${\rm SKS}104_{\rm MIDTERM2}$

Skasko_Stephen

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```
data <- read.csv("amazon.csv")
data</pre>
```

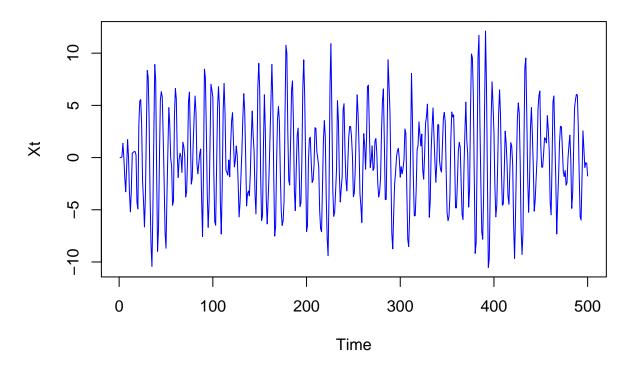
##		Quarter	Year	Sales
##	1	1	2000	573.889
##	2	2	2000	577.876
##	3	3	2000	637.858
##	4	4	2000	972.360
##	5	1	2001	700.356
##	6	2	2001	667.625
##	7	3	2001	639.281
##	8	4	2001	1115.171
##	9	1	2002	847.422
##	10	2	2002	805.605
##	11	3	2002	851.299
##	12	4	2002	1428.610
##	13	1	2003	1083.559
##	14	2	2003	1099.912
##	15	3	2003	1134.456
##	16	4	2003	1945.772
##	17	1	2004	1530.349
##	18	2	2004	1387.341
##	19	3	2004	1462.475
##	20	4	2004	2540.959
##	21	1	2005	1901.600
##	22	2	2005	1753.000
##	23	3	2005	1858.000
##	24	4	2005	2977.000
##	25	1	2006	2279.000
##	26	2	2006	2139.000
##	27	3	2006	2307.000
##	28	4	2006	3986.000
##	29	1	2007	3015.000
##	30	2	2007	2886.000
##	31	3	2007	3262.000
##	32	4	2007	5673.000
##	33	1	2008	4135.000
##	34	2	2008	4063.000
##	35	3	2008	4264.000
##	36	4	2008	6704.000
##	37	1	2009	4889.000

```
## 38
            2 2009 4651.000
## 39
            3 2009 5449.000
            4 2009 9519.000
## 40
            1 2010 7131.000
## 41
## 42
            2 2010 6566.000
## 43
            3 2010 7560.000
## 44
            4 2010 12948.000
            1 2011 9857.000
## 45
## 46
            2 2011 9913.000
            3 2011 10876.000
## 47
## 48
            4 2011 17431.000
            1 2012 13185.000
## 49
            2 2012 12834.000
## 50
## 51
            3 2012 13806.000
## 52
            4 2012 21268.000
## 53
            1 2013 16070.000
## 54
            2 2013 15704.000
## 55
           3 2013 17092.000
## 56
            4 2013 25587.000
## 57
            1 2014 19741.000
## 58
            2 2014 19340.000
  1.
# a) Roots are greater than 1, which does make it stationary.
coefficients_a \leftarrow c(1, -1.2, 0.85)
roots_a <- polyroot(coefficients_a)</pre>
roots_a
## [1] 0.7058824+0.8235294i 0.7058824-0.8235294i
# b) Roots are looking like they are greater than 1, which also makes it stationary but is a bit inconc
coefficients_b <- c(1, -11/6, 1, -1/6)
roots_b <- polyroot(coefficients_b)</pre>
roots_b
## [1] 1-0i 2+0i 3-0i
# c) Like b, there is uncertainty but one root is above 1, which implies potential for invertibility is
coefficients_c \leftarrow c(1, -15/4, 0, 1/4)
roots_c <- polyroot(coefficients_c)</pre>
roots_c
## [1] 0.2679492+0i -4.0000000-0i 3.7320508-0i
# d) The roots show greater than 1, which implies stationary and potential for invertibility.
coefficients_d \leftarrow c(1, -0.8, 0.15)
roots_d <- polyroot(coefficients_d)</pre>
coefficients_i_d <- c(1, 0.3)
roots_i_d <- polyroot(coefficients_i_d)</pre>
roots d
```

