta_1 \ll \eta_0$, thus significantly reducing the training time, as compared to the schemes in [3]–[5]. Finally, the matrix inversion operation for computing $\boldsymbol{\Psi}^{\dagger}$ and $\tilde{\boldsymbol{S}}^{\dagger}$ in general has a cubic time complexity in terms of $M+1$ and L, respectively, and may lead to suboptimal channel estimation due to the potential noise enhancement if either $\boldsymbol{\Psi}$ or $\tilde{\boldsymbol{S}}$ is ill-conditioned, which thus requires a proper joint design of $\boldsymbol{\Psi}$ and $\tilde{\boldsymbol{S}}$.

The minimum mean squared error (MSE) of the LS channel

[2] Specifically, the user sends back its estimated CSI to the BS, which designs the IRS reflection coefficients for data transmission and sends them to the IRS controller for implementation.

estimation in (5) is given by

$$\begin{aligned}\varepsilon_1 &= \frac{1}{L(M+1)}\mathbb{E}\left\{\left\|\left[\hat{\boldsymbol{d}},\hat{\boldsymbol{Q}}\right]-[\boldsymbol{d},\boldsymbol{Q}]\right\|_F^2\right\}\\ &= \frac{1}{L(M+1)}\mathbb{E}\left\{\left\|\tilde{\boldsymbol{S}}^{\dagger}\boldsymbol{V}\boldsymbol{\Psi}^{\dagger}\right\|_F^2\right\}\\ &= \frac{1}{L(M+1)}\mathrm{tr}\left\{\left(\boldsymbol{\Psi}^{\dagger}\right)^H\mathbb{E}\left\{\boldsymbol{V}^H(\tilde{\boldsymbol{S}}^{\dagger})^H\tilde{\boldsymbol{S}}^{\dagger}\boldsymbol{V}\right\}\boldsymbol{\Psi}^{\dagger}\right\}\\ &\overset{(a)}{=} \frac{\sigma^2}{L(M+1)}\mathrm{tr}\left\{\left(\boldsymbol{F}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{F}\right)^{-1}\right\}\mathrm{tr}\left\{\left(\boldsymbol{\Psi}\boldsymbol{\Psi}^H\right)^{-1}\right\} \end{aligned} \quad (6)$$

where (a) holds since $\mathbb{E}\left\{\boldsymbol{V}^H(\tilde{\boldsymbol{S}}^{\dagger})^H\tilde{\boldsymbol{S}}^{\dagger}\boldsymbol{V}\right\} = \sigma^2\mathrm{tr}\left\{\left(\tilde{\boldsymbol{S}}^H\tilde{\boldsymbol{S}}\right)^{-1}\right\}\boldsymbol{I}_{I_0} = \sigma^2\mathrm{tr}\left\{\left(\boldsymbol{F}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{F}\right)^{-1}\right\}\boldsymbol{I}_{I_0}$. As such, the MSE minimization problem in (6) can be equivalently decoupled into two sub-problems for minimizing $\mathrm{tr}\left\{\left(\boldsymbol{F}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{F}\right)^{-1}\right\}$ and $\mathrm{tr}\left\{\left(\boldsymbol{\Psi}\boldsymbol{\Psi}^H\right)^{-1}\right\}$, respectively, whose individual optimal values can be respectively achieved if and only if $\boldsymbol{S}^H\boldsymbol{S}=\gamma_1\boldsymbol{I}_{N_0}$ with γ_1 being the average sub-carrier power and $\boldsymbol{\Psi}\boldsymbol{\Psi}^H=(M+1)\boldsymbol{I}_{M+1}$ according to [9]. This indicates that the optimal short-OFDM symbol should be equipowered over all sub-carriers (e.g., $\boldsymbol{s}=\gamma_1\boldsymbol{1}_{N_0\times 1}$) and the optimal IRS training reflection pattern $\boldsymbol{\Psi}$ should be an orthogonal matrix with each entry satisfying the unit-modulus constraint (e.g., setting the IRS training reflection pattern $\boldsymbol{\Psi}$ as the $(M+1)\times(M+1)$ DFT matrix with each coefficient given by $\theta_m^{(i)} = e^{-j\frac{2\pi m(i-1)}{M+1}}$ with $m=1,\ldots,M$ and $i=1,\ldots,M+1$). Accordingly, we can obtain the minimum MSE of (6) as $\varepsilon_1^{\min}=\frac{\sigma^2}{\gamma_1(M+1)}$, and have $\tilde{\boldsymbol{S}}^{\dagger}=\frac{1}{\gamma_1}\tilde{\boldsymbol{S}}^H$ and $\boldsymbol{\Psi}^{\dagger}=\frac{1}{(M+1)}\boldsymbol{\Psi}^H$ without the need of matrix inversion operation, thus avoiding the high (cubic-order) complexity.

