

软件工程第一次上机

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Introduction

In a box bounded by $[-1,-1]$, given m ballons (they can't overlap) with variable radio r and position μ , find optimal value of r and μ which maximize $\sum r^2$

Algorithm

- 算法思路:贪心算法.每次找正方形中剩余空间中,内切圆最大的那个空间,放入该空间的内切圆.

方法:构造所有的相切的圆,然后按照半径排序.

构造方法:

设所求圆半径为 r ,圆心为 x,y ,已知圆的半径为 r_0 ,圆心为 x_0,y_0 ,以第一象限为例

1.一圆两个正方形边界所构成的区域内切圆为

构造出一个内切圆(内切上述区域)

则有方程 $r+r_0 = \sqrt{(x-x_0)^2+(y-y_0)^2}$ (圆外切性质)

$$r = 1-x$$

$$r = 1-y$$

2.两圆一个正方形边界所构成的区域内切圆为

构造出两个内切圆(内切上述区域) 且关于 $y=x$ 这条直线对称

以 $x=1$ 为正方形边界为例

则有方程 $r = 1-x$

$$r+r_0 = \sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$r+r_1 = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

同理可得 以 $y=1$ 为正方形边界时的解

3.三圆构成的区域内切圆为

根据外切圆的性质

$$r+r_0 = \sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$r+r_1 = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$r+r_2 = \sqrt{(x-x_2)^2+(y-y_2)^2}$$

构造出所有的圆之后,根据四个象限的对称性,可得当 m 取确定的值时,使得 $\sum (r_i^2)$ (i from 1 to m) 取得最大值时,所有圆的坐标和半径.

具体实现看如下代码

```
#include <iostream>
#include <complex>
#include <algorithm>
#include <cstdio>
```

```

#include <cstring>
#include <vector>
#include <queue>
#include <cmath>
// #define SHOW_CONSTRUCT
#define ITERATION
// #define NOLIMIT
#define LIMIT
using namespace std;
const double eps = 1e-6;
const double a = 2;
const int x[4] = {1,-1,1,-1};
const int y[4] = {1,1,-1,-1};
double ans = 0;
struct Balloon{
    double r;
    pair<double,double> mu;
    Balloon(){
        r = 0;
        mu.first = 0;
        mu.second = 0;
    }
    Balloon(double _r,pair<double,double> _mu){
        r = _r;
        mu = _mu;
    }
    //按照半径降序
    bool operator<(const Balloon& b) const{
        return this->r>b.r;
    }
};

Balloon getFirstSituation(double boundX,double boundY,Balloon a){ //第一种情况求解内切圆
    Balloon ans;
    double r = (sqrt(2)-sqrt(2)*a.mu.first-a.r)/(1+sqrt(2));
    double x,y;
    x = y = 1-r;
    ans.r = r;
    ans.mu.first = x;
    ans.mu.second = y;
    return ans;
};

void iterationMethod(double &r,double &y,Balloon a,Balloon b){
    r = 0;
    y = 0;
    double tmpr = 0;
    double tmpy = 0;
    int num = 0;
    while (num<1000){
        tmpy = y;
        tmpr = r;
        //r = (y-b.mu.second)*(y-b.mu.second)/(b.r+1-b.mu.first)+b.mu.first+1-b.r;
        y = sqrt((r+a.r)*(r+a.r)-(1.-r-a.mu.first)*(1.-r-a.mu.first))+a.mu.second;
        r = ((b.mu.first-1.)*(b.mu.first-1)-b.r*b.r+(y-b.mu.second)*(y-b.mu.second))/(b.r-b.mu.first+1.);
        r = 0.5*r;
        //cout<<y<< " "<<r<<endl;
        if(fabs(r-tmpr)<=eps&&fabs(y-tmpy)<=eps){
            break;
        }
        num++;
    }
}

Balloon getSecondSituation(double bound,Balloon a,Balloon b){ //第二种情况求解内切圆
    Balloon ans;
    double r,y;
#ifdef ITERATION
    iterationMethod(r,y,a,b);
    ans.mu.first = 1.-r;
    ans.mu.second = y;
    ans.r = r;
    if(isnan(ans.r)||isnan(ans.mu.first)||isnan(ans.mu.second)){
        ans.r = 0;
        ans.mu.first = 0;
        ans.mu.second = 0;
    }
    // cout<<r<<" -->r"<<endl;
#else
    ans.r = 0;
#endif
}

