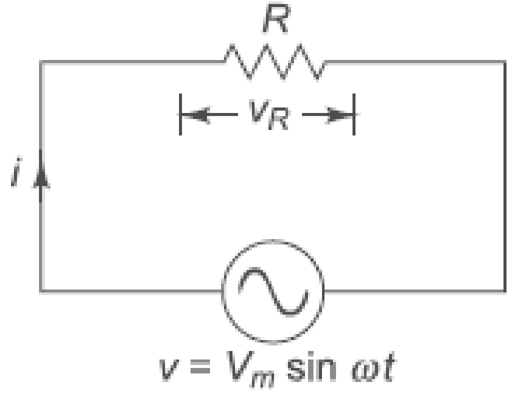
BEHAVIOUR OF A PURE RESISTOR IN AN AC CIRCUIT

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig.. Let the alternating voltage be $v = V_m \sin \omega t$.



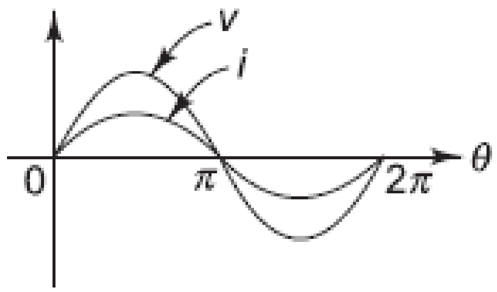
Purely resistive circuit

Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R} \right)$$

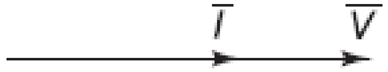
where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Waveforms The voltage and current waveforms are shown in Fig.



Waveforms

Phasor Diagram The phasor diagram is shown in Fig.. The voltage and current phasors are drawn in phase and there is no phase difference.



Phasor diagram

Impedance It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

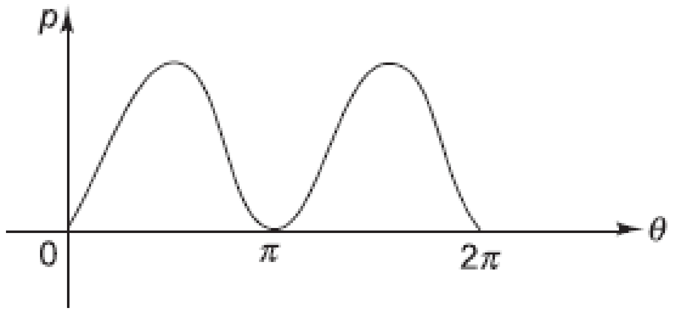
$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{Power factor} = \cos\phi = \cos(0°) = 1$$

Power Instantaneous power p is given by

$$\begin{aligned} p &= vi \\ &= V_m \sin\omega t\, I_m \sin\omega t \\ &= V_m I_m \sin^2\omega t \\ &= \frac{V_m I_m}{2}(1 - \cos 2\omega t) \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t \end{aligned}$$



Power waveform

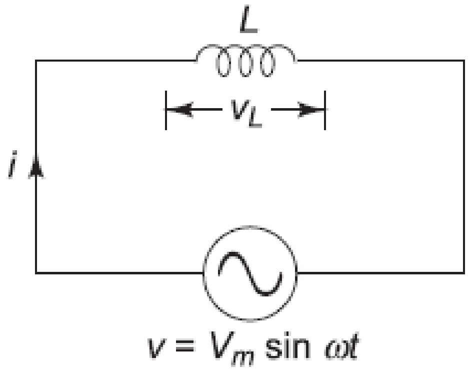
The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$. The frequency of the fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

BEHAVIOUR OF A PURE INDUCTOR IN AN AC CIRCUIT

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. Let the alternating voltage be $v = V_m \sin \omega t$.



Current The alternating current i is given by

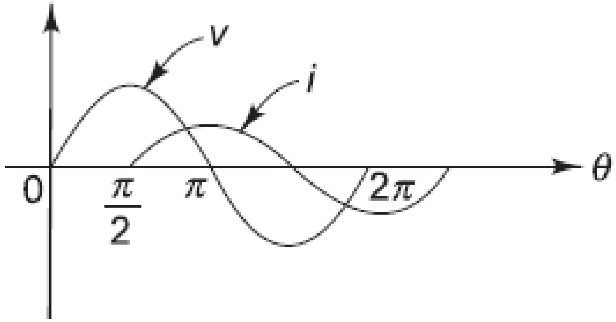
$$\begin{aligned} i &= \frac{1}{L}\int v\, dt \\ &= \frac{1}{L}\int V_m \sin \omega t\, dt \\ &= \frac{V_m}{\omega L}(-\cos \omega t) \end{aligned}$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

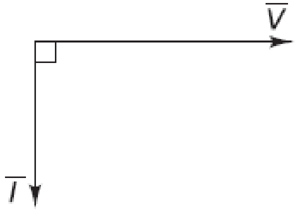
$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \cdots \left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms The voltage and current waveforms are shown in Fig.



Phasor Diagram The phasor diagram is shown in Fig.. Here, voltage V is chosen as reference phasor. Current I is drawn such that it lags behind V by 90°.



Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$

Thus, an inductor acts as a short circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous powers p is given by

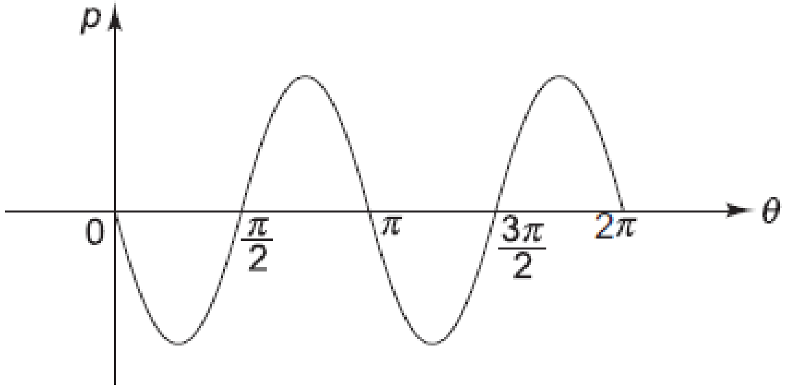
$$\begin{aligned} p &= vi \\ &= V_m \sin\omega t\, I_m \sin\left(\omega t - \frac{\pi}{2}\right) \end{aligned}$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely inductive circuit is zero.



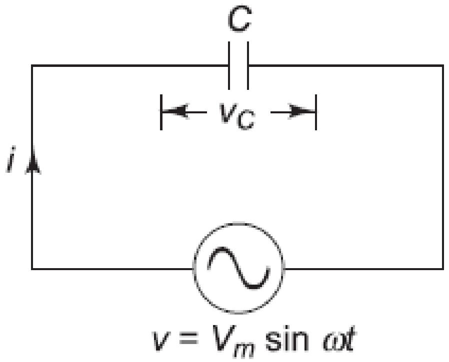
Power waveform

During first $\frac{\pi}{2}$ duration of cycle, the power is negative and power flows from the inductor to source. During the $\frac{\pi}{2}$ to π duration of cycle, the power is positive and power flows from the source to the inductor. The same cycle repeats from π to 2π. Hence, the resultant power over one cycle (upto $\theta = 2\pi$) is zero, i.e., the pure inductor consumes no power.

When power is positive, the energy is supplied from the source to build up the magnetic field in the inductor. When power is negative, the energy is returned to the source and magnetic field collapses. Hence, power circulates in the purely inductive circuit. The circulating power is called as reactive power.

BEHAVIOUR OF A PURE CAPACITOR IN AN AC CIRCUIT

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig.
Let the alternating voltage be $v = V_m \sin \omega t$.



Current The alternating current i is given by

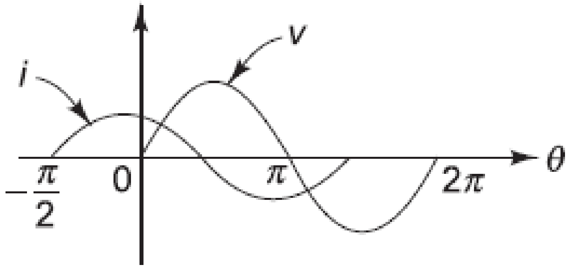
$$i = C \frac{dv}{dt}$$

$$= C \frac{d}{dt}(V_m \sin \omega t)$$

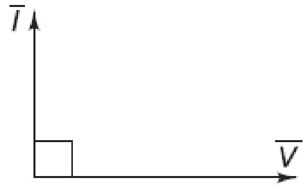
$$= \omega C V_m \cos \omega t$$
$$= \omega C V_m \sin (\omega t + 90°)$$
$$= I_m \sin (\omega t + 90°) \quad \ldots (I_m = \omega C V_m)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

Waveforms The voltage and current waveforms are shown in Fig.



Phasor Diagram The phasor diagram is shown in Fig.. Here, voltage V is chosen as reference phasor. Current I is drawn such that it leads V by 90°.



Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

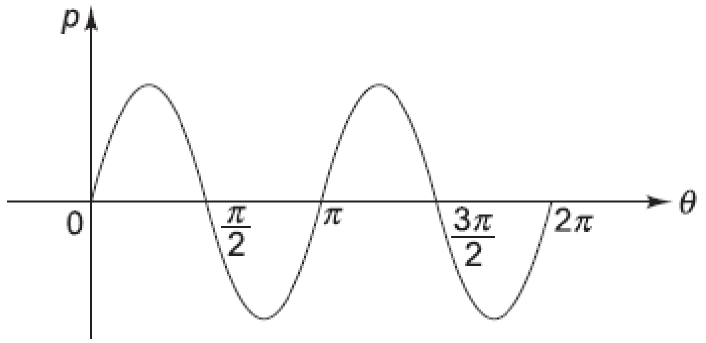
The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_C = \infty$

Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$



Power waveform

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous power p is given by

$$p = vi$$

$$= V_m \sin \omega t \, I_m \sin(\omega t + 90)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely capacitive circuit is zero.