IV. Channel Estimation Based on Sampling-Wise Varying IRS Reflection

In this section, we consider the typical scenario with single-path IRS→user channel (i.e., $L_2=1$) due to their practically short distance and propose another fast channel estimation scheme (referred to as Scheme 2) without changing the conventional OFDM symbol duration/structure.

For the purpose of exposition, we consider the channel estimation at the user with only one OFDM pilot symbol by assuming $N\geq L(M+1)$ in the rest of this letter.[3] As such, the training time for Scheme 2 in terms of sampling periods is $\eta_2=N+L_{cp}$. Let $\boldsymbol{x}\triangleq[x_0,x_1,\ldots,x_{N-1}]^T$ denote the OFDM pilot symbol (without CP) sampled in the time domain. Note that during the transmission of this OFDM symbol, the effective channel $\boldsymbol{h}$ is made artificially time-varying by tuning the IRS training reflection coefficients $\boldsymbol{\theta}$ over different sampling periods within one OFDM symbol to facilitate the cascaded channel estimation. Accordingly, the resultant ALTV channel at sampling period n, denoted by $\boldsymbol{h}^{(n)}$, is given by

$$\underbrace{\begin{bmatrix}h_0^{(n)}\\h_1^{(n)}\\\vdots\\h_{L-1}^{(n)}\end{bmatrix}}_{\boldsymbol{h}^{(n)}} = \underbrace{\begin{bmatrix}d_1\\d_2\\\vdots\\d_{L-1}\end{bmatrix}}_{\boldsymbol{d}} + \underbrace{\begin{bmatrix}q_{1,0}&q_{2,0}&\cdots&q_{M,0}\\q_{1,1}&q_{2,1}&\cdots&q_{M,1}\\\vdots&\vdots&&\vdots\\q_{1,L-1}&q_{2,L-1}&\cdots&q_{M,L-1}\end{bmatrix}}_{\boldsymbol{Q}}\underbrace{\begin{bmatrix}\theta_1^{(n)}\\\theta_2^{(n)}\\\vdots\\\theta_M^{(n)}\end{bmatrix}}_{\boldsymbol{\theta}^{(n)}} \quad (7)$$

where $\boldsymbol{q}_m=[q_{m,0},q_{m,1},\ldots,q_{m,L-1}]^T$ and $\theta_m^{(n)}$ denotes the phase shift of sub-surface m at sampling period n with $m\in\mathcal{M}$ and $n\in\{0,1,\ldots,N-1\}$. After CP removal at the user, the equivalent baseband signal reflected by sub-surface m in the time domain is given by (8) at the top of this page, and the equivalent baseband signal through the direct link in the time domain is $\boldsymbol{r}_0=\boldsymbol{X}\boldsymbol{d}$. Based on (8), the equivalent baseband received signal in the time domain can be rewritten as

$$\boldsymbol{y}=\underbrace{\boldsymbol{X}\boldsymbol{d}}_{\boldsymbol{r}_0}+\sum_{m=1}^{M}\underbrace{\boldsymbol{\Theta}_m\boldsymbol{X}\boldsymbol{q}_m}_{\boldsymbol{r}_m}+\tilde{\boldsymbol{v}}=\boldsymbol{\Xi}\boldsymbol{\lambda}+\tilde{\boldsymbol{v}} \quad (9)$$

