Problem Set 1

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1 Quantitative Macroeconomics, Part 2: Problem set 1

The file that contains all the functions is K_smith, it will be refered to most of the time.

1.1 Question 1

1. Value function

You can find the guesses for the value function in the function $Start\_VFI$. $V_{e,z}$ as it is defined in the code is the value of the agent if his or her employement status is e and the aggregate shock is z. $V_{e,z}(K,k) = \frac{U(\alpha z \frac{K}{L(z)}^{1-\alpha} k + (1-\alpha) z (\frac{K}{L(z)})^{\alpha} e - \delta k)}{1-\beta}$ When $e \in \{0,1\}$ and $z \in \{z_g, z_b\}$.

2. System of equations

I will refere in this section to $P(z^{'}, e^{'}|z,e)$ as $\pi_{z^{'},e^{'}|z,e}$.
$\pi_{b,0|g,0} + \pi_{b,1|g,0} = \pi_{b|g}$
$\pi_{b,0|g,1} + \pi_{b,1|g,1} = \pi_{b|g}$
$\pi_{g,0|b,0} + \pi_{g,1|b,0} = \pi_{g|b}$
$\pi_{g,0|b,1} + \pi_{g,1|b,1} = \pi_{g|b}$
$\pi_{g,0|g,0} + \pi_{g,1|g,0} = \pi_{g|g}$
$\pi_{g,0|g,1} + \pi_{g,1|g,1} = \pi_{g|g}$
$\pi_{b,0|b,0} + \pi_{b,1|b,0} = \pi_{b|b}$
$\pi_{b,0|b,1} + \pi_{b,1|b,1} = \pi_{b|b}$

We also need to make sure that unemployement in good shocks is 1.5 quarters and and 2.5 quarters in bad shocks. After some calculations we obtain:
$\pi_{g,0|g,0} = \frac{7}{24}$
$\pi_{b,0|b,0} = \frac{21}{40}$

We need also,
$\frac{\pi_{b,0|g,0}}{\pi_{b|g}} = 1.25\frac{\pi_{b,0|b,0}}{\pi_{b|b}}$ implies that $7\pi_{b,0|g,0} = 1.25\pi_{b,0|b,0}$
$\frac{\pi_{g,0|b,0}}{\pi_{g|b}} = 0.75\frac{\pi_{g,0|g,0}}{\pi_{g|g}}$ implies that $7\pi_{g,0|b,0} = 0.75\pi_{g,0|g,0}$

Using the fact that $\pi_{0|g} = 0.04$ and $\pi_{0|b} = 0.1$

$$\begin{pmatrix} \pi_{g,0|g,0} & \pi_{g,0|g,1} & \pi_{g,0|g,0} & \pi_{g,0|b,1} \\ \pi_{b,0|g,0} & \pi_{b,0|g,1} & \pi_{b,0|b,0} & \pi_{b,0|b,1} \end{pmatrix} \begin{pmatrix} 0.02 \\ 0.48 \\ 0.05 \\ 0.45 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.05 \end{pmatrix}$$

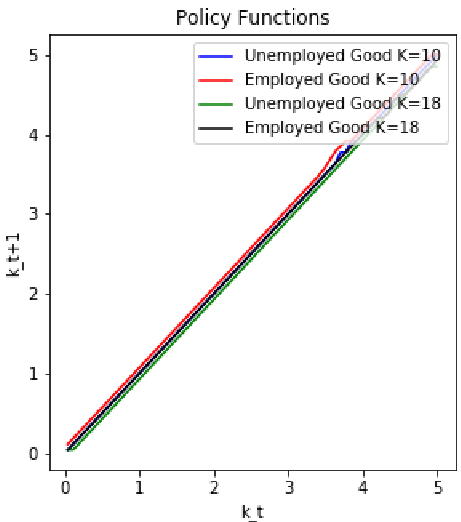
Finally,
$\pi_{g,0|g,1} = 0.005$
$\pi_{b,0|b,1} = 0.02$

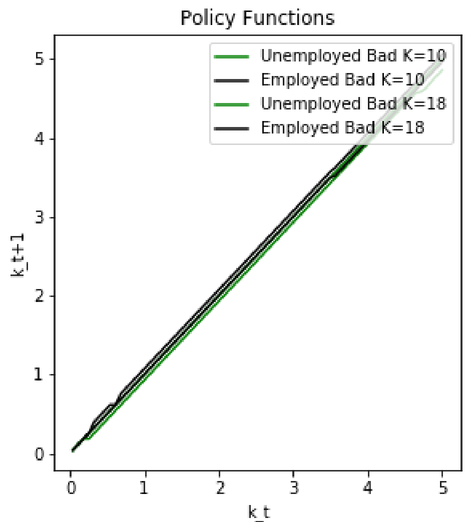
After solving the system of equations, we obtain the following transition matrix:

$$\begin{pmatrix} \pi_{g,0|g,0} & \pi_{g,1|g,0} & \pi_{b,0|b,0} & \pi_{b,1|g,0} \\ \pi_{g,0|g,1} & \pi_{g,1|g,1} & \pi_{b,0|g,1} & \pi_{b,1|g,1} \\ \pi_{g,0|b,0} & \pi_{g,1|b,0} & \pi_{b,0|b,0} & \pi_{b,1|b,0} \\ \pi_{g,0|b,1} & \pi_{g,1|b,1} & \pi_{b,0|b,1} & \pi_{b,1|b,1} \end{pmatrix}$$

