# Problem Set 5

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## 1 Quantitative Macroeconomics: Problem set 5

### 1.1 A simple wealth model

To solve this question I used the endogenous grid method, that you explained in class. You can refere to 2 files for this part $Agent1\_bis1.py$ and $Pset5\_main1.py$. I felt that the endogenous grid method was adequate to this exercise since we had to make sure that assets were over a lower bound. The only trouble I ran into was when I solved my economy by backward induction I was not able to set up the initial values for assets that I wanted. The solution that I found is to put an upper endogenous upper bound on future assets that depends on initial assets. I used the fact that:

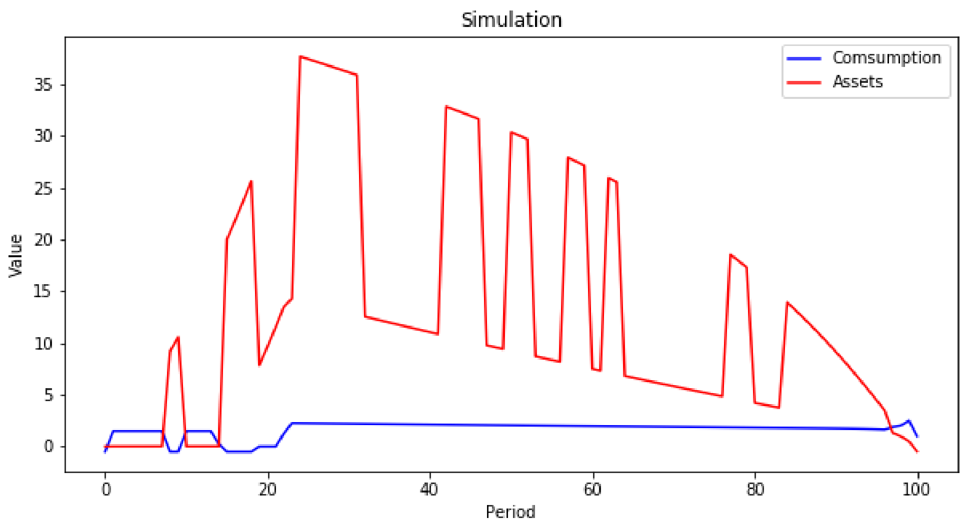
$a_{t+1} + \sum_{i=0}^{t} c_i = a_0(1+r)^{t+1} + \sum_{i=0}^{t} y_i$

$a_{t+1}$ is the highest when consumption is zero forever. Hence:

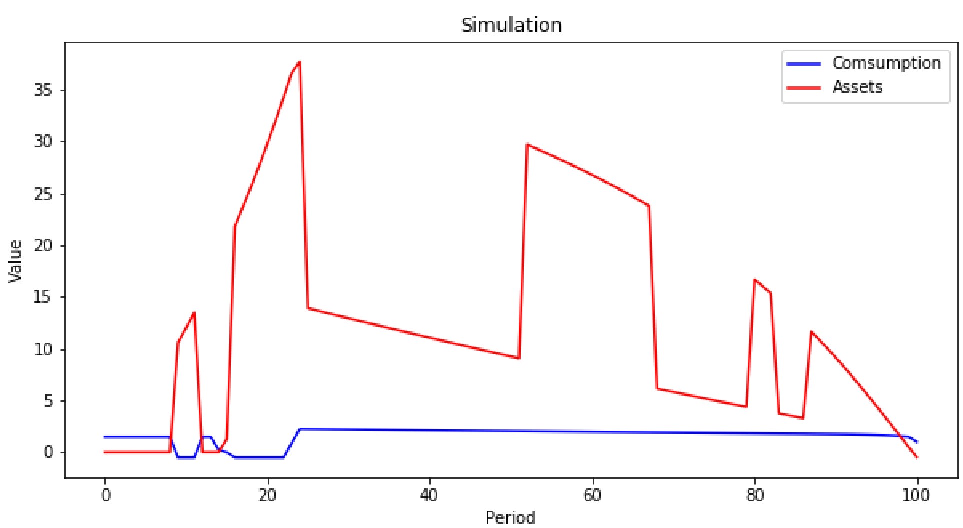
$\bar{a} = a_0(1+r)^{t+1} + \sum_{i=0}^{t} y_i$

I performed a simulation for 100 periods with $a_0 = 0$. Here are the results.

