Term Paper: Partial Default

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1 Introduction

In this paper I will solve a version of the model of default that we sow in class. In Aguiar and Amador (2014) there is a government that trades a bond with an outside investor, the government has the opportunity to default on the entire debt if it is beneficial. In the model that I solve, the government will be able to choose if it want to default or not but also the amount of the default. The impact of the default on the economy will be proportional to the amount defaulted. This specification of the model might match better some features of the data since full defaults are not observed very often. Usually, countries recive haircuts or are allowed to reschedule the their payements. Which is specifically what I am modeling.

2 Environment

In the model a government decides the consumption, and has preferences that are ordered by the following objective function; $E_0(\sum_{t=0}^{+\infty} \beta^t u(c_t))$. As I previously said, the government can trade a one period bond with a risk neutral investors.

Timing within period t

1. The government observes the outstanding debt b_t and the output realization y_t

2. The government decides to use default or not. $D \in \{0, 1\}$, if $D = 1$ default will be used, if $D = 0$ then default will not be used. Investors observe the variable D and d and adjust their behaviour accordingly. Where d is the proportion of default relative to the debt.

3. If $D = 0$, the government can issue new bonds b_{t+1} at price $q(b_{t+1}, y_t)$. If $D = 1$ then the government starts the period having no access to financial markets. Then it can gain access back with some probability. I suggest the following appraoch:

As I said, every period the government faces the risk of staying excluded from financial markets. The probability of staying excluded is:

$Pr(x) = \lambda f(b_{t-1}, d_t)$

$\lambda \in [0, 1]$ is an exogenous parameter, Remerber that d_t is the share of the debt the government defaulted on.

$$
\begin{aligned}
&Default_t = d_t b_{t-1}\\
&f(b_{t-1}, d_t) = \frac{Default_t}{b_{t-1}}\\
&f(b_{t-1}, d_t) = d_t\\
&Hence\\
&Pr(x) = \lambda d_t
\end{aligned} \tag{1}
$$

Notice, that every period spent in default state allows the government to default on some share of the remaining debt but it could gain access back to the market with probability 1, by setting d_t to zero. This is a behaviour that we might want to rule out including some persistence in the effect of partial default on the economy. This would however increase the dimension of the state space.

If default were not to be used then the government would issue debt and pay the outstanding debt;

$$c_t + b_{t-1} = y_t + q_t b_t$$

If default were to be used, then the government would not have access to financial markets and would suffer from a loss in productivity exacerbated by the function $f(d_t)$. Moreover, the value of the debt would drop by the amount defaulted on in every period the government is in the default state.

$$
\begin{aligned}
c_t &= y_t(1 - \tau f(d_t))\\
b_t &= (1 - d_t) b_{t-1}
\end{aligned}
$$

On the other hand, the investors first order condition can be expressed as follow.

$$q_t = E_t(\frac{1 - D_{t+1}d_{t+1}}{R})$$
$$q(b^{'}, y) = E(\frac{1 - D(b^{'}, y^{'})d(b^{'}, y^{'})}{R}|y)$$

Finally, we can write the recurcive formulation. Let $V^{Nd}(b, y)$ be the value of the government if it decides not to use default.

$$V^{Nd}(b, y) = max_{b'}\{u(y + q(b^{'}, y)b^{'} - b) + \beta E(V(b^{'}, y^{'})|y)\} \tag{2}$$

Let $V^d(b, y)$.

$$V^d(b, y) = max_d\{u(y(1 - \tau d)) + \lambda d\beta E(V^d(b(1 - d), y^{'})|y) +$$
$$(1 - \lambda d)\beta E(V^{Nd}(b(1 - d), y^{'})|y)\} \tag{3}$$

Let $V^{Nd}(b, y)$ be the value function.

$$V(b, y) = max_{D\in\{0,1\}}\{(1 - D)V^{Nd}(b, y) + DV^d(b, y)\} \tag{4}$$

3 Results

Figure 1:

