Problem Set 2

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1 Quantitative Macroeconomics, Part 2: Problem set 2

1.1 Solution to the model without aggregate risk

1. Household problem

1.

$$U(c,n) = log(c) - \Gamma\frac{n^{1+\gamma}}{1+\gamma}$$

$$V(k,\epsilon) = \max_{c,n,k'}\{U(c,n) + \beta E_{\epsilon'}\{V(k^{'},\epsilon^{'})\}\}$$

s.t

$$c + k^{'} = wn\epsilon + (1+r)k$$

$$V(k,\epsilon) = \max_{n,k'}\{U(wn\epsilon + (1+r)k - k^{'}, n) + \beta E_{\epsilon'}\{V(k^{'},\epsilon^{'})\}\}$$

F.O.C

$$\{k^{'}\} : -U_c(c,n) + \beta E_{\epsilon'}\{V_k(k^{'},\epsilon^{'})\}$$
$$\{n\} : w\epsilon U_c(c,n) + U_n(c,n) = 0$$

Let $g_k(k,\epsilon)$ and $g_n(k,\epsilon)$ be the policy functions for labor supply and capital.

$$V(k,\epsilon) = U(wg_n(k,\epsilon)\epsilon + (1+r)k - g_k(k,\epsilon), g_n(k,\epsilon)) + \beta E_{\epsilon'}\{V(g_k(k,\epsilon),\epsilon^{'})\}$$
$$V(k,\epsilon) = (wg_n^{'}(k,\epsilon)\epsilon + (1+r) - g_k^{'}(k,\epsilon))U_c(c,n) + g_n^{'}(k,\epsilon)U_n(c,n) + \beta g_k^{'}(k,\epsilon)E_{\epsilon'}\{V(k^{'},\epsilon^{'})\}$$
$$V(k,\epsilon) = g_n^{'}(k,\epsilon)(w\epsilon U_c(c,n) + U_n(c,n)) + ((1+r)U_c(c,n) + g_k^{'}(k,\epsilon))(\beta E_{\epsilon'}\{V(k^{'},\epsilon^{'})\} - U_c(c,n))$$
$$V(k,\epsilon) = (1+r)U_c(c,n)$$

Hence we obtain the following optimality conditions:

$$U_c(c,n) = (1+r)U_c(c^{'},n^{'})$$
$$-\frac{U_n(c,n)}{U_c(c,n)} = w\epsilon$$

Using the specific utility function:

$$c^{'} = \beta(1+r)c$$
$$\Gamma n^{\gamma} c = w\epsilon$$

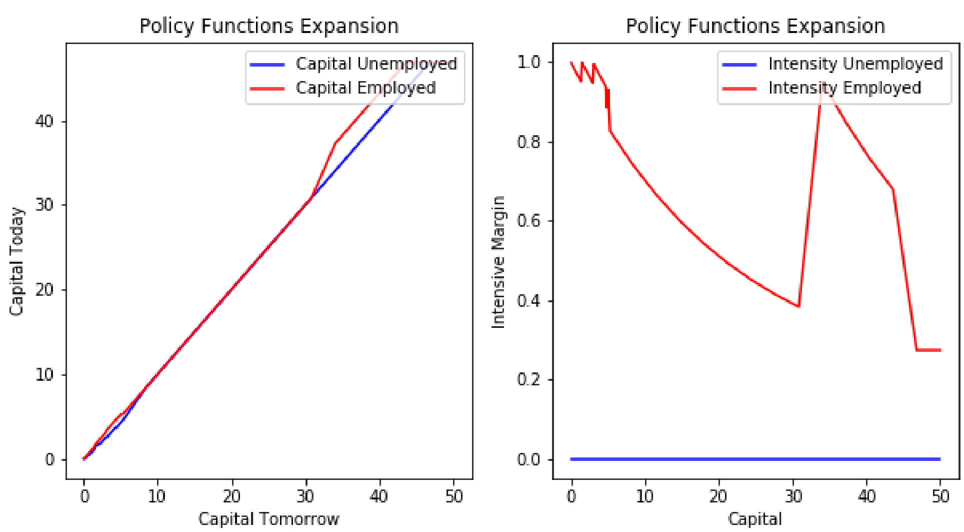
2.

