Problem Set 2

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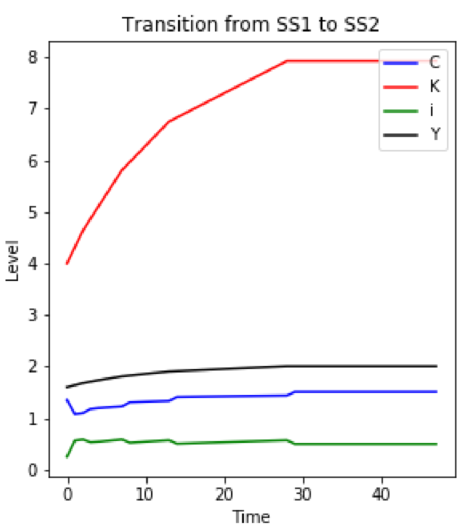
October 10, 2018

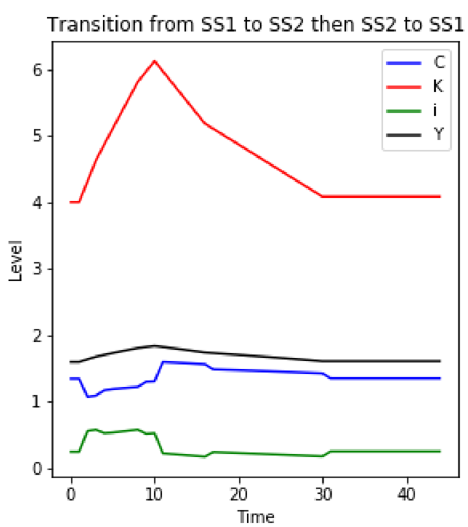
1 Quantitative Macroeconomics

1.1 Problem set 3

Question 1

This question is solved by value function iteration. The two rootine that describe the procedure are Rep-agent-tr.py and Question1-t.py. In this exercise we focus on two steady states, one in which K is 4 and the other in which K is 8. In order to compute the transition from the first to the second steady state, I need first to estimate the value function and the policy function in steady state 1. Lets call it V_1^*. If we double z as it was specified in the instructions, we can estimate the transition for the value and policy functions. The procedure consists of updating the value function and the policy function until observing convergence to V_2^*.





The graph at the left represents the transition from the first steady state to the second. The convergence happened in 30 periods. The graph at the right represents the unexpected shock after 1à periods.

Question 2

Question2 part 1 was performed in two ways. The first time I solved sequentially, seing that my results were not convincing I decided to do it by brute force and solving the maximization problem with a grid search.
First let me describe the procedure I follow for the sequencial resolution of the problem in order to see what might have been the problem.
I use 3 first order condition and the budget constraints and end up with three equations and three unknowns:

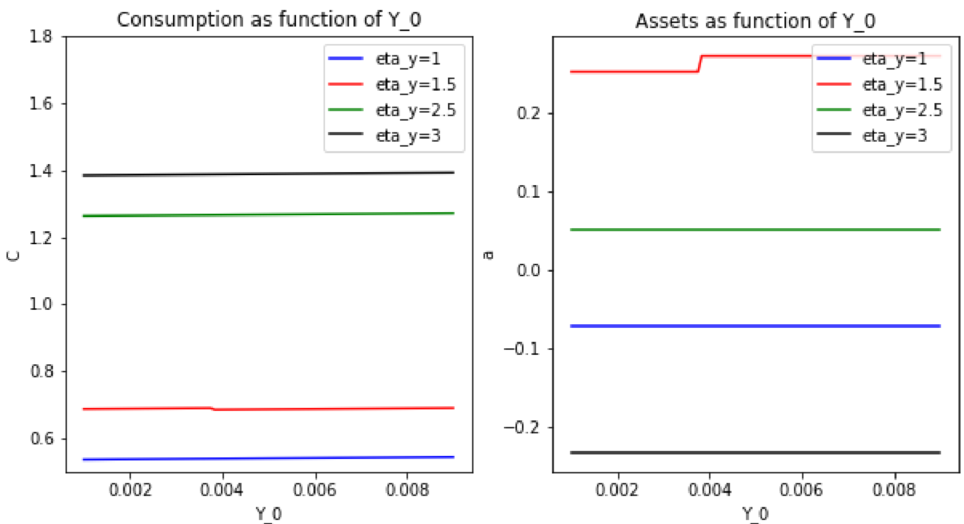
$$res1(a,h,h^{'}) = U_1((1-\tau)wh + y_0 + T_1 - a, h) - \beta(1+r)E(U_1((1-\tau)w^{'}h^{'} + (1+r)a + T2, h^{'}))$$

$$res2(a,h,h^{'}) = (1-\tau)wU_1((1-\tau)wh + (1+r)a + T2, h) + U_2((1-\tau)wh + (1+r)a + T2, h)$$

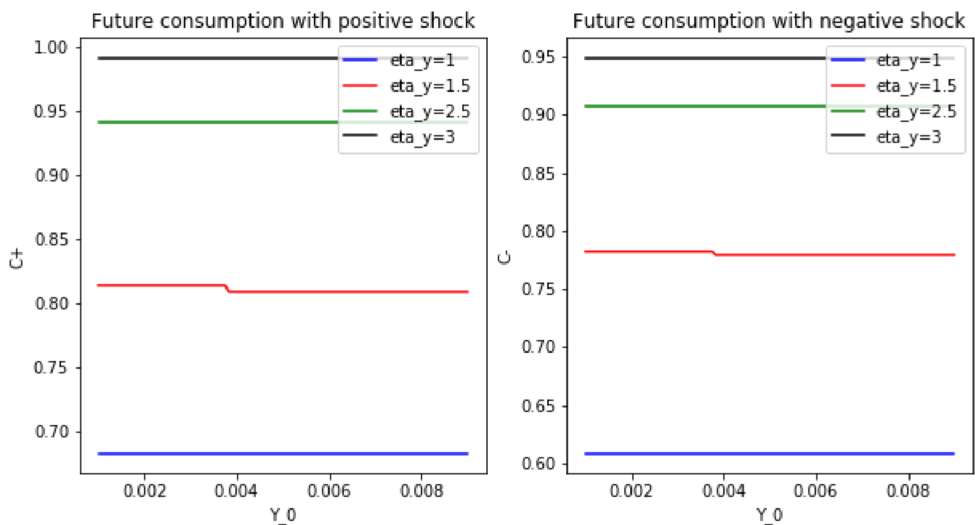
$$res3(a,h,h^{'}) = (1-\tau)E(w^{'}U_1((1-\tau)w^{'}h^{'} + (1+r)a + T2, h^{'}) + E(U_2((1-\tau)w^{'}h^{'} + (1+r)a + T2, h^{'}))$$

In order to solve this system I start with a naive guess: $G \equiv [a,h,h^{'}] = [0, 0.5, 0.5]$. The algorithm I construct in my code does the following steps:
I first look for the a that solves $res1(a, G[2], G[3]) = 0$, if the result is a_1 I update my guess G. G is now $[a_1, 0.5, 0.5]$.
Next, I solve $res2(G[1], h, G[3]) = 0$ and update my believe for h.
Finally, I update $h^{'}$ solving for $res2(G[1], G[2], h^{'}) = 0$. I do these steps until G^t and G^{t+1} are arbitraly close from each other.
The results for this part will be presented at the end of the file since the relevence of the results is questionable. Instead, let me focus on the second resolution of the problem.
The procedure consists of creating a grid of choices for a, h, and $h^{'}$, then iterate through every combination of $(a, h, h^{'}$ and find the one that maximizes the utility of the agent. This procedure is applied to every agent in the economy and for a set of interest rates.
Let $A, H, H^{'}$ be grids of size n containing the elements that variables a, h, and $h^{'}$ can take. Let O be a matrix of ones of dimenssion (n,n) and o a vector of ones (n,1). I construct the matrix of choices $A_e, H_e, H_e^{'}$ such that:
$A_e = Kron(O, Ao.T)$,
$H_e = Kron(O, oH.T)$,
$H_e^{'} = Kron(oH.T, O)$.
The set $\{(a,h,h^{'}) : a = (A_e)_{i,j}, h = (H_e)_{i,j}, h^{'} = (H_e^{'})_{i,j}, \forall i,j\}$ is the set of all possible combinations of $a, h, h^{'}$ that can be done from their grids.

