assignment_5_solution

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```r
# Q1.Consider that X is the time (in minutes) that a person has to wait in order to
# take a flight. If each flight takes off each hour X = U(0, 60).
# Find the probability that (a) waiting time is more than 45 minutes, and
# (b) waiting time lies between 20 and 30 minutes.
punif(45, min=0, max=60, lower.tail=FALSE)
```

```
## [1] 0.25
```

```r
#or 1-punif(45, min=0, max=60)
punif(30, min=0, max=60)-punif(20, min=0, max=60)
```

```
## [1] 0.1666667
```

```r
# Q2.The time (in hours) required to repair a machine is an exponential distributed
# random variable with parameter lambda = 1/2.
# (a) Find the value of density function at x = 3.
# (b) Plot the graph of exponential probability distribution for 0 <= x <= 5.
# (c) Find the probability that a repair time takes at most 3 hours.
# (d) Plot the graph of cumulative exponential probabilities for 0 <= x <= 5.
# (e) Simulate 1000 exponential distributed random numbers with lambda = 1/2 and plot
# the simulated data.
lambda <- 1/2
x <- 3
f_x <- dexp(x, rate = lambda)
print(paste("Density function at x=3, ",f_x))
```

```
## [1] "Density function at x=3,  0.111565080074215"
```

```r
x_vals <- seq(0, 5, by = 0.01)
pdf_vals <- dexp(x_vals, rate = lambda)
plot(x_vals, pdf_vals, type = "l", col = "blue", lwd = 2,
     main = "Exponential PDF (lambda = 1/2)",
     xlab = "x", ylab = "Density")
```
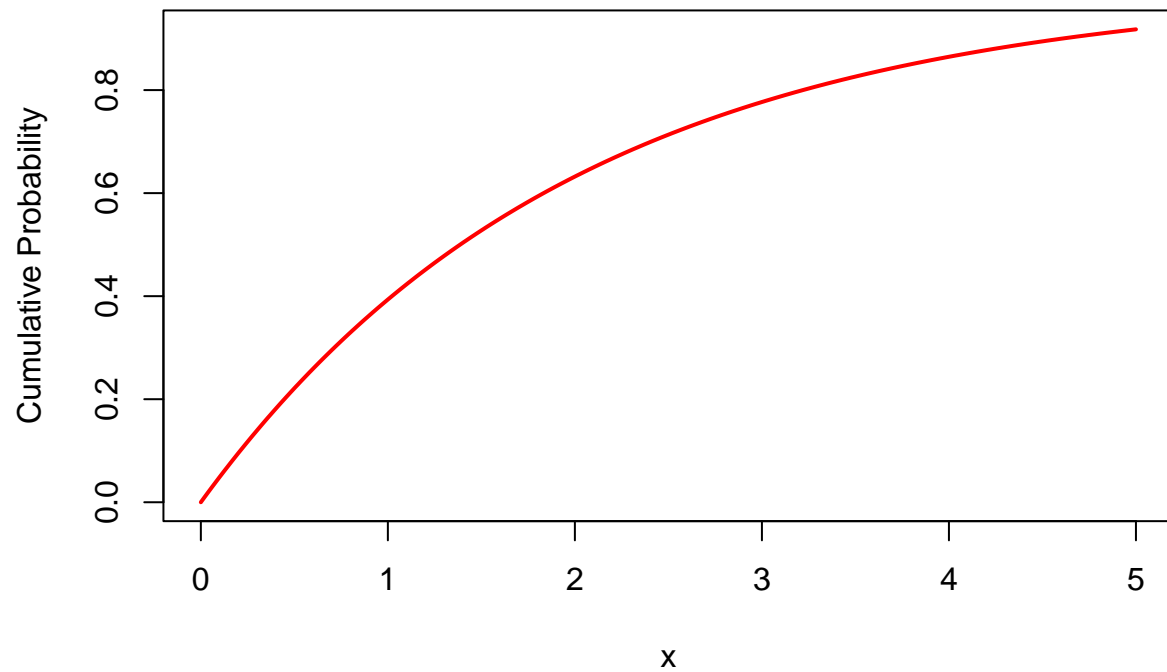
Exponential PDF (lambda = 1/2)
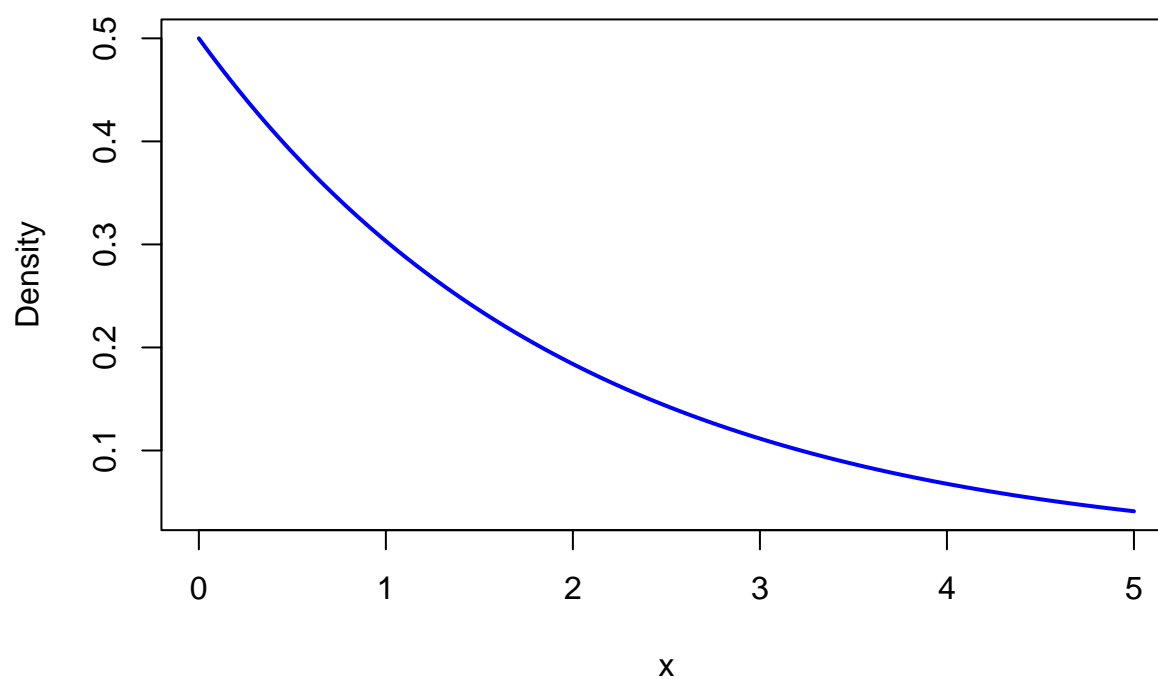