where $\boldsymbol{\Xi}\triangleq[\boldsymbol{\Theta}_0\boldsymbol{X},\boldsymbol{\Theta}_1\boldsymbol{X},\ldots,\boldsymbol{\Theta}_M\boldsymbol{X}]$ with $\boldsymbol{\Theta}_0=\boldsymbol{I}_N$, $\boldsymbol{\lambda}\triangleq\left[\boldsymbol{d}^T,\boldsymbol{q}_1^T,\ldots,\boldsymbol{q}_M^T\right]^T$, and $\tilde{\boldsymbol{v}}\sim\mathcal{N}_c(\boldsymbol{0},\sigma^2\boldsymbol{I}_N)$ is the AWGN vector at the user. It is noted that $\boldsymbol{\lambda}$ includes all the required CSI of the direct channel $\boldsymbol{d}$ and the cascaded channels $\{\boldsymbol{q}\}_{m=1}^M$. With $\boldsymbol{\Xi}^{\dagger}=\left(\boldsymbol{\Xi}^H\boldsymbol{\Xi}\right)^{-1}\boldsymbol{\Xi}^H$ denoting the left pseudo-inverse of $\boldsymbol{\Xi}$ and left-multiplying $\boldsymbol{y}$ in (9) by $\boldsymbol{\Xi}^{\dagger}$, we obtain the LS estimate of $\boldsymbol{\lambda}$ as

$$\hat{\boldsymbol{\lambda}}=\boldsymbol{\Xi}^{\dagger}\boldsymbol{y}=\boldsymbol{\lambda}+\boldsymbol{\Xi}^{\dagger}\tilde{\boldsymbol{v}}. \quad (10)$$

Note that for the LS channel estimation in (10), $N\geq L(M+1)$ is the necessary condition to ensure the existence of $\boldsymbol{\Xi}^{\dagger}$, implying that the total number of sampling periods should be no less than that of channel coefficients. Similarly, since the matrix inversion operation for computing $\boldsymbol{\Xi}^{\dagger}$ generally has a cubic time complexity in terms of $L(M+1)$, it requires a proper design of $\boldsymbol{\Xi}$ to reduce such complexity as well as achieve the minimum MSE of channel estimation based on (10), as will be shown in the next.

The MSE of the LS channel estimation in (10) is given by

$$\begin{aligned}\varepsilon_2 &= \frac{1}{L(M+1)}\mathbb{E}\left\{\left\|\hat{\boldsymbol{\lambda}}-\boldsymbol{\lambda}\right\|^2\right\}=\frac{1}{L(M+1)}\mathbb{E}\left\{\left\|\boldsymbol{\Xi}^{\dagger}\tilde{\boldsymbol{v}}\right\|^2\right\}\\ &= \frac{1}{L(M+1)}\mathrm{tr}\left\{\boldsymbol{\Xi}^{\dagger}\mathbb{E}\left\{\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}^H\right\}\left(\boldsymbol{\Xi}^{\dagger}\right)^H\right\}\\ &= \frac{\sigma^2}{L(M+1)}\mathrm{tr}\left\{\left(\boldsymbol{\Xi}^H\boldsymbol{\Xi}\right)^{-1}\right\}\end{aligned} \quad (11)$$

where $\mathbb{E}\left\{\tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}^H\right\}=\sigma^2\boldsymbol{I}_N$. For the MSE in (11), we have

$$\begin{aligned}\boldsymbol{\Xi}^H\boldsymbol{\Xi}&=[\boldsymbol{\Theta}_0\boldsymbol{X},\boldsymbol{\Theta}_1\boldsymbol{X},\ldots,\boldsymbol{\Theta}_M\boldsymbol{X}]^H[\boldsymbol{\Theta}_0\boldsymbol{X},\boldsymbol{\Theta}_1\boldsymbol{X},\ldots,\boldsymbol{\Theta}_M\boldsymbol{X}]\\ &\overset{(b)}{=}\begin{bmatrix}\boldsymbol{X}^H\boldsymbol{X}&\boldsymbol{X}^H\boldsymbol{\Theta}_0^H\boldsymbol{\Theta}_1\boldsymbol{X}&\cdots&\boldsymbol{X}^H\boldsymbol{\Theta}_0^H\boldsymbol{\Theta}_M\boldsymbol{X}\\ \boldsymbol{X}^H\boldsymbol{\Theta}_1^H\boldsymbol{\Theta}_0\boldsymbol{X}&\boldsymbol{X}^H\boldsymbol{X}&\cdots&\boldsymbol{X}^H\boldsymbol{\Theta}_1^H\boldsymbol{\Theta}_M\boldsymbol{X}\\ \vdots&\vdots&\ddots&\vdots\\ \boldsymbol{X}^H\boldsymbol{\Theta}_M^H\boldsymbol{\Theta}_0\boldsymbol{X}&\boldsymbol{X}^H\boldsymbol{\Theta}_M^H\boldsymbol{\Theta}_1\boldsymbol{X}&\cdots&\boldsymbol{X}^H\boldsymbol{X}\end{bmatrix}\end{aligned} \quad (12)$$

where (b) holds since $\boldsymbol{\Theta}_m^H\boldsymbol{\Theta}_m=\boldsymbol{I}_N,\forall m\in\mathcal{M}$. In particular, the OFDM pilot symbol $\boldsymbol{x}$ and the IRS training reflection coefficients $\left\{\theta_m^{(n)}\right\}$ should be jointly designed to achieve $\boldsymbol{\Xi}\boldsymbol{\Xi}^H=c\boldsymbol{I}_{L(M+1)}$ for the MSE minimization in (11), which