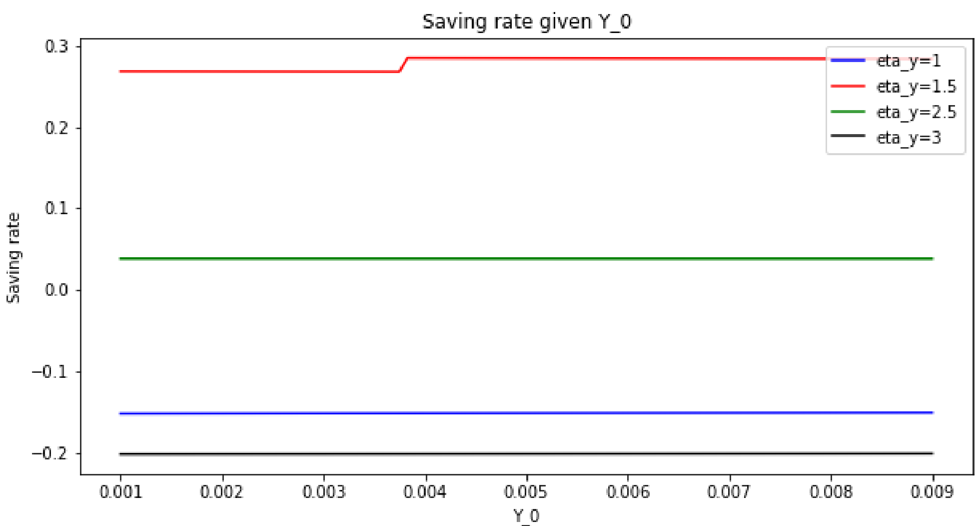
Part 1

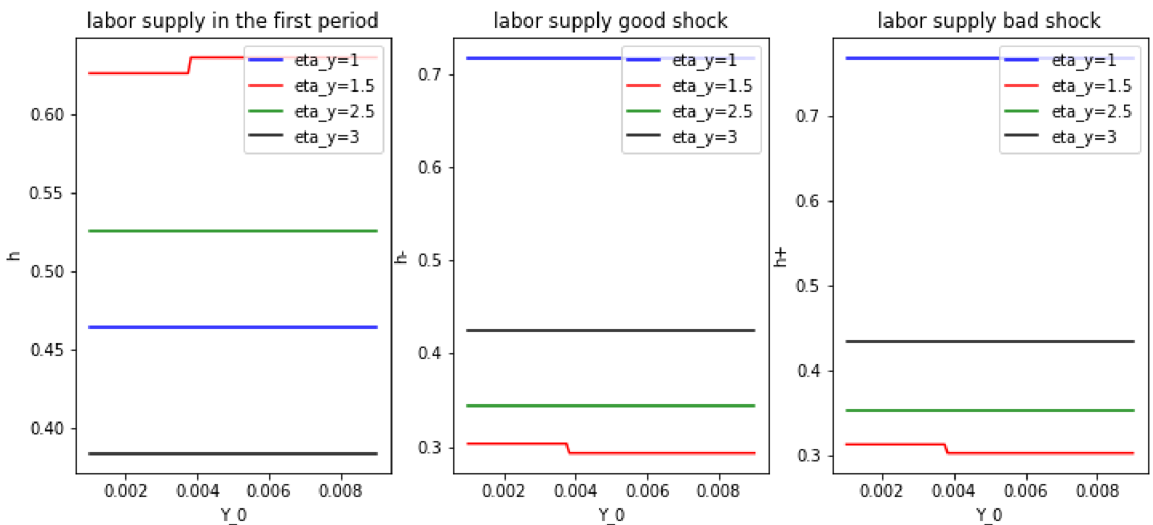


The most productive agent and the least productive borrow for 2 reasons. The most productive can afford to borrow because the productivity is higher and therefore can payback in the future. While the least productive agent borrows today because consumption today is more valuable than consumption tomorrow.

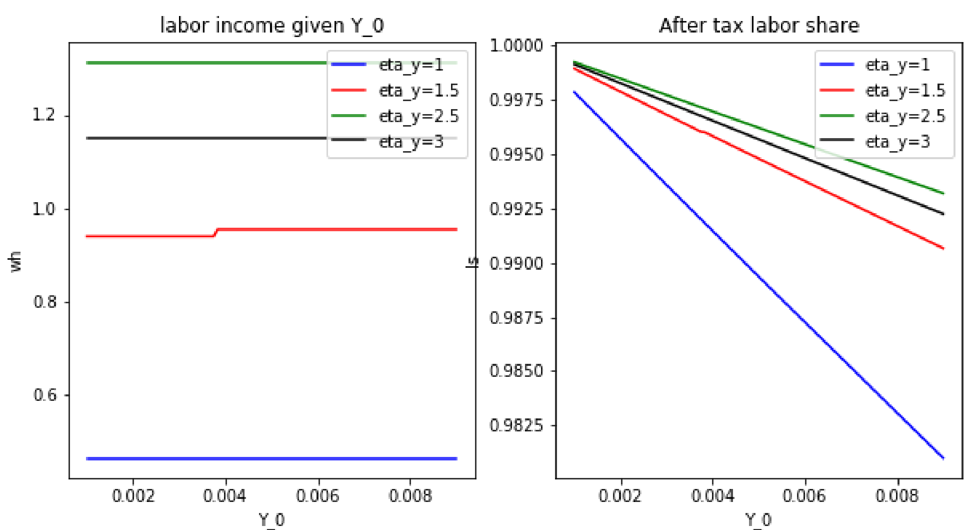


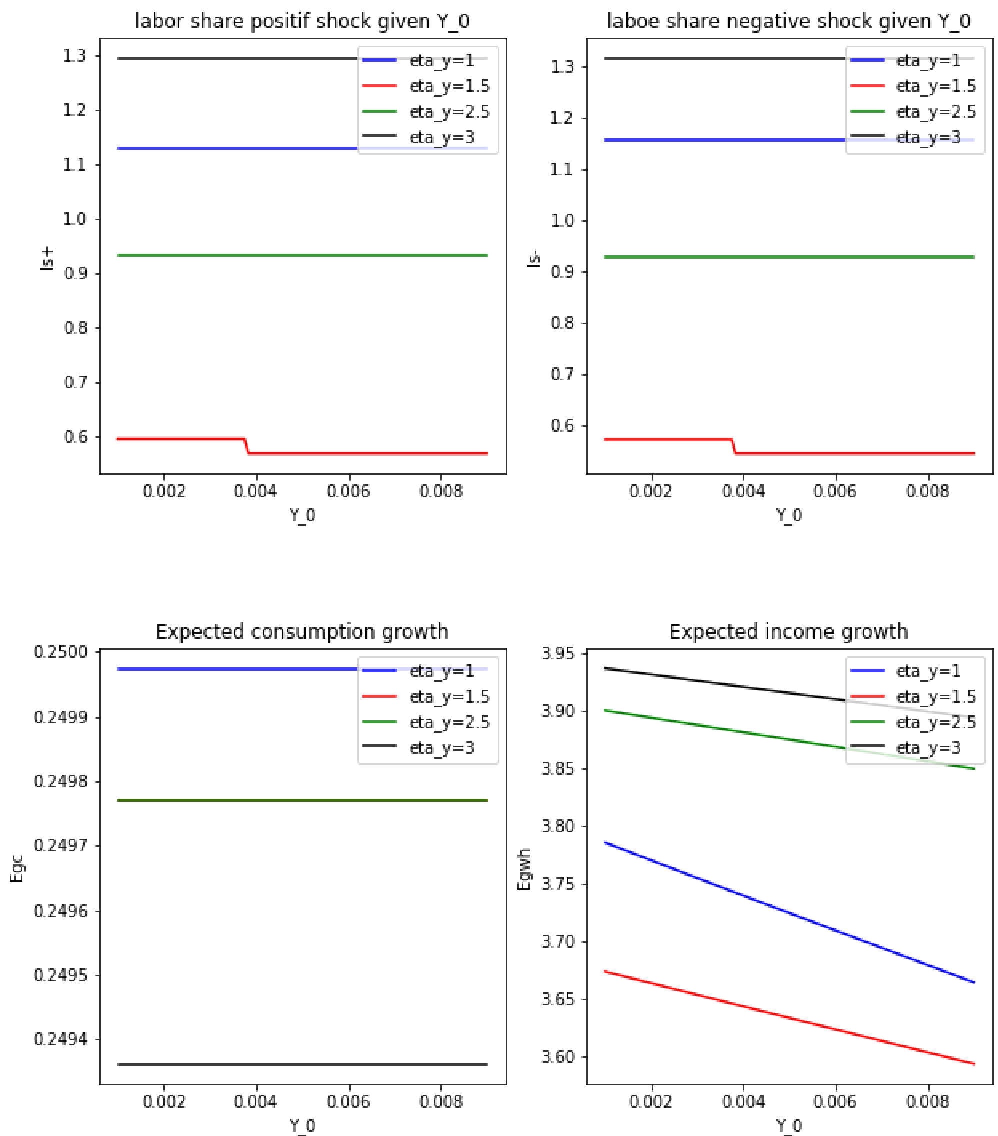
The least product agents smooths consumption better.