**CRRA**




**Quadratic with $\bar{C} = 100$**



## 1.2 Solving ABHI

### 1. The recurcive formulation

$V(a,y) = \max_{\{a',c\}\in\Gamma(a,y)} U(c) + \beta E(V(a^{'},y^{'}))$
$L = U(a(1+r) + y - a^{'}) + \beta E(V(a^{'},y^{'})) + \lambda a^{'}$

**FOC**

$U_c(c) = \beta E(V_a(a^{'},y^{'})) + \lambda$

$\lambda = 0$ if $a^{'} > -A$

$V(a,y) = U(a(1+r) + y - g_a(a)) + \beta E(V(g_a(a),y^{'})) + \lambda g_a(a)$

$V_a(a,y) = (1+r)U_c(c) - g_a^{'}(a)U(c) + g_a^{'}(a)\beta E(V(a^{'},y^{'})) + \lambda g_a^{'}(a)$

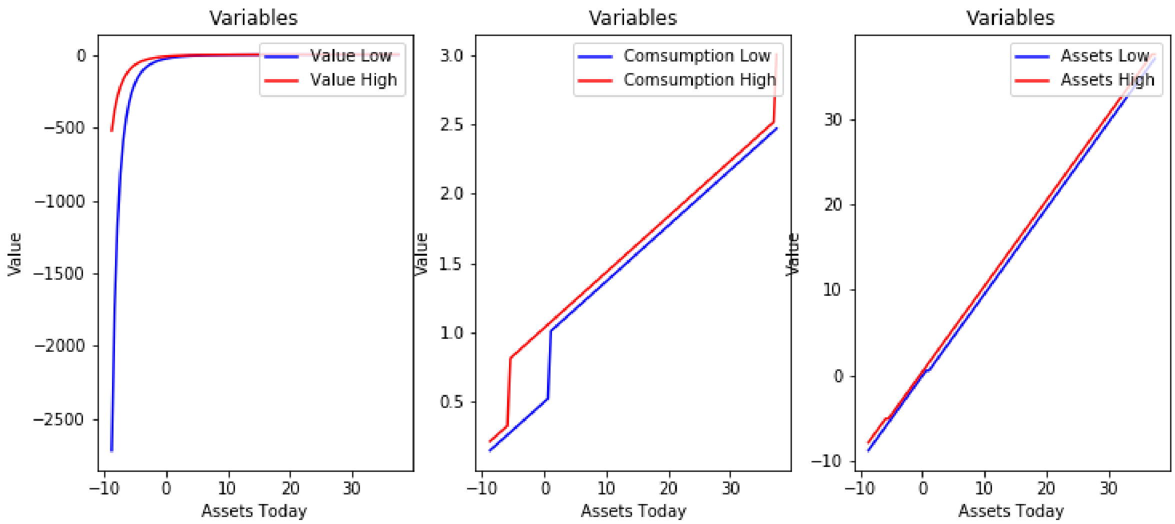
$V_a(a,y) = (1+r)U_c(c)$