```
## [1] 2.000000-0i 3.333333+0i
roots_i_d
## [1] -3.333333+0i
  2.
library(forecast)
## Registered S3 method overwritten by 'quantmod':
     method
                        from
     as.zoo.data.frame zoo
##
# a)
set.seed(1000)
n <- 500
Wt \leftarrow rnorm(n, mean = 0, sd = sqrt(4))
Xt <- numeric(n)</pre>
for (t in 3:n) {
 Xt[t] \leftarrow 1.2 * Xt[t-1] - 0.85 * Xt[t-2] + Wt[t]
```

Simulated Time Series

plot(Xt, type = "1", col = "blue", xlab = "Time", ylab = "Xt", main = "Simulated Time Series")



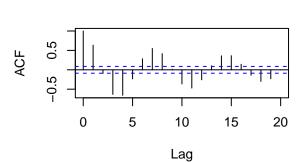
```
# b) The sample ACF plot shows a quick up-and-down pattern, suggesting a dynamic time series with decre
acf_ARMA11 <- ARMAacf(ar = c(1.2, -0.85), lag.max = 20)
pacf_ARMA11 <- ARMAacf(ar = c(1.2, -0.85), lag.max = 20, pacf = TRUE)

sample_acf_ARMA11 <- acf(Xt, lag.max = 20, plot = FALSE)
sample_pacf_ARMA11 <- pacf(Xt, lag.max = 20, plot = FALSE)

par(mfrow = c(2, 2))

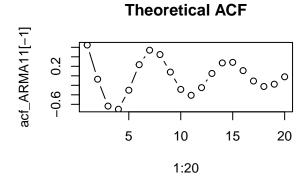
# Create sample ACF plot
plot(sample_acf_ARMA11, main = "Sample ACF")
plot(1:20, acf_ARMA11[-1], type = "b", main = "Theoretical ACF")

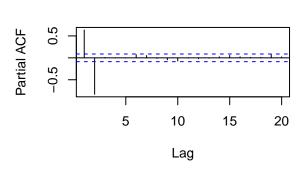
# Create sample PACF plot
plot(sample_pacf_ARMA11, main = "Sample PACF")
plot(sample_pacf_ARMA11, type = "b", main = "Theoretical PACF")
plot(1:20, pacf_ARMA11, type = "b", main = "Theoretical PACF")</pre>
```

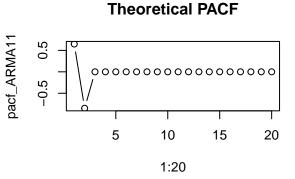


Sample ACF

Sample PACF







3.

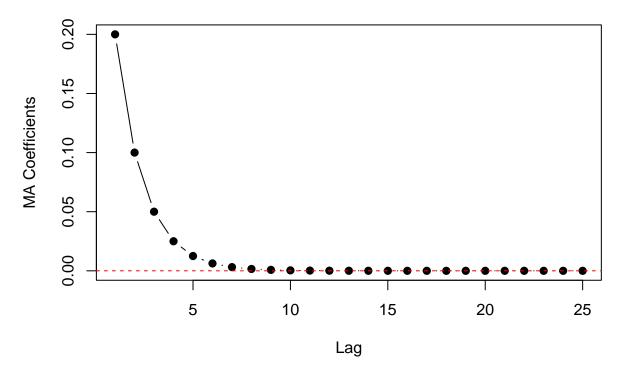
```
# a)
phi <- 0.5
theta <- -0.3

ma_coefs <- ARMAtoMA(ar = phi, ma = theta, lag.max = 25)
ma_coefs</pre>
```

```
## [6] 6.250000e-03 3.125000e-03 1.562500e-03 7.812500e-04 3.906250e-04
## [11] 1.953125e-04 9.765625e-05 4.882813e-05 2.441406e-05 1.220703e-05
## [16] 6.103516e-06 3.051758e-06 1.525879e-06 7.629395e-07 3.814697e-07
## [21] 1.907349e-07 9.536743e-08 4.768372e-08 2.384186e-08 1.192093e-08
# b) The shape of the plot shows a decay in coefficients rapidly and consistence with a consistent poin
plot(1:25, ma_coefs, pch = 19,type = "b", xlab = "Lag", ylab = "MA Coefficients", main = "MA Coefficients" abline(h = 0, col = "red", lty = 2)
```

MA Coefficients vs Lags

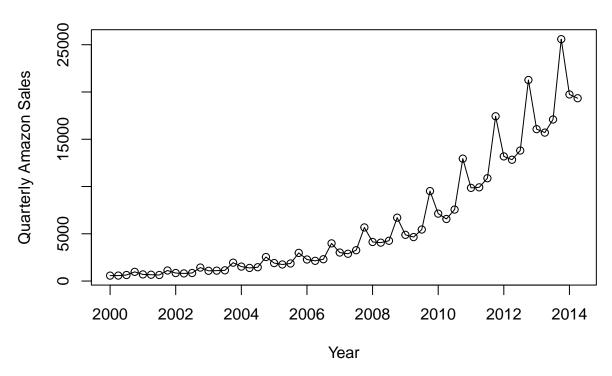
[1] 2.000000e-01 1.000000e-01 5.000000e-02 2.500000e-02 1.250000e-02



4.

