

# assignment\_4\_solution

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# Q1.The probability distribution of X, the number of imperfections per 10 meters
# of a synthetic fabric in continuous rolls of uniform width, is given as
# x = 0,1,2,3,4 with p(x)= 0.41, 0.37, 0.16, 0.05, 0.01
# Find the average number of imperfections per 10 meters of this fabric.
# (Try functions sum( ), weighted.mean( ), c(a %*% b) to find expected value/mean
x <- c(0,1,2,3,4)
px <- c( 0.41,0.37,0.16,0.05,0.01)
avg <- sum(x*px)
print(paste('Average using sum() = ',avg))
```

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## [1] "Average using sum() = 0.88"
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print(paste('Average using weighted.mean() = ',weighted.mean(x,px)))
```

```
## [1] "Average using weighted.mean() = 0.88"
```

```
print(paste('Average using c(a %*% b) = ',c(x %*% px)))
```

```
## [1] "Average using c(a %*% b) = 0.88"
```

```
# Q2.The time T, in days, required for the completion of a contracted project is
# a random variable with probability density function  $f(t) = 0.1 e^{-0.1t}$  for
#  $t > 0$  and 0 otherwise. Find the expected value of T.
# Use function integrate( ) to find the expected value of continuous random variable T.
f <- function(t){
  t*0.1*exp(-0.1*t)
}
ans <- integrate(f, lower = 0, upper = Inf)
print(ans)
```

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## 10 with absolute error < 6.7e-05
```

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# Q3.A bookstore purchases three copies of a book at $6.00 each and sells them
# for $12.00 each. Unsold copies are returned for $2.00 each.
# Let  $X = \{\text{number of copies sold}\}$  and  $Y = \{\text{net revenue}\}$ . If the probability mass
# function of X is  $x = 0,1,2,3$  with  $p(x) = 0.1,0.2,0.2,0.5$ 
# Find the expected value of Y.
x <- c(seq(0,3))
```

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px <- c(0.1,0.2,0.2,0.5)
y <- (10*x-12)
e <- sum(y*px)
print(e)
```

```
## [1] 9
```

```
# Q4.Find the first and second moments about the origin of the random variable X
# with probability density function  $f(x) = 0.5e^{-|x|}$ ,  $1 < x < 10$  and 0 otherwise.
# Further use the results to find Mean and Variance.
# (kth moment =  $E(X^k)$ , Mean = first moment and Variance = second moment - Mean^2
f1 <- function(x){
  x*0.5*exp(-abs(x))
}
f2 <- function(x){
  (x^2)*0.5*exp(-abs(x))
}
e1 <- integrate(f1,1,10)
print(paste("Value of first moment/mean", e1$value))
```

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## [1] "Value of first moment/mean 0.367629741557749"
```

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e2 <- integrate(f2,1,10)
print(paste("Value of second moment", e2$value))
```

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## [1] "Value of second moment 0.916929207213094"
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v <- (e2$value) - ((e1$value)^2)
print((paste("Value of variance", v)))
```

```
## [1] "Value of variance 0.781777580335277"
```

```
# Q5.Let X be a geometric random variable with probability distribution
#  $f(x) = (3/4) * ((1/4)^{x-1})$ ,  $x=1,2,3,\dots$ 
# Write a function to find the probability distribution of the random variable
#  $Y = X^2$  and find probability of Y for  $X = 3$ . Further, use it to find the
# expected value and variance of Y for  $X = 1,2,3,4,5$ .
f <- function(y){
  (3/4) * ((1/4)^(sqrt(y)-1))
}
x <- c(1,2,3,4,5)
y <- x^2
py <- f(y)
print(py)
```

```
## [1] 0.750000000 0.187500000 0.046875000 0.011718750 0.002929688
```

```
ey1 <- sum(y*py)
ey2 <- sum((y^2)*py)
v <- ey2 - (ey1^2)
print(paste("Mean of Y is ",ey1))
```

```
## [1] "Mean of Y is 2.1826171875"
```

```
print(paste("Variance of Y is ",v))
```

```
## [1] "Variance of Y is 7.61411190032959"
```