$U_c(c) = \beta(1+r)E(U_c(c^{'})) + \lambda$

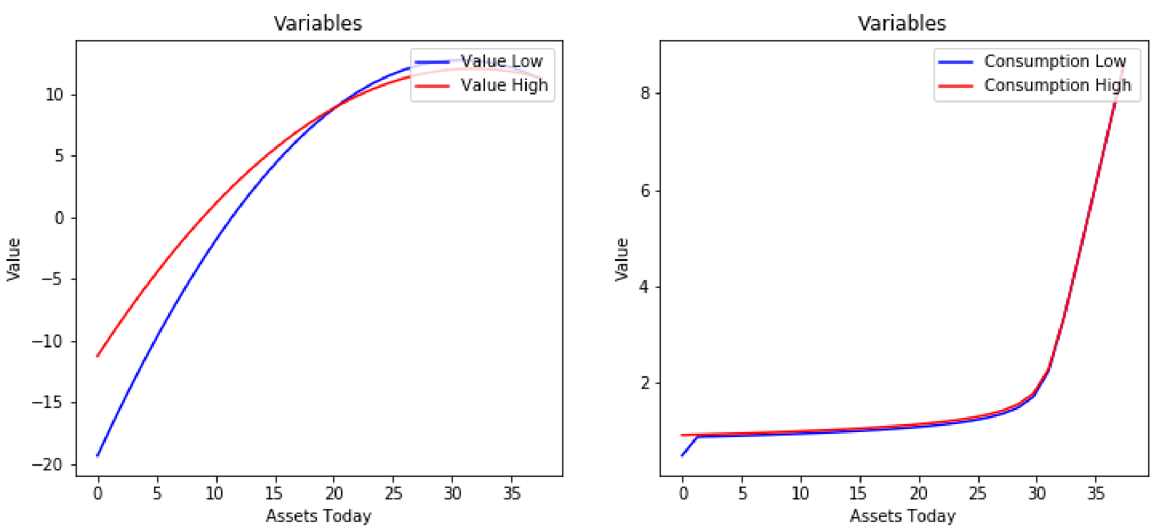
$U_c(c) \geq \beta(1+r)E(U_c(c^{'}))$

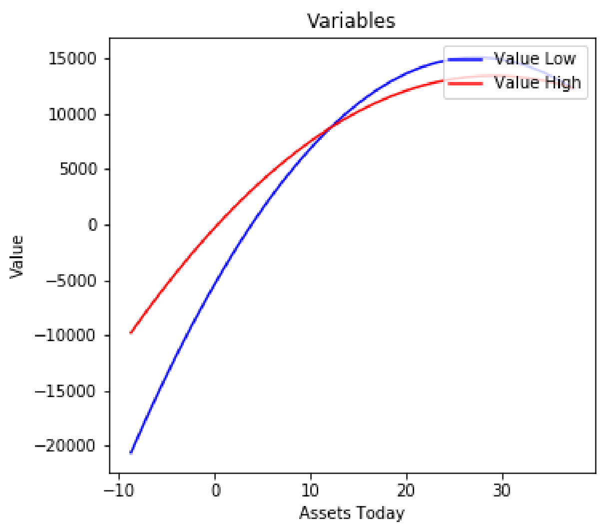
### 2. Infinitly lived economy

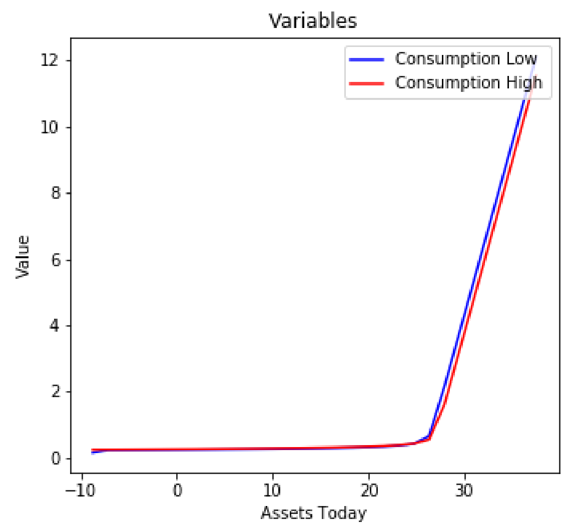
The files that you can refer to for this part are: *Aiyag_disc_1.py* and $Pset5_main2.py$. You can see the policy functions and the Value functions on the following graph for the discrete case.

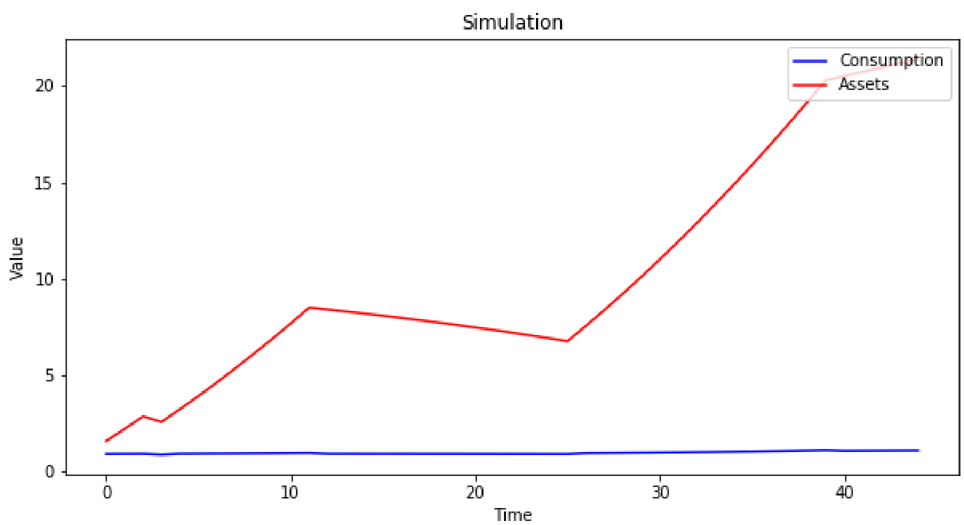



For the continuous case the files that you can refer to for this part are: *Aiyag_cont_1.py* and *Pset5_main2_bis1.py*.



