Non-Stationary Time Series - SARIMA (Contd.)

STAT 1321/2320

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Outline

Specifying the Order - ACF/PACF

SAR(1) with s = 12.

- Example 1: Consider a SARIMA $(0, 0, 0) \times (1, 0, 0)_{12}$ model

$$X_t = 0.5X_{t-12} + W_t \implies (1 - 0.5B^{12})X_t = W_t$$
$$\implies \Phi_1(B^{12})X_t = W_t$$

1 seasonal lag dep.

- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

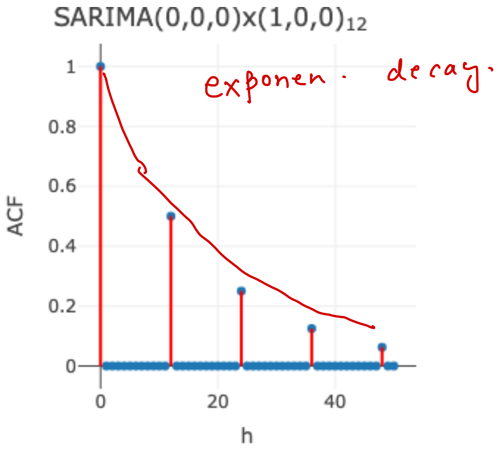
```
sar1<- ARMAacf(ar = c(rep(0,11), 0.5), lag.max=50) #ACF
sar1_p <- ARMAacf(ar = c(rep(0,11), 0.5), lag.max=50,
                                    pacf=T) #PACF
```

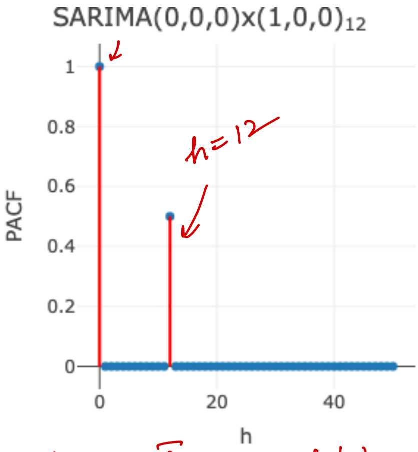
$(p, q) \times (P, Q)$.

$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots \quad + 0.5X_{t-12} + W_t$.

Example 1

SAR(1).





$X_t = \Phi X_{t-12} + w_t$.

Specifying the Order - ACF/PACF

SMA(1), s = 4.

- Example 2: Consider a SARIMA $(0,0,0) \times (0,0,1)_4$ model

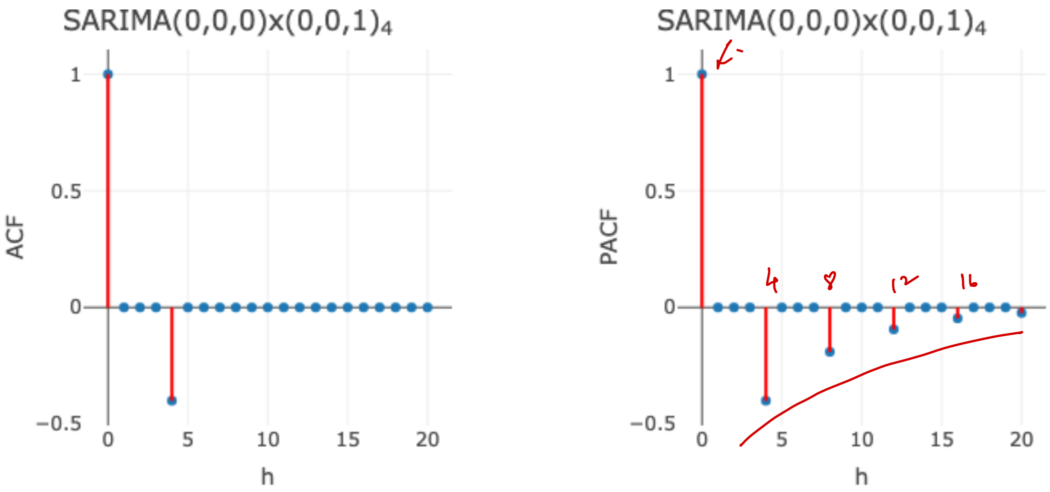
$$X_t = W_t - 0.5W_{t-4} = (1 - 0.5B^4)W_t = \Theta_1(B^4)W_t$$

- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

$w_{t-1}, w_{t-2}, w_{t-3}$ w_{t-4}

```
sma1<- ARMAacf(ma = c(rep(0,3), -0.5), lag.max=20) #ACF
sma1_p <- ARMAacf(ma = c(rep(0,3), -0.5), lag.max=20,
                                   pacf=T) #PACF
```

Example 2 $X_t = w_t - 0.5\, w_{t-4}$.



Specifying the Order - ACF/PACF

SMA(2), s = 4.

- Example 3: Consider a SARIMA $(0,0,0) \times (0,0,2)_4$ model

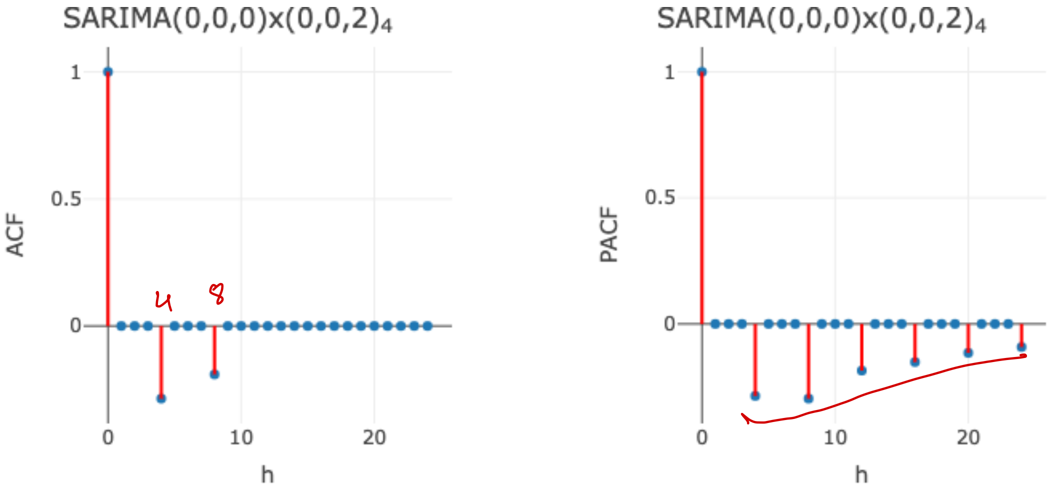
$$X_t = \underline{W_t} - 0.5\underline{W_{t-4}} - 0.25\underline{W_{t-8}} = (1 - 0.5B^4 - 0.25B^8)W_t = \Theta_2(B^4)W_t$$

- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

1-3 4 5-7 8

```
sma2<- ARMAacf(ma = c(rep(0,3), -0.5, rep(0,3), -0.25),
                                        lag.max=24) #ACF
sma2_p <- ARMAacf(ma = c(rep(0,3), -0.5, rep(0,3), -0.25),
                                        lag.max=24, pacf=T) #PACF
```