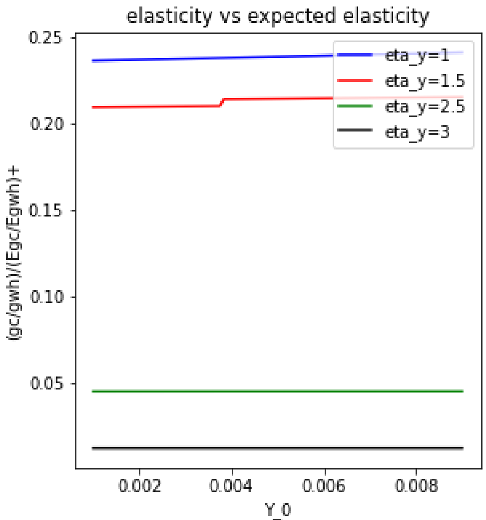


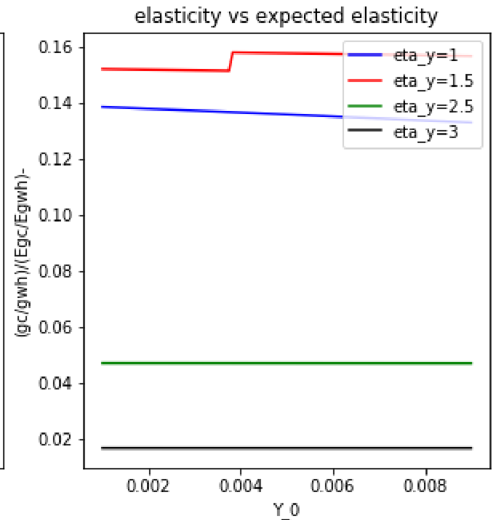


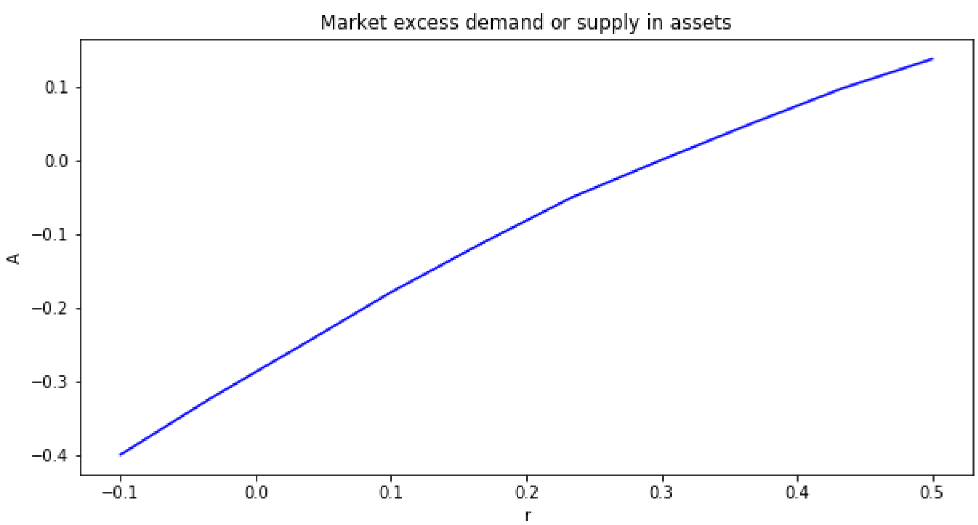
The labor supply is low for the productive agent because

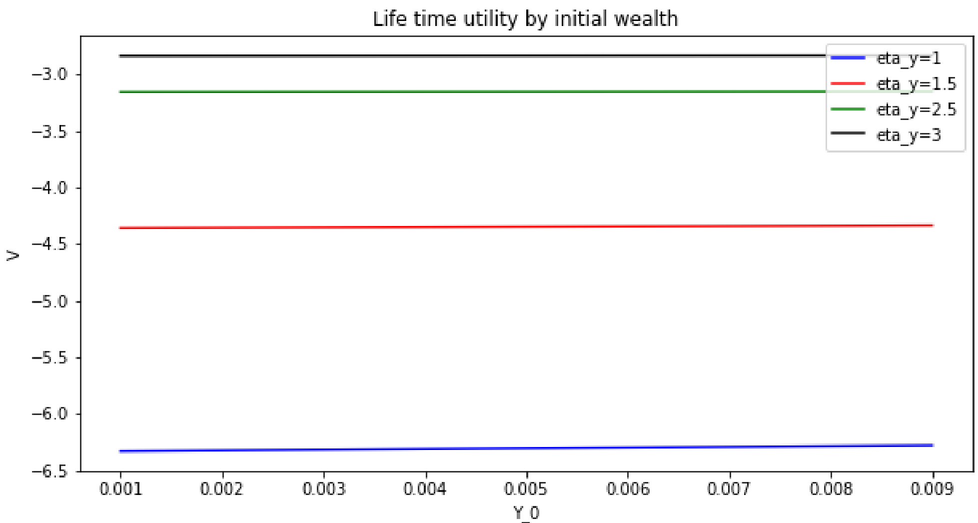



labor share positif shock given Y_0
ls+
Y_0
laboe share negative shock given Y_0
ls-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
Expected consumption growth
Egc
Y_0
Expected income growth
Egwh
Y_0


elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)+
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
0.25
0.20
0.15
0.10
0.05
0.002
0.004
0.006
0.008