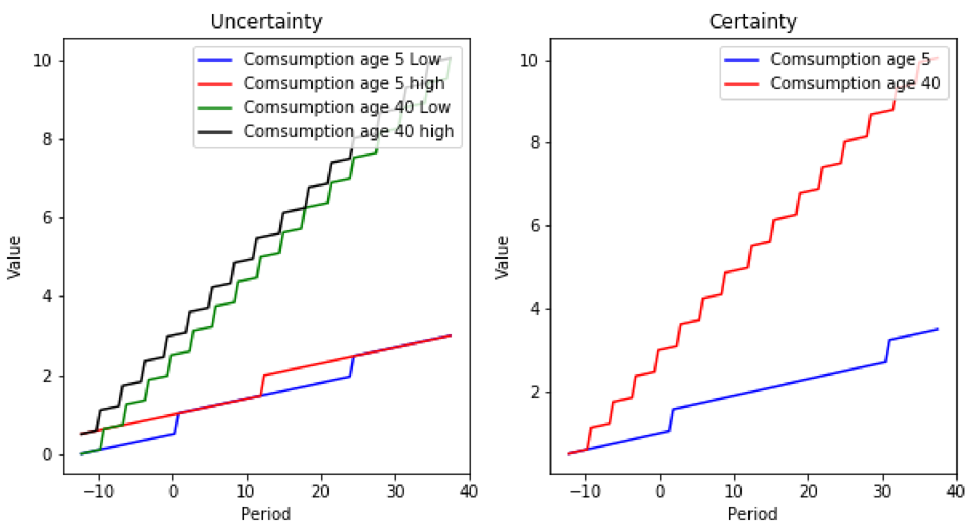

Variables
Value Low
Value High
Value
Assets Today


Variables
Consumption Low
Consumption High
Value
Assets Today


Simulation
Consumption
Assets
Value
Time

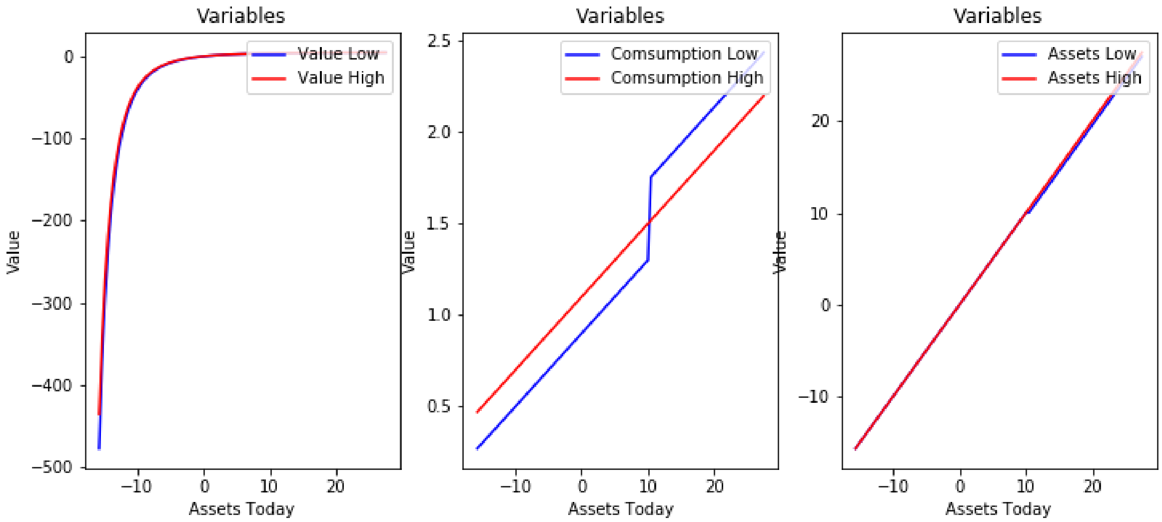
**3. and 4.1. The life cycle economy**

The files that you can refer to for this part are: $Agent1\_23.py$ and $Pset5\_main\_23.py$.