```
# (a)
data <- read.csv("amazon.csv")
sales_ts <- ts(data$Sales, start = c(data$Year[1], data$Quarter[1]), frequency = 4)
# (b)
plot(sales_ts, type = "o", xlab = "Year", ylab = "Quarterly Amazon Sales", main = "Time Series Plot")</pre>
```

Time Series Plot

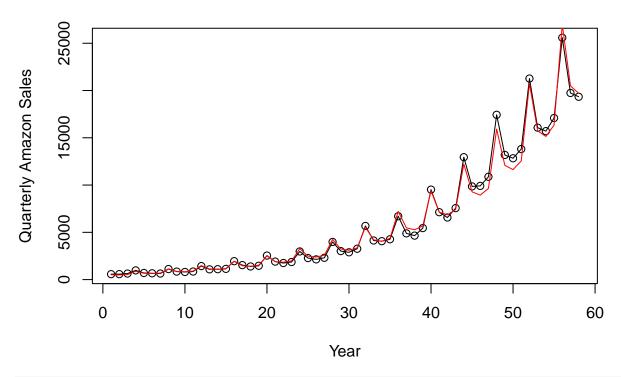


(c) The model appears to provide a close fit qdto the original data, with fitted values closely resem
time <- 1:length(sales_ts)
model <- lm(log(sales_ts) ~ time + as.factor(data\$Quarter))
summary(model)</pre>

```
##
## lm(formula = log(sales_ts) ~ time + as.factor(data$Quarter))
##
## Residuals:
##
       Min
                 1Q
                      Median
                                    3Q
                                            Max
  -0.12512 -0.04523 -0.01658 0.04419
##
## Coefficients:
##
                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                             6.168183
                                       0.025352 243.303 < 2e-16 ***
## time
                             0.065954
                                       0.000578 114.103 < 2e-16 ***
## as.factor(data$Quarter)2 -0.104492
                                       0.026903
                                                 -3.884 0.000287 ***
## as.factor(data$Quarter)3 -0.094190
                                       0.027373
                                                 -3.441 0.001138 **
## as.factor(data$Quarter)4 0.340925
                                       0.027379
                                                12.452 < 2e-16 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 0.07366 on 53 degrees of freedom
## Multiple R-squared: 0.9961, Adjusted R-squared: 0.9958
## F-statistic: 3357 on 4 and 53 DF, p-value: < 2.2e-16
```

```
fitted_values <- exp(predict(model))
plot(time, sales_ts, type = "o", xlab = "Year", ylab = "Quarterly Amazon Sales", main = "Fitted Values of lines(time, fitted_values, col = "red")</pre>
```

Fitted Values vs Observed Values



```
# (d)
rsquared <- summary(model)$r.squared
aic <- AIC(model)
bic <- BIC(model)
cat("R-squared:", rsquared, "\nAIC:", aic, "\nBIC:", bic, "\n")</pre>
```

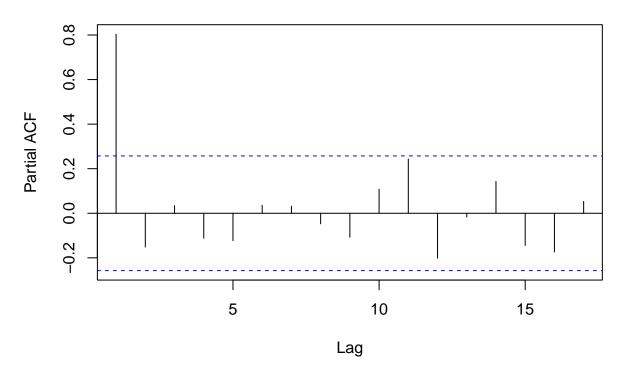
R-squared: 0.9960685 ## AIC: -131.1925 ## BIC: -118.8299

(e) The Ljung-Box test result (p-value = 2.049e-13) suggests that there is strong evidence of pattern residuals_check <- checkresiduals(model, test = "LB", LjungBox = TRUE)

Residuals 0.2 -0.1 0.0 --0.1 **-**10 20 30 50 Ö 40 60 15 **-**0.6 oft\$30 -0.3 -5 -0.0 -0.3 **-**10 15 5 -0.2 0.0 0.2 -0.10.1 Lag residuals ## ## Ljung-Box test ## ## data: Residuals ## Q* = 81.983, df = 10, p-value = 2.049e-13 ## ## Model df: 0. Total lags used: 10 $residuals_check$ ## Ljung-Box test ## ## data: Residuals ## Q* = 81.983, df = 10, p-value = 2.049e-13 # (f) library(tseries) adf_test <- adf.test(residuals(model))</pre> cat("ADF Test p-value:", adf_test\$p.value, "\n")

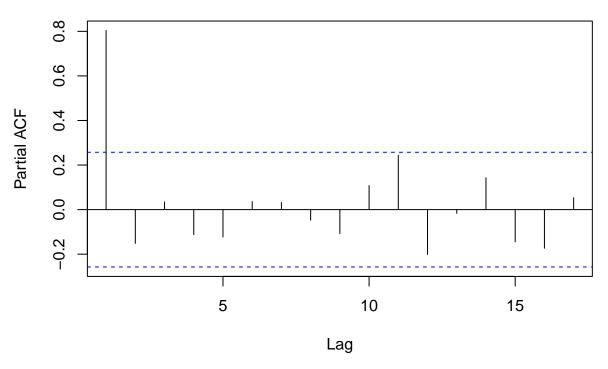
ADF Test p-value: 0.09728275

Series residuals(model)



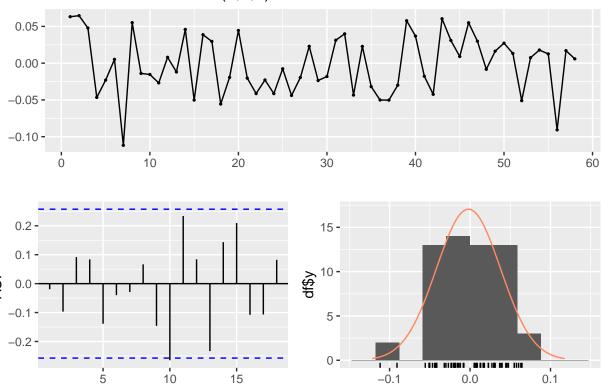
plot(pacf_res, main = "Partial Autocorrelation Function for Residuals")

Partial Autocorrelation Function for Residuals



