Homework 6

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Graph Theory

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Problem 1 See attached sheet for Male and and Female perfect matchings

Problem 2

Claim All $k-$regular bipartite graphs satisfy Tutte's Condition.

Proof. Suppose G is a $k-$regular bipartite graph. If $k = 1$, then our bipartite graph is a collection of disjoint paths. For any arbitrary $S \subseteq V(G)$ the number of odd components in G is equal to the number of vertices taken from unique paths in G. We do not add any odd components to $G - S$ if we take both vertices contained in a path or leave paths. As a result, $o(G - S) \leq |S|$.

Lemma A. Let $k \geq 2$. Suppose there are $u, v \in V(G)$ with cut edge uv that lie in different components of $G - uv$. Let $u \in A$ that is a bipartite subgraph of G and $v \in B$. Every vertex in A has degree k besides u that has degree $k - 1$. This is not possible so we reach a contradiction. Hence, there are no cut edges in any $k-$regular bipartite graphs where $k \geq 2$.

Let $k \geq 2$ and G be a connected $k-$regular bipartite graph. By *Lemma A* there cannot be any cut edges in any connected components of G.
Note, all components in G are $K_{k,k}$ and have $2k$ vertices. Suppose G consists of n components where each component is a $k-$regular bipartite graph. Deleting any number of vertices from G will □

Problem 3

Claim Suppose G is a $7-$regular connected graph that remains connected after deleting 5 edges. Then G has a perfect matching.

Proof. Let G' be the graph that results from deleting e_1, e_2, e_3, e_4, e_5 edges from G. Since G' is still connected we know that G did not contain any cut edges. So $o(G) \leq 1$. □

Problem 4

Claim There exists a $5-$regular simple connected graph that remains connected after deleting any 2 edges but does not have a perfect matching.

Proof. See attached sheet for drawing. □

Problem 5

Claim

Proof. □