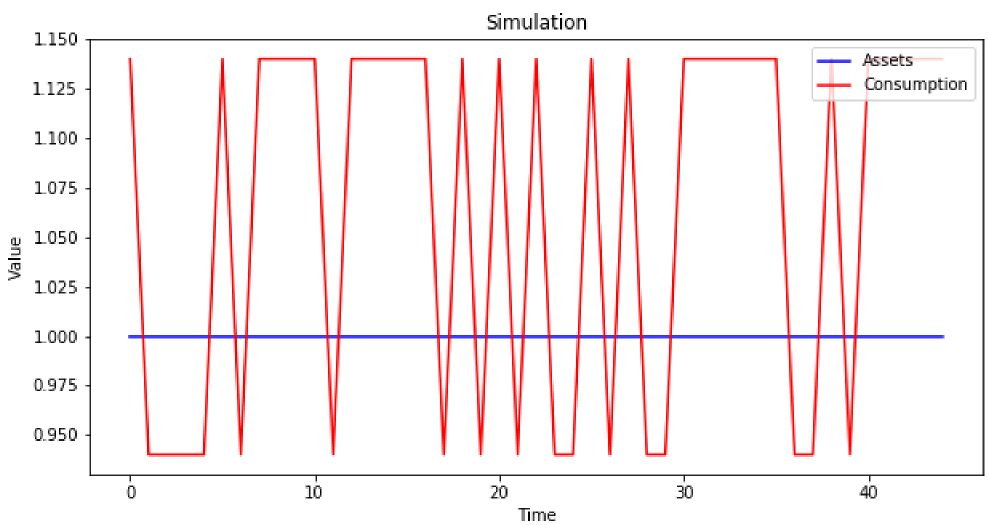



**4.2** $\gamma = 0$ **and** $\sigma_y = 0.1$

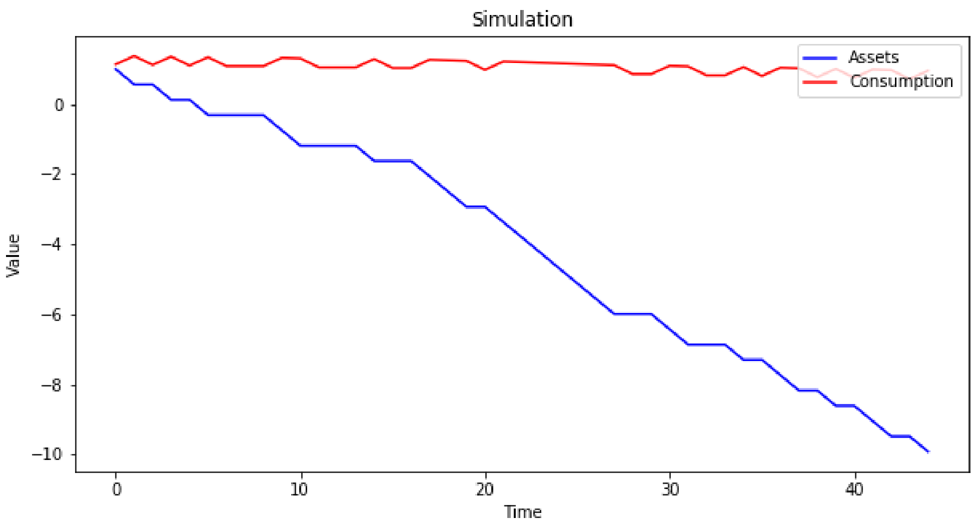
The files that you can refer to for this part are: $Aiyag\_disc\_1.py$ and $Pset5\_main2.py$.