```
# h) Sure, same model.
library(forecast)
residuals_model <- residuals(model)</pre>
auto.arima(residuals_model)
## Series: residuals_model
## ARIMA(1,0,0) with zero mean
##
## Coefficients:
##
            ar1
##
         0.8303
## s.e. 0.0726
## sigma^2 = 0.001616: log likelihood = 104.03
## AIC=-204.06 AICc=-203.85 BIC=-199.94
#wont work after 7 horus sorry
#armasubsets(residuals_model, nar = 2, nma = 2)
arma_model <- arima(residuals(model), order = c(1, 0, 1))</pre>
checkresiduals(arma_model)
```

Residuals from ARIMA(1,0,1) with non-zero mean



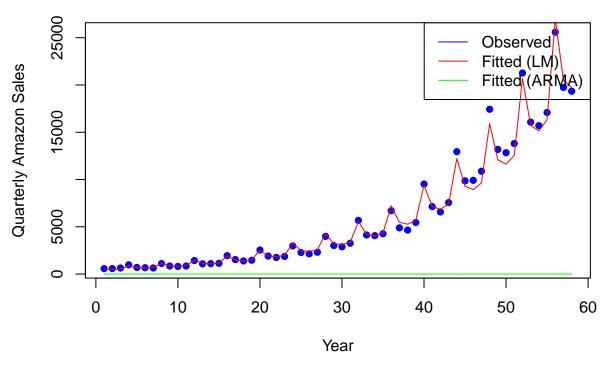
```
##
## Ljung-Box test
##
## data: Residuals from ARIMA(1,0,1) with non-zero mean
## Q* = 9.9234, df = 8, p-value = 0.2704
##
## Model df: 2. Total lags used: 10
```

Lag

```
# (j) The model fits quite well with the model with some anomalies.
fitted_lm <- exp(predict(model))
fitted_arma <- fitted(arma_model)
plot(time, as.vector(sales_ts), type = "p", col = "blue", pch = 16, xlab = "Year", ylab = "Quarterly Am lines(time, fitted_lm, col = "red", type = "l")
lines(time, fitted_arma, col = "green", type = "l")
legend("topright", legend = c("Observed", "Fitted (LM)", "Fitted (ARMA)"), col = c("blue", "red", "greend")</pre>
```

residuals

Observed vs Fitted Values



```
# (k) Yes, very similar.
future_time <- max(time) + 1:2
lm_future_data <- data.frame(time = future_time, Quarter = rep(1:4, times = 2))
#lm_forecast <- exp(predict(model, newdata = lm_future_data))
future_data <- data.frame(Quarter = rep(1:4, times = 2), Year = rep(max(data$Year) + 1, each = 4))
future_predictors <- data.frame(time = rep(future_time, each = 4), Quarter = rep(1:4, times = 2))
arma_forecast <- forecast(arma_model, h = 2)
arma_forecast</pre>
```

```
## Point Forecast Lo 80 Hi 80 Lo 95 Hi 95
## 59 -0.012743141 -0.06347563 0.03798935 -0.09033179 0.06484551
## 60 -0.008721917 -0.07818796 0.06074413 -0.11496107 0.09751724
```