assignment_4_solution

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```r
# Q1.The probability distribution of X, the number of imperfections per 10 meters
# of a synthetic fabric in continuous rolls of uniform width, is given as
# x = 0,1,2,3,4 with p(x)= 0.41, 0.37, 0.16, 0.05, 0.01
# Find the average number of imperfections per 10 meters of this fabric.
# (Try functions sum( ), weighted.mean( ), c(a %*% b) to find expected value/mean
x <- c(0,1,2,3,4)
px <- c( 0.41,0.37,0.16,0.05,0.01)
avg <- sum(x*px)
print(paste('Average using sum() = ',avg))
```

```
## [1] "Average using sum() =  0.88"
```

```r
print(paste('Average using weighted.mean() = ',weighted.mean(x,px)))
```

```
## [1] "Average using weighted.mean() =  0.88"
```

```r
print(paste('Average using c(a %*% b) = ',c(x %*% px)))
```

```
## [1] "Average using c(a %*% b) =  0.88"
```

```r
# Q2.The time T, in days, required for the completion of a contracted project is
# a random variable with probability density function f(t) = 0.1 e(-0.1t) for
# t > 0 and 0 otherwise. Find the expected value of T.
# Use function integrate( ) to find the expected value of continuous random variable T.
f <- function(t){
  t*0.1*exp(-0.1*t)
}
ans <- integrate(f, lower = 0, upper = Inf)
print(ans)
```

```
## 10 with absolute error < 6.7e-05
```

```r
# Q3.A bookstore purchases three copies of a book at $6.00 each and sells them
# for $12.00 each. Unsold copies are returned for $2.00 each.
# Let X = {number of copies sold} and Y = {net revenue}. If the probability mass
# function of X is x = 0,1,2,3 with p(x)= 0.1,0.2,0.2,0.5
# Find the expected value of Y.
x <- c(seq(0,3))
```

```r
px <- c(0.1,0.2,0.2,0.5)
y <- (10*x+6)
e <- sum(y*px)
print(e)
```

```
## [1] 27
```

```r
# Q4.Find the first and second moments about the origin of the random variable X
# with probability density function f(x) = 0.5e^-|x|, 1 < x < 10 and 0 otherwise.
# Further use the results to find Mean and Variance.
# (kth moment = E(X^k), Mean = first moment and Variance = second moment - Mean^2
f1 <- function(x){
  x*0.5*exp(-abs(x))
}
f2 <- function(x){
  (x^2)*0.5*exp(-abs(x))
}
e1 <- integrate(f1,1,10)
print(paste("Value of first moment/mean", e1$value))
```

```
## [1] "Value of first moment/mean 0.367629741557749"
```

```r
e2 <- integrate(f2,1,10)
print(paste("Value of second moment", e2$value))
```

```
## [1] "Value of second moment 0.916929207213094"
```

```r
v <- (e2$value) - ((e1$value)^2)
print((paste("Value of variance", v)))
```

```
## [1] "Value of variance 0.781777580335277"
```

```r
# Q5.Let X be a geometric random variable with probability distribution
# f(x) = (3/4) * ((1/4)^x-1) , x=1,2,3,...
# Write a function to find the probability distribution of the random variable
# Y = X^2 and find probability of Y for X = 3. Further, use it to find the
# expected value and variance of Y for X = 1,2,3,4,5.
#Method 1
f <- function(y){
  (3/4) * ((1/4)^(sqrt(y)-1))
}
x <- c(1,2,3,4,5)
y <- x^2
py <- f(y)
print(py)
```

```
## [1] 0.750000000 0.187500000 0.046875000 0.011718750 0.002929688
```

```r
ey1 <- sum(y*py)
ey2 <- sum((y^2)*py)
v <- ey2 - (ey1^2)
print(paste("Mean of Y is ",ey1))
```

```
## [1] "Mean of Y is  2.1826171875"
```

```r
print(paste("Variance of Y is ",v))
```

```
## [1] "Variance of Y is  7.61411190032959"
```

```r
#Method 2
# PMF of the geometric distribution for X
f_x <- function(x) {
  return((3/4) * (1/4)^(x-1))
}

# Function to compute Y = X^2 and the corresponding probabilities
f_y <- function(x) {
  y <- x^2  # Y = X^2
  prob <- f_x(x)  # Probability of X
  return(list(y = y, prob = prob))  # Return Y and its probability
}

# Compute probabilities for X = 1, 2, 3, 4, 5
x_values <- 1:5  # Values of X
y_probabilities <- lapply(x_values, f_y)

# Convert the result into a data frame
y_probs <- data.frame(
  X = x_values,
  Y = sapply(y_probabilities, function(x) x$y),  # Extract Y values
  Probability = sapply(y_probabilities, function(x) x$prob)  # Extract probabilities
)

# Print the table of probabilities
print(y_probs)
```

```
##   X  Y Probability
## 1 1  1 0.750000000
## 2 2  4 0.187500000
## 3 3  9 0.046875000
## 4 4 16 0.011718750
## 5 5 25 0.002929688
```

```r
# Calculate the expected value E[Y]
expected_value <- sum(y_probs$Y * y_probs$Probability)
expected_square <- sum((y_probs$Y^2) * y_probs$Probability)

# Calculate the variance Var(Y)
variance <- expected_square - expected_value^2
```

```r
# Print the expected value and variance
cat("Expected Value/Mean of Y:", expected_value, "\n")
```

```
## Expected Value/Mean of Y: 2.182617
```

```r
cat("Variance of Y:", variance, "\n")
```

```
## Variance of Y: 7.614112
```