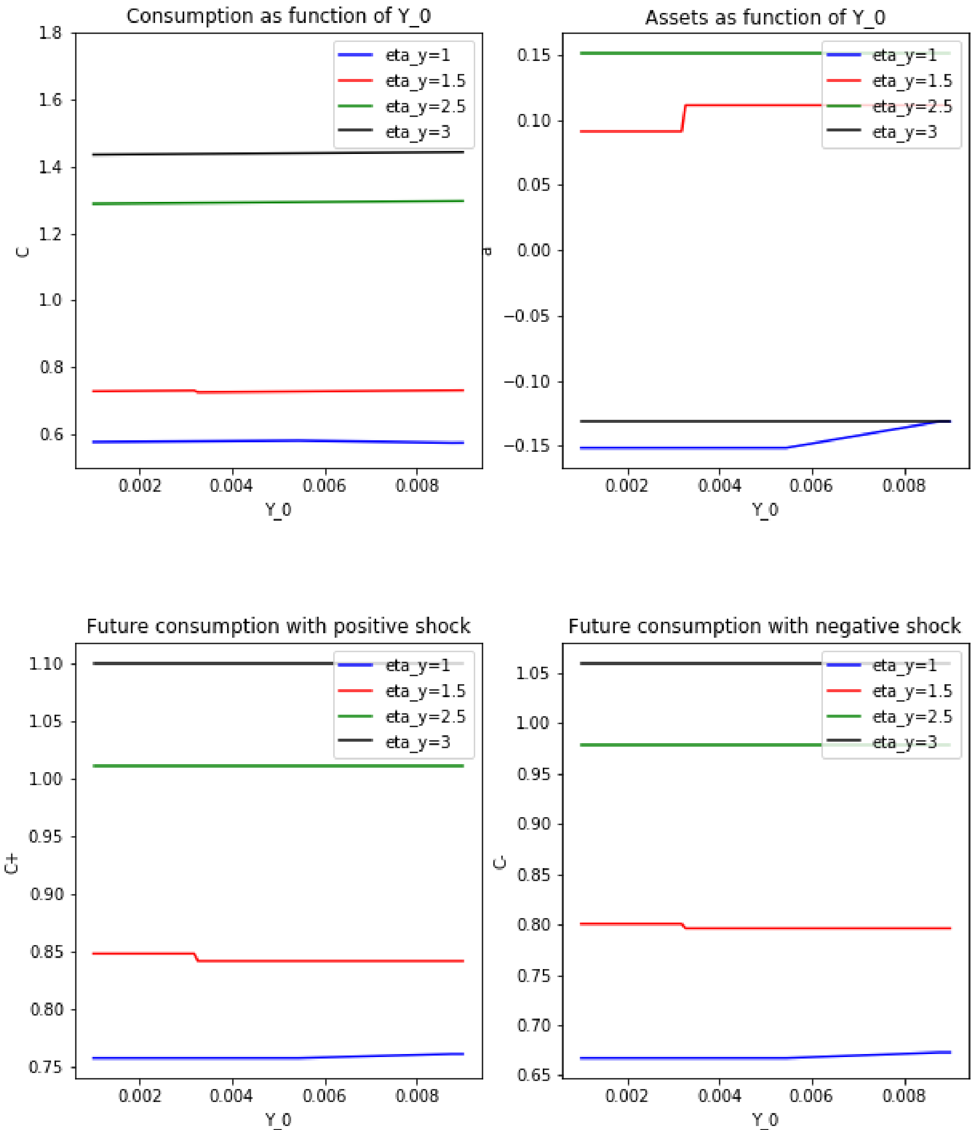

elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.002
0.004
0.006
0.008

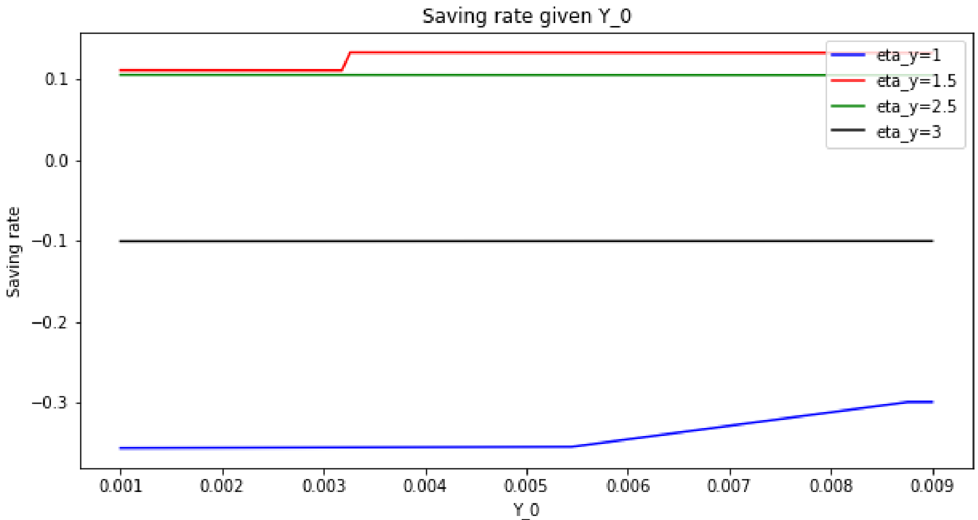

Market excess demand or supply in assets
A
r
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.1
0.0
0.1
0.2
0.3
0.4
0.5