Variables
Value Low
Value High
Value
Assets Today
0
−100
−200
−300
−400
−500
−10
0
10
20
Variables
Comsumption Low
Comsumption High
Value
Assets Today
2.5
2.0
1.5
1.0
0.5
Variables
Assets Low
Assets High
Value
Assets Today

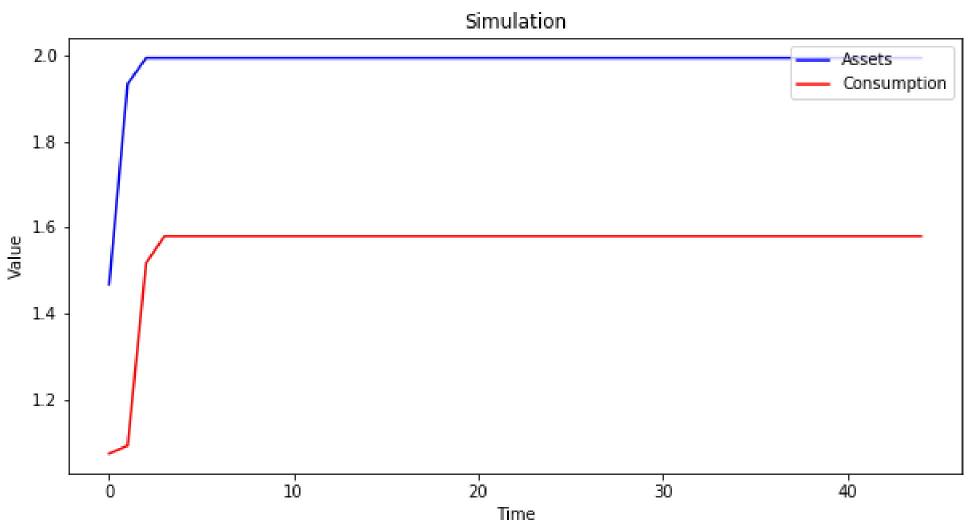
CASE $\sigma = 5$


Simulation
Assets
Consumption
Value
Time
1.150
1.125
1.100
1.075
1.050
1.025
1.000
0.975
0.950
0
10
20
30
40