Example 3


SARIMA(0,0,0)x(0,0,2)$_4$
ACF
1
0.5
0
0
10
20
h
4
8
SARIMA(0,0,0)x(0,0,2)$_4$
PACF
1
0.5
0
0
10
20
h

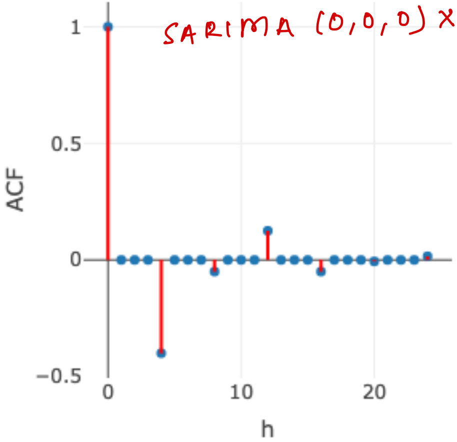
Guess the order - I

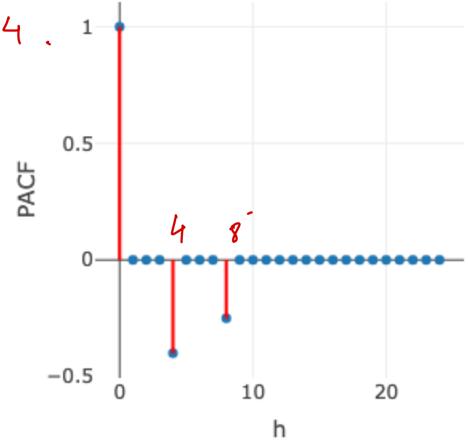
SAR (2)

exp. decay

$x_t = \Phi_1 x_{t-4} + \Phi_2 x_{t-8} + w_t$.

SARIMA $(0,0,0) \times (2,0,0)_4$.





Specifying the Order - ACF/PACF

$p=1, \ P=1, \ s=12.$

- Example 4: Consider a SARIMA $(1,0,0)\times(1,0,0)_{12}$ model

$\Phi(B^{12})$

$\phi(B)$

$$X_t = 0.5X_{t-1} + 0.8X_{t-12} - 0.4X_{t-13} + W_t \implies (1-0.5B)(1-0.8B^{12})X_t = W_t$$

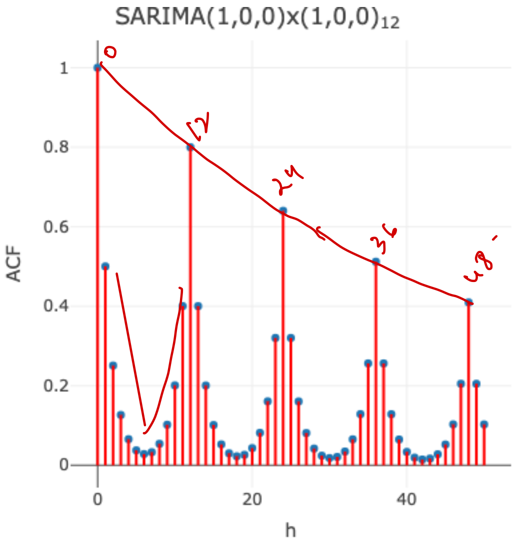
$$\implies \phi(B)\Phi_1(B^{12})X_t = W_t$$

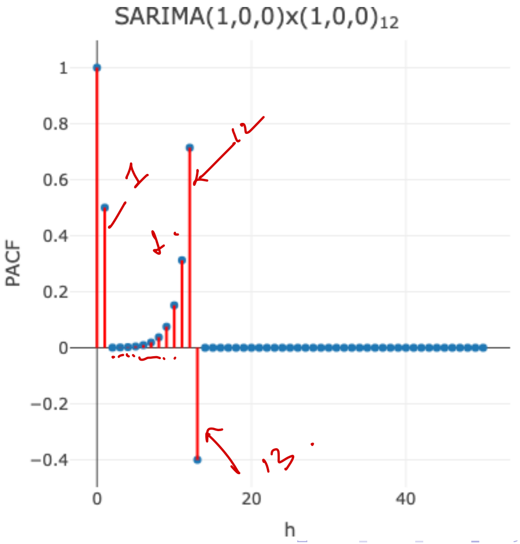
- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

