

# SJÄLVSTÄNDIGA ARBETEN I MATEMATIK

MATEMATISKA INSTITUTIONEN, STOCKHOLMS UNIVERSITET

Gröbner Bases and Elimination in Macaulay 2

av

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# Gröbner Bases and Elimination in Macaulay 2

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Självständigt arbete i matematik 15 högskolepoäng, grundnivå

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## **Abstract**

Your summary goes here



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# 1 Introduction

Hello world

[[DAC10](#)]

[[LM21](#)]

## 2 The Endless Hunger of Algebra

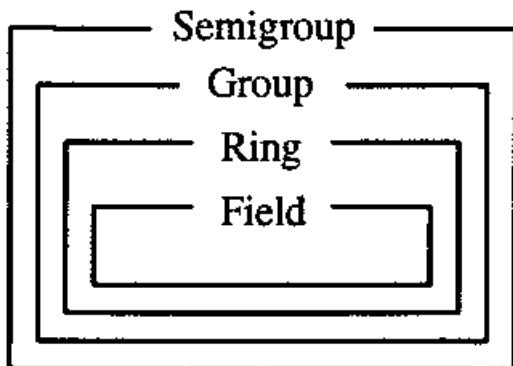
Algebra is the mathematical expression of the human need for structure and abstraction. Through algebra, humans try to capture more of the mathematical, and natural, domain under their pen. The endless hunger for knowledge can express itself in various ways. When using algebra as a tool, creating abstract models that capture generic structure is favored. However, these abstract models can be difficult to understand. The ability to navigate between levels of abstraction is a vital tool for any mathematician interested in applying algebraic methods.

This paper assumes a degree of familiarity with algebraic structures, but the uninitiated reader is kindly directed towards the easy-going introduction of "Introduction to Modern Algebra" [Wei70] with bite size exercises, or alternatively, the more rigorous "Abstract Algebra" [DF03] for a deeper understanding.

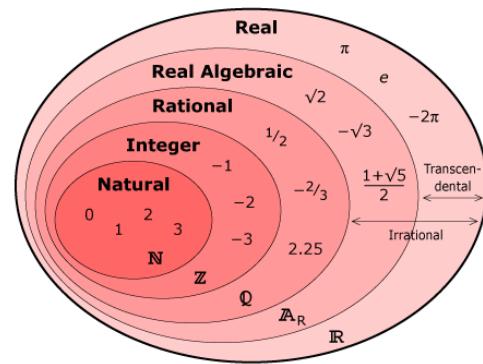
### 2.1 Levels of Abstraction

Abstract algebra traverses through layers of abstraction as an exercise. As an example, if we are interested in how groups act when supplemented with commutativity (Abelian groups), it would be useful to start at groups and traverse the layers of figure 1a downwards. Note that climbing upwards may not be as fruitful for this particular case.

Choosing an Abelian group that stays between groups and rings is  $(\mathbb{R}, +)$ . Here the set of real numbers is only considered with the binary operator  $+$ , and commutativity follows. Analyzing this group and all the properties that are associated with it is of interest. However, much use can be extracted from climbing further down the abstraction ladder and analyzing rings a. Keeping it concrete with numbers, one can consider the abelian ring  $\mathbb{Z}$ . Time can be spent on this layer of abstraction and knowledge can be obtained through leveraging  $\mathbb{Z}$ 's special properties. Going even further down the ladder, to a field, one could analyze  $\mathbb{Q}$ . At each layer, new intuition can be gained. Note also, that while traversing downwards in the group abstraction, we were climbing up and down in the generalization of numbers. Starting at the top with  $\mathbb{R}$ , we jumped all the way down to  $\mathbb{Z}$ , and then up again to  $\mathbb{Q}$ . It is important to consider this, as different layers of abstraction do not necessarily have to correspond with another. Caution is required when dealing with this iterative abstract process.



(a) Abstractions within Groups [Lat]



(b) Generalization of Numbers [Diand]

Figure 1: Traversing the ladder of Abstraction

## 2.2 Polynomials

## 2.3 Affine Varieties

## 2.4 Rings and Ideals

## 2.5 Ordering Polynomials

## 2.6 An Analytic Bridge

## 2.7 Hilbert Strong Nullstellensatz

### **3 Gröbner Bases**

**3.1 Hilbert Bases**

**3.2 Gröbner Bases**

**3.3 Properties of Gröbner Bases**



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