```r
print(paste("Probability that a repair time takes at most 3 hours: ",pexp(3, rate=1/2)))
```

```
## [1] "Probability that a repair time takes at most 3 hours:  0.77686983985157"
```

```r
cdf_vals <- pexp(x_vals, rate = lambda)
plot(x_vals, cdf_vals, type = "l", col = "red", lwd = 2,
     main = "Exponential CDF (lambda = 1/2)",
     xlab = "x", ylab = "Cumulative Probability")
```
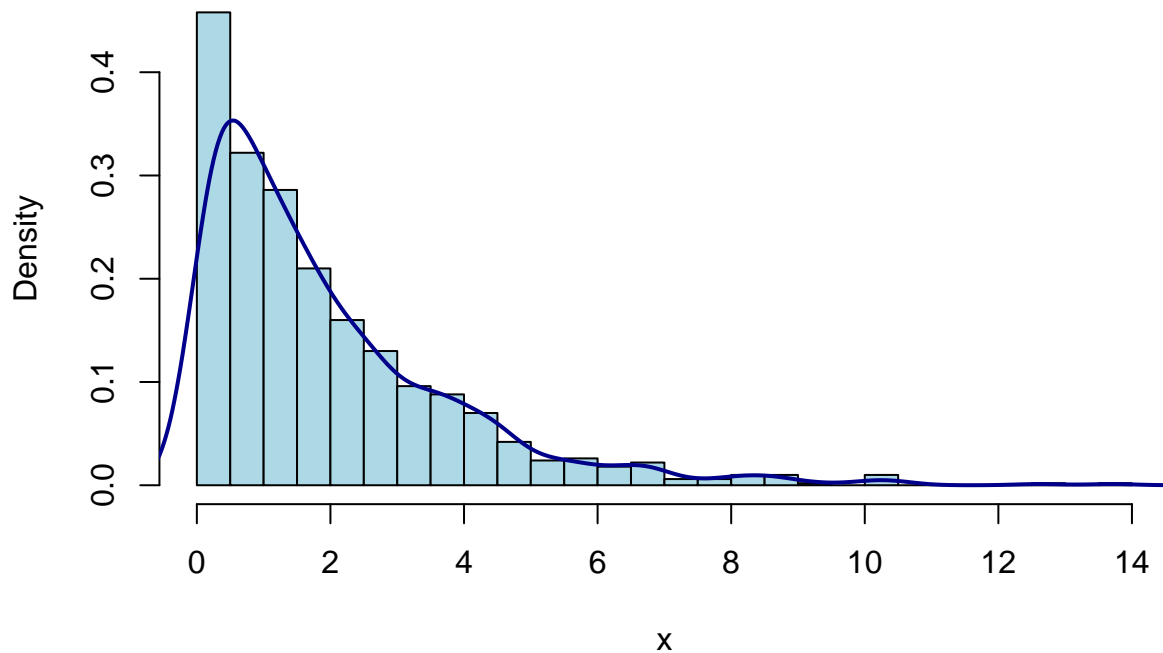
Exponential CDF (lambda = 1/2)


```r
n=1000
sim_data <- rexp(n, rate = lambda)
hist(sim_data, probability = TRUE, col = "lightblue",
     main = "Histogram of Simulated Exponential Data",
     xlab = "x", breaks = 30)

# Add the theoretical density line on top of the histogram
lines(density(sim_data), col = "darkblue", lwd = 2)
```

Histogram of Simulated Exponential Data


```r
# Q3.The lifetime of certain equipment is described by a random variable X that
# follows Gamma distribution with parameters alpha = 2 and beta = 1/3.
# (a) Find the probability that the lifetime of equipment is (i) 3 units of time,
# and (ii) at least 1 unit of time.
# (b) What is the value of c, if P(X <= c) >= 0.70? (Hint: qgamma())
alpha <- 2  # Shape parameter
beta <- 1/3  # Scale parameter

p_at_3 <- dgamma(3, shape = alpha, scale = beta)
print(paste("Probability that the lifetime of 3 units: ",p_at_3 ))
```

```
## [1] "Probability that the lifetime of 3 units:  0.00333206471034035"
```

```r
p_at_least_1 <- 1 - pgamma(1, shape = alpha, scale = beta)
print(paste("Probability that the lifetime of at least 1 unit: ",p_at_least_1 ))
```

```
## [1] "Probability that the lifetime of at least 1 unit:  0.199148273471456"
```

```r
c_value <- qgamma(0.70, shape = alpha, scale = beta)
print(paste("Value of c for P(X <= c) >= 0.70 : ",c_value))
```

```
## [1] "Value of c for P(X <= c) >= 0.70 :  0.813072161093401"
```