1   2 − 11   12   13.

```
ar1_s<- ARMAacf(ar = c(0.5, rep(0,10), 0.8, -0.5*0.8),
                      lag.max=50) #ACF
ar1_sp <- ARMAacf(ar =c( 0.5, rep(0,10), 0.8,-0.5*0.8),
                      lag.max=50, pacf=T) #PACF
```

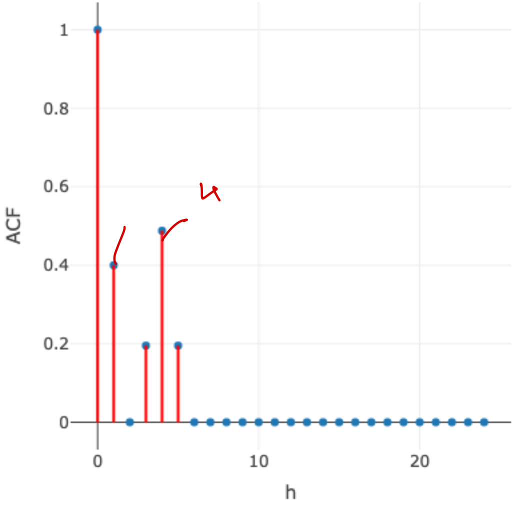
Example 4

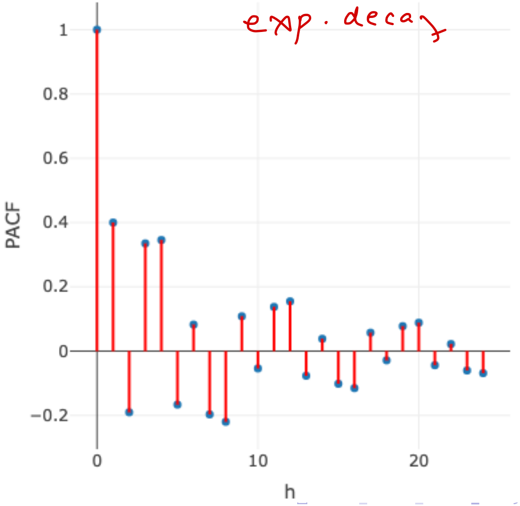

SARIMA(1,0,0)x(1,0,0)$_{12}$
ACF
h
0
12
24
36
48


SARIMA(1,0,0)x(1,0,0)$_{12}$
PACF
h
1
12
13

Guess the Order - II

SARIMA (0,0,1) × (0,0,1)$_4$





Specifying the Order - ACF/PACF

AR(1) , SMA(1)

S = 12

SMA(1)$_{12}$

- Example 5: Consider a SARIMA $(1, 0, 0) \times (0, 0, 1)_{12}$ model

AR(1)

$$X_t = 0.8X_{t-1} + W_t - 0.5W_{t-12} \implies (1 - 0.8B)X_t = (1 - 0.5B^{12})W_t$$
$$\implies \phi(B)X_t = \Theta_1(B^{12})W_t$$

- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

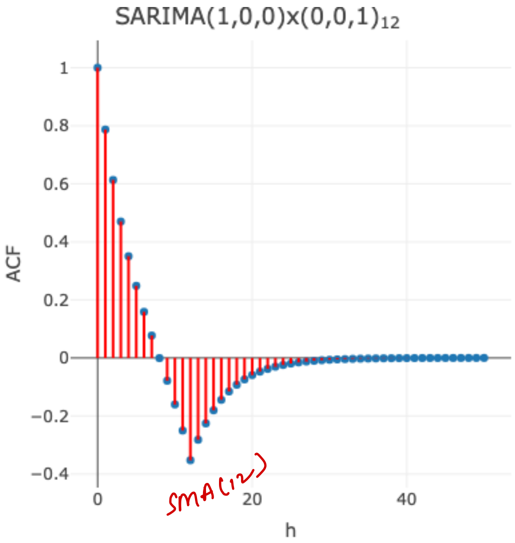
```
arsma1<- ARMAacf(ar = 0.8, ma = c(rep(0, 11), -0.5),
                        lag.max = 50) #ACF

arsma1_p <- ARMAacf(ar = 0.8, ma = c(rep(0, 11), -0.5),
                        lag.max = 50, pacf=T) #PACF
```