[3] For the general case with arbitrary L and M, the minimum number of OFDM pilot symbols for this scheme to estimate all the channel coefficients is $\left\lceil\frac{L(M+1)}{N}\right\rceil$, where $\lceil\cdot\rceil$ denotes the ceiling function.

TABLE I
COMPARISON OF TWO PROPOSED CHANNEL ESTIMATION SCHEMES FOR IRS-ASSISTED OFDM

	Complexity (number of multiplications)	Training time (sampling periods)	Channel condition	Minimum MSE	Processing domain
Scheme 1	$N_0L(I_0+1)+LI_0(M+1)$	$(M+1)(N_0+L_{cp})$	Arbitrary channels	$\frac{\sigma^2}{\gamma_1(M+1)}$	Frequency
Scheme 2	$NL(M+1)$	$N+L_{cp}$	Single-path IRS→user channel ($L_2=1$)	$\frac{\sigma^2}{\gamma_2 N}$	Time

is equivalent to the following conditions:

$$\boldsymbol{X}^H\boldsymbol{X}=c\boldsymbol{I}_L \tag{13}$$

$$\boldsymbol{X}^H\boldsymbol{\Theta}_m^H\boldsymbol{\Theta}_{m'}\boldsymbol{X}=\boldsymbol{0}_{L\times L} \tag{14}$$

where $m\neq m', \forall m,m'\in\{0,1,\ldots,M\}$ and c is a positive constant to be determined later.

Let $\boldsymbol{C}\triangleq[\boldsymbol{x}_0,\boldsymbol{x}_{-1},\ldots,\boldsymbol{x}_{-N+1}]$ denote the circulant matrix generated from the OFDM pilot symbol $\boldsymbol{x}$, where $\boldsymbol{x}_{-n}$ is the circularly shifted version of $\boldsymbol{x}$ by n steps in the downward direction with $n=0,1,\ldots N-1$. As shown in (8), $\boldsymbol{X}$ is an $N\times L$ matrix consisting of the first L columns of the circulant matrix $\boldsymbol{C}$, i.e., $\boldsymbol{X}=[\boldsymbol{x}_0,\boldsymbol{x}_{-1},\ldots,\boldsymbol{x}_{-L+1}]$. The condition in (13) indicates that any two columns in $\boldsymbol{X}$ are orthogonal, i.e., $\boldsymbol{x}_{-l}^H\boldsymbol{x}_{-l'}=0$ with $l\neq l', \forall l,l'\in\{0,1,\ldots,L-1\}$, which requires that the auto-correlation of $\boldsymbol{x}$ should be zero. This can be achieved by setting the OFDM pilot symbol $\boldsymbol{x}$ as a Zadoff-Chu sequence [10] with each element given by

$$x_n=\gamma_2\cdot e^{-j\frac{\pi\omega n^2}{N}}, \quad n=0,1,\ldots N-1 \tag{15}$$