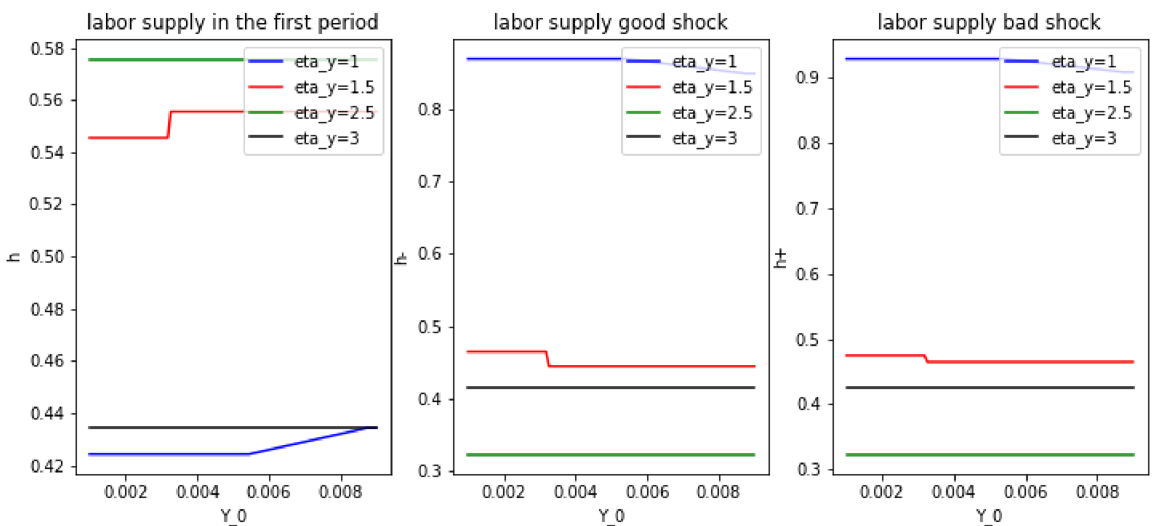


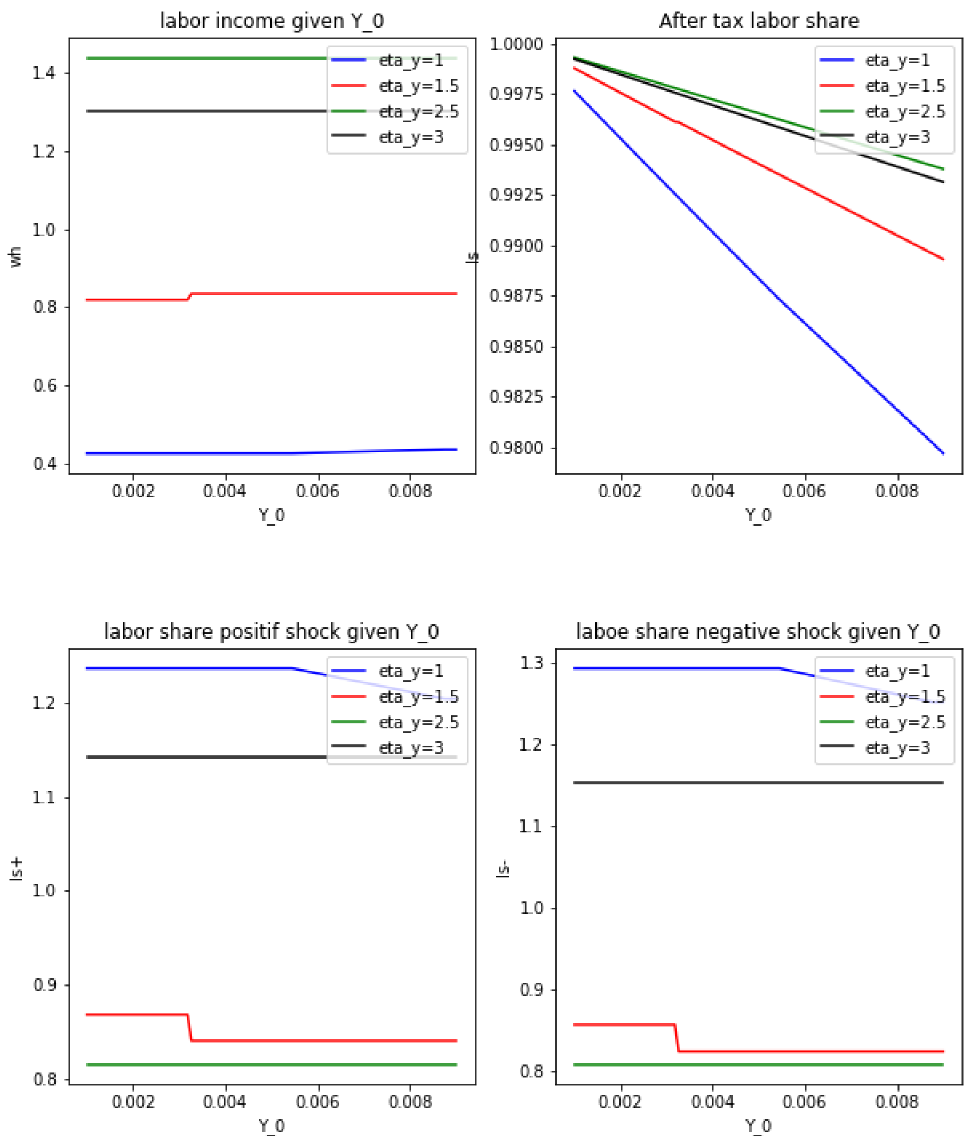
Part 2

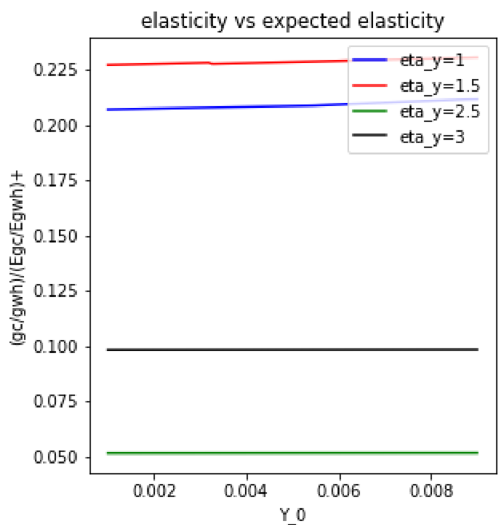
The problem with taxes is solved in two step I optimize the model without taxes, then compute the optimal values of $(c, h, h^{'})$ given that h and $h^{'}$ are known I can compute the taxes collected for every agent hence I can also compute transfers. Ones I have the transfers from the first step I plug them back into the maximization problem compute the optimal choices and the new transfers again. This method suffers probably from errors in estimation of the transfers, but the alternative of iterating through different levels of transfers was extremly slow.

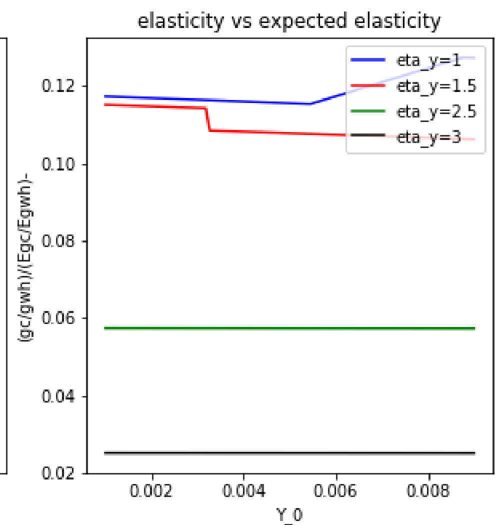

Consumption as function of Y_0
Assets as function of Y_0
Future consumption with positive shock
Future consumption with negative shock
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
Y_0
C
a
C+
C-

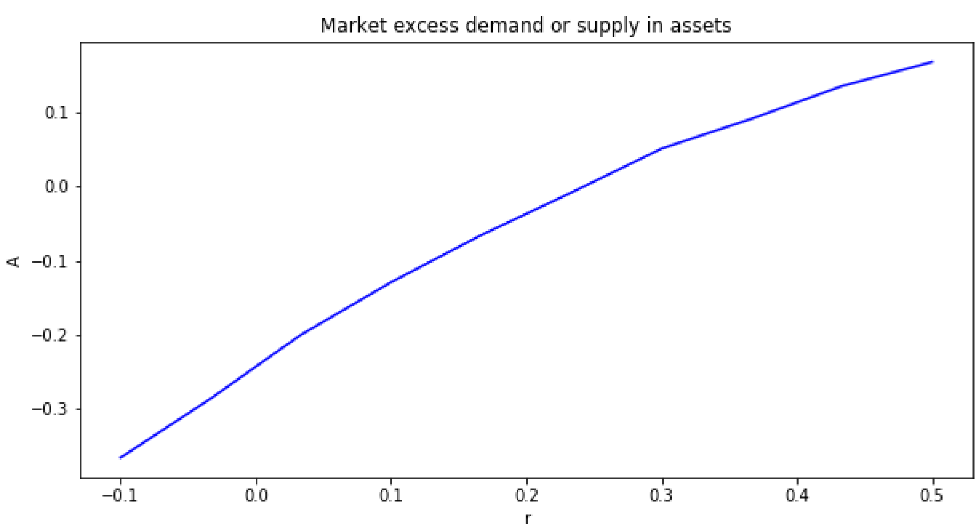

Saving rate given Y_0
Saving rate
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
0.1
0.0
−0.1
−0.2
−0.3
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009

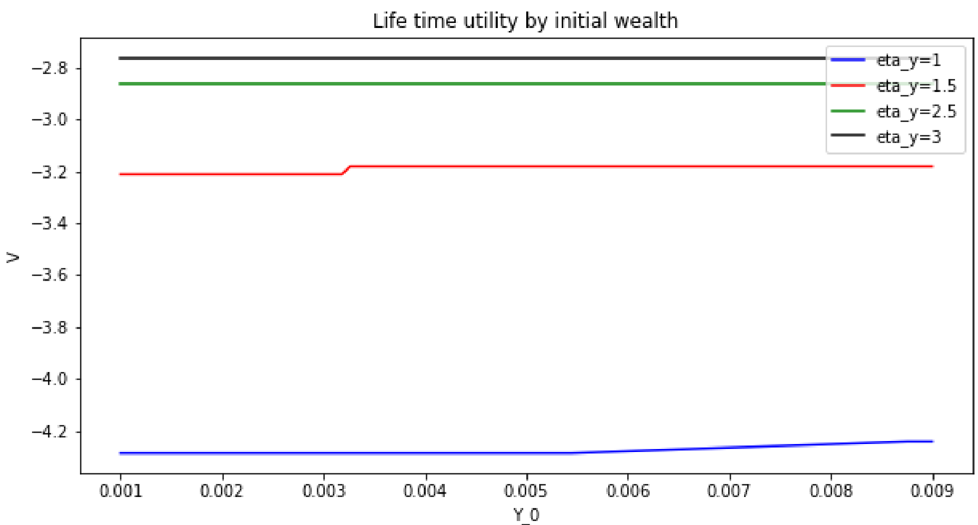

labor supply in the first period
labor supply good shock
labor supply bad shock
h
h-
h+
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
0.002
0.004
0.006
0.008


labor income given Y_0
wh
Y_0
After tax labor share
ls
Y_0
labor share positif shock given Y_0
ls+
Y_0
laboe share negative shock given Y_0
ls-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3


elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)+
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

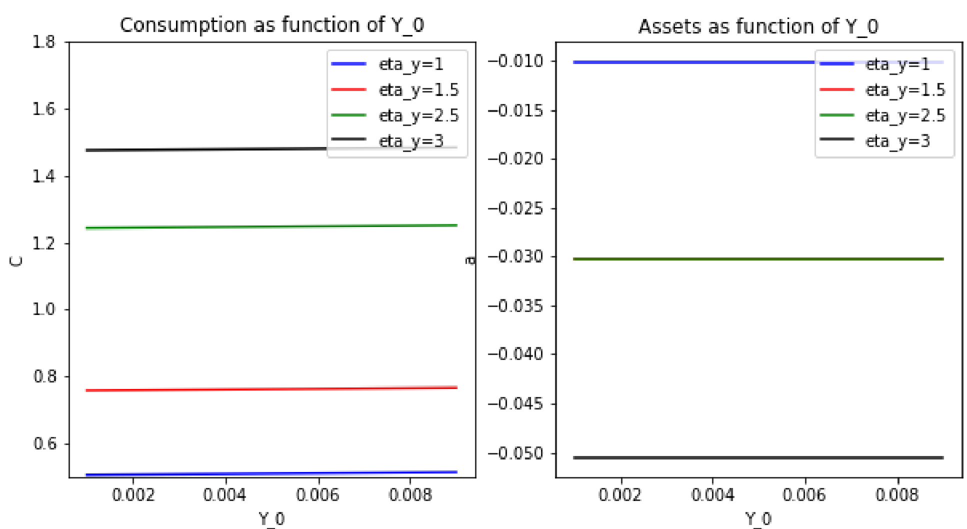

elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3


Market excess demand or supply in assets
A
r

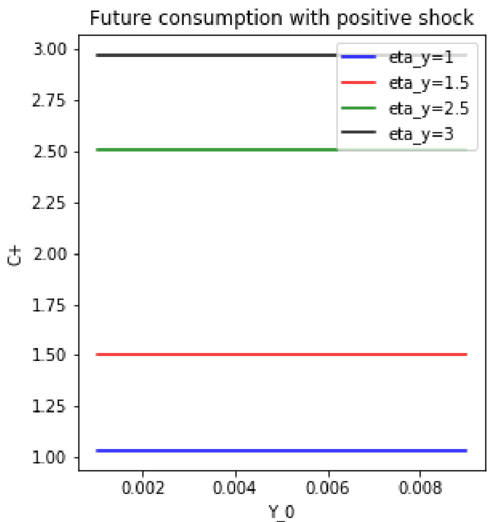


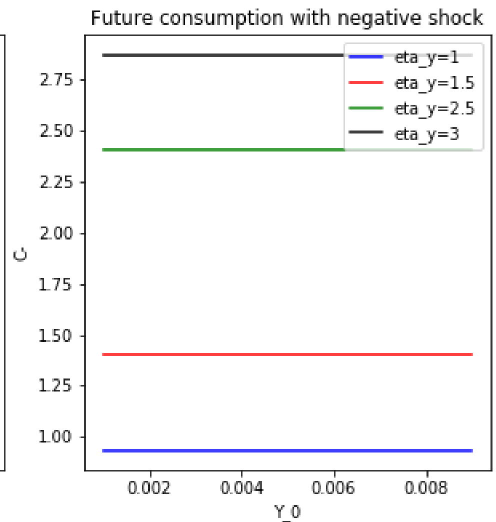
Life time utility by initial wealth
V
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