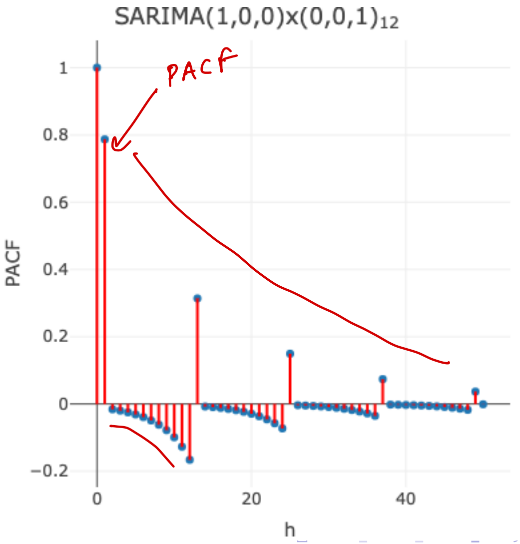
Example 5

AR(1) : exp. decay.

MA -

AR(1) :





Specifying the Order - ACF/PACF

- Example 6: Consider a SARIMA $(1, 0, 1) \times (0, 0, 1)_{12}$ model

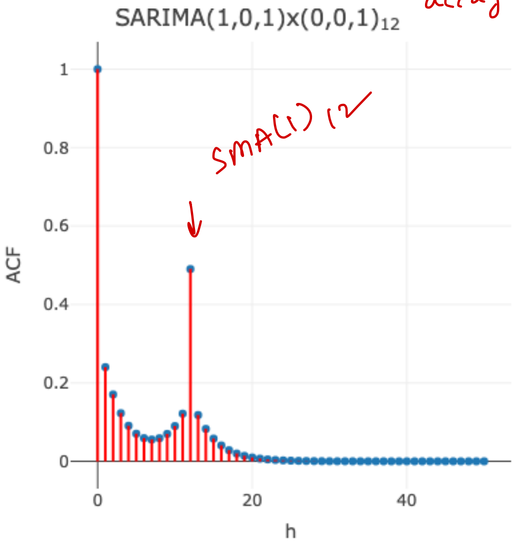
$$X_t = 0.7X_{t-1} + W_t - 0.5W_{t-1} + 0.8W_{t-12} - 0.4W_{t-13}$$
$$\implies (1 - 0.7B)X_t = (1 - 0.5B)(1 + 0.8B^{12})W_t$$
$$\implies \phi(B)X_t = \theta(B)\Theta_1(B^{12})W_t$$

- Use the **ARMAacf()** fucntion to calculate theoretical ACF and PACF for the model

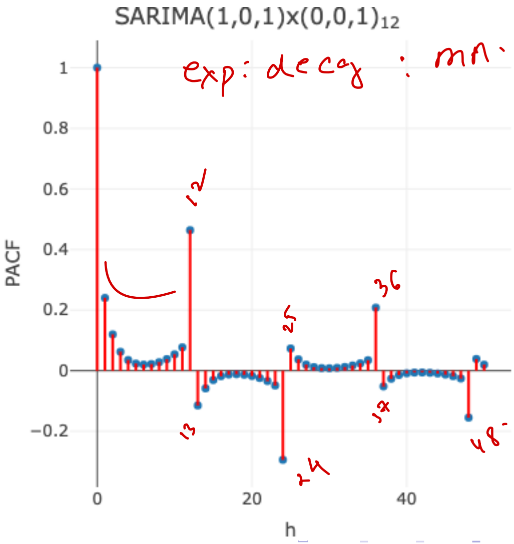
```
arsma2<- ARMAacf(ar = 0.7, ma = c(-0.5, rep(0, 10), 0.8,
                         -0.5*0.8), lag.max = 50) #ACF
arsma2_p <- ARMAacf(ar = 0.7, ma = c(-0.5, rep(0, 10), 0.8,
                      -0.5*0.8), lag.max = 50, pacf=T) #PACF
```