where ω is an integer relatively prime to N and γ_2 denotes the average power at each sampling period. Note that as compared to the design of $\boldsymbol{x}$ to achieve (13), it is much more challenging to jointly design the IRS training reflection coefficients $\left\{\theta_m^{(n)}\right\}$ to satisfy the condition in (14). Fortunately, we notice that given $\boldsymbol{x}$ in (15), the N cyclically shifted versions of $\boldsymbol{x}$ in $\boldsymbol{C}$ are pairwise orthogonal to each other; however, only L of them are involved in $\boldsymbol{X}$ (see (8)), while the remaining $N-L$ cyclically shifted versions of $\boldsymbol{x}$ have not been exploited yet. Inspired by this, we let $\bar{\boldsymbol{X}}_m\triangleq\boldsymbol{\Theta}_m\boldsymbol{X}$ and disjointly assign the remaining $N-L$ cyclically shifted versions of $\boldsymbol{x}$, i.e., $\{\boldsymbol{x}_{-n}\}_{n=L}^{N-1}$ for each $\bar{\boldsymbol{X}}_m$ to achieve pairwise orthogonality, i.e., $\bar{\boldsymbol{X}}_m^H\bar{\boldsymbol{X}}_{m'}=\boldsymbol{0}_{L\times L}$ with $m\neq m', \forall m,m'\in\{0,1,\ldots,M\}$. For example, we can set $\bar{\boldsymbol{X}}_m=\left[\boldsymbol{x}_{-mL},\boldsymbol{x}_{-mL-1},\ldots,\boldsymbol{x}_{-(m+1)L+1}\right]$ with $m\in\{0,1,\ldots,M\}$, and thus have $\boldsymbol{\Theta}_m=\mathrm{diag}(\boldsymbol{x}_{-mL-l})\left(\mathrm{diag}(\boldsymbol{x}_{-l})\right)^{-1}, \forall l\in\{0,1,\ldots,L-1\}$, in which the corresponding IRS training reflection coefficients $\left\{\theta_m^{(n)}\right\}$ are given by

$$\theta_m^{(n)}=\frac{x_{n-mL}}{x_n}=\frac{e^{-j\frac{\pi\omega(n-mL)^2}{N}}}{e^{-j\frac{\pi\omega n^2}{N}}}=e^{j\frac{\pi\omega(2n-mL)mL}{N}} \tag{16}$$

with $m\in\mathcal{M}, n\in\{0,1,\ldots,N-1\}$. Based on (15) and (16), we obtain $c=\gamma_2 N$ and thus the minimum MSE of (11) is given by $\varepsilon_2^{\min}=\frac{\sigma^2}{\gamma_2 N}$. Moreover, we have $\boldsymbol{\Xi}^\dagger=\frac{1}{\gamma_2 N}\boldsymbol{\Xi}^H$, which avoids the matrix inversion operation of cubic-order complexity.

Finally, we illustrate the OFDM symbol structures (short vs. long) and IRS reflection variations (symbol-wise vs. sampling-wise) for the two proposed channel estimation schemes in Fig. 2 and summarize their comparison in Table I. Note that owing to the perfect orthogonality of the joint training design for the pilot signal and IRS reflection pattern, each proposed channel estimation scheme achieves its minimum MSE with low complexity (linear with respect to M and/or L). Generally speaking, Scheme 1 requires lower channel estimation complexity than Scheme 2, but incurs more training time as well as higher minimum MSE (as will be shown by simulations in Section V) for the case of $L_2=1$. Furthermore, the MSE gain of Scheme 2 over Scheme 1 in dB for the case of $L_2=1$ is given by

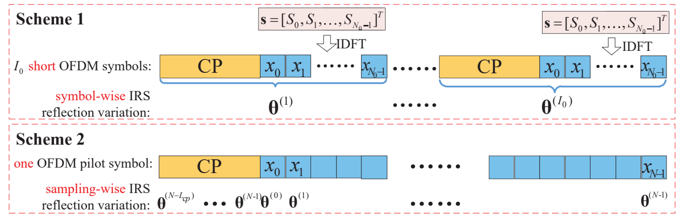

Fig. 2. Illustration of OFDM symbol structures and IRS reflection variations for the two proposed channel estimation schemes.

$$G=-10\log_{10}\left(\frac{\varepsilon_2^{\min}}{\varepsilon_1^{\min}}\right)=10\log_{10}\frac{\gamma_2 N}{\gamma_1(M+1)}. \tag{17}$$

The MSE gain in (17) is due to two factors: one is the average power ratio (which is associated with the training time ratio given the same total power budget, i.e., $\gamma_2/\gamma_1=\eta_1/\eta_2$); the other is the IRS reflection variation ratio (sampling-wise vs. symbol-wise) between the two channel estimation schemes, i.e., $N/(M+1)$.