When power is positive, i.e., voltage increases across the plates of capacitor, energy is supplied form source to build up the electrostatic field between the plates of capacitor and the capacitor is energized. When power is negative, i.e., voltage decreases, the collapsing electrostatic field returns the stored energy to the source. This circulating power is called as reactive power.

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution $R = 10\ \Omega$

$V = 230$ V

$f = 50$ Hz

(i) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23 \text{ A}$$

(ii) Power consumed

$$P = VI = 230 \times 23 = 5290 \text{ W}$$

(iii) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$\text{pf} = \cos\phi = \cos(0°) = 1$$

(iv) Voltage and current equations

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27 \text{ V}$$

$$I_m = \sqrt{2}\,I = \sqrt{2} \times 23 = 32.53 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$

$$v = V_m \sin\omega t = 325.27 \sin 314.16\,t$$

$$i = I_m \sin\omega t = 32.53 \sin 314.16\,t$$

A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current.

An inductive coil having negligible resistance and 0.1 henry inductance is connected across a 200 V, 50 Hz supply. Find (i) inductive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

The voltage and current through circuit elements are

$$v = 100 \sin (314\, t + 45°) \text{ volts}$$

$$i = 10 \sin (314\, t + 315°) \text{ amperes}$$

(i) Identify the circuit elements. (ii) Find the value of the elements. (iii) Obtain an expression for power.

Solution

$$v = 100 \sin (314\, t + 45°)$$

$$i = 10 \sin (314\, t + 315°)$$

$$= 10 \sin (314\, t + 315° - 360°)$$

$$= 10 \sin (314\, t - 45°)$$

(i) Identification of elements

From voltage and current equations, it is clear that the current i lags behind the voltage by 90°. Hence, the circuit element is an inductor.

(ii) Value of elements

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega$$

$$X_L = \omega L$$

$$10 = 314\, L$$

$$L = 31.8\ \text{mH}$$

(iii) Expression for power

$$p = -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{100 \times 10}{2} \sin (2 \times 314 t) = -500 \sin 628\, t$$

SERIES R-L CIRCUIT

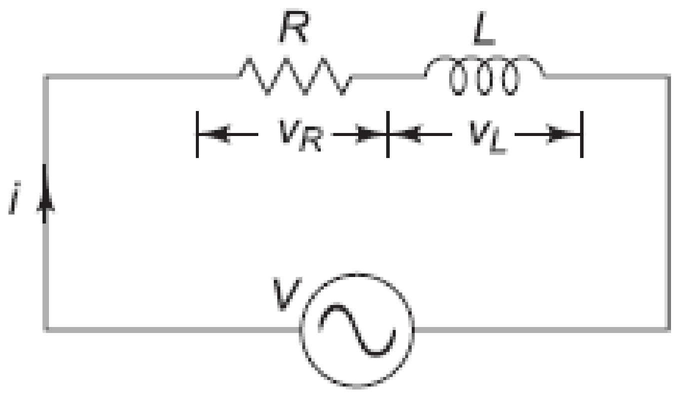
Figure shows a pure resistor R connected in series with a pure inductor L across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90°.



$v = V_m \sin \omega t$

Series R-L circuit

Phasor Diagram

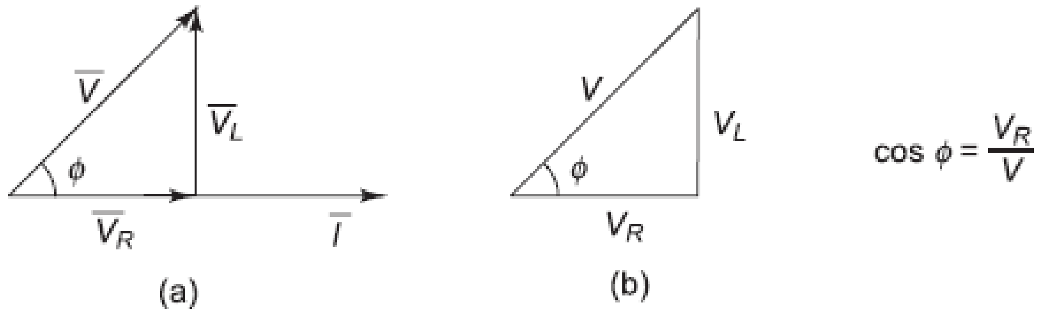
Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\overline{I}$ is taken as reference phasor.
2. Draw $\overline{V}_R$ in phase with $\overline{I}$.
3. Draw $\overline{V}_L$ such that it leads $\overline{I}$ by 90°.
4. Add $\overline{V}_R$ and $\overline{V}_L$ by triangle law of vector addition such that

$$\overline{V} = \overline{V}_R + \overline{V}_L$$

5. Mark the angle between $\overline{I}$ and $\overline{V}$ as ϕ.

The phasor diagram is shown in Fig.



(a) Phasor diagram (b) Voltage triangle

It is clear from phasor diagram that current $\overline{I}$ lags behind applied voltage $\overline{V}$ by an angle ϕ ($0° < \phi < 90°$).

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_L = R\bar{I} + jX_L\,\bar{I} = (R + jX_L)\,\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R + jX_L = \bar{Z}$$

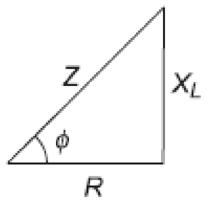
$$\bar{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the R-L circuit.

Impedance Triangle The impedance triangle is shown in Fig.



$\cos\phi = \dfrac{R}{Z}$

Impedance triangle

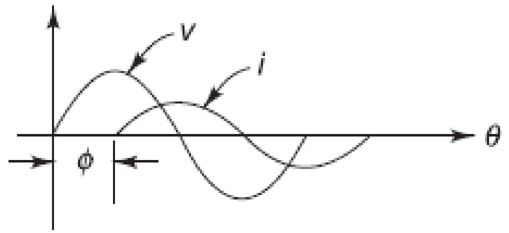
Current From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin\,\omega t$ then the current equation will be

$$i = I_m \sin\,(\omega t - \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Waveforms The voltage and current waveforms are shown in Fig.



Waveforms

Power Instantaneous power p is given by

$$\begin{aligned} p &= v\,i \\ &= V_m \sin \omega t \, I_m \sin (\omega t - \phi) \\ &= V_m I_m \sin \omega t \sin (\omega t - \phi) \\ &= V_m I_m \left[\frac{\cos \phi - \cos (2\omega t - \phi)}{2} \right] \\ &= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t - \phi) \end{aligned}$$

Thus, power consists of a constant part $\frac{V_m I_m}{2}$ cos ϕ and a fluctuating part $\frac{V_m I_m}{2}$ cos $(2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

Average power $P = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = VI \cos\phi$

Thus, power is dependent upon the in-phase component of the current. The average power is also called *active power* and is measured in watts.

We know that a pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called *reactive power*. It is a product of the voltage and reactive component of the current, i.e., $I \sin\phi$ and is measured in VAR (volt–ampere-reactive).

Reactive power $Q = VI \sin\phi$.

The product of voltage and current is known as *apparent power* (S) and is measured in volt–ampere (VA).

$$S = \sqrt{P^2 + Q^2}$$

Power Triangle In terms of circuit components,

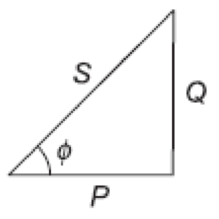
$$\cos\phi = \frac{R}{Z}$$

and $V = ZI$

$$P = VI\cos\phi = ZII\frac{R}{Z} = I^2R$$

$$Q = VI\sin\phi = ZII\frac{X_L}{Z} = I^2X_L$$

$$S = VI = ZII = I^2Z$$



$\cos\phi = \frac{P}{S}$

Power triangle

The power triangle is shown in Fig.

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle, $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an *R-L* series circuit, the power factor is lagging in nature since the current lags behind the voltage by an angle ϕ.

An alternating voltage of 80 + j60 V is applied to a circuit and the current flowing is 4 – j2 A. Find the (i) impedance, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution $\bar{V} = 80 + j60 \text{ V}$

$\bar{I} = 4 - j2 \text{ A}$

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87^\circ}{4.47\angle -26.56^\circ} = 22.37\ \angle 63.43^\circ\ \Omega$$

$$Z = 22.37\ \Omega$$

(ii) Phase angle

$$\phi = 63.43^\circ$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(63.43^\circ) = 0.447 \text{ (lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 100 \times 4.47 \times 0.447 = 199.81 \text{ W}$$

An rms voltage of 100 $\angle 0°$ V is applied to a series combination of Z_1 and Z_2 when $Z_1 = 20 \angle 30° \Omega$. The effective voltage drop across Z_1 is known to be 40 $\angle -30°$ V. Find the reactive component of Z_2.

Solution

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{Z}_1 = 20\ \angle 30°\ \Omega$$

$$\bar{V}_1 = 40\ \angle -30°\ \text{V}$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{40\angle -30^{o}}{20\angle 30^{o}} = 2\angle -60°\ \text{A}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{100\angle 0°}{2\angle -60°} = 50\ \angle 60° = 25 + j43.3\ \Omega$$

$$\bar{Z}_1 = 20\ \angle 30° = 17.32 + j10\ \Omega$$

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$

$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 25 + j43.3 - 17.32 - j10 = 7.68 + j33.3\ \Omega$$

Reactive component of $\bar{Z}_2 = 33.3\ \Omega$

A voltage $v(t) = 177 \sin(314t + 10°)$ is applied to a circuit. It causes a steady-state current to flow, which is described by $i(t) = 14.14 \sin(314t - 20°)$. Determine the power factor and average power delivered to the circuit.