CASE $\sigma = 2$

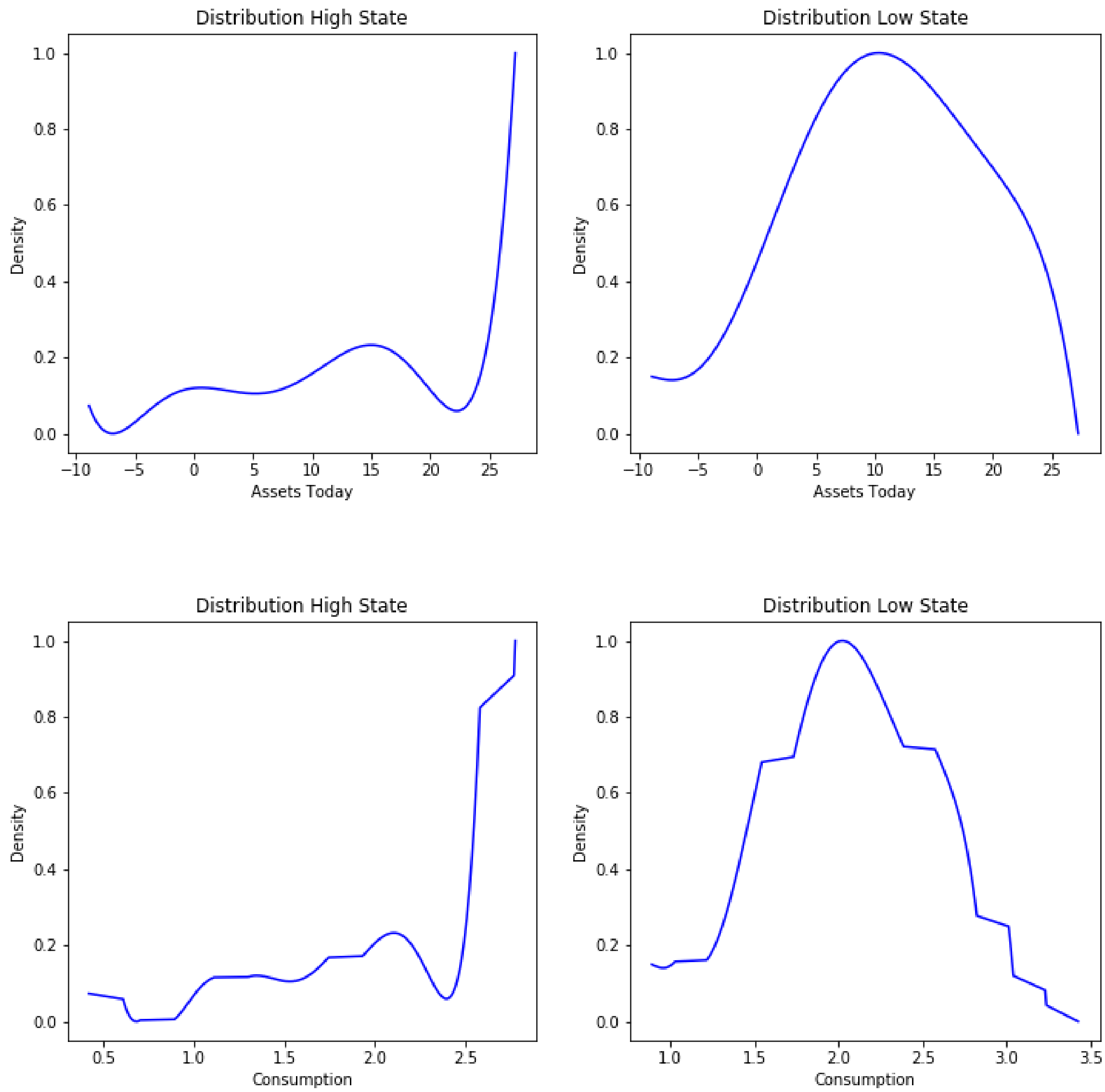



### 4.2.5



### 5.1 General Equilibrium

For this part you can refer to *ABHI.py* and *Pset5_main_ABHI.py*




The distributions that you see in the graph were smoothed by chebychev regression. It is important to know which procedure I used to compute the invariant distribution in this economy, which is exactly what we are looking for. For this we need to combine endogenous choices with exogenous states and get

the transition matrix, to do so let me introduce some notation.
Let $\Phi(j)$ be the transition matrix for the endogenous choice given state j, $\Phi(j)_{i,k} = 1$ when $a_k$ is the optimal choice given that we previously saved $a_i$ in state $j$, $\Phi(j)_{i,k} = 0$ otherwise.
From the class in $ABHI.py$ we get the optimal choice of the agent in each state, therefore, we can easily build $\Phi(j)$. Lets construct Tendog and Texog such that:

$$Tendog = \begin{pmatrix} \Phi(1)' & 0 & \cdots & 0 \\ 0 & \Phi(2)' & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \Phi(J)' \end{pmatrix}, \text{ with J the number of states}$$

$$Texog = \begin{pmatrix} \Pi_{1,1} I_{N_a} & \Pi_{2,1} I_{N_a} & \cdots & \Pi_{J,1} I_{N_a} \\ \Pi_{1,2} I_{N_a} & \Pi_{2,2} I_{N_a} & \cdots & \Pi_{J,2} I_{N_a} \\ \vdots & \vdots & \vdots & \vdots \\ \Pi_{1,J} I_{N_a} & \Pi_{2,J} I_{N_a} & \cdots & \Pi_{J,J} I_{N_a} \end{pmatrix} = Kron(\Pi', I_{N_a})$$

