Theorem 1 *If* $x, y, m \in \mathbb{Z}$ *then* $gcd(x, y) = gcd(x, y - mx)$

Lemma A. $p|q \wedge p|r \implies p|q + r$
Assume p, q, r are all integers such that $p, q, r \neq 0$
Let $p|q$ and $p|r$ evenly
Since $p|q \implies (\exists k_1)(q = pk_1)$
$p|r \implies (\exists k_2)(r = pk_2)$
$q + r = p(k_1 + k_2)$
$q + r = p(k_3)$ for some integer $k_3 = k_1 + k_2$
By definition of divisibility $p|q + r$

Lemma B. Let a, b, c be integers such that $a, b, c \neq 0$
$a|b \implies a|bc$
$b = ar$ where $r \in \mathbb{Z}$
$bc = arc = (ar)c$
Since rc is an integer $a|bc$

PROOF There are two cases that must be considered for this proof.

Case 1 where $x = y = 0$
Let $d_0|x \wedge y$

$$\begin{aligned} gcd(0,0) &= gcd(0, 0 - 0) \\ &= gcd(0,0) \end{aligned}$$

Hence, it must be true for this case.

Case 2 Assume at least one of x, y is non zero
Suppose $d_1|x$ and $d_1|y$
Let $n = -mx$
By Lemma B it is true that $d_1|n$
Since $d_1|x \wedge d_1|n$ by Lemma A it must be true that $d|y + n$
Hence $d_1|y + n$
$d_1|y - mx$