Solution $v(t) = 177 \sin (314t + 10°)$

$$i(t) = 14.14 \sin (314t - 20°)$$

(i) Power factor

Current $i(t)$ lags behind voltage $v(t)$ by 30°.

$$\phi = 30°$$

$$\text{pf} = \cos \phi = \cos (30°) = 0.866 \text{ (lagging)}$$

(ii) Average power

$$P = VI \cos \phi = \frac{177}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times 0.866 = 1083.7 \text{ W}$$

When a sinusoidal voltage of 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by $i(t) = 28.3 \sin(314t - \phi)$. *Find the circuit resistance and inductance.*

Solution

$$V = 120 \text{ V}$$

$$P = 1200 \text{ W}$$

$$i(t) = 28.3 \sin(314t - \phi)$$

(i) Resistance

$$I = \frac{28.3}{\sqrt{2}} = 20.01 \text{ A}$$

$$P = VI \cos \phi$$

$$1200 = 120 \times 20.01 \times \cos \phi$$

$$\cos \phi = 0.499$$

$$\phi = 60.02°$$

$$Z = \frac{V}{I} = \frac{120}{20.01} = 6 \, \Omega$$

$$\bar{Z} = Z \angle\phi = 6 \angle 60.02° = 3 + j5.2 \, \Omega$$

$$R = 3 \, \Omega$$

(ii) Inductance

$$X_L = 5.2 \, \Omega$$

$$X_L = \omega L$$

$$5.2 = 314 \times L$$

$$L = 0.0165 \text{ H}$$

In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$ *and* $v(t) = 20\sin\left(314t + \frac{5\pi}{6}\right)$. *(i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the average power drawn by the circuit?*

Solution $$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$

$$v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$$

(i) Impedance

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{20}{5} = 4\ \Omega$$

(ii) Power factor, resistance and inductance

Current $i(t)$ lags behind voltage $v(t)$ by an angle $\phi = 150° - 120° = 30°$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

$$\bar{Z} = 4\angle 30° = 3.464 + j2\ \Omega$$

$$R = 3.464\ \Omega$$

$$X_L = 2\ \Omega$$

$$X_L = \omega L$$

$$2 = 314 \times L$$

$$L = 6.37 \text{ mH}$$

(iii) Average power

$$P = VI\cos\phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3 \text{ W}$$

Two coils are connected in series across a 200 V, 50 Hz ac supply, The power input to the circuit is 2 kW and 1.15 kVAR. If the resistance and the reactance of the first coil are 5 Ω and 8 Ω respectively, calculate the resistance and reactance of the second coil. Calculate the active power and reactive power for both the coils individually.

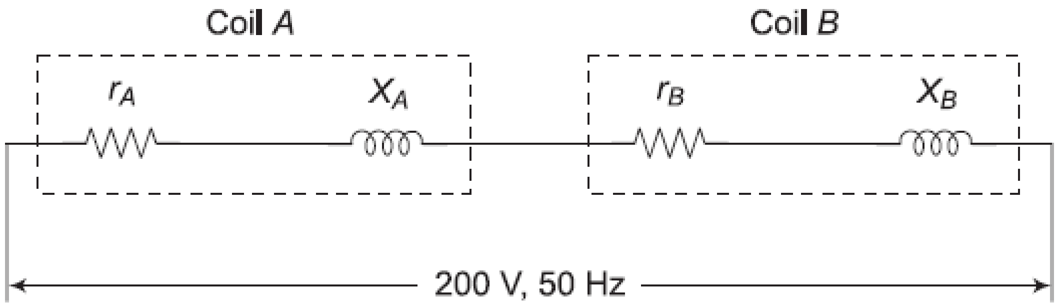
Solution

$$P_T = 2 \text{ kW}$$

$$Q_T = 1.15 \text{ kVAR}$$

$$r_A = 5\ \Omega$$

$$X_A = 8\ \Omega$$



(i) Resistance and reactance of the coil B

$$P_T = VI\cos\phi$$

$$Q_T = VI\sin\phi$$

$$\tan\phi = \frac{Q_T}{P_T} = \frac{1.15}{2} = 0.575$$

$$\phi = 29.9°$$

$$P_T = VI\cos\phi$$

$$2000 = 200 \times I \times \cos(29.9°)$$

$$I = 11.54\text{ A}$$

$$P_T = I^2(r_A + r_B)$$

$$2000 = (11.54)^2(5 + r_B)$$

$$r_B = 10.02\ \Omega$$

$$Q_T = I^2(X_A + X_B)$$

$$1.15 \times 10^3 = (11.54)^2(8 + X_B)$$

$$X_B = 0.64\ \Omega$$

(ii) Active power and reactive power for both the coils individually

$$P_A = I^2 r_A = (11.54)^2 \times 5 = 665.86\text{ W}$$

$$Q_A = I^2 X_A = (11.54)^2 \times 8 = 10.65.37\text{ VAR}$$

$$P_B = I^2 r_B = (11.54)^2 \times 10.02 = 1334.38\text{ W}$$

$$Q_B = I^2 X_B = (11.54)^2 \times 0.64 = 82.53\text{ VAR}$$

SERIES R-C CIRCUIT

Figure shows a pure resistor R connected in series with a pure capacitor C across an alternating voltage v.

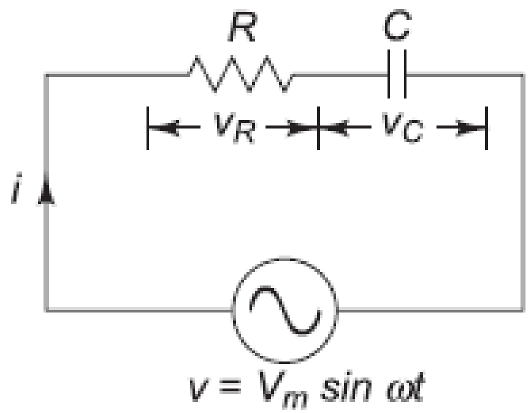
Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_C$$



Series R-C circuit

Phasor Diagram

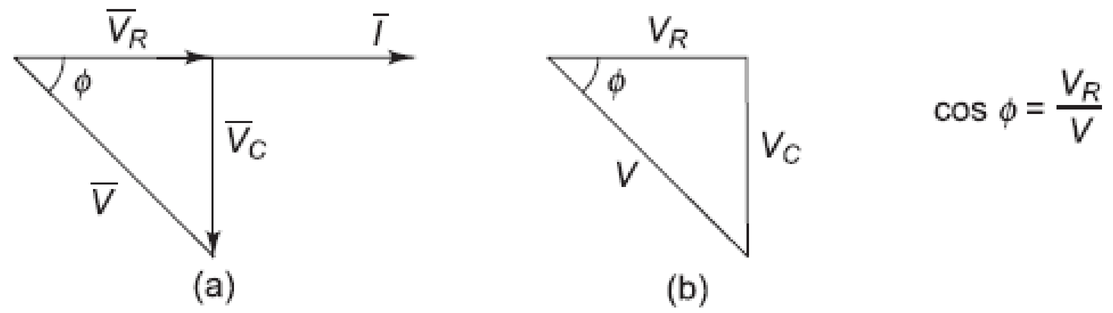
Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\overline{I}$ is taken as reference phasor.
2. Draw $\overline{V}_R$ in phase with $\overline{I}$.
3. Draw $\overline{V}_C$ such that it lags behind $\overline{I}$ by 90°.
4. Add $\overline{V}_R$ and $\overline{V}_C$ by triangle law of addition such that

$$\overline{V} = \overline{V}_R + \overline{V}_C$$

5. Mark the angle $\overline{I}$ and $\overline{V}$ as ϕ.

The phasor diagram is shown in Fig. It is clear from phasor diagram that current $\overline{I}$ leads applied voltage $\overline{V}$ by an angle ϕ $(0° < \phi < 90°)$.



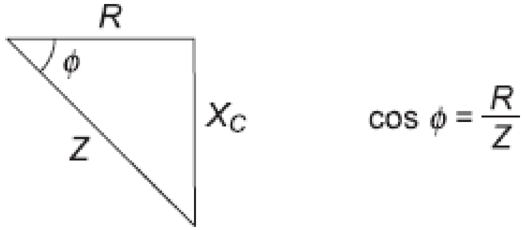
(a) Phasor diagram (b) Voltage triangle

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_C$$
$$= R\bar{I} - jX_C\bar{I}$$
$$= (R - jX_C)\bar{I}$$
$$\frac{\bar{V}}{\bar{I}} = R - jX_C = \bar{Z}$$
$$\bar{Z} = Z\angle -\phi$$
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

The quantity $\bar{Z}$ is called the *complex impedance* of the *R-C* circuit.

Impedance Triangle The impedance triangle is shown in Fig.



Impedance triangle

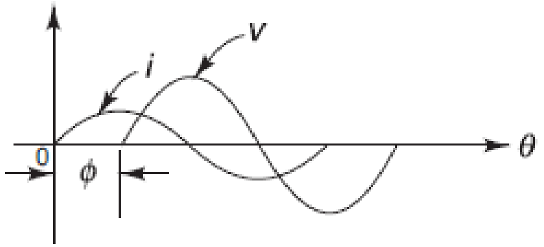
Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Waveforms The voltage and current waveforms are shown in Fig.



Waveforms

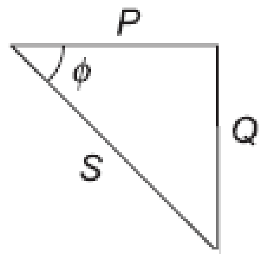
Power

Active power $P = VI \cos \phi = I^2 R$

Reactive power $Q = VI \sin \phi = I^2 X_C$

Apparent power $S = VI = I^2 Z$

Power Triangle The power triangle is shown in Fig.



$$\cos\phi = \frac{P}{S}$$

Power triangle

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an *R-C* series circuit, the power factor is leading in nature since the current leads the voltage by an angle ϕ.

The voltage applied to a circuit is e = 100 sin (ωt + 30°) and the current flowing in the circuit is i = 15 sin (ωt + 60°). Determine impedance, resistance, reactance, power factor and power.

Solution $e = 100 \sin(\omega t + 30°)$

$$i = 15 \sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$

$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}}\angle 30°}{\frac{15}{\sqrt{2}}\angle 60°} = 6.67 \angle -30° = 5.77 - j3.33 = R - j X_C$$

$$Z = 6.67\ \Omega$$

(ii) Resistance

$$R = 5.77\ \Omega$$

(iii) Reactance

$$X_C = 3.33\ \Omega$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (leading)}$$

(v) Power

$$P = EI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5\ \text{W}$$

A series circuit consumes 2000 W at 0.5 leading power factor, when connected to 230 V, 50 Hz ac supply. Calculate (i) current, (ii) kVA, and (iii) kVAR.