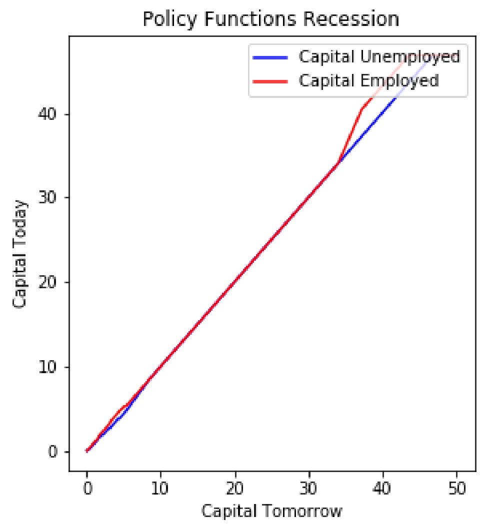
$$c^{'} = \beta(1+r)c$$
$$n = (\frac{w\epsilon}{\Gamma c})^{\frac{1}{\gamma}}$$

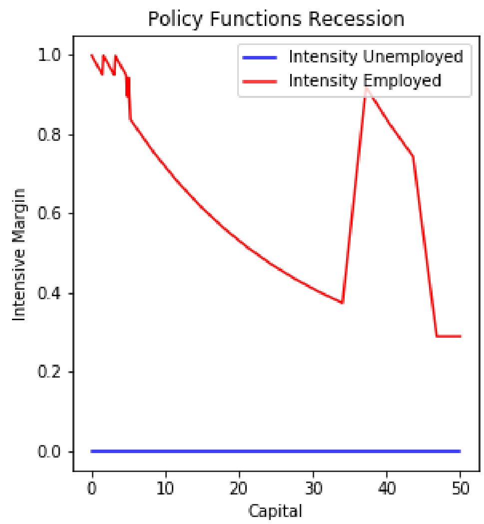
$$c + k^{'} = wn\epsilon + (1+r)k$$
$$c + k^{'} = w\epsilon(\frac{w\epsilon}{\Gamma c})^{\frac{1}{\gamma}} + (1+r)k$$

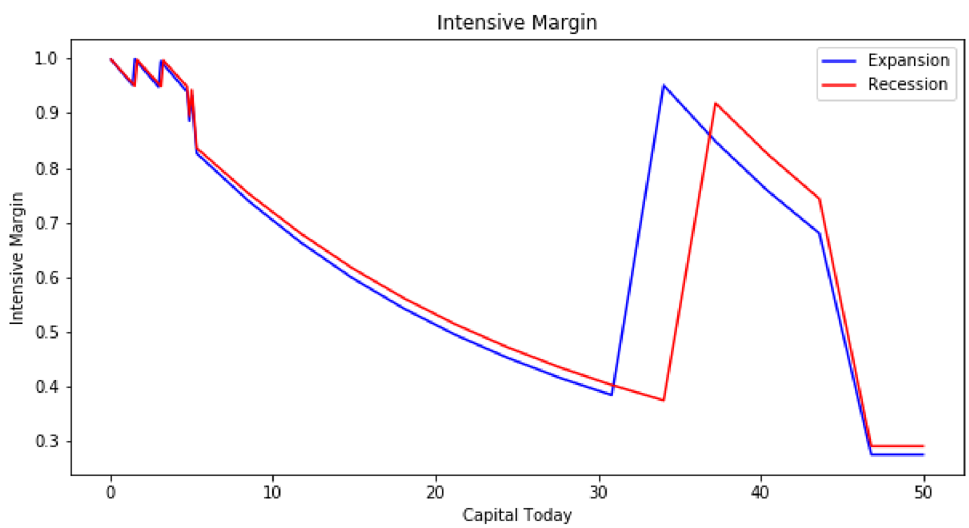
$$c = (1+r)k - k^{'} if \epsilon = 0$$
$$c = fsolve(c - \frac{w^{\frac{\gamma+1}{\gamma}}}{\Gamma^{\frac{1}{\gamma}}} c^{\frac{1}{\gamma}} - (1+r)k + k^{'})$$

3.









2. Equilibrium

1.