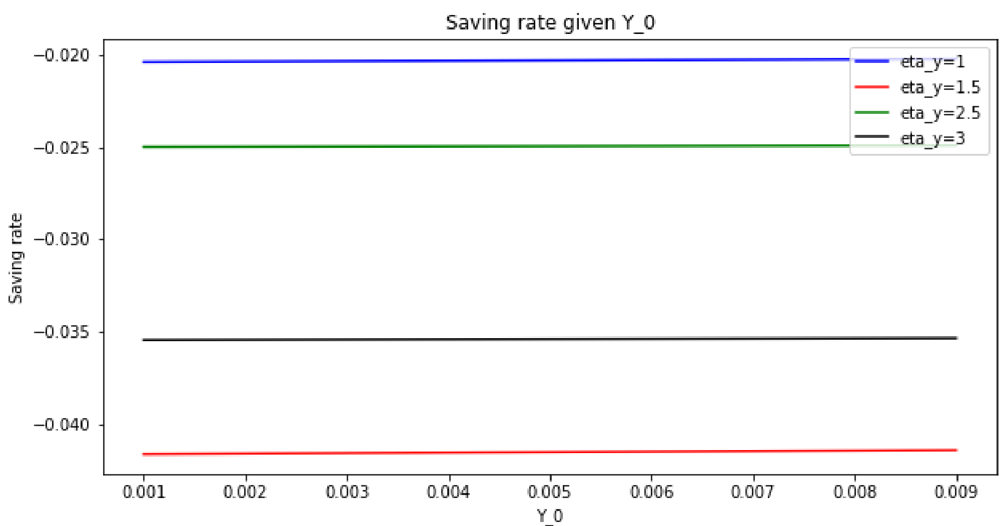
Part 3

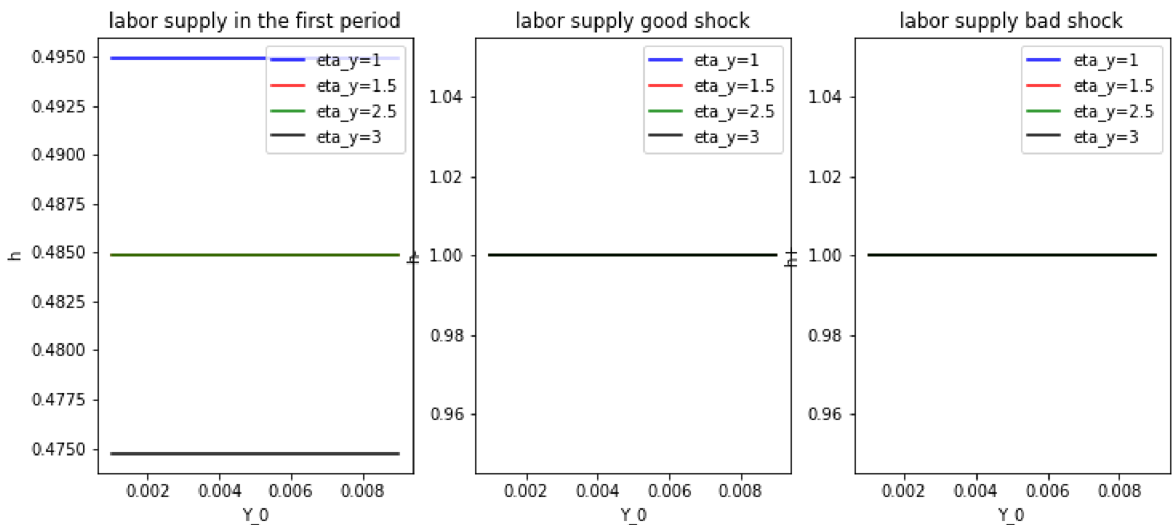


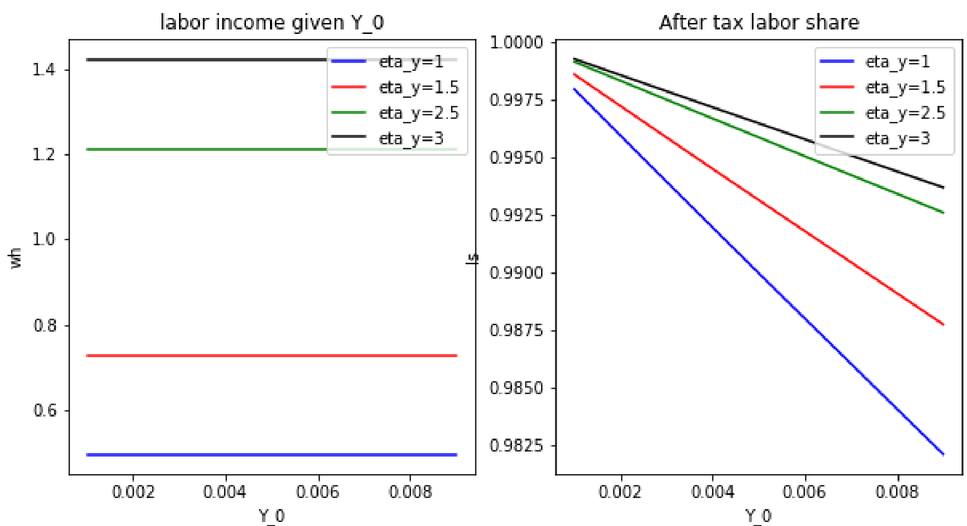
Consumption as function of Y_0
C
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
Assets as function of Y_0
a
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