Solution $P = 2000 \text{ W}$

$$\text{pf} = 0.5 \text{ (leading)}$$

$$V = 230 \text{ V}$$

(i) Current

$$P = VI \cos \phi$$

$$2000 = 230 \times I \times 0.5$$

$$I = 17.39 \text{ A}$$

(ii) Apparent power

$$S = VI = \frac{P}{\cos\phi} = \frac{2000}{0.5} = 4 \text{ kVA}$$

(iii) Reactive power

$$\phi = \cos^{-1}(0.5) = 60°$$

$$Q = VI \sin \phi = 230 \times 17.39 \times \sin(60°) = 3.464 \text{ kVAR}$$

A resistor R in series with a capacitor C is connected to a 240 V, 50 Hz ac supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum stored energy in C.

Solution

$$V = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$P = 300 \text{ W}$$
$$V_R = 100 \text{ V}$$

(i) Value of C

$$P = \frac{V_R^2}{R}$$

$$300 = \frac{(100)^2}{R}$$

$$R = 33.33\ \Omega$$
$$P = I^2 R$$
$$300 = I^2 \times 33.33$$
$$I = 3 \text{ A}$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$72.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 43.77\ \mu\text{F}$$

(ii) Maximum charge

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17\ \text{V}$$

$$V_{C\max} = 218.17 \times \sqrt{2} = 308.54\ \text{V}$$

$$Q_{\max} = CV_{C\max} = 43.77 \times 10^{-6} \times 308.54 = 0.0135\ \text{C}$$

(iii) Maximum stored energy

$$E_{\max} = \frac{1}{2}\, C\,(V_{C\max})^2 = \frac{1}{2} \times 43.77 \times 10^{-6} \times (308.54)^2 = 2.08\ \text{J}$$

A capacitor of 35 μF is connected in series with a variable resistor. The circuit is connected across 50 Hz mains. Find the value of the resistor for a condition when the voltage across the capacitor is half the supply voltage.

Solution

$$C = 35\ \mu\text{F}$$

$$f = 50\ \text{Hz}$$

$$V_C = \frac{1}{2}\text{V}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 35 \times 10^{-6}} = 90.946\ \Omega$$

$$V_C = \frac{1}{2}\text{V}$$

$$X_C I = \frac{1}{2} Z I$$

$$X_C = \frac{1}{2} Z$$

$$Z = 2X_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$(2X_C)^2 = R^2 + X_C^2$$

$$R^2 = 3X_C^2 = 3 \times (90.946)^2 = 24813.35$$

$$R = 157.5\ \Omega$$

A resistor and a capacitor are connected across a 250 V supply. When the supply frequency is 50 Hz, the current drawn is 5 A. When the frequency is increased to 60 Hz, it draws 5.8 A. Find the values of R and C and power drawn in the second case.

Solution $V = 250\text{ V}$

$f_1 = 50\text{ Hz}$

$I_1 = 5\text{ A}$

$f_2 = 60\text{ Hz}$

$I_2 = 5.8\text{ A}$

(i) Values of R and C

For $f_1 = 50\text{ Hz},$

$$Z_1 = \frac{V}{I_1} = \frac{250}{5} = 50\ \Omega$$

$$Z_1 = \sqrt{R^2 + \left(\frac{1}{2\pi f_1 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{100\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 2500 \qquad (1)$$

For $f_2 = 60$ Hz,

$$Z_2 = \frac{V}{I_2} = \frac{250}{5.8} = 43.1\ \Omega$$

$$Z_2 = \sqrt{R^2 + \left(\frac{1}{2\pi f_2 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{120\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{120\pi C}\right)^2 = 1857.9\ \Omega \qquad (2)$$

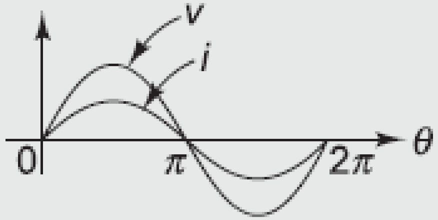
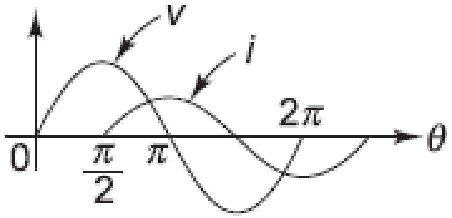
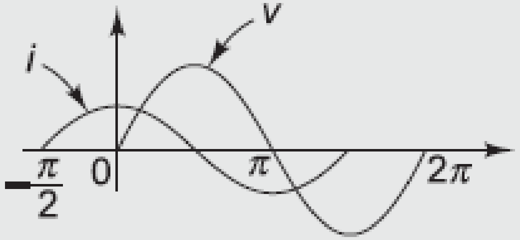
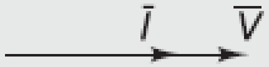
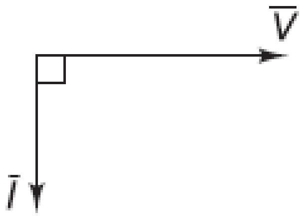
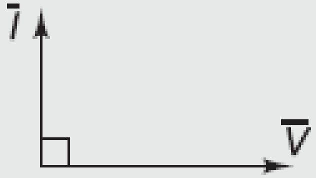
Solving Eqs (1) and (2),

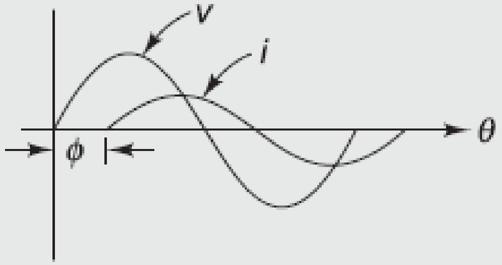
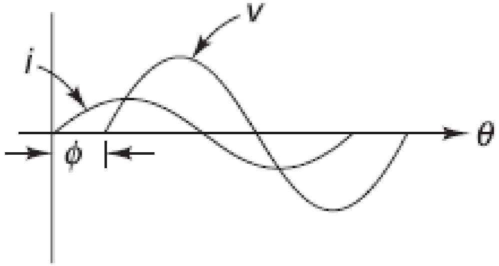
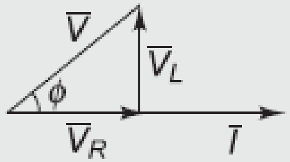
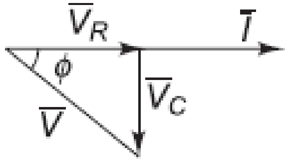
$$R = 19.96\ \Omega$$

$$C = 69.4\ \mu\text{F}$$

(ii) Power drawn in the second case

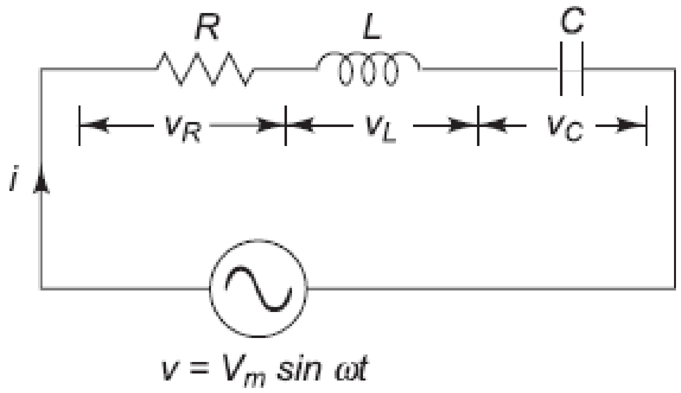
$$P_2 = I_2^2 R = (5.8)^2 \times 19.96 = 671.45\ \text{W}$$

	R	L	C
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin \omega t$	$I_m \sin (\omega t - 90°)$	$I_m \sin (\omega t + 90°)$
Waveform			
Phasor Diagram			
Impedance	R	$j\omega L$	$\dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C}$
Phase Difference	0°	90°	90°
Power Factor	1	0	0
Power	VI	0	0

	Series R-L Circuit	*Series R-C Circuit*
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin (\omega t - \phi)$	$I_m \sin (\omega t + \phi)$
Waveform		
Phasor Diagram		
Impedance	$R + jX_L, Z \angle \phi$	$R - jX_C, Z \angle -\phi$
Phase Difference	$0° < \phi < 90°$	$0° < \phi < 90°$
Power Factor	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$
Power	$P = VI \cos \phi = I^2R$ $Q = VI \sin \phi = I^2X_L$ $S = VI = I^2Z$	$P = VI \cos \phi = I^2R$ $Q = VI \sin \phi = I^2X_C$ $S = VI = I^2Z$

SERIES R-L-C CIRCUIT

Figure shows a pure resistor R, pure inductor L and pure capacitor C connected in series across an alternating voltage v.



Series R-L-C circuit

Let V and I be the rms values of the applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

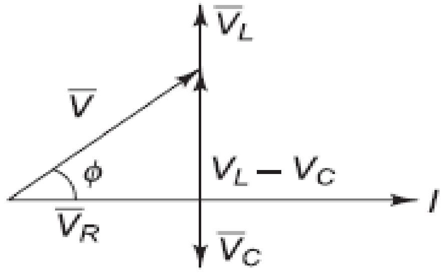
The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$, the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90° and the voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Phasor Diagram Since the same current flows through R, L and C, the current I is taken as a reference phasor.

Case (i) $X_L > X_C$

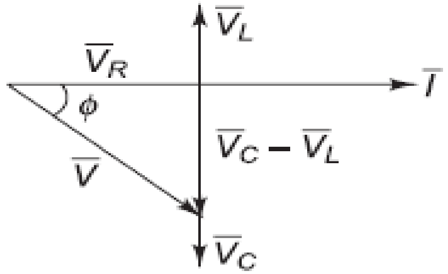
The reactance X will be inductive in nature and the circuit will behave like an R-L circuit.