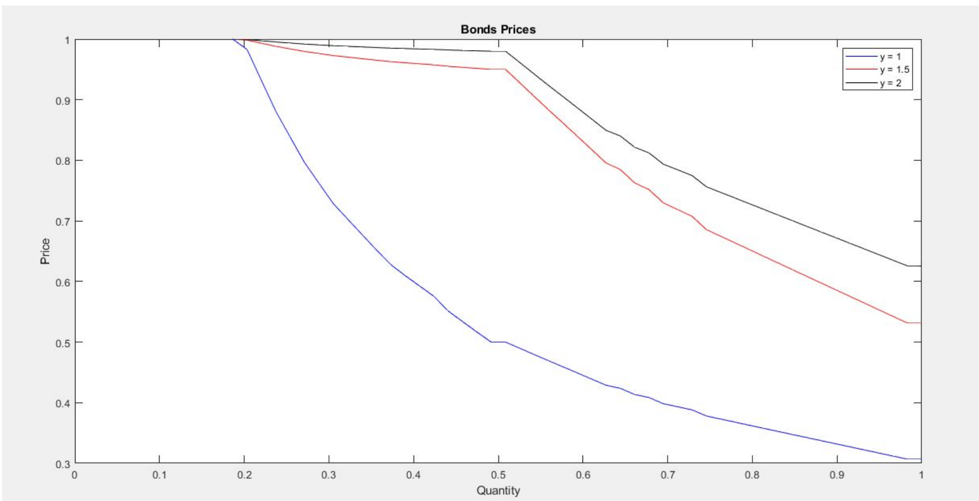


Figure 1, represents the evolution of bond prices relative to the bond supply. Comparing this graph to the one we obtained in the full default economy. It is clear that the price schedule is richer, in the sense that we have here a variety of prices that was not found when full default was mandatory. The feature that is interesting is that for a supply of bonds higher than a certain level, the price of the bond will decreases continuously. What happens is that the higher is the level of debt the higher is the incentive of government to default on a higher share of the debt, which drives the price of the bond down.

Figure 2:

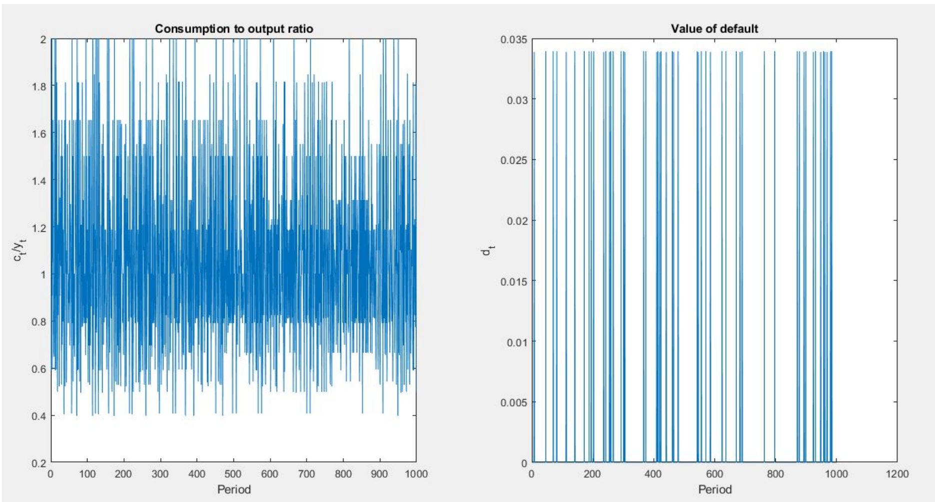
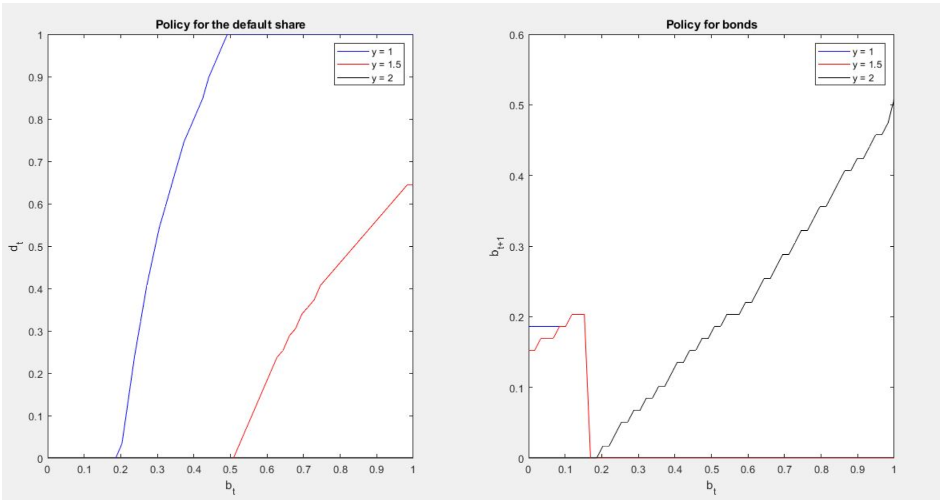