Example 6

exponential decay → AR(1)



exp: decay : MA



Specifying the Order

- For seasonal AR(P) models, the ACF tends to tail off (decay toward zero) at lags ks, for k = 1, 2, $\cdots$.
- For seasonal AR(P) models, the PACF tends to cut off (become zero) after lag Ps.
- For seasonal MA(Q) models, the ACF tends to cut off after lag Qs.
- For seasonal MA(Q) models, the PACF tends to tail off at lags ks.
- For seasonal ARMA(P;Q) models, both the ACF and the PACF tend to to tail off at lags ks, so the ACF and PACF are not so useful for specifying the seasonal orders of the full SARMA model.

Example - AirPassengers

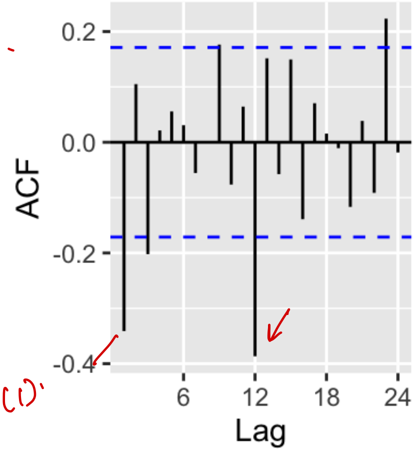
What we did earlier?

- Fit a quadratic trend plus seasonal means model to the log transformed time series (multiplicative).
- Analyze residuals - they were stationary but not white noise. Model suggestions for residuals:
  - auto.arima: MA(3)
  - armasubsets: AR(1) (we chose this!)
- We can now just using SARIMA to model the data thereby skipping the regression steps.
  - Let's try to figure out the order of the model by plotting ACF and PACF of the twice differenced log transformed series.

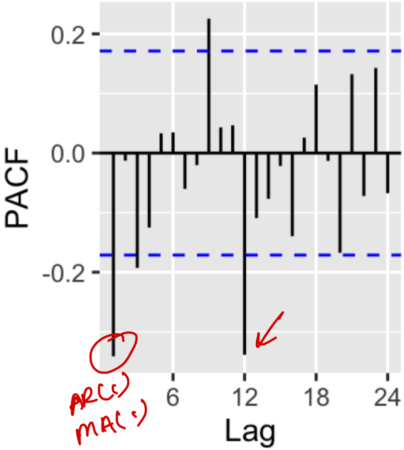
Example - AirPassengers (ACF/PACF of Differenced Log Transformed Series)

seasonal : 1 lag 1
trend : 1

stabilizing the variance.



MA(1).



AR(1)
MA(1)

$$SARIMA(p, d, q) \times (P, D, Q)_s$$

$d = 1$

$D = 1$, $s = 12$

$q = 1$, $Q = 1$ $P + Q \leq 2$

$p = 0, 1$ $P = 0, 1$

Example - AirPassengers

- R Code:

```
# Access AirPassengers data
data("AirPassengers")
auto.arima(log(AirPassengers))
```

- The model recommends a first order differencing along with a seasonal (lag 12) differencing.
- The suggested model is an MA(1) and seasonal MA(1).
- How do you interpret this?

```
Series: log(AirPassengers)
ARIMA(0,1,1)(0,1,1)[12]
```

p, d q   P, D Q   s

```
Coefficients:
          ma1      sma1
      -0.4018   -0.5569
s.e.   0.0896    0.0731

sigma^2 = 0.001371:  log
likelihood = 244.7
AIC=-483.4   AICc=-483.21
BIC=-474.77
```