Phasor diagram

Case (ii) $X_C > X_L$

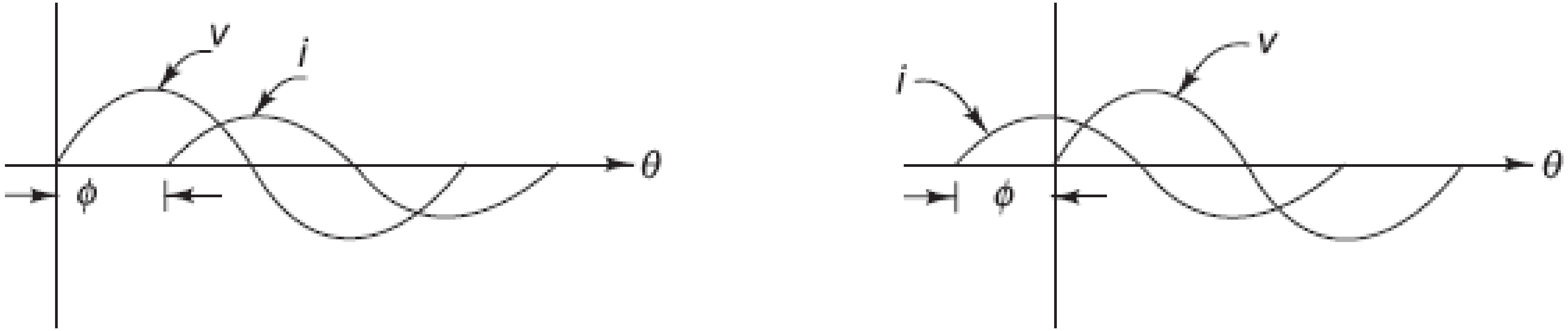
The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit.



Phasor diagram

Waveforms The voltage and current waveforms are shown in Fig.

Case (i) $X_L > X_C$ Case (ii) $X_C > X_L$



Waveforms

Power

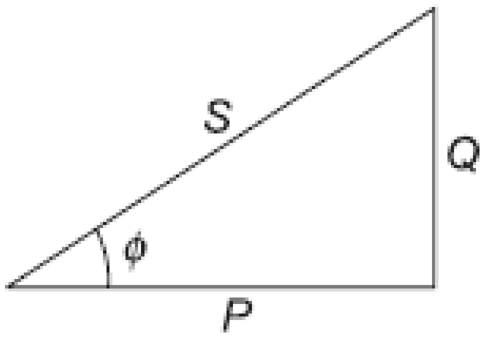
Average power $P = VI \cos \phi = I^2R$

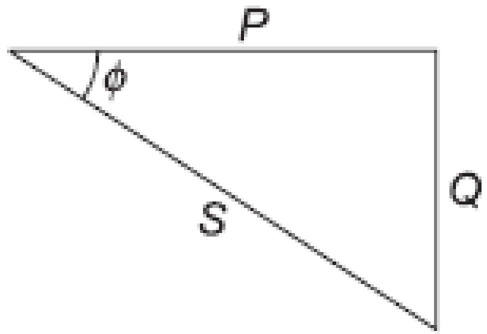
Reactive power $Q = VI \sin \phi = I^2X$

Apparent power $S = VI = I^2Z$

Power Triangles Power triangles are shown in Fig.

Case (i) $X_L > X_C$ Case (ii) $X_C > X_L$





Power triangles

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

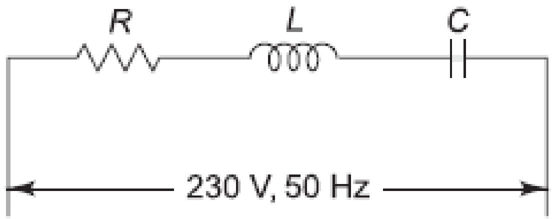
$$\text{pf} = \cos\phi$$

$$\text{pf} = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

Solution

$$R = 20\ \Omega$$
$$L = 0.05\ \text{H}$$
$$C = 50\ \mu\text{F}$$
$$V = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$



(i) Impedance

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = R + jX_L - jX_C$$
$$= 20 + j15.71 - j63.66$$
$$= 51.95 \angle -67.36°\ \Omega$$
$$Z = 51.95\ \Omega$$

(ii) Phase difference

$$\phi = 67.36°$$

(iii) Current

$$I = \frac{V}{Z} = \frac{230}{51.95} = 4.43 \text{ A}$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(67.36°) = 0.385 \text{ (leading)}$$

(v) Active power

$$P = VI\cos\phi = 230 \times 4.43 \times 0.385 = 392.28 \text{ W}$$

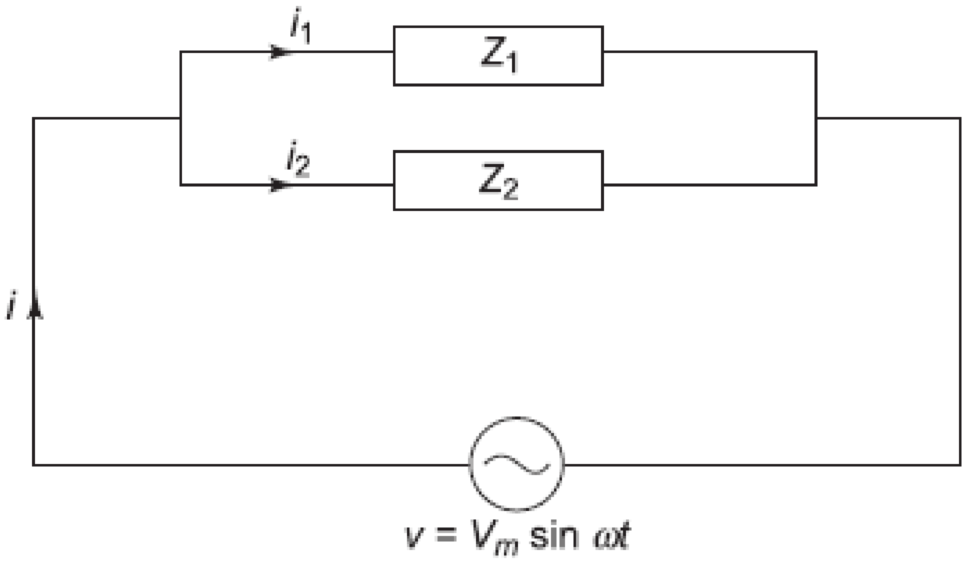
(vi) Reactive power

$$Q = VI\sin\phi = 230 \times 4.43 \times \sin(67.36°) = 940.39 \text{ VAR}$$

PARALLEL AC CIRCUITS

In parallel circuits, resistor, inductor and capacitor or any combination of these elements are connected across same supply. Hence the voltage is same across each branch of the parallel ac circuit. The total current supplied to the circuit is equal to the phasor sum of the branch currents.

For the parallel ac circuit shown in Fig.



Parallel ac circuit

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2$$

where Y represents the admittance of the circuit and is defined as the reciprocal of impedance. The real part of admittance is called conductance (G) and the imaginary part is called susceptance (B), and these are measured in mhos (℧) or siemens (S).

If $\quad \overline{Z}_1 = R + jX_L,$ and $\overline{Z}_2 = -jX_C$

then,
$$\frac{1}{\overline{Z}} = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

$$= G + jB$$

where,
$$G = \frac{R}{R^2 + X_L^2}$$

$$B = \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}$$

The current in the parallel ac circuit can be found as the phasor sum of the branch currents,

i.e., $\quad \overline{I} = \overline{I}_1 + \overline{I}_2$

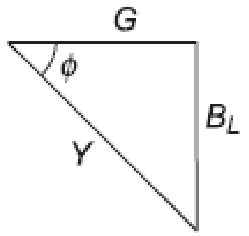
Note: 1. For a series R-L circuit,

$$\overline{Z} = R + jX_L$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$= \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2} = G - jB_L$$

where $\quad G = \dfrac{R}{R^2 + X_L^2} \quad$ and $\quad B_L = \dfrac{X_L}{R^2 + X_L^2}$



Admittance triangle

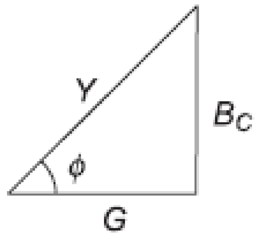
2. For a series R-C circuit,

$$\overline{Z} = R - jX_C$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2} = \frac{R}{R^2 + X_C^2} + j\frac{X_C}{R^2 + X_C^2}$$

$$= G + jB_C$$

where $\quad G = \dfrac{R}{R^2 + X_C^2} \quad$ and $\quad B_C = \dfrac{X_C}{R^2 + X_C^2}$



Admittance triangle

A resistance of 10 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 200 V, 50 Hz supply. Find the total current and power factor.

Solution

$$V = 200\text{ V}$$

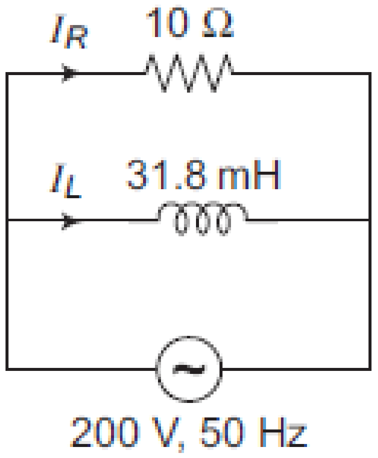
$$R = 10\ \Omega$$

$$L = 31.8\text{ mH}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} \approx 10\ \Omega$$

$$I_R = \frac{V}{R} = \frac{200}{10} = 20\text{ A}$$

$$\bar{I}_L = \frac{V}{jX_L} = \frac{200}{j10} = \frac{200}{10\angle 90^\circ} = 20\angle -90^\circ\text{ A}$$



(i) Total current

$$\bar{I} = \bar{I}_R + \bar{I}_L = 20\angle 0^\circ + 20\angle -90^\circ\text{ A} = 28.28\angle -45^\circ\text{A}$$

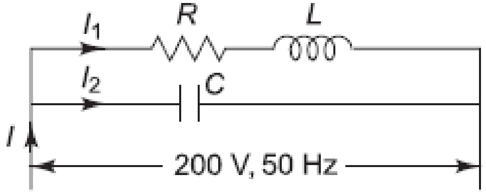
(ii) Power factor

$$\text{pf} = \cos\phi = \cos(45^\circ) = 0.707\text{ (lagging)}$$

A coil having a resistance of 50 Ω and an inductance of 0.02 H is connected in parallel with a capacitor of 25 μF across a single-phase 200 V, 50 Hz supply. Calculate the current in coil and capacitance. Calculate also the total current drawn, total pf and total power consumed by the circuit.