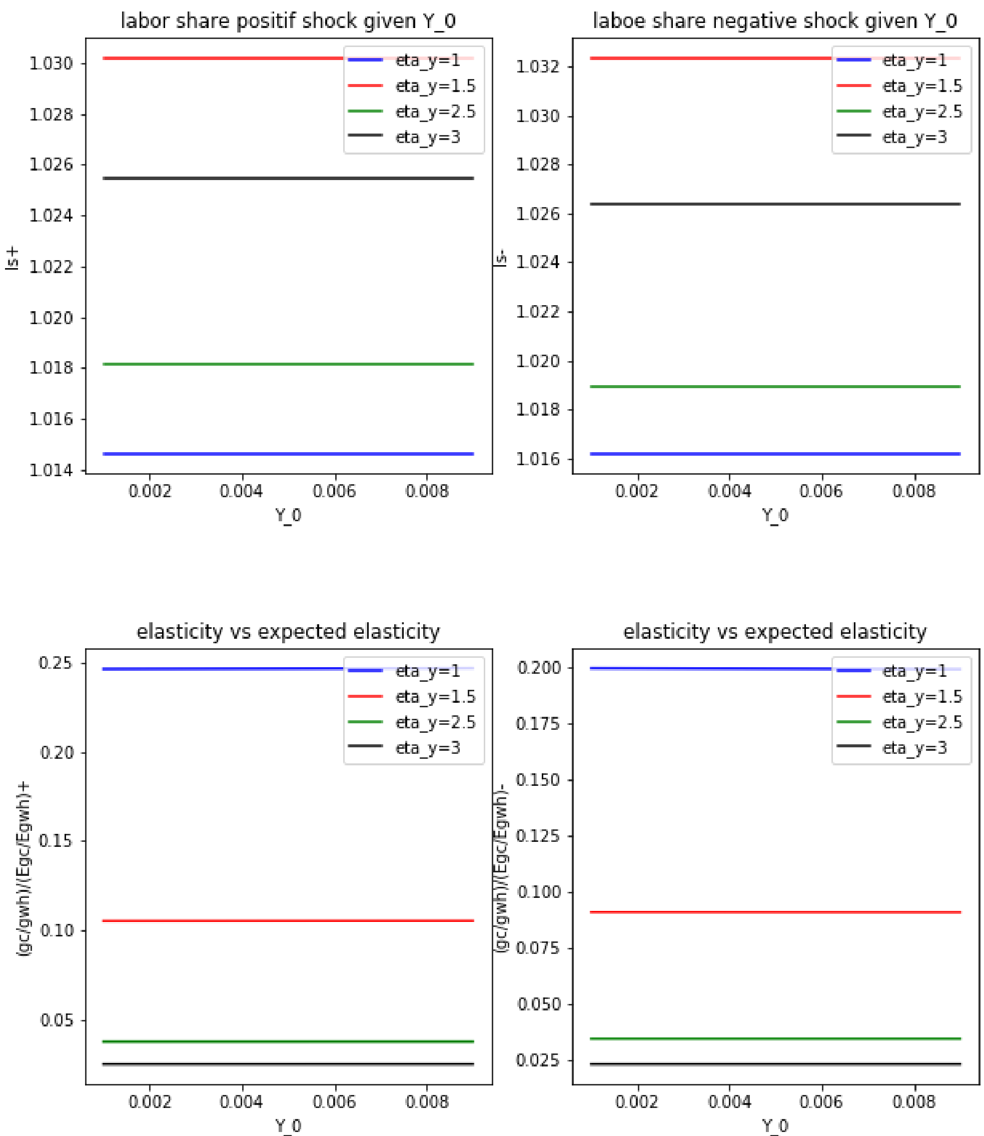

Future consumption with positive shock
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
C+
0.002
0.004
0.006
0.008
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

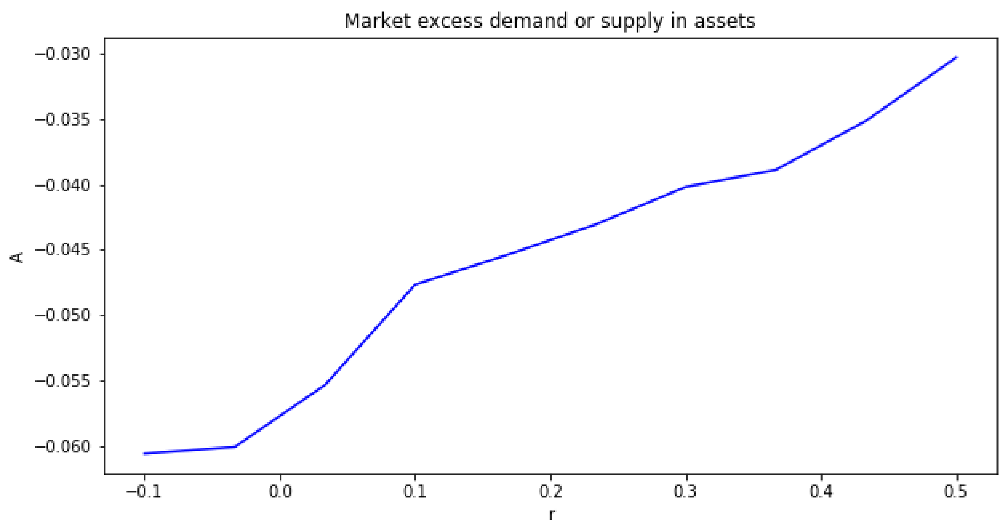

Future consumption with negative shock
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
C-
0.002
0.004
0.006
0.008
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

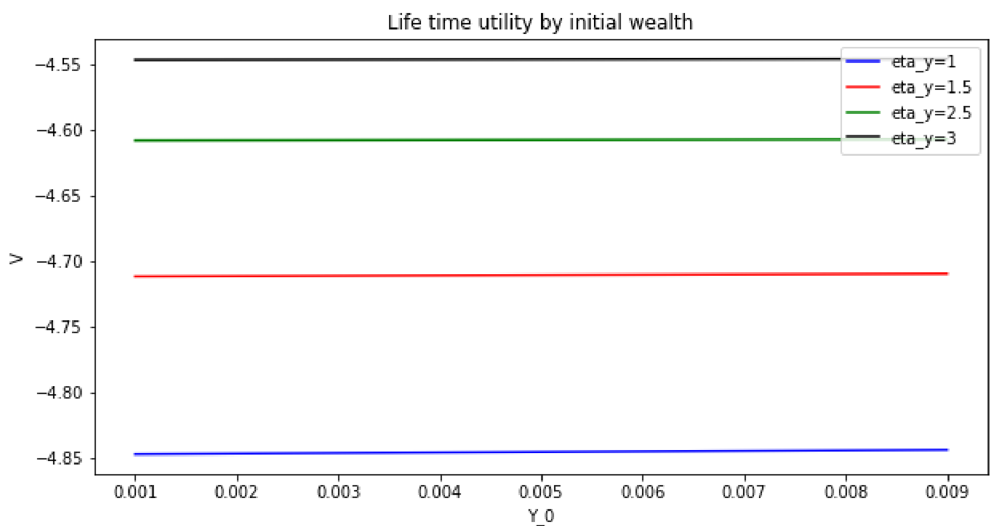

Saving rate given Y_0
−0.020
−0.025
−0.030
−0.035
−0.040
Saving rate
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3


labor supply in the first period
labor supply good shock
labor supply bad shock
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
h
Y_0


labor income given Y_0
After tax labor share
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
wh
ls
Y_0

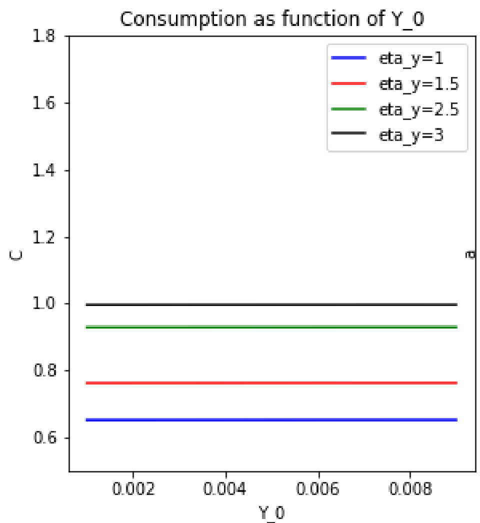

labor share positif shock given Y_0
ls+
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
laboe share negative shock given Y_0
ls-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)+
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
elasticity vs expected elasticity
(gc/gwh)/(Egc/Egwh)-
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3

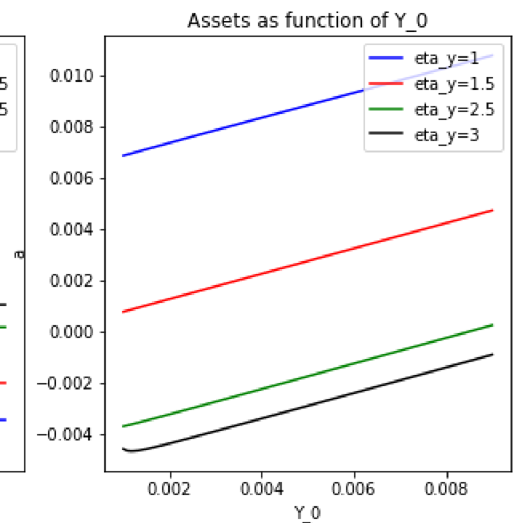

Market excess demand or supply in assets
−0.030
−0.035
−0.040
−0.045
−0.050
−0.055
−0.060
A
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
r