1.2 Worker's problem and simulation

1. Policy function





In good times employed agents are going to save more than unemployed ones. Also, we can see that a decrease in aggregate capital increases savings. The latest fact can be explained by the decrease in interest rate observed after a decrease in aggregate capital. In bad times a similar explanation can be made however, it seems that policy function are closer in distance between each other. This is the result of the fact that good and bad shocks follow a markov chain, therefore given that agents are in the bad state they predict to be in it for long, therefore they also expect to face unemployement more frequently which makes their policy function more similar to the one of unemployed agents.

2. Simulation

The simulation will be performed in part 3 for the estimation of the parametric expectation. You can also refere to the functions $Simulation\_U\_Z$ and $simulation\_Ag\_K$ that contribute to the simulation of the aggregates states of the economy. The function $simulation\_Ag\_K$ stores aggregate capital.

1.3 Solution of the Model

1. Parameters Value

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Parameter Updated: 9
Parameter Error: 0.014986747816742535
Parameters: [1.27922835 0.87045513 4.08681785 0.58687713]
R_squared good 0.7334030471737427
R_squared bad 0.28078283865108977

In [22]:
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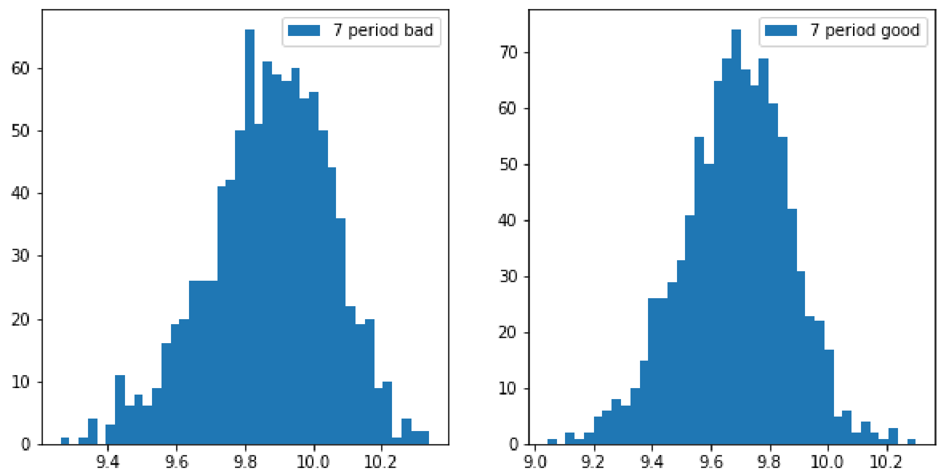
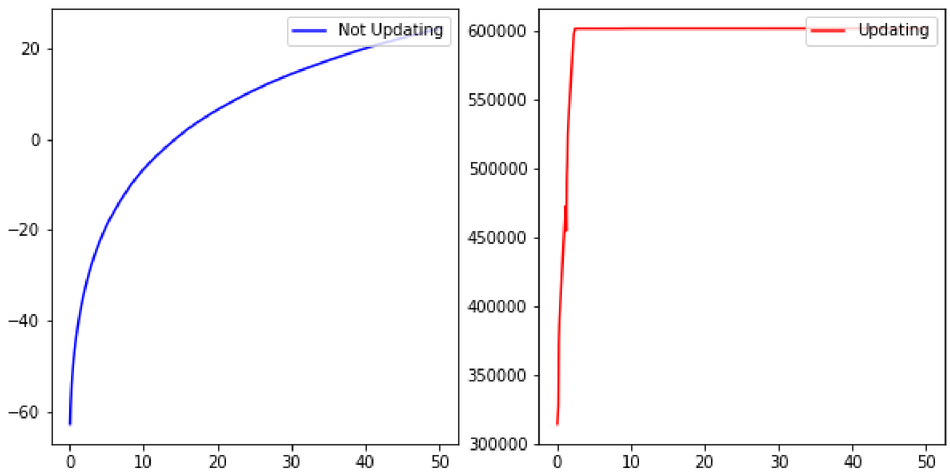
These are the results of the estimation. The accuracy seems to be high for the estimation of the parametrized expectation under good shock however, it is not the case for the bad shock. Some changes can be implemented to improve the results. I reduced the grid of aggregate capital to 10 elements and reduced the tolerance to make the algorithm converge faster. This measures certainly affected the results.



Here are the 2 distributions with the converged parametrized expectation. The good shock distribution is a bit skewed to the right, meaning that agents save more. Wheras the bad shock distribution is more centered and also flater which means that agents are consuming their capital.

2. Welfare Evaluation



These 2 graphs are computed by calculating the value function for the 2 types of agents. The conclusion is that the welfare of the agent that updates his or her believe is much higher than the welfare of the one that does not update, which should be expected since one type has an advantage on the other one. However, I suspect some issue in the computation of the value function for the agent that updates the parametric expectation. The range of the function seems to be too wide.