Solution

$$R = 50\ \Omega$$
$$L = 0.02\ \text{H}$$
$$C = 25\ \mu\text{F}$$
$$V = 200\ \text{V}$$
$$f = 50\ \text{Hz}$$



(i) Current in coil and capacitance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\ \Omega$$

$$\bar{Z}_1 = R + jX_L = 50 + j\,6.28 = 50.39\ \angle 7.16°\ \Omega$$

$$\bar{Z}_2 = -jX_C = -j127.32 = 127.32\ \angle{-90°}\ \Omega$$

$$\bar{V} = 200\ \angle 0° = 200\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{50.39\angle 7.16°} = 3.97\ \angle{-7.16°}\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{127.32\angle -90°} = 1.57\ \angle 90°\ \text{A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 3.97\ \angle{-7.16°} + 1.57\ \angle 90° = 4.08\ \angle 15.27°\ \text{A}$$

(iii) Total pf

$$\text{pf} = \cos\phi = \cos(15.27°) = 0.965\ \text{(lagging)}$$

(iv) Total power consumed

$$P = VI\cos\phi = 200 \times 4.08 \times 0.965 = 787.44\ \text{W}$$

Two impedances $\bar{Z}_1 = 30\angle 45°\ \Omega$ and $\bar{Z}_2 = 45\angle 30°\ \Omega$ are connected in parallel across a single-phase 230 V, 50 Hz supply. Calculate (i) current drawn by each branch, (ii) total current, and (iii) overall power factor.

Also draw the phasor diagram indicating the current drawn by each branch and the total current, taking the supply voltage as reference.

Solution

$$\bar{Z}_1 = 30\angle 45°\ \Omega$$

$$\bar{Z}_2 = 45\angle 30°\ \Omega$$

$$V = 230\ \text{V}$$

(i) Current drawn by each branch

Let $$\bar{V} = 230\,\angle 0^\circ = 230\text{ V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{230}{30\angle 45^\circ} = 7.67\,\angle{-45^\circ}\text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{230}{45\angle 30^\circ} = 5.11\,\angle{-30^\circ}\text{ A}$$

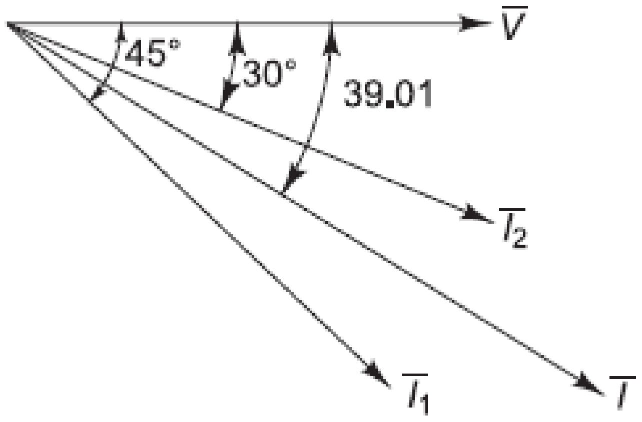
(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 7.67\,\angle{-45^\circ} + 5.11\,\angle{-30^\circ} = 12.67\,\angle{-39.01^\circ}\text{ A}$$

(iii) Overall power factor

$$\text{pf} = \cos\phi = \cos(39.01^\circ) = 0.777 \text{ (lagging)}$$

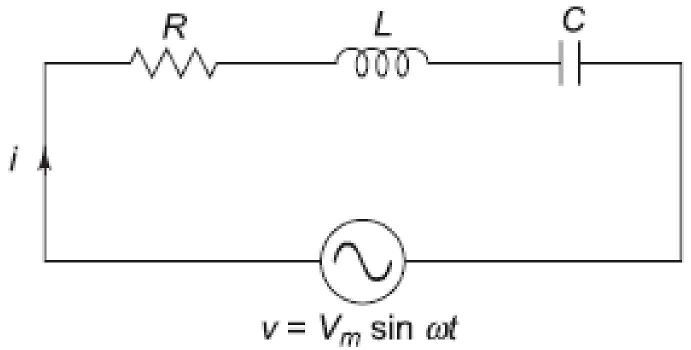
(iv) Phasor diagram



A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series R-L-C circuit as shown in Fig. The impedance of the circuit is

$$\begin{aligned}\bar{Z} &= R + jX_L - jX_C \\ &= R + j\omega L - j\frac{1}{\omega C} \\ &= R + j\left(\omega L - \frac{1}{\omega C}\right)\end{aligned}$$



Series circuit

At resonance, Z must be resistive. Therefore, the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Power Factor

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

At resonance $Z = R$

$$\text{Power factor} = \frac{R}{R} = 1$$

Current Since impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such, an *R-L-C* circuit under resonance is called an *acceptor circuit.*

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

Voltage At resonance,

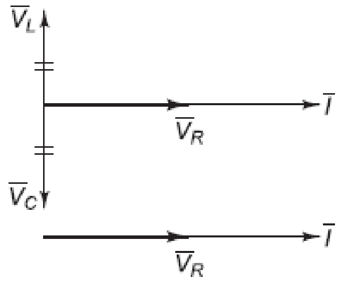
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

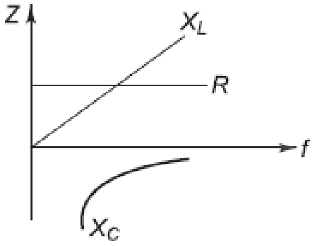
Thus, potential difference across inductor equal to potential difference across capacitor being equal and opposite cancel each other. Also, since I_0 is maximum, V_{L_0} and V_{C_0} will also be maximum. Thus, voltage magnification takes place during resonance. Hence, it is also referred to as voltage magnification circuit.

Phasor Diagram The phasor diagram is shown in Fig. .



Phasor diagram

Behaviour of R, L and C with Change in Frequency Resistance remains constant with the change in frequencies. Inductive reactance X_L is directly proportional to frequency f. It can be drawn as a straight line passing through the origin. Capacitive reactance X_C is inversely proportional to the frequency f. It can be drawn as a rectangular hyperbola in the fourth quadrant.



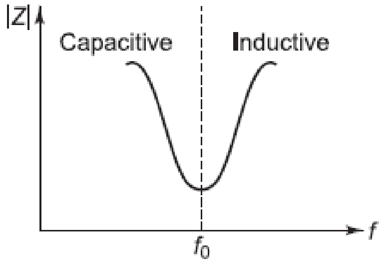
Behaviour of R, L and C with change in frequency

Total impedance $\overline{Z} = R + j\,(X_L - X_C)$

(a) When $f < f_0$, impedance is capacitive and decreases up to f_0. The power factor is leading in nature.

(b) At $f = f_0$, impedance is resistive. The power factor is unity.

(c) When $f > f_0$, impedance is inductive and goes on increasing beyond f_0. The power factor is lagging in nature.



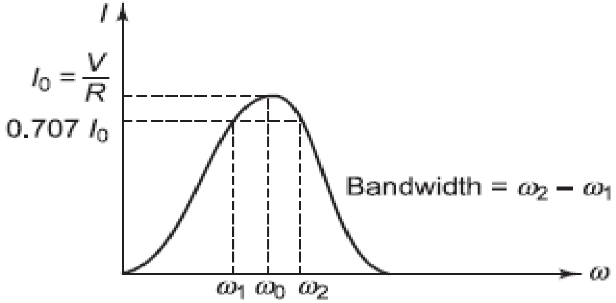
Impedance

Bandwidth For the series R-L-C circuit, bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance. From the shape of the resonance curve, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance. For this reason, these frequencies are referred as those corresponding to the half-power points. The magnitude of the current at each half-power point is the same.

Hence,
$$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point. It follows then that

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$



Resonance curve

Accordingly, the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance. In Fig. the bandwidth is $\omega_2 - \omega_1$.

Expression for Bandwidth Generally, at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad .1)$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But
$$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \quad .2)$$

From Eqs 1 and 2

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both the sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by, $$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm\frac{R}{2L} + \omega_0$$ (considering only positive sign of ω_0)

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

and $$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

and $$f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

Quality Factor It is a measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R}$$

$$= \frac{X_{L_0}}{R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right) L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

A series R-L-C circuit has the following parameter values: R = 10 Ω, L = 0.01 H, C = 100 μF. Compute the resonant frequency, bandwidth, and lower and upper frequencies of the bandwidth.

Solution $R = 10\ \Omega$

$L = 0.01\ \text{H}$

$C = 100\ \mu\text{F}$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times 100\times 10^{-6}}} = 159.15\,\text{Hz}$$

(ii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times 0.01} = 159.15\,\text{Hz}$$

(iii) Lower frequency of bandwidth

$$f_1 = f_0 - \frac{BW}{2} = 159.15 - \frac{159.15}{2} = 79.58\ \text{Hz}$$

(iv) Upper frequency of bandwidth

$$f_2 = f_0 + \frac{BW}{2} = 159.15 + \frac{159.15}{2} = 238.73\ \text{Hz}$$

A series RLC circuit has the following parameter values: R = 10 Ω, L = 0.014 H, C = 100 μF. Compute the resonant frequency, quality factor, bandwidth, lower cut-off frequency and upper cut-off frequency.

