

assignment_5_solution

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```
# Q1.Consider that X is the time (in minutes) that a person has to wait in order to
# take a flight. If each flight takes off each hour  $X = U(0, 60)$ .
# Find the probability that (a) waiting time is more than 45 minutes, and
# (b) waiting time lies between 20 and 30 minutes.
punif(45, min=0, max=60, lower.tail=FALSE)
```

```
## [1] 0.25
```

```
#or 1-punif(45, min=0, max=60)
punif(30, min=0, max=60)-punif(20, min=0, max=60)
```

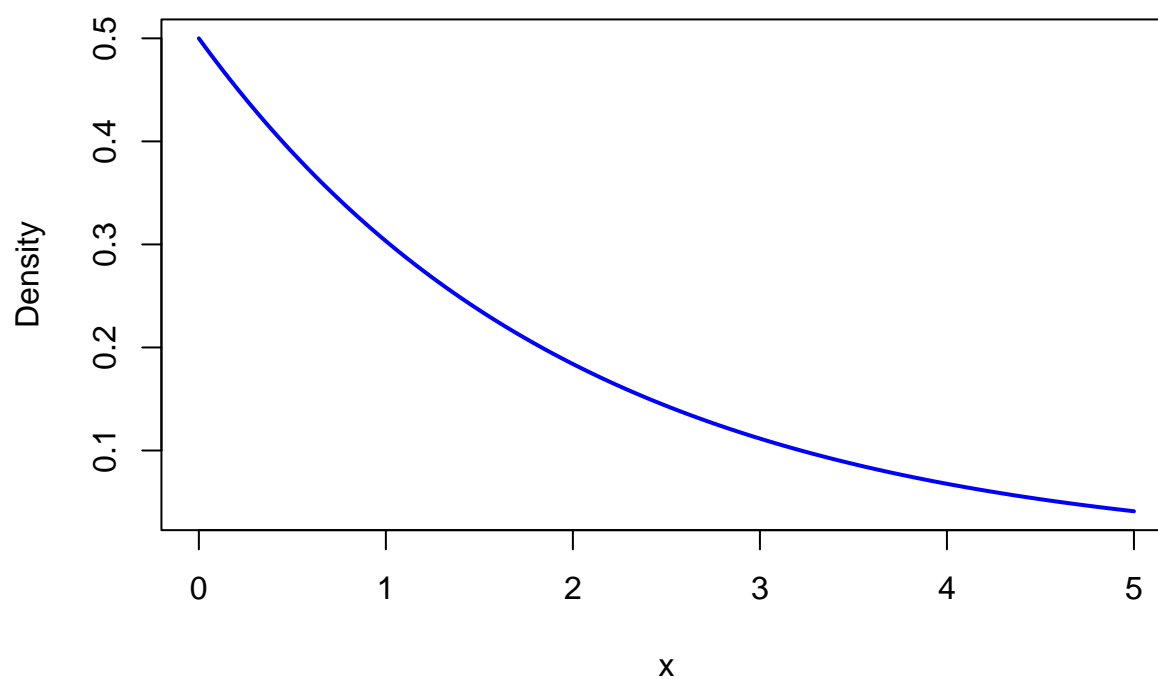
```
## [1] 0.1666667
```

```
# Q2.The time (in hours) required to repair a machine is an exponential distributed
# random variable with parameter  $\lambda = 1/2$ .
# (a) Find the value of density function at  $x = 3$ .
# (b) Plot the graph of exponential probability distribution for  $0 \leq x \leq 5$ .
# (c) Find the probability that a repair time takes at most 3 hours.
# (d) Plot the graph of cumulative exponential probabilities for  $0 \leq x \leq 5$ .
# (e) Simulate 1000 exponential distributed random numbers with  $\lambda = 1/2$  and plot
# the simulated data.
lambda <- 1/2
x <- 3
f_x <- dexp(x, rate = lambda)
print(paste("Density function at x=3, ",f_x))
```

