SJÄLVSTÄNDIGA ARBETEN I MATEMATIK

MATEMATISKA INSTITUTIONEN, STOCKHOLMS UNIVERSITET

Gröbner Bases and Elimination in Macaulay 2

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Abstract

Your summary goes here

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1 Introduction

Hello world
[DAC10]
[LM21]

2 The Endless Hunger of Algebra

Algebra is the mathematical expression of the human need for structure and abstraction. Through algebra, humans try to capture more of the mathematical, and natural, domain under their pen. The endless hunger for knowledge can express itself in various way. When using algebra as a tool, creating abstract models that capture generic structure is favored. However, these abstract models can be difficult to understand. The ability to navigate between levels of abstraction is a vital tool for any mathematician interested in applying algebreic methods.

This paper assumes a degree of familiarity with algebreic structures, but the uninitiated peruser is kindly directed towards the easy-going introduction of "Introducetion to Modern Algebra" [Wei70] with bite size exercises, or alternativly, the more rigorous "Abstract Algebra" [DF03] for a deeper understanding.

2.1 Levels of Abstraction

Abstract algebra traverses through layers of abstraction as an exercise.

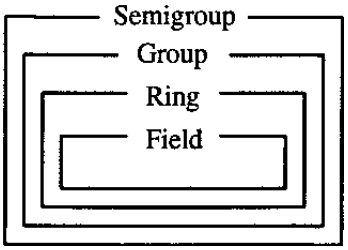

Figure 1: Layers of Abstraction in Lattice Algebra [Lat]

2.2 Polynomials

2.3 Affine Varieties

2.4 Rings and Ideals

2.5 Ordering Polynomials

2.6 An Analytic Bridge

2.7 Hilbert Strong Nullstellensatz

3 Gröbner Bases

3.1 Hilbert Bases

3.2 Gröbner Bases

3.3 Properties of Gröbner Bases

References

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[DF03] David S. Dummit and Richard M. Foote. *Abstract Algebra*. Wiley, Hoboken, NJ, USA, 3 edition, 2003.

[Lat] Lattice and boolean algebra 2.1 algebra 2.2 lattice. URL: https://api.semanticscholar.org/CorpusID:18433539.

[LM21] Antonio Tornambè Laura Menini, Corrado Possieri. *Algebraic Geometry for Robotics and Control Theory*. WORLD SCIENTIFIC (EUROPE), 2021. arXiv:https://www.worldscientific.com/doi/pdf/10.1142/q0308.

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