```

```

    ans.mu.first = 0;
    ans.mu.second = 0;
#endif
    return ans;
}
Balloon getThirdSituation(Balloon a,Balloon b,Balloon c){ //第三种情况求解内切圆
    Balloon ans;
    ans.r = 0;
    ans.mu.first = 0;
    ans.mu.second = 0;
    return ans;
}
vector<Balloon> res;
vector<Balloon> conv; // 构造序列
vector<pair<double,double> >limPoint;
void construct(int m){
    // conv.clear();
#ifdef NOLIMIT
    conv.push_back(Balloon(1,make_pair(0.,0.)));
    Balloon preFisrtSituation = conv[0];
    for(int i = 0;i<=m;i++){
        Balloon tmpFirst = getFirstSituation(1,1,preFisrtSituation);
        preFisrtSituation = tmpFirst;
        conv.push_back(tmpFirst);
        Balloon tmpSecond = getSecondSituation(1,tmpFirst,preFisrtSituation);
        conv.push_back(tmpSecond);
        conv.push_back(Balloon(tmpSecond.r,make_pair(tmpSecond.mu.second,tmpSecond.mu.first)));
        Balloon tmpThirid = getThirdSituation(tmpFirst,tmpSecond,preFisrtSituation);
        conv.push_back(tmpThirid);
    }
#endif

#ifdef LIMIT
    int times = 0;
    for(int i = 0;i<conv.size()&&times<20;i++){
        Balloon tmpfirst = getFirstSituation(1,1,conv[i]); //与四个边界进行构造
        Balloon tmpsecond = getFirstSituation(-1,-1,conv[i]);
        conv.push_back(tmpfirst);
        times++;
        // conv.push_back(tmpsecond);
    }
#endif
    sort(conv.begin(),conv.end());
}
double getSumrArea(vector<Balloon> vec){
    double ans = 0;
    for(int i = 0;i<vec.size();i++){
        ans+=vec[i].r*vec[i].r;
    }
    return ans;
}
void solve(){
    res.clear();
    res.push_back(Balloon(1,make_pair(0.,0.))); //当m==1时,为正方形的内切圆
}
void showConstruct(){
    for(int i = 0;i<conv.size();i++){
        cout<<"r = "<<conv[i].r<<" pos ( "<<conv[i].mu.first<<" , "<<conv[i].mu.second<<" ) "<<endl;
    }
}
void showAns(int m){
    ans = 0;
    if(m>=1){
        cout<<"r = "<<conv[0].r<<" pos ( "<<conv[0].mu.first<<" , "<<conv[0].mu.second<<" ) "<<endl;
        ans+=conv[0].r*conv[0].r;
    }
    int tmpm = m-1;
    int tmpre = tmpm%4;
    tmpm/=4;
    for(int i = 1;i<=tmpm;i++){
        cout<<"r = "<<conv[i].r<<" pos ( "<<conv[i].mu.first<<" , "<<conv[i].mu.second<<" ) "<<endl;
        cout<<"r = "<<conv[i].r<<" pos ( "<<-conv[i].mu.first<<" , "<<conv[i].mu.second<<" ) "<<endl;
        cout<<"r = "<<conv[i].r<<" pos ( "<<conv[i].mu.first<<" , "<<-conv[i].mu.second<<" ) "<<endl;
        cout<<"r = "<<conv[i].r<<" pos ( "<<-conv[i].mu.first<<" , "<<-conv[i].mu.second<<" ) "<<endl;
        ans+=4*conv[i].r*conv[i].r;
    }
}