```
## [1] "Density function at x=3, 0.111565080074215"
```

```
x_vals <- seq(0, 5, by = 0.01)
pdf_vals <- dexp(x_vals, rate = lambda)
plot(x_vals, pdf_vals, type = "l", col = "blue", lwd = 2,
     main = "Exponential PDF ( $\lambda = 1/2$ )",
     xlab = "x", ylab = "Density")
```

Exponential PDF (lambda = 1/2)

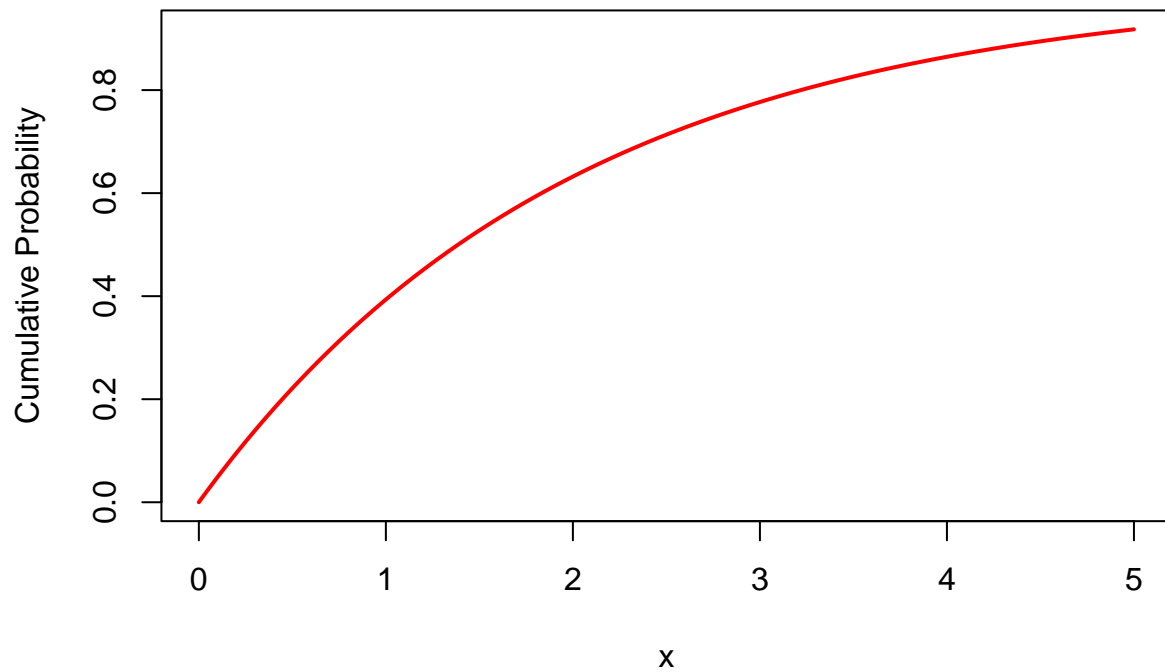


```
print(paste("Probability that a repair time takes at most 3 hours: ",pexp(3, rate=1/2)))
```

```
## [1] "Probability that a repair time takes at most 3 hours: 0.77686983985157"
```