$$X_t \longrightarrow y_t = \log(X_t)$$

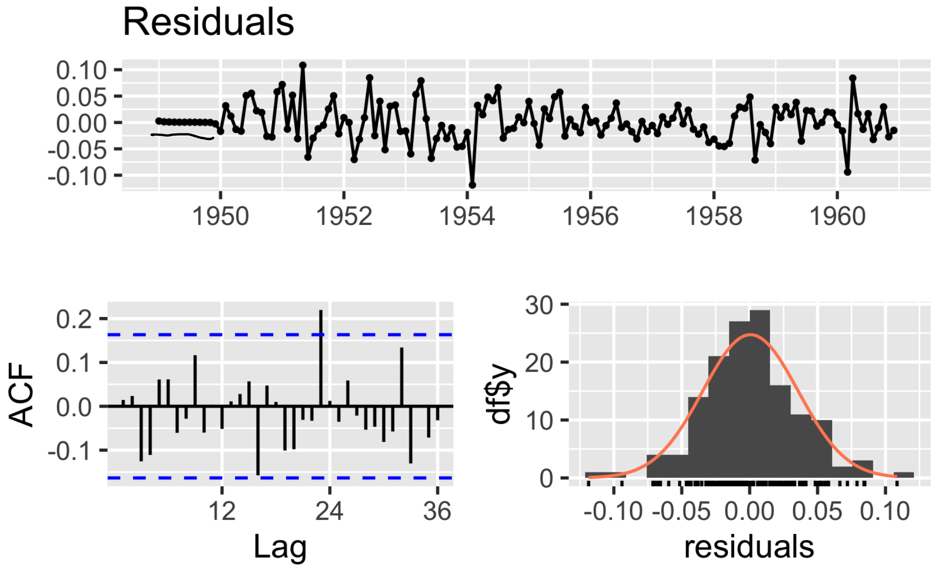
$$(1-B)(1-B^{12})\,y_t = (1-\Theta B^{12})(1-\theta B)\,w_t$$

T.D   S.D   SMA(1)   MA(1)

$$y_t - y_{t-1} - y_{t-12} + y_{t-13} = w_t - \theta w_{t-1} - \Theta w_{t-12} + \theta \Theta w_{t-13}$$

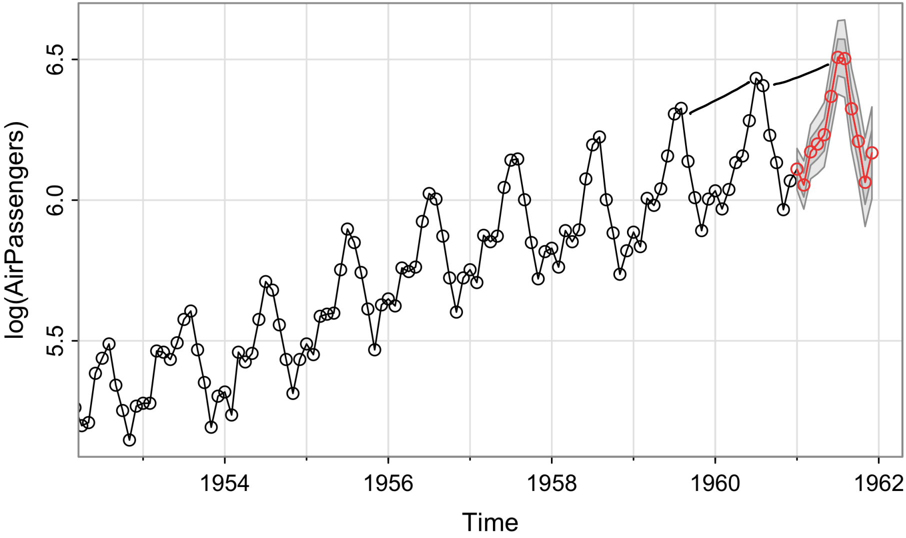
random shocks.