Figure 2, is documenting a simulation for consumption and the share of default. Notice, that something singular is happening in the model. From what I can see the government seems to issue debt and default by the same share when it needs to. Notice also that it gains access to financial markets endogenously by setting d_t to zero in the next period. This type of behaviour is optimal in the model, nevertheless I would like to avoid it since the dynamics are not really interesting. In fact what happens here is that defaulting today will not have a significant impact on the continuation value of the government since it can get out of the default state endogenously.

Figure 3:



4 Extension of the model

I will proceed to a single modification in the model that will introduce more dynamics and increase the dimension of the state space by one. Let the new probability of staying excluded from financial markets to be;

$Pr(x) = \lambda(\omega_1 d_t + \omega_2 d_{t-1})$, where ω_1 and ω_2 are parameters of the model such that $\omega_1 + \omega_2 = 1$. Notice that the government builds a reputation, meaning that the share defaulted in the previous period has an impact on current output and the current probability to access to credit. Because this aspect of reputation of the government is present I would expect what I saw in figure 2 not to happen.

Lets write down the equations of the model:

$$q(d^{'}, b^{'}, y) = E(\frac{1 - D(d^{'}, b^{'}, y^{'})d(d^{'}, b^{'}, y^{'})}{R}|y)$$

$$V^{Nd}(d, b, y) = max_{b'}\{u(y + q(0, b^{'}, y)b^{'} - b) + \beta E(V(0, b^{'}, y^{'})|y)\}$$

$$V^{d}(d, b, y) = max_{d}\{u(y(1 - \tau(\omega_1 d^{'} + \omega_2 d))) + \lambda(\omega_1 d^{'} + \omega_2 d)\beta E(V^{d}(d^{'}, b(1 - d^{'}), y^{'})|y)+$$

$$(1 - \lambda(\omega_1 d^{'} + \omega_2 d))\beta E(V^{NS}(d^{'}, b(1 - d^{'}), y^{'})|y)\}$$

$$V(d, b, y) = max_{D\in\{0,1\}}\{(1 - D)V^{Nd}(d, b, y) + DV^{d}(d, b, y)\}$$

Figure 4 represents bond prices for a set of elements in the state space. My findings are consistent with the theory and don't contradict the results from the first model. However, it is important to say that d_t seem to increase prices, which is quite surprising. The previous fact is explained by figure 6. In fact, an increase in d_t seems to lower d_{t+1}, hence since default is expected to decrease in the future, bond prices should increase.

Figure 4:

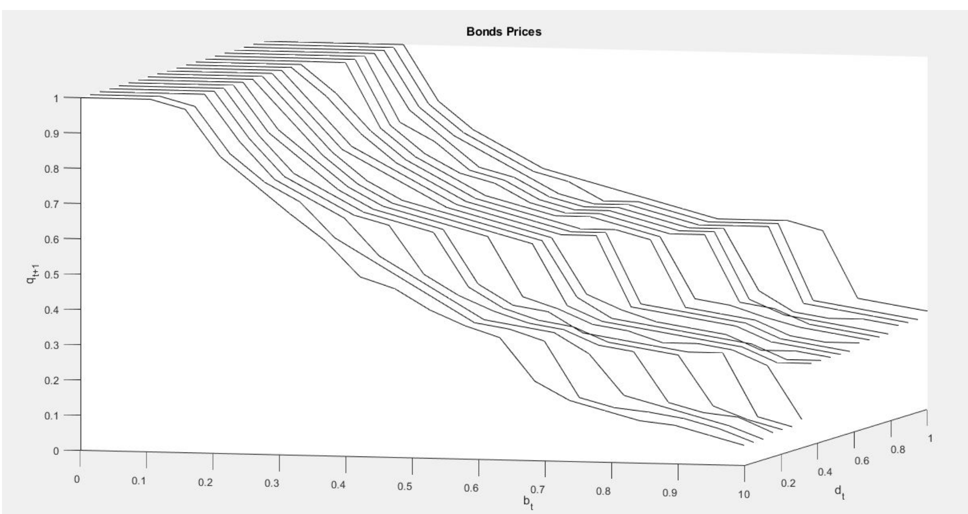


Figure 5, shows that I was successful modeling the dynamics, I failed to simulate in the first model. As you can see comparing with figure 2. The share defaulted on takes 4 different values, because now governments have reputation, it is more difficult for them to reintegrate financial markets, and therefore the strategy that consisted in a default followed by not defaulting is not optimal anymore, because not defaulting doesn't necessarly allow you to issue debt in the next period.