```
cdf_vals <- pexp(x_vals, rate = lambda)
plot(x_vals, cdf_vals, type = "l", col = "red", lwd = 2,
     main = "Exponential CDF (lambda = 1/2)",
     xlab = "x", ylab = "Cumulative Probability")
```

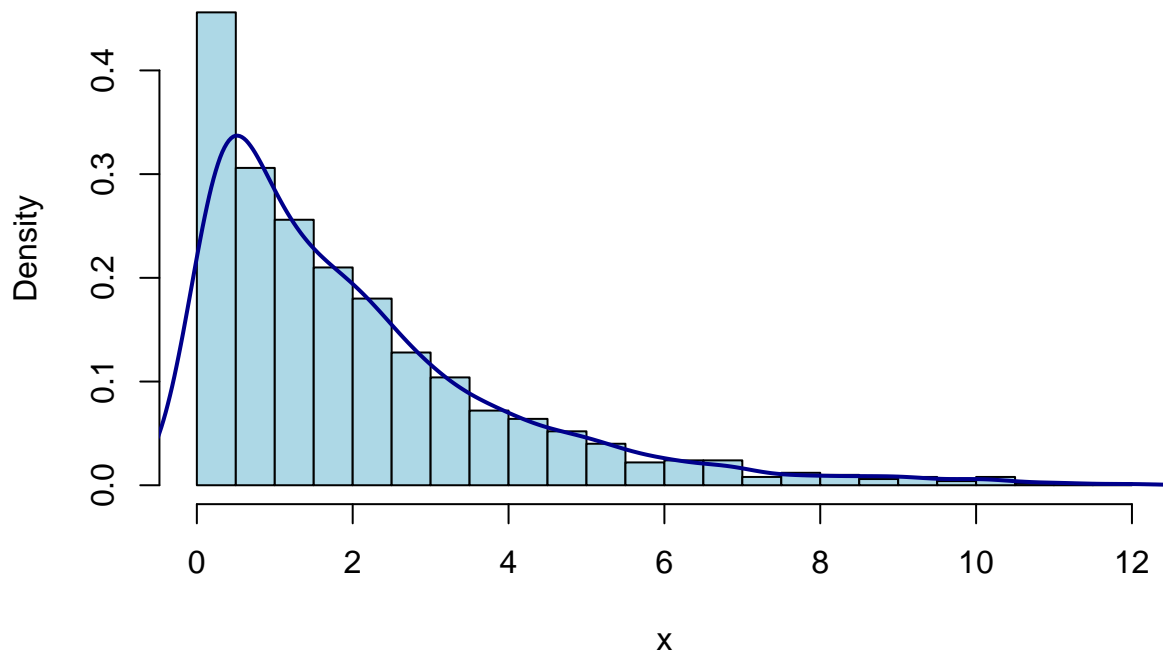
Exponential CDF ($\lambda = 1/2$)



```
n=1000
sim_data <- rexp(n, rate = lambda)
hist(sim_data, probability = TRUE, col = "lightblue",
     main = "Histogram of Simulated Exponential Data",
     xlab = "x", breaks = 30)

# Add the theoretical density line on top of the histogram
lines(density(sim_data), col = "darkblue", lwd = 2)
```

Histogram of Simulated Exponential Data



```
# Q3. The lifetime of certain equipment is described by a random variable  $X$  that  
# follows Gamma distribution with parameters  $\alpha = 2$  and  $\beta = 1/3$ .  
# (a) Find the probability that the lifetime of equipment is (i) 3 units of time,  
# and (ii) at least 1 unit of time.
```

```
# (b) What is the value of  $c$ , if  $P(X \leq c) \geq 0.70$ ? (Hint: qgamma())
```

```
alpha <- 2 # Shape parameter
```

```
beta <- 1/3 # Rate parameter
```

```
p_at_3 <- dgamma(3, shape = alpha, rate = beta)
```

```
print(paste("Probability that the lifetime of 3 units: ", p_at_3 ))
```

```
## [1] "Probability that the lifetime of 3 units: 0.122626480390481"
```

```
p_at_least_1 <- 1 - pgamma(1, shape = alpha, rate = beta)
```

```
print(paste("Probability that the lifetime of at least 1 unit: ", p_at_least_1 ))
```

```
## [1] "Probability that the lifetime of at least 1 unit: 0.955375080765052"
```

```
c_value <- qgamma(0.70, shape = alpha, rate = beta)
```

```
print(paste("Value of  $c$  for  $P(X \leq c) \geq 0.70$  : ", c_value))
```

```
## [1] "Value of  $c$  for  $P(X \leq c) \geq 0.70$  : 7.31764944984061"
```