Example - AirPassengers



Example - AirPassengers

Sarima.for(   )



x_t, t
$\sigma^2_{x_t}$

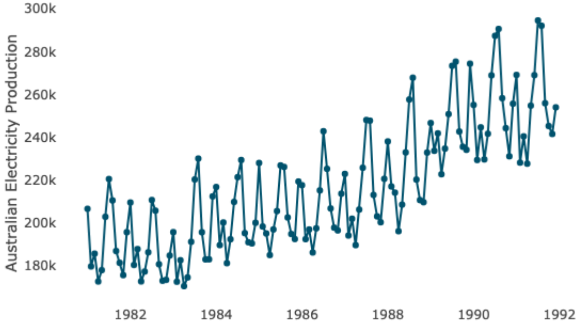
Example - AirPassengers

```
# Access the Air Passengers data
data("AirPassengers")
# Find the best model
auto.arima(log(AirPassengers))
# Fit the recommended model
fit <- arima(log(AirPassengers), order=c(0,1,1),
                                   seasonal = c(0,1,1))
# Analyze residulas
checkresiduals(fit$residuals)
# Forecast next 12 values
sarima.for(log(AirPassengers), n.ahead=12, p=0, d=1,q=1,
                              P=0, D=1, Q=1, S=12)
```

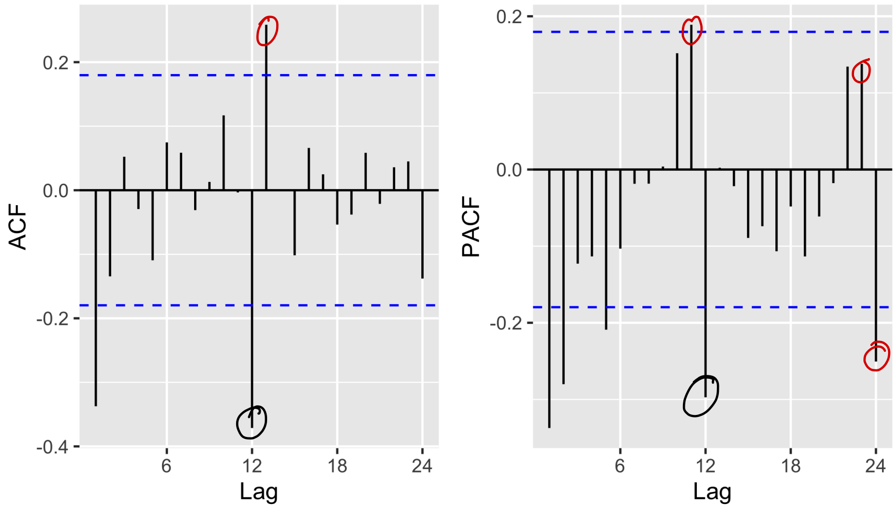
Example - Electricity Production

SARIMA (d) × (D) 12

- Data on monthly electricity production in Australia is available in the **electricity** ts object in the **TSA** package.
- I am selecting a window from January 1981 to December 1991 for the analysis.



Example - Electricity Production (Twice Differenced)



Example - Electricity Production

```
auto.arima(elec, ic="aic")

Series: elec
ARIMA(1,1,1)(0,1,1)[12]
```

d q D Q

```
Coefficients:
         ar1      ma1     sma1
      0.3197  -0.7752  -0.8289
s.e.  0.1358   0.0862   0.1077

sigma^2 = 4.6e+07:  log
likelihood = -1224.29
AIC=2456.58   AICc=2456.93
BIC=2467.7
```

```
auto.arima(elec, ic="bic")

Series: elec
ARIMA(0,1,1)(0,1,1)[12]
```

d q D Q -

```
Coefficients:
          ma1     sma1
      -0.5472  -0.8055
s.e.   0.1146   0.1013

sigma^2 = 48026429:  log
likelihood = -1226.64
AIC=2459.27   AICc=2459.48
BIC=2467.61
```

Example - Electricity Production

```
sarima.for(elec, p=1,d=1,q=1,P=0,D=1,Q=1,S=12, n.ahead = 12)
```