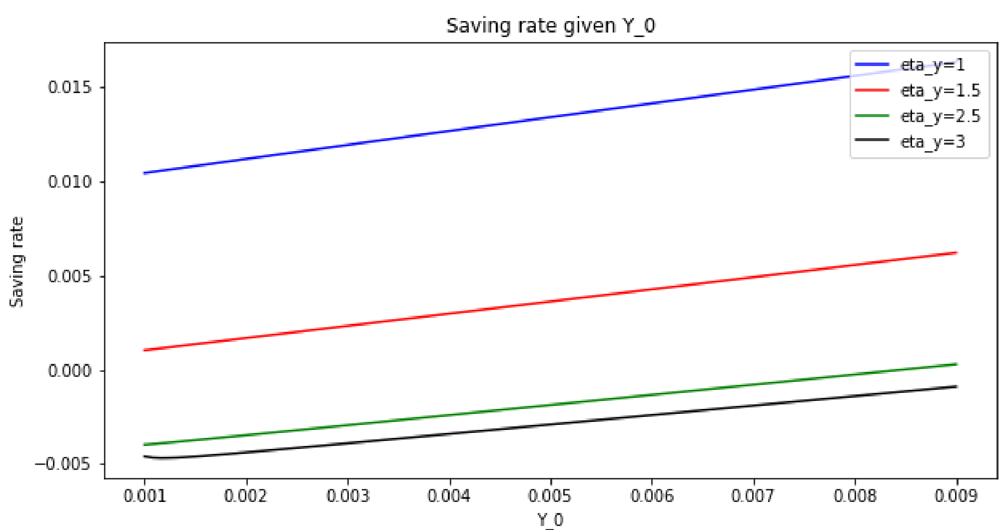

Life time utility by initial wealth
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3
−4.55
−4.60
−4.65
−4.70
−4.75
−4.80
−4.85
V
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
Y_0

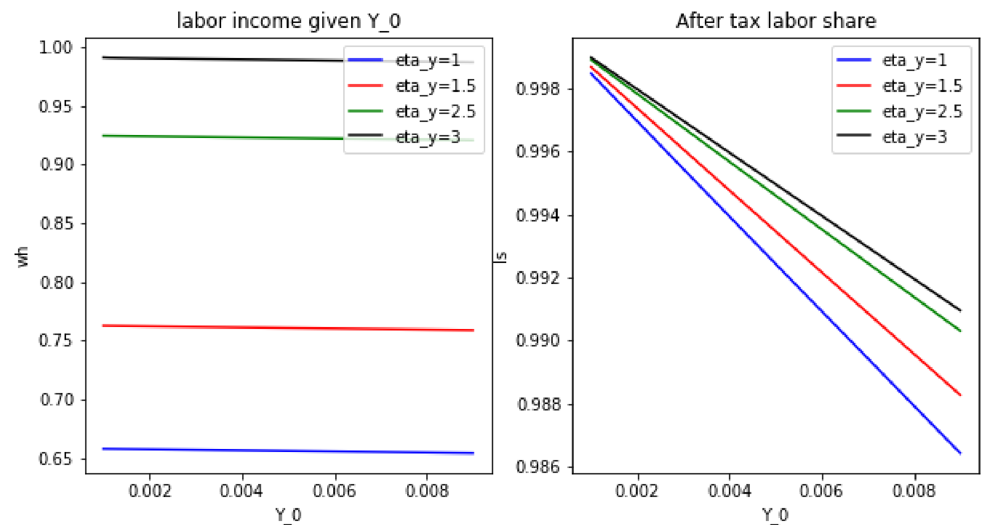
Part 4

This part is dedicated to the sequential resolution of the problem. As you can see I obtain some results that do make sens. However, some graphs indicated me that something in the procedure I followed was off, for instance if you look at the labor share and the lifetime wealth variables are quite volatile which doesn t make sens.

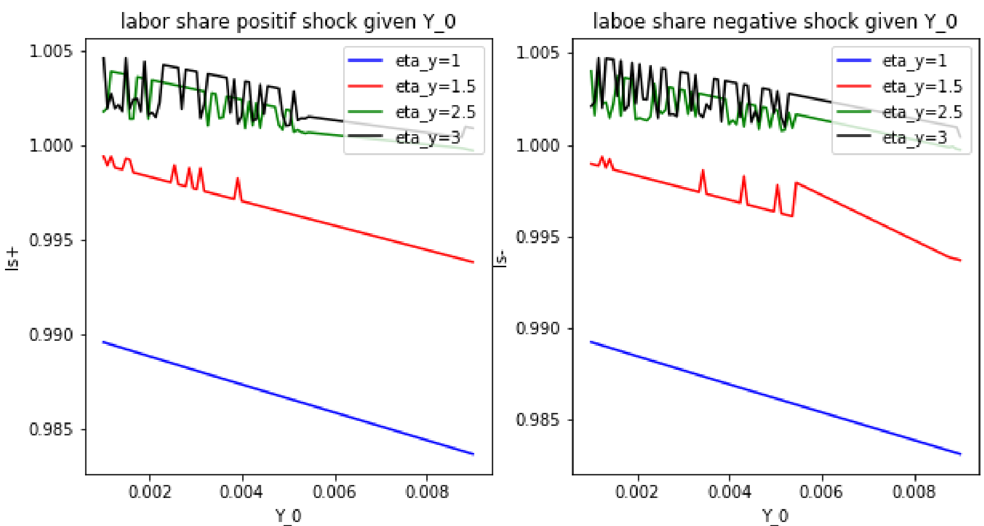


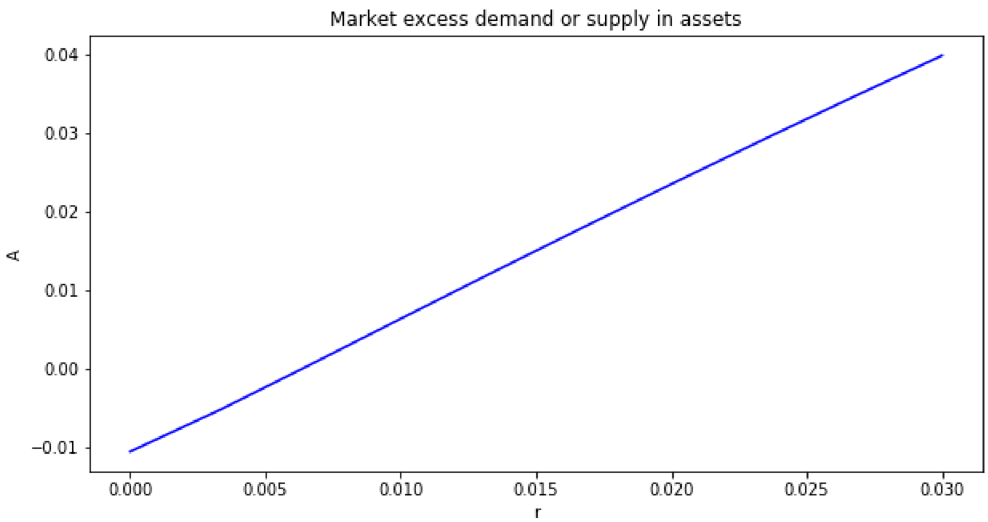


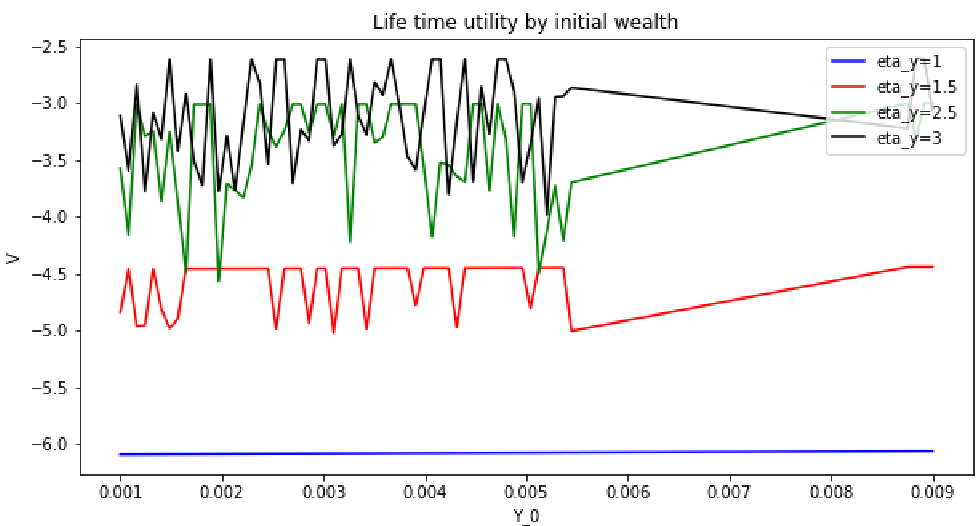




This is an example of what didn't make sens in my graphs.




Market excess demand or supply in assets
0.04
0.03
0.02
0.01
0.00
−0.01
A
0.000
0.005
0.010
0.015
0.020
0.025
0.030
r


Life time utility by initial wealth
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
V
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
Y_0
eta_y=1
eta_y=1.5
eta_y=2.5
eta_y=3