# Seminar On Bounded Sets

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**Question 4.** Find the lub and glb of the following sets.

**Exercise** $A = \{x | x = 2^{-p} + 3^{-q}, \forall p, q \in \mathbb{N}\}$**.** let $(p, q) = (1, 1)$
$\therefore x = 2^1 + 3^2 = 5$
$lub(A) = 5$

let $p, q = b$
$x = \lim_{b\to\infty} 2^{-b} + 3^{-b}$
$x = 0$
$\therefore$ glb(A) $= 0$

**Exercise** $B = \{x | (\, x \in (0, 1)\,) \wedge (x \in \mathbb{R})\}$**.** $glb(B) = 0$
$lub(B) = 1$

**Question 6.** Which of the following statements are true and which are false? Give adequate reasons for you answer.

**Exercise B.** $(\forall r \in \mathbb{R})(\exists B \subset Q)(r = glb(B))$
False.
Since $\mathbb{R}$ is uncountable by Cantor's Diagonal argument and $\mathbb{Q}$ is countable also by Cantor's Diagonal argument. It cannot be true that there is a map from all real numbers to a set in which the glb of that set is a real number.

**Exercise D.** If the greatest lower bound of a set of real numbers exists but is not a member of the set, then the set must be infinite, and have a subsequence that converges to its greatest lower bound.

Let $A$ be an infinite set and $A_n$ be a subsequence
$(\alpha = glb(A)) \wedge (\alpha \notin A) \implies (|A| = \infty) \wedge (\lim_{An \to \infty} = glb(A))$
True.
By definition of glb, $(glb(A) = \alpha)(\forall x \in A)(\alpha \leq x)$ however, it is not necessarily true that $(\alpha \in A)$
If $(|A| = n)$ where $(n \in \mathbb{Z})$ and is finite then $\alpha \in A$
If this is not the case then there must exist some subsequence $A_n$ such that as $A_n \to \infty = L$ where $L = \alpha$

**Question 7.** Prove that the cubic equation $x^3 - x - 1 = 0$ has a real root by showing that any root of the equation is the lub of a suitable set.

let $A = \{x | x^3 - x - 1 < 0\}$, $c = glb(A)$, $f(x) = x^3 - x - 1$

Since $f$ is continuous on the interval $(-\infty, \infty)$ and $(f(0) = -1) \wedge (f(2) = 3)$ by the Intermediate value theorem we know that f has is a real root somewhere on the interval of [0,2]

Choose this 'suitable set' to be $S$ where $S = \{x | x^3 - x - 1 < 0\}$ and $c$ is the lub of $S$. We can use binary search on different values of $x$ where $x \in \mathbb{R}$ to determine the value of $C$.