# Homework 7

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Graph Theory

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**Problem 1**

**Claim** If $G$ is a simple graph $G$ with $\Delta(G) = 3$, then $\kappa'(G) = \kappa(G)$

**Problem 2**

**Claim** A simple connected graph $G$ with at least 3 vertices is $2-$connected if and only if $\forall x, y \in V(G)$ and any $e \in E(G)$. $G$ has an $x, y$ path through $e$.

*Proof.* □

**Problem 3**

**Claim** A graph $G$ is 2-connected if and only if $\forall u, v \in V(G)$ there is a $u, v$ necklace in $G$.

*Proof.* □

**Problem 4**

**Claim** For $k \geq 2$, a graph G with at least $k + 1$ vertices is $k-$connected if and only if for all subsets $S, T$ of $V(G)$ such that $|T| = 2, |S| = kandT \subseteq S$, there is a cycle in $G$ that contains $T$ and avoids $S - T$.

*Proof.* □

**Problem 5**

**Claim** Let $xy$ be an edge in a digraph $G$.

*Proof.* □