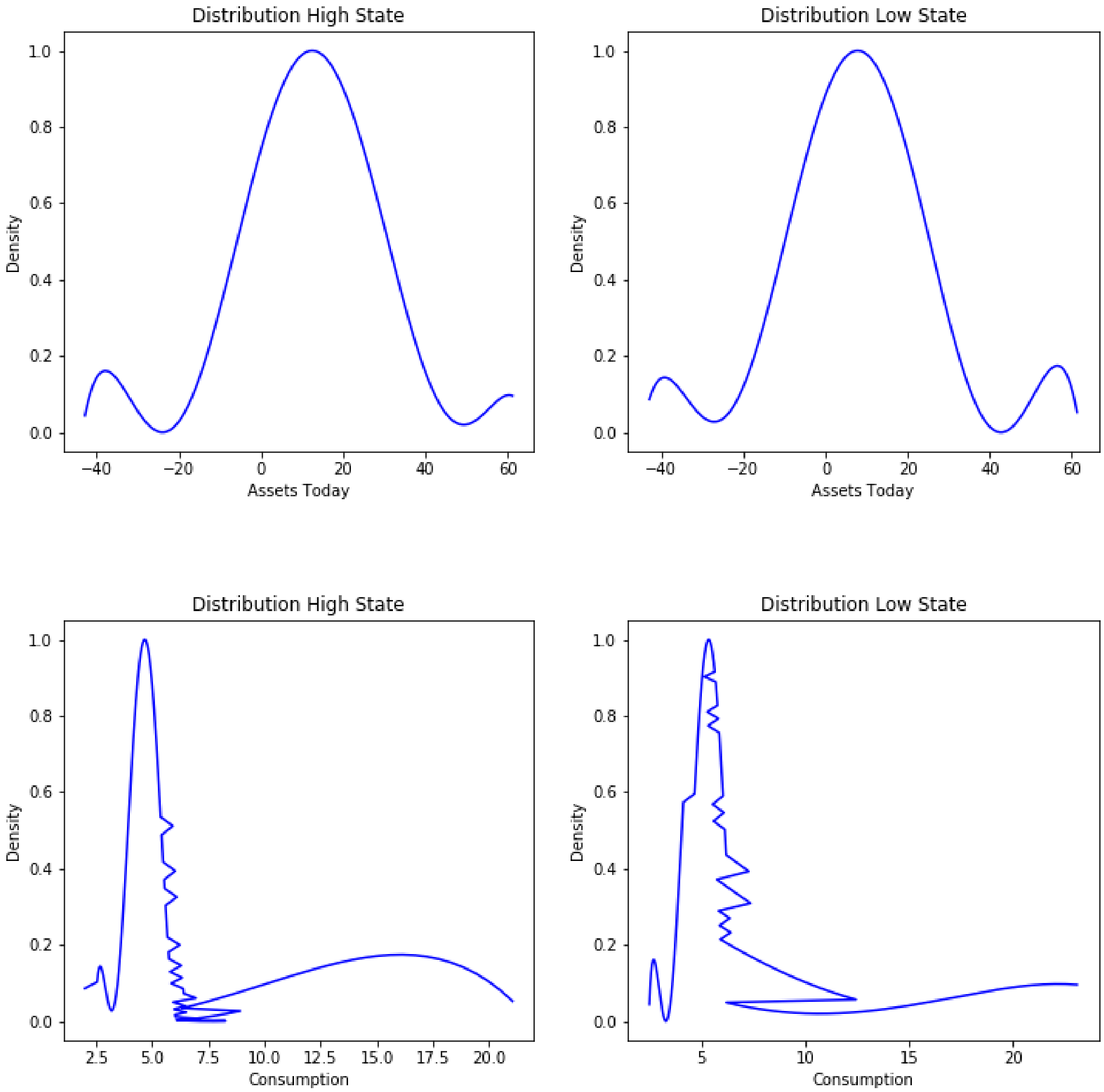
The final transition matrix is $T = Texog.Texog$

$$T = \begin{pmatrix} \Pi_{1,1}\Phi(1)' & \Pi_{2,1}\Phi(1)' & \cdots & \Pi_{J,1}\Phi(1)' \\ \Pi_{1,2}\Phi(2)' & \Pi_{2,2}\Phi(2)' & \cdots & \Pi_{J,2}\Phi(2)' \\ \vdots & \vdots & \vdots & \vdots \\ \Pi_{1,J}\Phi(J)' & \Pi_{2,J}\Phi(J)' & \cdots & \Pi_{J,J}\Phi(J)' \end{pmatrix}$$

Finally we obtain T of dimension $(N_a.J, N_a.J)$. It is important when we have the transition matrix to compute the invariant distribution, there is different methods available, I did the following:
Guess a distribution $O(\frac{1}{N_a.J})$ Where O is a vector of ones of length $N_a J$. Then iterate the recurcive algorithm $P_{t+1} = T.P_t$ until $P_{t+1}$ and $P_t$ are arbitrarly close from each other.
The class in $ABHI.py$ computes the interest rate that clears the market, I found $r = 0.054$ with the parameters I set up.

### 5.2 General Equilibrium

$\rho = 0.6$ $\mu = 3$ this is one of the cases in the table 2 of Aiyagari 1994.
Both distributions are much more alike because first agents have more income for the same standard deviation which will create similarities between the high and the low state. I report an interest rate of 7.1 percent and a saving rate of 4.5. It is quite diffent from the paper because it has seven states and I only have 2.


Distribution High State
Density
Assets Today
Distribution Low State
Density
Assets Today
Distribution High State
Density
Consumption
Distribution Low State
Density
Consumption