$K = \int g_k(k,\epsilon) d_{\mu(k,\epsilon)}$, $L_z = \int g_n(k,\epsilon) d_{\mu(k,\epsilon)}$

$C + H(K) = (1 + r(K))K + wL_z$, where consistency implies $H(K) = \int g_k(k,\epsilon) d_{\mu(k,\epsilon)}$

2.

Most of the questions are done in the file *K_S_Labor_Q2.py*

f.
In my code I create a grid of 200 elements for aggregate capital aggregate labor, I will refere to them as $grid_K$ and $grid_L$. My first guesses K, L are the elements in the midle of the grid for both of them. With this first guess I can solve the model, get the policy functions and the invariant distribution $\mu_{k,\epsilon}$. I compute $New_K = \int g_k(k,\epsilon) d_{\mu_{k,\epsilon}}$ and $New_L = \int g_n(k,\epsilon) d_{\mu_{k,\epsilon}}$. I update capital the following way $Knext = K(1-\eta) + New_K\eta$ and $Lnext = L(1-\eta) + New_L\eta$, where $\eta = 0.1$. Then I find the element in the grid that is the closest to $Knext$ and $Lnext$.

$$I_k = argmin_{\{i\}}(|Knext - grid_K[i]|)$$

$$I_l = argmin_{\{i\}}(|Lnext - grid_L[i]|)$$

Finally set up $K = grid_K[I_k]$ and $L = grid_L[I_l]$

4.

```
    error: 0.111596060171557
RESULT: 12
Update Capital Grid: [32.67676767676768, array(70, dtype=int64)]
Excess Capital: 0.43233263800212285
Update Intensive Grid: [0.908080808080808, array(86, dtype=int64)]
Excess Labor: 0.017225121075577166
Wage: 2.3015288444374273
Interest: 0.03597695108257963
RESULT CALIBRATION: 0
Error: -0.0007070707070707671
Gamma: 0.5625
gamma: 2.0

In [5]: |
```

This is the result of the calibration. The only parameter that has been modified is Γ. γ is fixed to it's orginal value 2.

3. Solution to aggregate risk

1.
a.

$$V(k,\epsilon,z,\mu_z(k,\epsilon)) = max_{(k^{'},c,n)\in\Gamma(k,\epsilon,z,\mu_z(k,\epsilon))}\{U(c,n)+\beta\sum_{z'}\sum_{\epsilon'}V(k^{'},\epsilon^{'},z^{'},\mu_{z'}(k^{'},\epsilon^{'}))\pi_{z^{'},\epsilon^{'}|z,\epsilon}\}$$

$\Gamma(k,\epsilon,z,\mu_z(k,\epsilon)) = \{(k^{'},c,n) \in \mathbb{R}^2_+ \times [0,1] : c + k^{'} \leq (1 + r(z,\mu_z(k,\epsilon)))k + w(z,\mu_z(k,\epsilon))\epsilon n\}$

b.

$\mu_z(k,\epsilon)$ needs to be specified as a state variable now, when it was not necessary in Krusell and Smith. It is the case because now agents need to anticipate agregate labor which depend on decisions made in the current period. Wheras in krusell and Smith the distribution was not necessary because capital is a state variable and therefore the distribution $\mu_z(k,\epsilon)$ does not affect agregate capital since it is decided before the distribution is realized.

2.

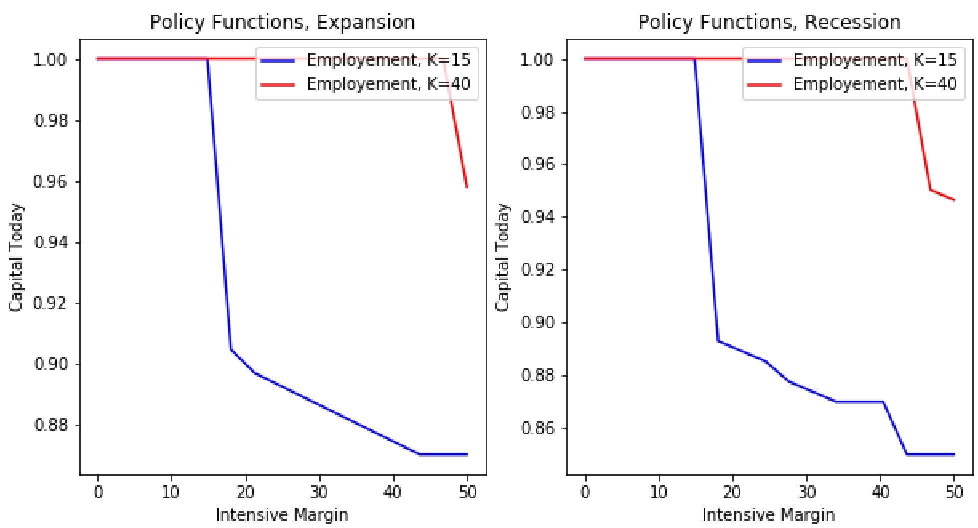
a.

