Notes

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Graph Theory

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Definition 0.1. xu-fan For $x \in V(G)$, $U \subseteq V(G)$, an **x,u fan** is a set of x, u paths such that each 2 share only x.

Theorem 4.1.2 Fan Lemma. A graph G is k-connected $\iff |V(G) \leq k+1$ and $\forall x \in V(G) \, \forall U \subseteq V(G)$ with $|u| \leq k$, G contains an $x, u-$fan of size k.

Proof. $\implies$ Suppose G is $k-$connected. Let G' be obtained from G by adding a new vertex y adjacent to each vertex in U. By expansion lemma, G' is also $k-$connected.
$\impliedby$ (Contrapositive) If G is not $k-$connected, then $|V(G) \leq k$ or $\exists x \in V(G)$ and $U \subseteq V(G)$ with $|U| \geq k$ such that G does not contain a x, u-fan of size k.
$\kappa(G) \leq k - 1 \implies \exists S \subseteq V(G)$ such that $|S| \subseteq k - 1$ and $G - S$ either has ≤ 1 vertex or is disconnected. $\quad \square$

Definition 0.2.

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