Theorem 1 *If* $x, y, m \in \mathbb{Z}$ *then* $gcd(x, y) = gcd(x, y - mx)$

PROOF Given $m \in \mathbb{Z}$
It must be true that
$(y - mx) = (y \mod x)$
Thus we are really trying to prove..

$$\begin{aligned} gcd(x, y) &= gcd(x, y - mx) \\ &= gcd(x, ymodx) \end{aligned}$$

That being true, there are two cases that must be considered for this proof.

Case 1 where $x = y = 0$

$$\begin{aligned} gcd(0,0) &= gcd(0,0-0) \\ &= gcd(0,0) \end{aligned}$$

Case 2 Assume at least one of x, y is non zero
Suppose $d|x$ and $d|y$
We now must prove that $d|y - mx$
Since $(d|x \wedge y)(\exists k_0, k_1)$ such that $(x = d * k_0)(y = d * k_1)$ given $(k_0, k_1 \in Z)$

$$\begin{aligned} gcd(x, y) &= gcd(x, y - mx) \\ &= gcd(x, ymodx) \\ &= d \\ &= ax + by \\ &= ax + y(\end{aligned}$$