```

```

for(int i = 0;i<tmpre;i++){
    ans+=conv[tmpm+1].r*conv[tmpm+1].r;
    cout<<"r = "<<conv[tmpm+1].r<<" pos ( "<<x[i]*conv[tmpm+1].mu.first<<" , "<<y[i]*conv[tmpm+1].mu.second<<" ) "
<<endl;
}
}
void inputLimit(int n){
    conv.clear();
    pair<double,double> lim;
    Balloon limBallon;
    for(int i = 0;i<n;i++){
        cin>>lim.first>>lim.second;
        limPoint.push_back(lim);
        limBallon = Balloon(eps,lim);
        conv.push_back(limBallon);
    }
}
int main() {
    //ios_base::sync_with_stdio(false);
    //cin.tie(NULL);
    int m,n;
    cout<<"input m the number of balloon and n the limit points "<<endl;
    while (cin>>m>>n){
        cout<<"m = "<<m<<endl;
        inputLimit(n);
        construct(m);

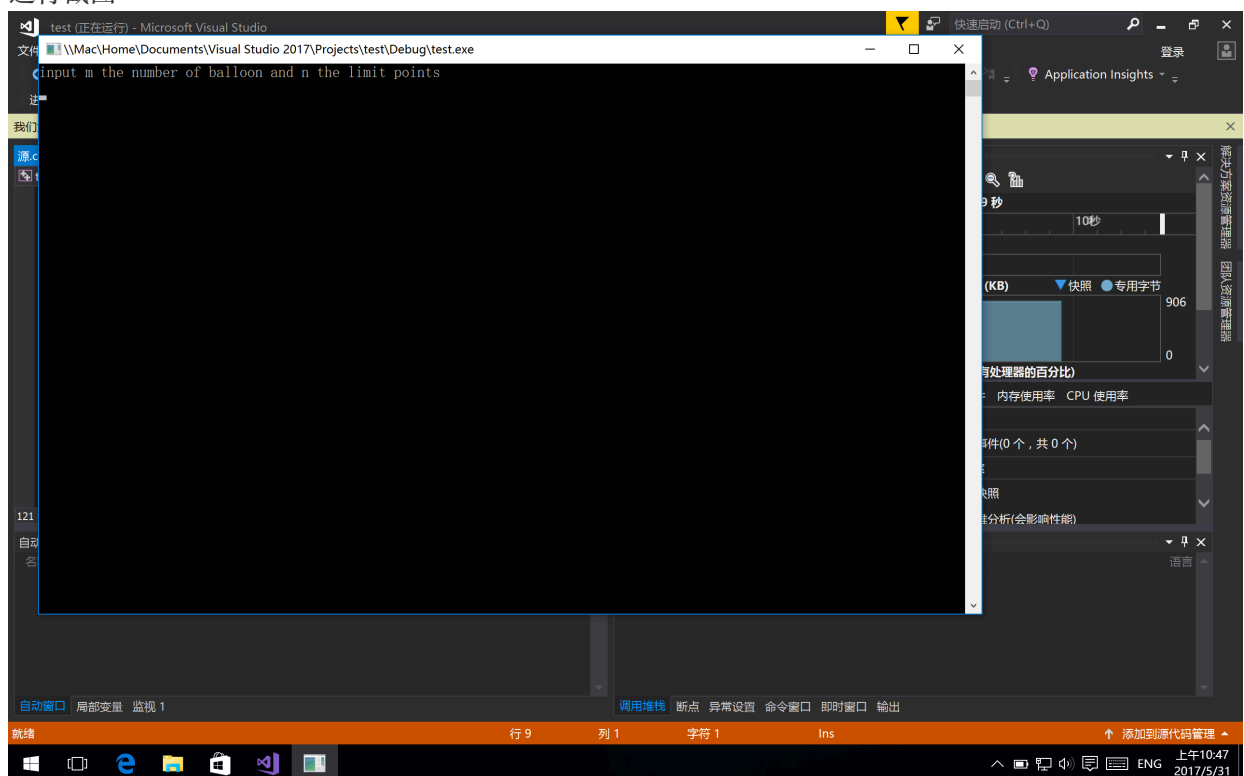
#ifdef SHOW_CONSTRUCT
        showConstruct();
#endif

        showAns(m);
        //cout<<"the max sum r^2 is "<<ans<<endl;
        printf("the max sum r^2 is %.10lf\n", ans);
        cout<<"\ninput m the number of balloon and the n the limit points"<<endl;
    }
    return 0;
}

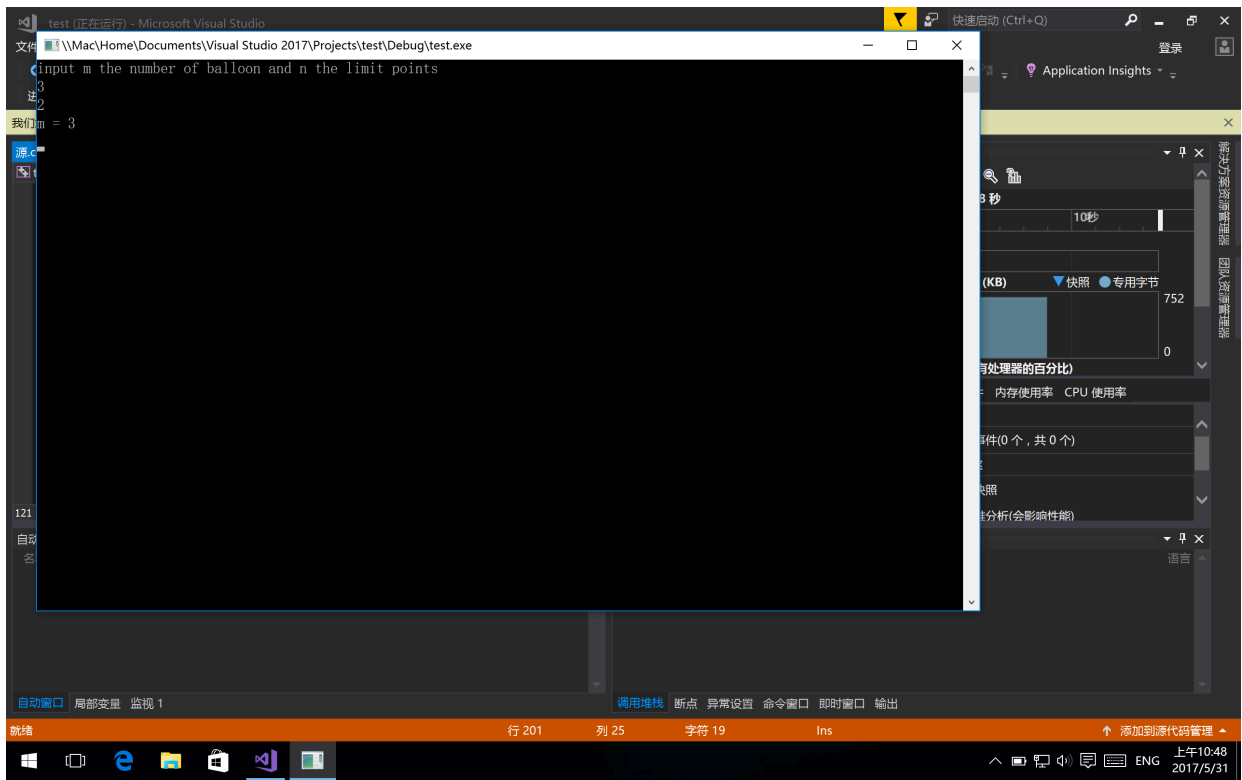
```

Test

- 运行截图



- 输出结果



- 注：m是结果

Conclusion

通过练习这个算法题，我对数学建模的认识更加深刻，

git log