V. SIMULATION RESULTS

In this section, we present simulation results to validate the effectiveness of our proposed channel estimation schemes. The IRS consists of $M_0=15\times 9=135$ reflecting elements with half-wavelength spacing, which are divided into $M=15$ subsurfaces, each with $\mu=M_0/M=9$ elements. The maximum delay spreads of both the direct BS→user channel and the cascaded BS→IRS→user channel are set as $L_r=L_d=L=8$, while the exact settings of L_1 and L_2 for the BS→IRS and IRS→user channels will be specified later depending on the scenarios. For the BS→user and BS→IRS links, their frequency-selective Rayleigh fading channels are characterized by an exponentially decaying power delay profile with a root-mean-square delay spread and a spread power decaying factor of 2. For the IRS→user link modeled by the frequency-selective Rician fading channel (i.e., $L_2\geq 1$ in general), the first tap is set as the LoS component and the remaining taps are Non-LoS (NLoS) Rayleigh fading components, with κ being the Rician factor. The distance-dependent channel path loss is modeled as $\gamma=\gamma_0/D^\alpha$, where γ_0 is the reference path loss at a distance of 1 meter (m), D is the individual link distance, and α is the path loss exponent. Moreover, the distance between

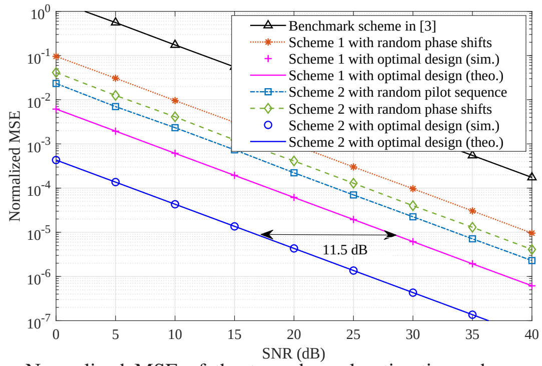

Fig. 3. Normalized MSE of the two channel estimation schemes vs. SNR with $L_1 = 8$ and $L_2 = 1$.

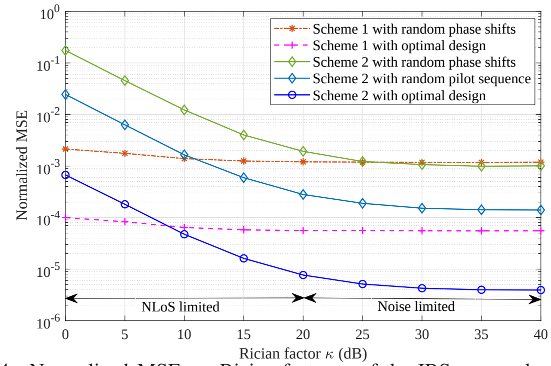

Fig. 4. Normalized MSE vs. Rician factor κ of the IRS$\rightarrow$user channel with SNR $= 20$ dB, $L_1 = 7$, and $L_2 = 2$.

the BS and IRS is 50 m and the user is located in the proximity of the IRS with a distance of 1.5 m.

For Scheme 1, since $N_0 \geq L$ and $L_{cp} \geq L$, it requires at least $\eta_{1,\min} = 2L(M+1)$ sampling periods for channel training; while for Scheme 2, by letting $N = L(M+1)$ and $L_{cp} = L$, the minimum training time in terms of sampling periods is $\eta_{2,\min} = L(M+1)+L = L(M+2)$. Thus, Scheme 2 only requires about half of the training time of Scheme 1. For fair comparison, we consider the same total power budget P for the channel training such that we have $\gamma_1 = \frac{P}{\eta_{1,\min}}$ and $\gamma_2 = \frac{P}{\eta_{2,\min}}$ for the two proposed schemes, respectively. The SNR is defined as the ratio between the average power of the received signal at each sampling period and the noise power at the user, which is given by

$$\text{SNR} = \mathbb{E}\left\{\frac{P\|\boldsymbol{d}+\boldsymbol{Q}\boldsymbol{\theta}\|^2}{\sigma^2(N+L_{cp})}\right\} = \frac{P(M_0\gamma_0^2 D_1^{-\alpha_1} D_2^{-\alpha_2}+\gamma_0 D_3^{-\alpha_3})}{\sigma^2(N+L_{cp})}$$

where D_1, D_2, and D_3 denote the distances of the IRS$\rightarrow$user, BS$\rightarrow$IRS, and (direct) BS$\rightarrow$user links, respectively; α_1, α_2, and α_3 denote the path loss exponents of these links, which are set as 2.2, 2.4, and 3.6, respectively; the path loss at the reference distance is set as $\gamma_0 = -30$ dB for each individual link; and the noise power is set as $\sigma^2 = -80$ dBm. We calculate the normalized MSE (with respect to the overall channel gain, i.e., $\|[\boldsymbol{d}, \boldsymbol{Q}]\|_F^2$ or $\|\boldsymbol{\lambda}\|^2$) over 10000 independent fading channel realizations.

