STAT 1321/2320: Final Theory

Instructions:

- You need to show your work (formula setup, calculations, etc.) and state any properties or rules you need to solve the problems to get full credit.

1. (8 points) Identify the order (ARIMA or SARIMA) of the following models.
   (a) $(1-B)(1-B^4)(1-0.43B^4)X_t = (1+0.22B)(1+0.88B^4)W_t$
   (b) $(1+0.6B)(1-B)X_t = (1-0.9B)^2W_t$
   (c) $(1-0.8B)X_t = (1-1.6B+0.64B^2)W_t$
   (d) $(1-0.3B+1.1B^2)X_t = (1-0.5B)W_t$

2. (8 points) Suppose that W_t is a white noise process with variance σ^2. Consider the time series model:
$$X_t = \beta_0 + \beta_1 t + Y_t$$
   where $Y_t = Y_{t-1} + W_t - \theta W_{t-1}$ and β_0 and β_1 are constants.
   (a) Give the expression of the first order differenced series ∇X_t .
   (b) Identify the process ∇X_t and comment on its stationarity.

3. (14 points) Suppose that W_t is a white noise process with variance σ^2. Consider the time series model:
$$X_t = 0.5X_{t-1} + W_t - 0.2W_{t-1} - 0.15W_{t-2}$$
   (a) Write this model using the backshift notation.
   (b) Determine whether this model is stationary and/or invertible.
   (c) Identify this model as an ARIMA(p, d, q) process; that is, specify p, d, and q.

4. (15 points) Suppose that X_t is a seasonal model given as
$$X_t = W_t - 0.5W_{t-1} - 0.5W_{t-4} + 0.25W_{t-5}$$
   where W_t is white noise with variance σ_w^2
   (a) Derive expressions for the mean function μ_t and variance $\gamma_t(0)$
   (b) Derive the autocovariance function for lags $h = 1, 2, 3, \ldots$
   (c) Using parts (a) and (b), comment on the stationarity of the process.