Solution

$$R = 10\ \Omega$$
$$L = 0.014\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.014\times 100\times 10^{-6}}} = 134.51\ \text{kHz}$$

(ii) Qualify factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100\times 10^{-6}}} = 1.18$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times 0.014} = 113.68\ \text{Hz}$$

(iv) Lower cut-off frequency (f_1)

$$f_1 = f_0 - \frac{BW}{2} = 134.51 - \frac{113.68}{2} = 77.67\ \text{Hz}$$

(v) Upper cut-off frequency (f_2)

$$f_2 = f_0 + \frac{BW}{2} = 134.51 + \frac{113.68}{2} = 191.35\ \text{Hz}$$

An R-L-C series circuit with a resistance of 10 Ω, inductance of 0.2 H and a capacitance of 40 μF is supplied with a 100 V supply at variable frequency. Find the following w.r.t. the series resonant circuit:

(i) *frequency at which resonance takes place*
(ii) *current*
(iii) *power*
(iv) *power factor*
(v) *voltage across R-L-C at that time*
(vi) *quality factor*
(vii) *half-power points*
(viii) *resonance and phasor diagrams*

Solution

$$R = 10\ \Omega$$
$$L = 0.2\ \text{H}$$
$$C = 40\ \mu\text{F}$$
$$V = 100\ \text{V}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = 56.3\ \text{Hz}$$

(ii) Current

$$I_0 = \frac{V}{R} = \frac{100}{10} = 10\ \text{A}$$

(iii) Power

$$P_0 = I_0^2\, R = (10)^2 \times 10 = 1000\ \text{W}$$

(iv) Power factor

$$\text{pf} = 1$$

(v) Voltage across R, L, C

$$V_{R_0} = R\,I_0 = 10 \times 10 = 100 \text{ V}$$

$$V_{L_0} = X_{L_0}\,I_0 = 2\pi f_0 L I_0 = 2\pi \times 56.3 \times 0.2 \times 10 = 707.5 \text{ V}$$

$$V_{C_0} = X_{C_0}\,I_0 = \frac{1}{2\pi f_0 C} I_0 = \frac{1}{2\pi \times 56.3 \times 40 \times 10^{-6}} \times 10 = 707.5 \text{ V}$$

(vi) Quality factor

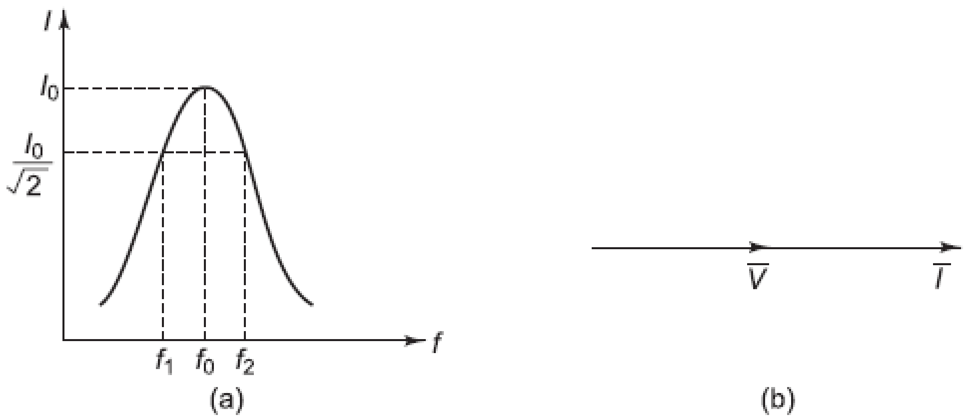
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.2}{40 \times 10^{-6}}} = 7.07$$

(vii) Half-power points

$$f_1 = f_0 - \frac{R}{4\pi L} = 56.3 - \frac{10}{4\pi \times 0.2} = 52.32 \text{ Hz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 56.3 + \frac{10}{4\pi \times 0.2} = 60.3 \text{ Hz}$$

(viii) Resonance and phasor diagram



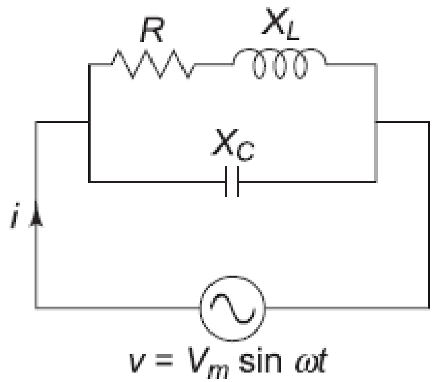
Consider a parallel circuit consisting of a coil and a capacitor as shown in Fig. impedances of two branches are

$$\bar{Z}_1 = R + jX_L$$

$$\bar{Z}_2 = -jX_C$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$



Parallel circuit

Admittance of the circuit $\bar{Y} = \bar{Y}_1 + \bar{Y}_2$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore, the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L then

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Dynamic Impedance of a Parallel Circuit At resonance, the circuit is purely resistive. The real part of admittance is $\frac{R}{R^2 + X_L^2}$. Hence, the dynamic impedance at resonance is given by

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,

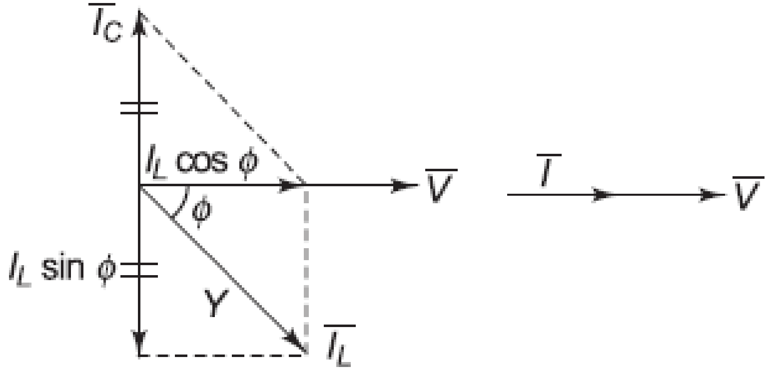
$$R^2 + X_L^2 = X_L X_C = \frac{L}{C}$$

$$Z_D = \frac{L}{CR}$$

Current Since impedance is maximum at resonance, the current is minimum at resonance.

$$I_0 = \frac{V}{Z_D} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

Phasor Diagram At resonance, power factor of the circuit is unity and the total current drawn by the circuit is in phase with the voltage. This will happen only when the current I_C is equal to the reactive component of the current in the inductive branch, i.e., $I_C = I_L \sin\phi$



Phasor diagram

Hence, at resonance

$$I_C = I_L \sin\phi$$

and

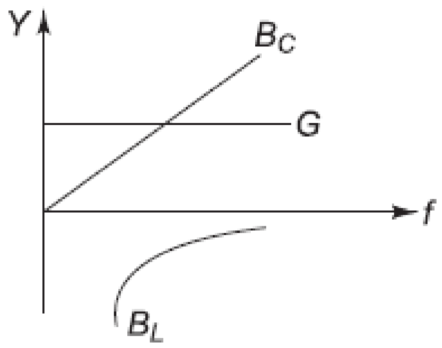
$$I = I_L \cos\phi$$

Behaviour of Conductance G, Inductive Susceptance B_L and Capacitive Susceptance with Change in Frequency Conductance remains constant with the change in frequencies.

Inductive susceptance B_L is

$$B_L = \frac{1}{jX_L} = -j\frac{1}{X_L} = -j\frac{1}{2\pi fL}$$

It is inversely proportional to the frequency. Thus, it decreases with the increase in the frequency. Hence, it can be drawn as a rectangular hyperbola in the fourth quadrant.



Behaviour of G, B_L and B_C with change in frequency

Capacitive susceptance B_C is

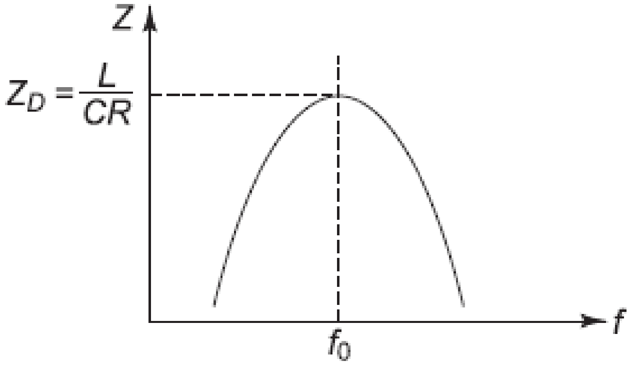
$$B_C = \frac{1}{-jX_C} = j\frac{1}{X_C} = j2\pi fC$$

It is directly proportional to the frequency. It can be drawn as a straight line passing through the origin.

(a) When $f < f_0$, inductive susceptance predominates. Hence, the current lags behind the voltage and the power factor is lagging in nature.

(b) When $f = f_0$, net susceptance is zero. Hence, the admittance is minimum and impedance is maximum. At f_0, the current is in phase with the voltage and the power factor is unity.

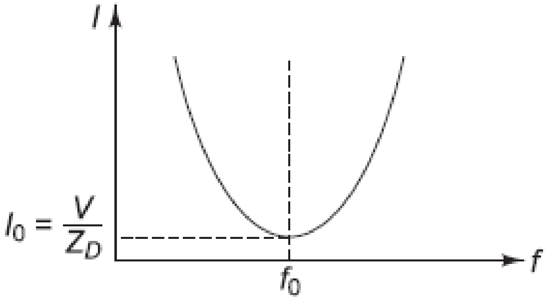
(c) When $f > f_0$, capacitive susceptance predominates. Hence, the current leads the voltage and power factor is leading in nature.



Impedance

Bandwidth The bandwidth of a parallel resonant circuit is defined in the same way as that for a series resonant circuit.

Quality Factor It is a measure of current magnification in a parallel resonant circuit.



