1.

a) $X_t = X_{t-4} + W_t + 0.5W_{t-1} - 0.25W_{t-2}$

NOT a SARIMA model

b) $X_t = 0.5X_{t-1} + X_{t-4} - 0.5X_{t-5} + W_t - 0.3W_{t-1}$

NOT a SARIMA model

c) $X_t = X_{t-1} + X_{t-12} - X_{t-13} + W_t - 0.5W_{t-1} - 0.5W_{t-12} + 0.25W_{t-13}$

yes, it is a SARIMA model

Backshift:

$$X_t = X_{t-1}(1-B) + W_t(1-B^{12}) - 0.5W_{t-1}(1-B)$$
$$- 0.5W_{t-12}(1-B^{12}) + 0.25W_{t-13}(1-B^{12})$$

2.

$$X_t = X_{t-4} + W_t$$

$W_t \sim iid\, N(0, \sigma^2)$

$t = 16$, if $X_0 = 0$

$$X_t = X_{t-4} + W_t$$

$$X_{16} = X_{16-4} + W_{16}$$

$$= X_8 + W_{12} + W_{16}$$

$$= X_4 + W_8 + W_{12} + W_{16}$$

$$= X_0 + W_4 + W_8 + W_{12} + W_{16}$$

$$Var(X_{16}) = 0 + \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2$$

$$= 4\sigma^2$$

3.

$$X_t = W_t - 0.5W_{t-4} \quad \text{where } W_t \sim \text{iid } N(0, \sigma^2)$$

$$\begin{aligned} m = E[X_t] &= E[W_t - 0.5W_{t-4}] \\ &= -0.5\,E[W_{t-4}] \\ &= -0.5 \cdot 0 \\ &= 0 \end{aligned}$$

Autocorvar:

$h=0$

$var(W_t - 0.5 W_{t-4})$

$= Var(W_t) + 0.25 var(W_{t-4})$

$= \sigma^2 + 0.25\sigma^2$

$= 1.25\sigma^2$

$h \neq 0$

$cov(W_t - 0.5 W_{t-4}, W_{t-h} - 0.5 W_{t-h-4})$

$= Cov(W_t, W_{t-h}) - 0.5 Cov(W_{t-4}, W_{t-h}) - 0.5 Cov(W_t, W_{t-h-4})$
$+ 0.25 cov(W_{t-4}, W_{t-h-4})$

$= -0.5 Cov(W_{t-4}, W_{t-h}) - 0.5 Cov(W_t, W_{t-h-4}) +$
$0.25 Cov(W_{t-4}, W_{t-h-4})$

$= -0.5 \cdot 0 - 0.5 \cdot 0 + 0.25 \cdot 0$

$= 0$

$h=4$

$Cov(W_t - 0.5 W_{t-4}, W_{t-4} - 0.5 W_{t-8})$

$= Cov(W_t, W_{t-4}) - 0.5 Cov(W_{t-4}, W_{t-8}) - 0.5 Cov(W_t, W_{t-12})$
$+ 0.25 cov(W_{t-4}, W_{t-12})$

$= 0 - 0.5 \cdot 0 - 0.5 \cdot 0 + 0.25 \cdot 0$

$= 0$

$$\begin{cases} 1.25\sigma^2 & h=0 \\ 0 & h \neq 0 \\ 0 & h=4 \end{cases}$$