FileCards Week 4

Cameron Dart

February 12, 2016

Question 1. What is the decimal expansion of $\frac{3}{7}$?

Proof. $\frac{3}{7} = 0.428571429$ $\square$

Question 2. What is $A * B$ in hexadecimal?

Proof. $A_{16} = (10 * 16^0) = 10_{10}$
$B_{16} = (11 * 16^0) = 11_{10}$
$(A * B)_{16} = 110_{10}$
$110_{10} = 96_{10} + 14_{10}$
$110_{10} = (6 * 16^1) + (14 * 16^0)$
$110_{10} = 6E_{16}$ $\square$

Question 3. Write the right-hand sides of the seven definitions above in good English, without symbols.

Function f:XY iff (xX)(! yY)(y=f(x))
injective: iff (a,bX)(f(a)=f(b)a=b)
surjective: iff (yY)(xX)(y=f(x))
bijective: iff (yY)(! xX)(y=f(x))
inverse of point: f-1(y)=xXf(x)=y
inverse of a subset: f-1(B)=xXf(x)B
image of a subset: f(A)=yY(xX)(y=f(x))

Proof. **Not positive about these**

Function- For all elements in the domain there does not exist an element in the codomain

Injective- For any two elements in the domain, if the function evaluated at a is equal to the function evaluated at b then those two elements are equal

Surjective- For all outputs there is one input

Bijective - Every element in the codomain is mapped to one element in the domain

Inverse of Point - the inverse function is the set of elements in the domain such that there is a function that maps elements from the domain to codomain

Inverse of Subset - the inverse of a set is the set containing elements of the domain such that there is a function that maps elements of that function to the inverse set

Image of Subset - The image of a subset is equal to elements of the codomain ST there exists an element in the domain that maps to an element in the codomain

□

Question 4. For the function just defined, write an explicit formula for $f^{-1}(z)\, where\, z \in \mathbb{Z}$

Proof. let $f^{-1}(z)\, where\, z \in \mathbb{Z}$
Solve $z = f(x) \implies f^{-1}(z) = \{f(odd) = \frac{y-1}{2}, f(even) = \frac{y}{2}\}$ □

Question 5. Show that the even whole numbers are also countably infinite.

Proof. **Countable Set** - A set with the same cardinality as a subset of the $\mathbb{N}$
Countably Infinite - A set is considered to be countably infinite if it has a one-to-one correspondence with the set of $\mathbb{N}$
Let $S = \{2k | k \in \mathbb{N}\}$ and $f(x) = 2x$
f is a bijection between from $\mathbb{N}$ to E since f is one to one and onto.
Surjective: let $t \in S$ then $\exists t = 2k$, for some $t \in \mathbb{N} \wedge f(k) = t$
Injective: let $f(n) = f(m) \implies 2n = 2m \therefore n = m$ It has been proven that there exists a bijection from $\mathbb{N}$ to S. QED. □