$V(k,\epsilon,z,K) = max_{(k^{'},c,n)\in\Gamma(k,\epsilon,z,K)}\{U(c,n)+\beta\sum_{z^{'}}\sum_{\epsilon^{'}}V(k^{'},\epsilon^{'},z^{'},K^{'})\pi_{z^{'},\epsilon^{'}|z,\epsilon}\}$

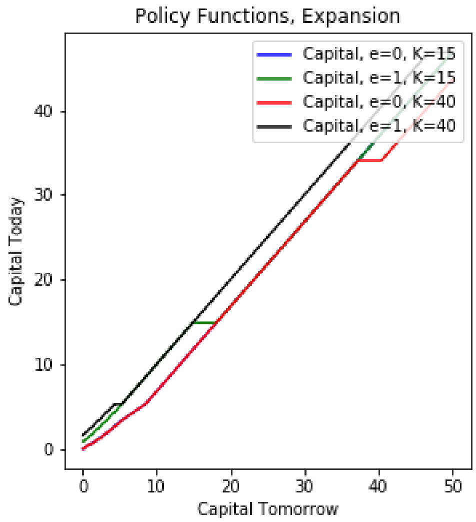
$\Gamma(k,\epsilon,z,K) = \{(k^{'},c,n) \in \mathbb{R}^2_+ \times [0,1] : c + k^{'} \leq (1+r(z,K,N))k + w(z,K,N)\epsilon n\}$

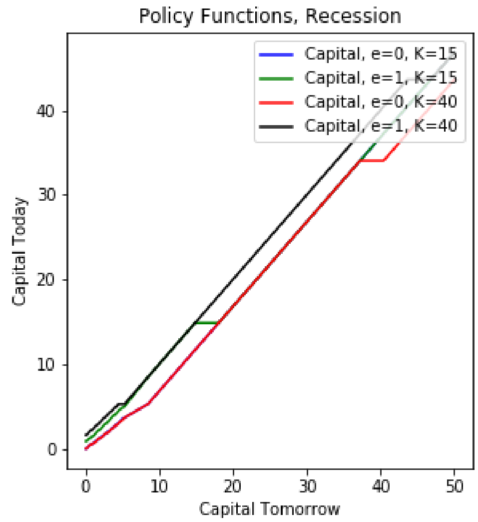
Where $H(K,z) = K^{'}$ and $N = G(K,z) = \sum_{z\in\{z_l,z_h\}} N_{z_i} 1_{\{z_i=z\}}$

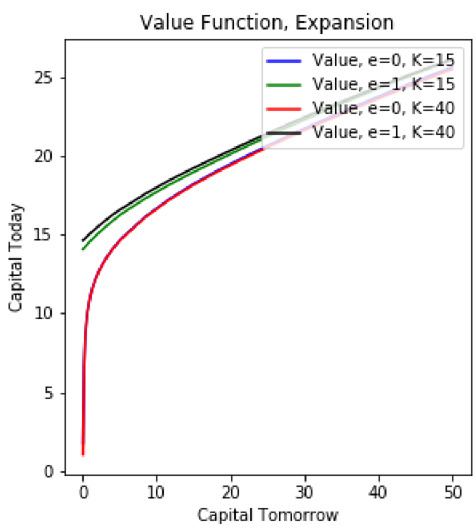
3.

