HW 3

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Problem 1. homomorphism $h : \epsilon \implies \Delta^*$
if $h(w) = \{\epsilon$, if $w = \epsilon$
$h(a)h(\mu)$, if $w = an$ where $a \in \Sigma$, $\mu \in \Sigma^*\}$
$h^*(L) = \{h(w)|w \in L\}$ where $L \subseteq \Sigma^*$
$h^{-1} = \{h(w)|h(w) \in L\}$

$\Sigma = \{a, b\}$, $h(a) = 01$
$\Delta = \{0, 1\}$, $h(b) = 10$

a)
$h^{-1}(\{0101\}) = (ab)$
$h^{-1}(\{00\}) = \emptyset$
$h^{-1}(\{001\}) = \emptyset$
$h^{-1}(\{1001\}) = (ba)$

b)
$L = L((00 + 1)^*)$
$h^{-1}(L) = (ba)^*$
$h(h^{-1}(L)) = h((ba)^*) = (1001)^*$

Problem 2. Regular language are closed under inverse homomorphic images
if $h : \Sigma \implies \Delta^*$ and $L \subseteq \Delta^*$ is regular than h^{-1} is regular
a) $w \in \Sigma^*$, $d_n^*(s^{'}w) = \delta_M^*(s, h(w))$

b) Prove the correctness statement by induction on the length w

Proof. We will do induction on $|w|$
Base Case:
When $|w|$, $w = \epsilon$
$\delta_M^*(s^{'}, w) = \delta_M^*(\delta, h(w))$
$\delta_M^*(s^{'}, w) = \delta_M^*(\delta, h(\epsilon))$

Since $s'^* = s$
$\delta_M^*(s, \epsilon) = \delta_M^*(s, h(\epsilon))$

By definition of a homomorphism
$h(\epsilon) = \epsilon$
$\delta^*_M(s, \epsilon) = \delta^*_M(s, \epsilon)$

Inductive Hypothesis: $\forall w$ where $|w| < n$, $\delta^*_M(s, w) = \delta^*_M(s', h(w))$
Induction Step:
Let w be a string such that $|w| = \mu$
And as we know $s = s'$ then
$\delta^*_M(s, w) = \delta^*_M(s, h(w)) =$
Let $w = an$, $a \in \Sigma$, $\mu \in \Sigma^*$
$\delta^*_M(s, a * m) = \delta^*_M(s, h(a) * h(\mu))$
$\delta^*_M(\delta(s, a), \mu) = \delta^*_M(\delta^*_M(s, h(a), h(\mu))$
$\delta^*_M(\delta(s, a), \mu) = \delta^*_M(\delta^*_M(s, a), \mu)$

$\square$