Resonance curve

$$Q_0 = \frac{\text{Current through inductor or capacitor}}{\text{Current at resonance}} = \frac{I_{C_0}}{I_0}$$

Substituting values of I_{C_0} and I_0,

$$Q_0 = \frac{\dfrac{V}{X_{C_0}}}{\dfrac{VCR}{L}} = \frac{\dfrac{1}{X_{C_0}}}{\dfrac{CR}{L}} = \frac{\omega_0 C}{\dfrac{CR}{L}} = \frac{\omega_0 L}{R}$$

Neglecting the resistance R, the resonant frequency ω_0 is given by

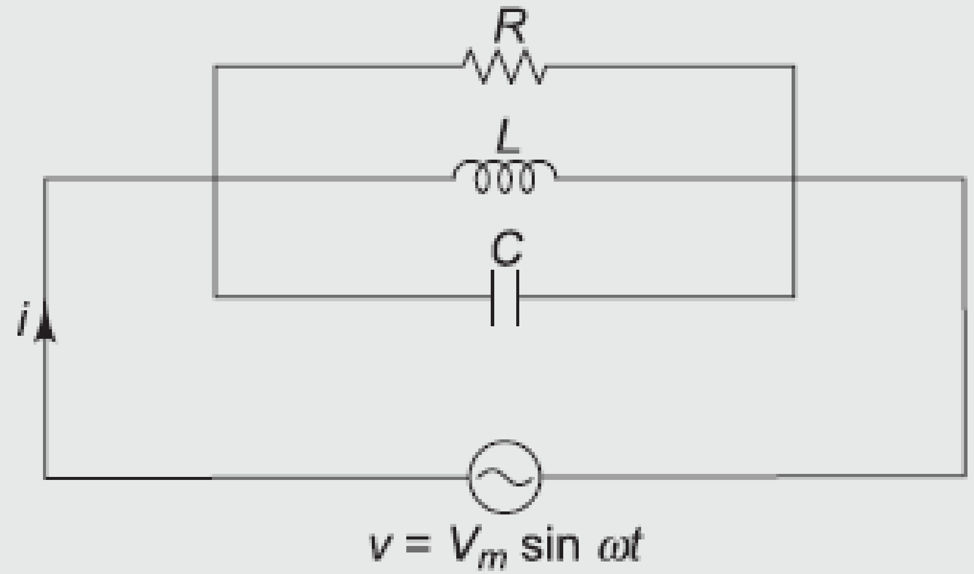
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\dfrac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

COMPARISON OF SERIES AND PARALLEL RESONANT CIRCUITS

Parameter	*Series Circuit*	*Parallel Circuit*
Current at resonance	$I = \dfrac{V}{R}$ and is maximum	$I = \dfrac{VCR}{L}$ and is minimum
Impedance at resonance	$Z = R$ and is minimum	$Z = \dfrac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$	$f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$
Q-factor	$Q = \dfrac{2\pi f_0 L}{R}$	$Q = \dfrac{2\pi f_0 L}{R}$
It magnifies	Voltage across L and C	Current through L and C

Derive the expression for resonant frequency for the parallel circuit shown in Fig. calculate the impedance and current at resonance.



Solution $\bar{Z}_1 = R$

$$\bar{Z}_2 = jX_L$$

$$\bar{Z}_3 = -jX_C$$

(i) Resonant frequency of the circuit

For parallel circuit

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$\bar{Y} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

(ii) Impedance at resonance

At resonance, the circuit is purely resistive. Hence, the imaginary part of $\overline{Y}$ is zero.

$$Y_D = \frac{1}{R}$$

$$Z_D = R$$

(iii) Current at resonance

$$I_0 = \frac{V}{Z_D} = \frac{V}{R}$$

A coil having an inductance of L henries and a resistance of 12 Ω is connected in parallel with a variable capacitor. At $\omega = 2.3 \times 10^6$ rad/s, resonance is achieved and at this instant, capacitance $C = 0.021\ \mu F$. Find the inductance of the coil.

Solution

$$R = 12\ \Omega$$

$$\omega_0 = 2.3 \times 10^6 \text{ rad/s}$$

$$C = 0.021\ \mu\text{F}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2.3 \times 10^6 = \sqrt{\frac{1}{L \times 0.021 \times 10^{-6}} - \frac{(12)^2}{L^2}}$$

$$L = 89.7\ \mu\text{H}$$

A coil of 20 Ω resistance has an inductance of 0.2 H and is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which this circuit will behave as a non-inductive resistance. Find also the value of dynamic resistance.

Solution $R = 20\ \Omega$

$L = 0.2\ \text{H}$

$C = 100\ \mu\text{F}$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.2\times 100\times 10^{-6}} - \left(\frac{20}{0.2}\right)^2} = 31.83\ \text{Hz}$$

(ii) Dynamic resistance

$$Z_D = \frac{L}{CR} = \frac{0.2}{100\times 10^{-6}\times 20} = 100\ \Omega$$

An inductive coil of 10 Ω resistance and 0.1 H inductance is connected in parallel with a 150 μF capacitor to a variable frequency, and 200 V supply. Find the resonance frequency at which the total current taken from the supply is in phase with the supply voltage. Also find value of this current. Draw the phasor diagram.

Solution

$$R = 10\ \Omega$$

$$L = 0.1\ \text{H}$$

$$C = 150\ \mu\text{F}$$

$$V = 200\ \text{V}$$

(i) Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 150 \times 10^{-6}} - \left(\frac{10}{0.1}\right)^2}$$

$$= 37.89\ \text{Hz}$$

(ii) Value of current

$$Z_D = \frac{L}{CR} = \frac{0.1}{150 \times 10^{-6} \times 10} = 66.67\ \text{A}$$

$$I = \frac{V}{Z_D} = \frac{200}{66.67} = 3\text{A}$$

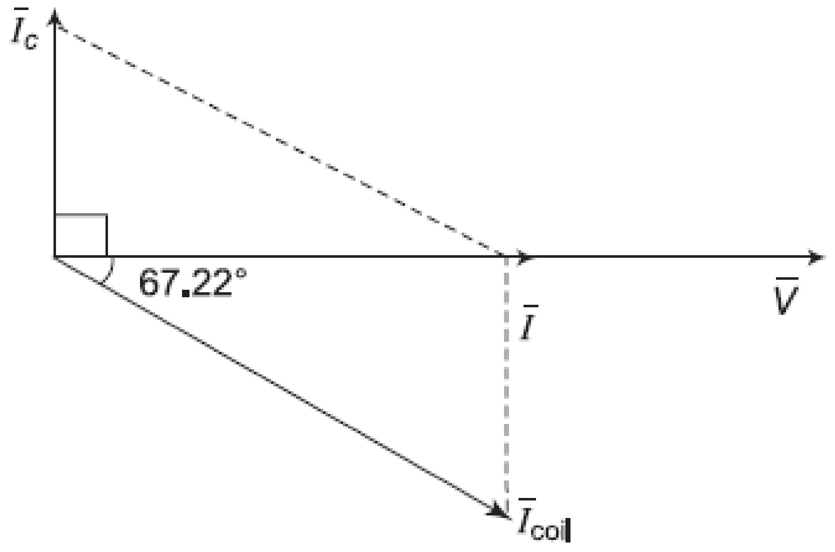
(iii) Phasor diagram

$$\bar{Z}_{\text{coil}} = 10 + j2\pi \times 37.89 \times 0.1 = 25.82\ \angle 67.22°\ \Omega$$

$$\bar{Z}_C = -j\frac{1}{2\pi \times 37.89 \times 150 \times 10^{-6}} = -j28 = 28\angle -90°\ \Omega$$

$$\bar{I}_{\text{coil}} = \frac{200}{25.82\angle 67.22°} = 7.75\angle -67.22°\ \Omega$$

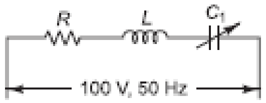
$$\bar{I}_C = \frac{200}{28\angle -90°} = 7.14\angle 90°\ \Omega$$



A series circuit consisting of a 12 Ω resistor, 0.3 H inductor and a variable capacitor is connected across a 100 V, 50 Hz ac supply. The capacitance value is adjusted to obtain maximum current. Find, the capacitance value and the power drawn by the circuit under the condition. Now, the supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of the capacitance C_1 to be connected across the above series circuit so that current drawn from the supply is minimum.

Solution

$$R = 12\ \Omega$$
$$L = 0.3\ \text{H}$$
$$V = 100\ \text{V}$$
$$f = 50\ \text{Hz}$$



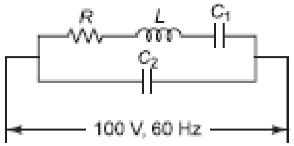
(i) Value of capacitance C_1

The resonance occurs at $f = 50$ Hz

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

$$50 = \frac{1}{2\pi\sqrt{0.3 \times C_1}}$$

$$C_1 = 33.77\ \mu\text{F}$$



(ii) Value of capacitance C_2

The resonance occurs at 60 Hz.

$$X_L = 2\pi f_0 L = 2\pi \times 60 \times 0.3 = 113.1\ \Omega$$

$$X_{C_1} = \frac{1}{2\pi f_0 C_1} = \frac{1}{2\pi \times 60 \times 33.77 \times 10^{-6}} = 78.55\ \Omega$$

$$\bar{Z}_1 = 12 + j\,113.1 - j\,78.55 = 36.57\ \angle 70.85\ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{36.57\angle 70.85°} = 0.027\ \angle 70.85°\ \mho = 8.86 \times 10^{-3} - j\,0.0255\ \mho$$

$$\bar{Y}_{req} = \bar{Y}_1 + \bar{Y}_2 = 8.86 \times 10^{-3} - j\,0.0255 + \bar{Y}_2$$

At resonance, the imaginary part of $\bar{Y}_{req}$ becomes zero.

$$\therefore \quad \bar{Y}_2 = j\,0.0255\ \mho$$

$$\bar{Y}_2 = \frac{1}{X_{C_2}} = 0.0255\ \mho$$

$$X_{C_2} = 39.22\ \Omega$$

$$X_{C_2} = \frac{1}{2\pi f_0 C_2}$$

$$39.22 = \frac{1}{2\pi \times 60 \times C_2}$$

$$C_2 = 67.63\ \mu\text{F}$$