Figure 5:

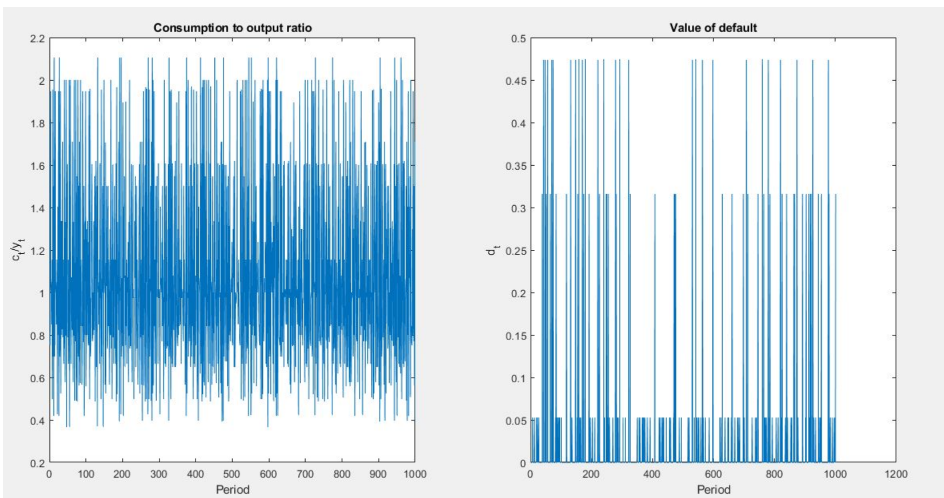
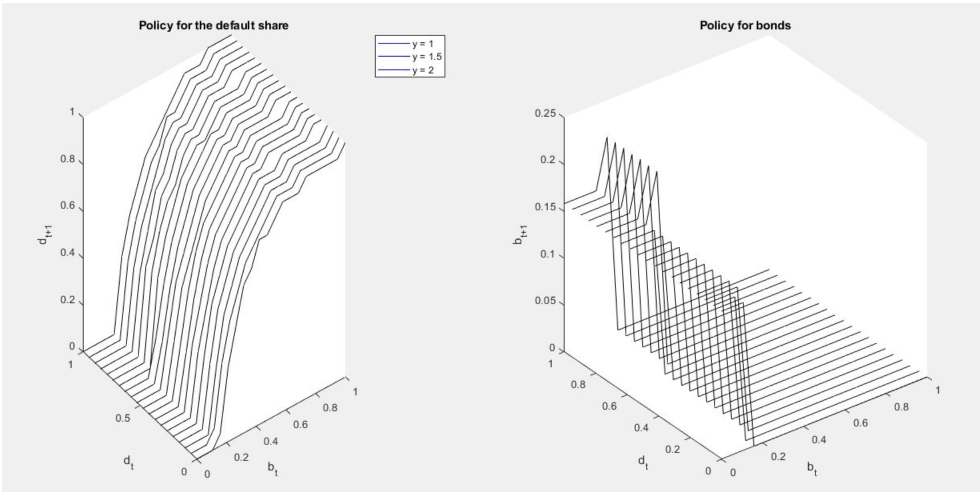


Figure 6:



5 Technical Challenge

The main technical challenge that I encountered in both versions of the model was to compute the expected value function. This step was challenging because for instance if I want to compute $E(V^{NS}(d^{'}, b(1-d^{'}), y^{'})|y)$ I can't really discretize the maximization problem, since it is very unlikely for $b(1-d^{'})$ to belong to my grid for debt. I instead decided to use collocation methods in order to approximate the value function on the entire domain. I decided to use a two dimensional chebychev interpolation that is performed by the rootine *cheby_interp_2d.m*. This allows me to evaluate the value function at any point in the state space, and therefore to compute the expected value.