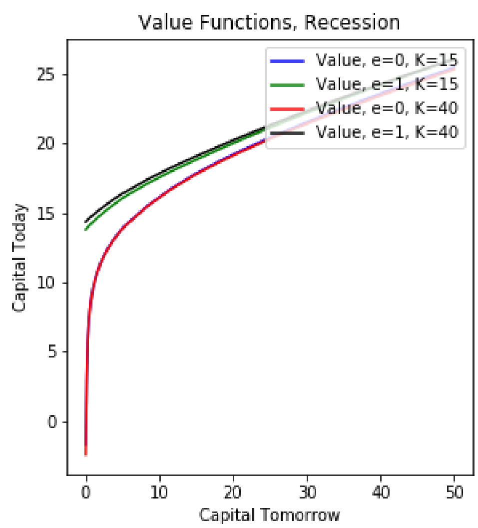


We can observe that agents work more in expansion than in recession. The wealth effect dominates the substitution effect, because the desutility from labor is low. The increase in wages generated by the positive TFP shock induces an increase in labor. We can also observe that low levels of aggregate capital imply a decrease in labor supply due to the fact that interest rates increased, therefore agents prefere to substitute labor by lesure since the return they get from their savings is large enough.

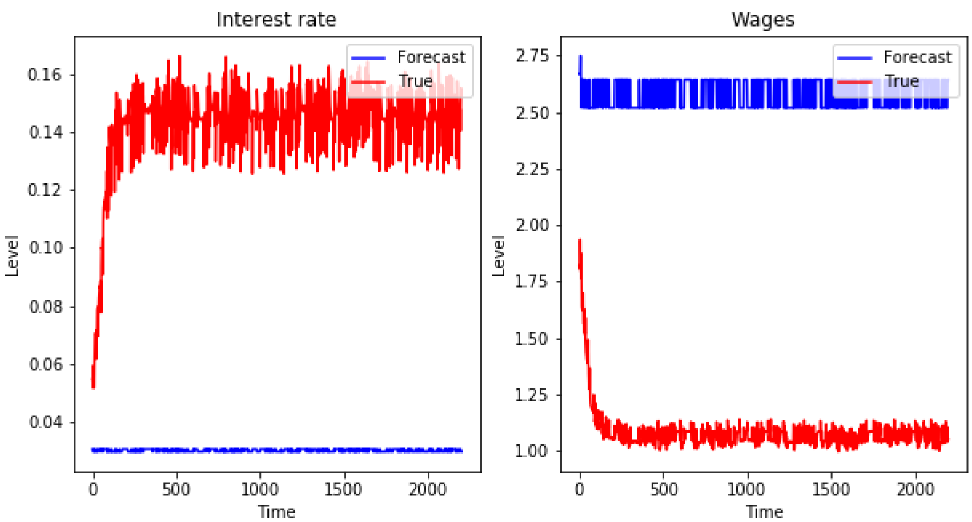

Policy Functions, Expansion
Capital, e=0, K=15
Capital, e=1, K=15
Capital, e=0, K=40
Capital, e=1, K=40
Capital Today
Capital Tomorrow


Policy Functions, Recession
Capital, e=0, K=15
Capital, e=1, K=15
Capital, e=0, K=40
Capital, e=1, K=40
Capital Today
Capital Tomorrow


Value Function, Expansion
Value, e=0, K=15
Value, e=1, K=15
Value, e=0, K=40
Value, e=1, K=40
Capital Today
Capital Tomorrow


Value Functions, Recession
Value, e=0, K=15
Value, e=1, K=15
Value, e=0, K=40
Value, e=1, K=40
Capital Today
Capital Tomorrow

5.



The results from the simulation show that the guesses for the functions G(.) and H(.) are not correct. It is necessary to update the parameters associated to the function in order to get more accurate results.

6.

During the simulation we obtained $K_s = \{K_i^{'} : K_i^{'} = H_s(K_i, z_i)\}$ and $N_s = \{N_i^{'} : N_i^{'} = G_s(K_i, z_i)\}$ but we also computed the true realization of aggregate labor and aggregate capital.

If our assumption is that:

$H(K, zi) = e^{\sum_i (\alpha_i + \beta_i log(K))zi}$, Where zi can take values 0 or 1.

$G(K, zi) = e^{\sum_i (\gamma_i + \delta_i log(K))zi}$

Then we just need to use the true realization of aggregate capital and labor to estimate the parameters $\alpha_i, \beta_i, \gamma_i, \delta_i \forall i \in \{0, 1\}$ and reiterate the processes until there is convergence of the estimator.