In Fig. 3, we compare the normalized MSE of the two proposed channel estimation schemes with $L_1 = 8$ and $L_2 = 1$ (i.e., the IRS$\rightarrow$user link with the LoS component only). It is observed that the theoretical analysis of MSE given in (6) and (11) is in perfect agreement with the simulation results. Moreover, Scheme 2 achieves up to 11.5 dB SNR gain over Scheme 1 (albeit Scheme 1 spends longer time for channel training), which corroborates the analytical MSE gain $G = 10\log_{10}\frac{\gamma_2 N}{\gamma_1(M+1)} = 11.53$ dB given in (17). Finally, we consider the benchmark designs where either the IRS training reflection coefficients or the pilot symbols are generated with random phase shifts following the uniform distribution within $[0, 2\pi)$ for comparison.[4] It is observed that the two proposed channel estimation schemes with optimal training designs significantly outperform their corresponding benchmark schemes.

In Fig. 4, we examine the effect of NLoS interference in the IRS$\rightarrow$user link on the channel estimation performance for Scheme 2 as compared to Scheme 1, by showing the normalized MSE vs. this channel Rician factor κ (dB) with SNR $= 20$ dB, $L_1 = 7$, and $L_2 = 2$. As the Rician factor κ increases, it is observed that the normalized MSE of Scheme 2 decreases drastically in the range of $\kappa \in [0, 20]$ dB, while it approaches an error floor in the range of $\kappa \in [20, 40]$ dB. This is due to the fact that given SNR $= 20$ dB, the channel estimation error is mainly attributed to the NLoS interference when its power is higher than the noise power (i.e., $\kappa < 20$ dB). In contrast, we observe that the performance of Scheme 1 is almost unaffected by the NLoS component power, since it is applicable to arbitrary frequency-selective fading channels.

VI. Conclusions

In this letter, we proposed two efficient channel estimation schemes for different channel setups in the IRS-assisted OFDM system. By exploiting the novel concept of sampling-wise varying IRS training reflection, Scheme 2 was shown to achieve much lower MSE with even less training time as compared to Scheme 1 under the condition of LoS-dominant IRS-user channel, but at the expense of slightly higher complexity. Both proposed schemes were shown to achieve their respective minimum MSE via jointly optimized IRS training reflection pattern and pilot signal design.

[4]The DFT-based IRS training reflection pattern for Scheme 1 is inapplicable to Scheme 2 since the resultant $\boldsymbol{\Xi}$ with the Zadoff-Chu pilot sequence is not of full rank.

References

[1] Q. Wu and R. Zhang, "Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network," *IEEE Commun. Mag.*, vol. 58, no. 1, pp. 106–112, Jan. 2020.

[2] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.

[3] Y. Yang, B. Zheng, S. Zhang, and R. Zhang, "Intelligent reflecting surface meets OFDM: Protocol design and rate maximization," *IEEE Trans. Commun.*, vol. 68, no. 7, pp. 4522–4535, Jul. 2020.

[4] B. Zheng and R. Zhang, "Intelligent reflecting surface-enhanced OFDM: Channel estimation and reflection optimization," *IEEE Wireless Commun. Lett.*, vol. 9, no. 4, pp. 518–522, Apr. 2020.

[5] B. Zheng, C. You, and R. Zhang, "Intelligent reflecting surface assisted multi-user OFDMA: Channel estimation and training design," *arXiv preprint arXiv:2003.00648*, 2020.

[6] T. L. Jensen and E. De Carvalho, "An optimal channel estimation scheme for intelligent reflecting surfaces based on a minimum variance unbiased estimator," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Barcelona, Spain, May 2020, pp. 5000–5004.

[7] J. Chen, Y.-C. Liang, H. V. Cheng, and W. Yu, "Channel estimation for reconfigurable intelligent surface aided multi-user MIMO systems," *arXiv preprint arXiv:1912.03619*, 2019.

[8] C. You, B. Zheng, and R. Zhang, "Channel estimation and passive beamforming for intelligent reflecting surface: Discrete phase shift and progressive refinement," *to appear in IEEE J. Sel. Areas Commun*, 2019. [Online]. Available: http://arxiv.org/abs/1912.10646

[9] S. M. Kay, *Fundamentals of statistical signal processing*. Prentice Hall PTR, 1993.

[10] D. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inf. Theory*, vol. 18, no. 4, pp. 531–532, Jul. 1972.